



**Discussion Papers in Economics**

SELF-STABILIZING FIRMS AND UNEMPLOYMENT  
PERSISTENCE

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# Self-Stabilizing Firms and Unemployment Persistence

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## **Abstract**

The question of why the unemployment rate takes a long time before it reverts back to its natural-rate following a negative exogenous shock has been the subject of unremitting interest in macroeconomics. This paper shows that the speed of adjustment to the steady-state unemployment and the degree of risk-aversion in firms are positively related. The reason is that risk-aversion in firms creates a self-adjusting mechanism whereby cautious firms try to vigorously regain the pre-shock employment levels in an attempt to minimize fluctuations in profits.

**JEL Classification:** E24, E27

**Keywords:** Unemployment, Persistence, Risk-Aversion.

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# 1 Introduction

The question of why the unemployment rate takes a long time before it reverts back to its natural-rate following a negative exogenous shock has been the subject of unremitting interest in macroeconomics. An insight to this question is crucial for understanding the rise in the unemployment rates in European countries in the past 25, while the US unemployment rates have remained stable. Much of the existing literature draws on two alternatives. The dominant view is the ‘structuralist’ theory (Phelps 1994; Phelps and Zoega, 1998) where the natural-rate has a time path that is generated by the structure of the economy: its technological factors, institutions, laws and individual and social preferences. Real disturbances in this model (or its variants) are met by changes to the structure of the economy (labour tax laws, working hours, etc.) which then prevent the natural-rate from returning to its pre-shock levels.

An alternative strand of thinking is that changes in unemployment are determined by past shocks, rather than changes in its fundamental determinants. For example Henry, Karanassou and Snower (2000) and Karanassou and Snower (1998 a, b) show that a chain of past transitory but long-lasting shocks to labour demand and supply and real wages delay the adjustment of unemployment to its steady-state levels. The economy, therefore, takes a while before it works its way through these networks of shocks. Bentolila and Bertola (1990) and Bertola (1990) find that hiring and firing cost can also create inertia in unemployment. Indeed, with hiring and firing costs as sunk costs the expected profitability per hiree has to be high enough to justify hiring. However, until this perceived profitability reaches the threshold that justifies hiring, unemployment rates may stagnate. The insider-outsider theory of Lindbeck and Snower (1988) and Blanchard and Summers (1986) argue that in the aftermath of a negative shock to unemployment, the remaining employees - the insiders - bargain for higher wages with little regards to the unemployed - the outsiders. The insiders have such bargaining power due to the presence of hiring and firing costs. The consequence of this process is that they are able to push their

wages high at the cost of creating employment and again delaying the recovery process. Another plausible reason for inertia is the shortages in capital stock that may prevent firms from hiring rapidly. Thus, so long as the lack of capital persists, so will the unemployment rates. Finally, Blanchard and Diamond (1994) and Layard and Nickell (1987) argue that unemployed workers gradually lose human capital during the unemployment period. This can either make them less attractive workers to hire or can reduce their effectiveness in the job search process. Either way, there will be inertia in employment. Amongst all these explanations, Bean (1994) argues that there is reasonable evidence to suggest that the hiring and firing cost and the duration of unemployment arguments are more successful in explaining the persistence problem.

Our paper contributes to this debate on the causes of unemployment persistence by raising the interesting theoretical possibility that a low degree of risk-aversion on the part of the firms can obstruct the speed at which employment reverts to its steady-state levels. From the perspective of the intertemporal-equilibrium model of endogenous natural-rate theory of Hoon and Phelps (1992) we show that in the aftermath of a shock that drives the economy below its steady-state, risk-aversion in firms accelerates the speed of adjustment to the steady-state employment. The reason is that risk-aversion in firms serves as a self-adjusting device whereby cautious firms try to vigorously regain the pre-shock employment levels in an attempt to minimize fluctuations in profits. The price paid to dampen these fluctuations shows up in the wage, training and hiring cost the firm incurs during the adjustment process. In this sense, risk aversion can be seen as to actually dampen sluggishness in the employment series irrespective of the levels of labour market rigidities characterized in the form of high training cost and low quit rates in our model<sup>1</sup>.

Our theory is independent of the prevailing theories on the speed of unemployment adjustment. A model that comes close to the ideas developed in this paper is Frank (1990) in that in both papers firms are risk-averse; but in contrast to Frank

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<sup>1</sup>Low quit rates and high training costs could both result from legislation in the labour markets.

our model is dynamic and focuses on adjustment speeds of unemployment (rather than the sustainability of equilibrium recessions).

But to what extent can we treat firms as being risk-averse? And if risk-aversion is empirically relevant, why should firms behave in this way? A potential answer to the latter question, and partly to the first, lies in the literature on the agency theory and its related topics. In particular, theories on incentive of procurement and regulation suggest that the degree of risk-aversion amongst decision-makers in firms may actually be generated by the incentive structure in that organization (Laffont and Tirole, 1993 and 1986). For example, take a large modern firm in which there is a separation between ownership and management and the managers maximize their lifetime incomes by pleasing the owners who favour steady income growth. In such a set up managers are likely to avoid risky situations and prefer steady income growth (Monsen and Downs, 1965). Hence, preferences for risk-aversion are endogenously determined by the structure and the incentive provided by the owners of the organization.

The reason why we believe this is interesting in the context of the unemployment debate is that the choice of one institutional structure and incentive schemes over another may emerge from the regulatory environment, political structures and individual and social values within which the firm operates. We would expect such features to differ across countries (also across time) hence different risk-aversion behaviour. Although, our intention in this paper is to explore analytically the relationship between the speed of adjustment of unemployment and risk-aversion, the issue of how the institutional structure of the economy might be related to risk-aversion in firms and how it may differ across countries awaits further scrutiny and future work.

To answer the first question on the empirical relevance of risk aversion in firms, this is a common assumption in labour market contracting models such as Grossman and Hart (1981). It can also be found in surveys where a large proportion of established-entrepreneurs who own their businesses desire a moderate level of

risk Mancuso (1975) and Brockhaus (1980). Interestingly, consider the following study where a randomly selected sample of top-level executives were asked to play a hypothetical role within a business context and make decisions under a variety of risky situations. It was found that executives had the tendency of avoiding risk or delaying decisions until more information was made available to them (see Wehrung and MacCrimmon, 1984 & 1990). Thus, it seems that risk-aversion in firms may be more commonplace than one would normally expect.

The remainder of this paper is organized as follows. In Section 2 we develop the basic structure of the model and solve for the steady-state. Section 3 analyzes, using both theory and numerical simulations, the dynamic properties of the model with respect to the firm risk-aversion and the speed of adjustment of unemployment. Section 4 concludes the paper.

## 2 The Basic Model

Production is carried out by  $n$  identical competitive firms each with labour,  $E/n$ , as the only factor of production and with constant returns to scale. Therefore, the production function is given by

$$Y = f\left(\frac{E}{n}\right) = \Lambda \frac{E}{n} \quad (1)$$

where  $Y$  is the firm's output,  $E$  is total employment and  $E/n$  is the stock of employees each firm has. The dynamics of total employment are

$$\dot{E} = H - q\left(\frac{\bar{w}}{w}\right)E; \quad q' > 0, \quad q'' \geq 0 \quad (2)$$

In (38), the change in employment is the difference between the total number of trainees,  $H$ , and number of people who quit,  $qE$ . The variables  $Y$ ,  $H$  and  $E$  are measured in per capita terms such that we normalize the workforce  $L \geq E$  at unity. This implies that  $E$  is the employment rate, while  $H$  is the number of trainees per capita. As in Salop (1979) and Hoon and Phelps (1992) the quit rate,  $q$ , is a concave function of the ratio of the expected average industry wage,  $\bar{w}$  to the firm's

own wage,  $w$ . When the wage prospects are better elsewhere the employees leave the firm at the rate of  $\frac{1}{w}q'(\cdot)$ , while when the employing-firm offers wages that are better than the industry average the employee quit rate drops by  $-\frac{\bar{w}}{w^2}q'(\cdot)$ .

At time  $\tau$ , the representative firm employs the proportion  $E/n^2$ , denoted by  $e$ , out of the total workforce and maximizes the expected utility of the present discounted value of its cash flow given by

$$\begin{aligned} V(\tau) &= \int_{\tau}^{\infty} u(\pi(t))\epsilon^{-r(t-\tau)}dt \\ \pi &= (\Lambda - w - T(h))e, \quad T(0) = 0, \quad T', T'' > 0 \end{aligned} \quad (3)$$

Firms are risk-averse and face a concave utility function in profits such that  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ . In the profit function,  $\pi$ , the first term is the output produced by the firm where prices have been suppressed to unity. The second term is the wage bill of the firm. The final term  $T(h)e$  is the total cost of training  $H$  new workers, where  $h = H/E$  is the firms' hiring rate. The convex function  $T(h)$  denotes the training cost per unit of labour and captures the output lost per worker at the hire rate  $h$ . The discount rate  $\epsilon^{-r(t-\tau)}$  is the rate at which output can be substituted intertemporally. The rate of interest  $r$  is exogenously given.

## 2.1 The Firm's Optimization Problem

At time,  $\tau$ , the firm maximizes  $V(\tau)$  given by (3) with respect to  $\{w(t), h(t)\}$ ,  $t \geq \tau$  subject to  $e(t)$  given by

$$\dot{e} = \left[ h - q\left(\frac{\bar{w}}{w}\right) \right] e \quad (4)$$

given  $e(0)$  and a terminal condition  $\lim_{t \rightarrow \infty} [e^{-rt}e(t)] \geq 0$ . Equation (4) shows how employment evolves at firm level and is obtained by dividing both sides of equation (38) by the number of firms  $n$ . Assume that  $n$  is constant.

To solve the firm's maximization problem, we define the current-value Hamilto-

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<sup>2</sup>The proportion of workers employed by each firm is  $\frac{E/L}{n}$ . However, given the assumption that  $L = 1$  we get that  $\frac{E}{n} = e$ .



nian given by

$$H = \epsilon^{-rt} \left[ u(f(e) - we - T(h)e) + \lambda \left( h - q \left( \frac{\bar{w}}{w} \right) \right) e \right] \quad (5)$$

where  $\lambda$  is the shadow value of the trained worker. The applying Pontryagin's Maximum Principle, the first-order necessary conditions are given by:

$$\frac{\partial H}{\partial h} = 0 \Rightarrow T'(h)u'(\pi) = \lambda \quad (6)$$

$$\frac{\partial H}{\partial w} = 0 \Rightarrow u'(\pi) = \frac{\lambda \bar{w}}{w^2} q' \left( \frac{\bar{w}}{w} \right) \quad (7)$$

$$\dot{\lambda} - r\lambda = -\frac{\partial H}{\partial e} = -u'(\pi) [f'(e) - w - T(h)] - \lambda \left[ h - q \left( \frac{\bar{w}}{w} \right) \right] \quad (8)$$

$$\frac{\partial H}{\partial \lambda} = 0 \Rightarrow \dot{e} = \left[ h - q \left( \frac{\bar{w}}{w} \right) \right] e \quad (9)$$

$$\lim_{t \rightarrow \infty} [e^{-rt} \lambda(t) e(t)] = 0 \quad (10)$$

In Equation (6) the right-hand-side is the value of a trained worker, while the left-hand-side is the sacrifice the firm makes – in the form of the output loss that occurs when trained workers spend time training the hires instead of actively producing and the marginal change in firm's profits evaluation that results from this investment – in having to train a newly hired worker. Thus, the equation shows that the firm will sacrifice resources on training until it brings the value of a marginal trainee at parity with the value of the trained employee. Interestingly, the intuition just spelled out is subtly different in comparison with Hoon and Phelps (1992) and other studies such as Orszag and Zoega (1996, 1995). Indeed, our model features the possibility that any variable that affects firm's current profitability also affects the valuation ( the utility ) the firm assigns to a profit level. This in turn affects the firm's perception of the value of its stock of employees. Consequently, we can infer from our first-order-condition equation (6) that an extra worker is worth substantially more at lower levels of profits compared with higher profit levels due to diminishing marginal utility in profits. Usually, the utility in profits is not cause for concern since the worth of an extra unit of profit is independent of the levels of

profits i.e., the usual assumption of risk-neutral firms with utility functions linear in profits.

Equation (7) tells that at the optimum the marginal cost of raising wages by a unit – in the form of the lower current profits due to the direct wage increases by the amount  $e$  and the fall in the marginal utility associated with these reduced profitability levels – equals to the marginal benefit – in the form of lower turnover costs due the retaining of employees whose value is given by  $\lambda$ .

Equation (8) says that shadow value of a worker evolves over time, i.e.  $\dot{\lambda}$ , according to the marginal benefits and marginal costs that result from raising employment by one unit. The marginal benefits – the first term – come in the form of the net marginal product of a newly employed worker and the immediate change in the marginal utility in profits that is associated with this extra product. The net marginal product is composed of the marginal product from extra employment,  $f'(e)$ , the loss in the output caused by our employee having to train trainees instead of actively producing,  $T(h)$ , and the savings in the marginal costs of training made possible by our new employee sharing the burden of training the trainees,  $hT'(h)$ . The marginal costs are composed of the direct wage cost of employing a worker and the marginal loss in the utility of profits that this extra cost brings,  $u'(\pi)w$ , the quit rate evaluated at the worker's shadow value,  $\lambda q(\frac{\bar{w}}{w})$ , and the opportunity cost of investing on the employee,  $r\lambda$ , evaluated at the shadow-value of the worker.

## 2.2 The Macroeconomic Equilibrium

Having set out the individual firm's optimization problem we endogenize the expected average industry wage as  $\bar{w} = \bar{w} \times \text{probability of employment} = wE$  following Calvo (1979), Salop(1979) and Hoon and Phelps (1992). To complete our macro-model of employment, hiring rate and wages we remind that our normalization of the workforce at unity implies that  $E$  and  $1 - E$  are the employment and unemployment rates respectively.

The analysis of the dynamic system (6) to (9) will be carried out in terms of

state variables  $e = \frac{E}{n}$  and  $\lambda$ . Upon substituting  $\bar{w} = Ew$  and dividing (6) by (7) we obtain wages as function of employment and the hiring rate

$$w = Eq'(E)T'(h) = \omega(E, h) \quad (11)$$

say. Substituting into (6) we then have

$$u'(f(\frac{E}{n}) - \omega(E, h)\frac{E}{n} - T(h)\frac{E}{n})T'(h) = \lambda \quad (12)$$

Hence solving (12) implicitly for  $h$  and substituting back into (11) we arrive at  $w$  and  $h$  expressed in terms of  $E$  and  $\lambda$ :

$$h = h(E, \lambda) \quad (13)$$

$$w = \omega(E, h(E, \lambda)) = w(E, \lambda) \quad (14)$$

say. Using (14), (13) and (6), the dynamic system (8) and (9) in  $(E, \lambda)$  space can now be written as

$$\dot{E} = [h(E, \lambda) - q(E)] E = F(E, \lambda) \quad (15)$$

$$\begin{aligned} \dot{\lambda} &= \left[ r + h(E, \lambda) - q(E) - \frac{1}{T'(h(E, \lambda))} (\Lambda \frac{E}{n} - w(E, \lambda) - T(h(E, \lambda))) \right] \lambda \\ &= G(E, \lambda) \end{aligned} \quad (16)$$

plus the transversality condition, (10). One state variable,  $E$ , is predetermined at time  $t$  with initial condition  $E(0)$  given exogenously. The second state variable,  $\lambda$ , is non-predetermined or 'free' and satisfies the terminal condition (10) with  $e(t) = \frac{E(t)}{n}$ .

### 2.3 The Steady State

In the steady state with  $\dot{E} = \dot{\lambda} = 0$  we have

$$h = q(E) \quad (17)$$

$$w = Eq'(E)T'(h) = Eq'(E)T'(q(E)) \quad (18)$$

$$r = \frac{1}{T'(h)} (\Lambda - w - T(h)) \quad (19)$$

giving us three equations for  $h$ ,  $w$  and  $E$ . Then  $\lambda$  is given by (6).

We first examine the wage-employment relationship. At the steady-state the wages depend on the investment made on the marginal worker – in the form of training costs – and their marginal quit propensity. Differentiating (18) with  $h = q(E)$ , we then have

$$w'(E) = (q'(E) + Eq''(E))T'(q(E)) + Eq'(E)T''(E) > 0 \quad (20)$$

since  $T', T'', q'(E), q''(E) \geq 0$ . The first term is the rise turnover as employment rises,  $q' + Eq''$ , and each quitting worker must be replaced at cost,  $T'$ . The second term is the rise training cost associated with higher employment. Thus, higher wages and higher (lower) employment (unemployment) go hand in hand and this relationship is concave (convex) from the origin. This ensures that our model is consistent with the Phillip's curve wage-employment (or wage-unemployment) relationship. To summarise:

**Proposition 1.**

**In the steady state, the wage is an increasing function of the employment rate.**

The steady-state is at the intersection of the  $\dot{E} = 0$  curve, (17), and the  $\dot{\lambda} = 0$  curve. The former is upward-sloping in  $(h, E)$  space since  $q' > 0$ . To show that the latter is downward-sloping, differentiate (19) implicitly to obtain along this curve

$$\frac{dh}{dE} = -\frac{w'(E)}{T' + rT''} < 0 \quad (21)$$

assuming the conditions for proposition 1 hold (in which case  $w'(E) > 0$ ), and recalling that  $T', T'' > 0$ . The derivative implies that, higher levels of steady-state employment are associated with lower levels of hiring rate. Hence we have shown that:

**Proposition 2.**

**There exists a unique steady state that is independent of the degree of risk aversion of the firm.**

Figure 1 shows the ‘E stationary’ curve ( $\dot{E} = 0$ ) and the ‘ $\lambda$ - stationary’ curve ( $\dot{\lambda} = 0$ ) in  $(h, E)$  space for functional forms and parameter values discussed in sec-

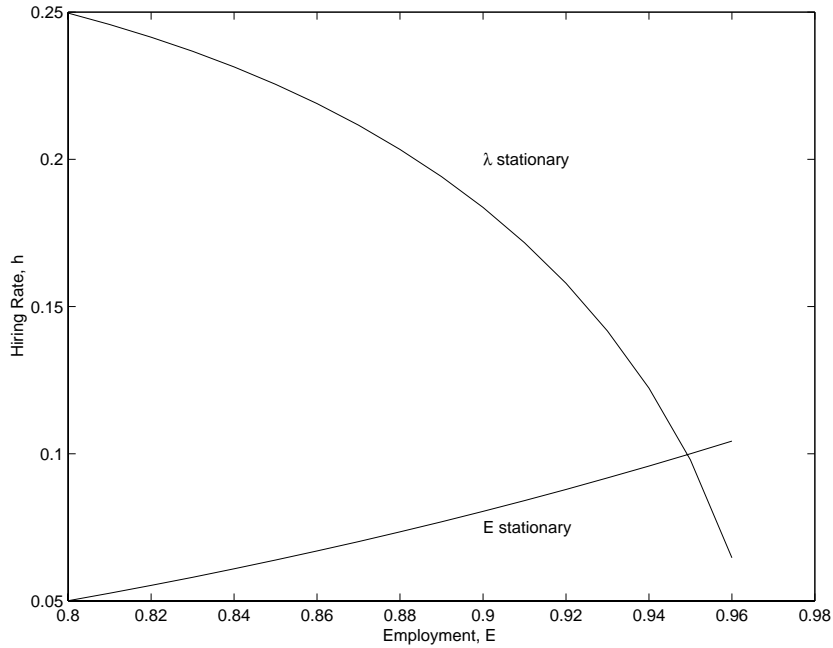


Figure 1: **The Steady State in h-E Space**

tion 3.1. Notice how the equilibrium unemployment rate stands as 5%.

### 3 Dynamic Analysis

#### 3.1 Saddlepath Stability

To analyze the local stability of the dynamic system (15) and (16) in the vicinity of the unique steady state, we evaluate the Jacobian

$$J = \begin{pmatrix} F_E & F_\lambda \\ G_E & G_\lambda \end{pmatrix} \quad (22)$$

where partial derivatives  $F_E \equiv \frac{\partial F}{\partial E}$ , etc are evaluated at the steady state. The eigenvalues  $\mu = \mu_1$  and  $\mu = \mu_2$  are given by the solutions to

$$\mu^2 - (F_E + G_\lambda)\mu + \Theta = 0 \quad (23)$$

where  $\Theta \equiv F_E G_\lambda - F_\lambda G_E$ . Since we have one predetermined and one non-predetermined variable, saddlepath stability requires that one root has real part positive and the other has real part negative.

Partially differentiating  $F(E, \lambda)$  and  $G(E, \lambda)$  defined in (15) and (16) at the steady state, we have

$$F_E = E(h_E - q') \quad (24)$$

$$F_\lambda = E h_\lambda \quad (25)$$

$$G_E = \lambda \left[ h_E - q' + \frac{1}{T'} (w_E + h_E (r T'' + T')) \right] \quad (26)$$

$$G_\lambda = \lambda \left[ h_\lambda + \frac{1}{T'} (h_\lambda T'' r + w_\lambda + T' h_\lambda) \right] \quad (27)$$

where  $q = q(E)$ ,  $T = T(h)$ ,  $f = f(E)$  and we omit the arguments in  $q'$ ,  $T'$ ,  $T''$  and  $f''$ . It follows that

$$\Theta \equiv F_E G_\lambda - F_\lambda G_E = \frac{E\lambda}{T'} [-q' h_\lambda (T' + T'' r) + w_\lambda (h_E - q') - h_\lambda w_E] \quad (28)$$

This expression can be simplified further by partially differentiating (11) to obtain

$$w_\lambda = E q' T'' h_\lambda \quad (29)$$

$$w_E = (E q'' + q') T' + E q' T'' h_E \quad (30)$$

These partial derivatives imply that, other things being equal, the higher the shadow-value of a worker or the employment rate the higher wages that has to be paid to the employees at the equilibrium. Such a policy will help deter workers from quitting and hence reduce firms' turnover and training costs.

Hence (24) becomes

$$G_\lambda = \lambda h_\lambda \left[ 2 + \frac{T''}{T'} (r + E q') \right] \quad (31)$$

Substituting back into (28) we arrive at

$$\Theta = \frac{E\lambda h_\lambda}{T'} [-q' (2T' + r T'' + E q' T'') - E q'' T'] \quad (32)$$

To complete the dynamic analysis we require partial derivatives

$$h_\lambda = \frac{1}{u'(\pi)T'' - \frac{u''(\pi)}{n}ET'(Eq'T'' + T')} \quad (33)$$

$$h_E = -\frac{u''(\pi)(T')^2(r - E(Eq'' + q'))}{u'(\pi)T'' - \frac{u''(\pi)}{n}ET'(Eq'T'' + T')} \quad (34)$$

We can now use these results to examine the saddlepath stability of the dynamic system. From (23) the eigenvalues are given by

$$\mu = \frac{F_E + G_\lambda \pm \sqrt{(F_E + G_\lambda)^2 - 4\Theta}}{2} \quad (35)$$

Hence one root ( $\mu = \mu_1$ ) is real and negative and one root ( $\mu = \mu_2$ ) is real and positive if  $\Theta < 0$ . From (34)  $h_\lambda > 0$ , and hence from (32), since  $\lambda > 0$ , we have that  $\Theta < 0$  iff  $[-q'(2T' + rT'' + Eq'T'') - Eq''T'] < 0$ . Since we have made the usual convexity assumption that  $q', q'', T', T'' > 0$ , all the terms in this expression are negative and hence we have shown that

**Proposition 3.**

**The dynamic system is saddlepath stable.**

### 3.2 Hysteresis and Risk Aversion

Up to this point we have shown that the firm's risk aversion neither affects the steady state nor the possibility of instability. However the degree of risk aversion does affect the size of the eigenvalues and hence the rate at which the system reverts to the steady state following an exogenous shock. From standard analysis of linear systems with non-predetermined variables (see, for example, Levine and Currie (1987)) the solution to the linearized dynamic system

$$\begin{bmatrix} \dot{E} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} F_E & F_\lambda \\ G_E & G_\lambda \end{bmatrix} \begin{bmatrix} E \\ \lambda \end{bmatrix} \quad (36)$$

is given by

$$\lambda - \bar{\lambda} = -M_{22}^{-1}M_{21}(E - \bar{E}) \quad (37)$$

$$\dot{E} = [F_E - F_\lambda M_{22}^{-1}M_{21}](E - \bar{E}) \quad (38)$$

where  $\bar{\lambda}$  and  $\bar{E}$  denote steady-state values and  $M$  is a matrix of eigenvectors defined by

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} F_E & F_\lambda \\ G_E & G_\lambda \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (39)$$

where  $\mu_1$  and  $\mu_2$  are the real negative and positive eigenvalues found in the previous subsection. Solving (39) we find that

$$M_{22}^{-1}M_{21} = \frac{\mu_2 - G_\lambda}{F_\lambda} = \frac{F_E - \mu_1}{F_\lambda} \quad (40)$$

Hence (38) becomes

$$\dot{E} = [F_E + G_\lambda - \mu_2]E = \mu_1(E - \bar{E}) \quad (41)$$

using  $\text{trace}(J) \equiv F_E + G_\lambda = \mu_1 + \mu_2$ .

We have now established that the rate at which employment returns to the steady steady following either a temporary shock to employment itself (in which case the economy reverts to the same steady state) or to a permanent the natural rate of employment (then reversion is to a new steady state) is given by the absolute size of the negative eigenvalue  $\mu_1$ :

$$\mu_1 = \frac{F_E + G_\lambda - \sqrt{(F_E + G_\lambda)^2 - 4\Theta}}{2} \quad (42)$$

Provided the dynamic system is saddlepath stable, which we have shown to be the case, then the dynamics of employment is given by

$$E(t) = E(0)e^{\mu_1 t} \quad (43)$$

To see how  $\mu_1$  changes as risk aversion increases consider a constant coefficient of relative risk aversion profit function

$$\begin{aligned} u(\pi) &= \frac{\pi^{1-\rho}}{1-\rho}; \quad \rho > 0, \rho \neq 1 \\ &= \log(\pi); \quad \rho = 1 \end{aligned} \quad (44)$$

where  $\rho = -\frac{\pi u''}{u'}$  is the constant coefficient of risk aversion. In the steady state, using (19) we have that economy-wide profits are  $n\pi = rT'E$ . Then in terms of  $\rho$ ,



from (32), (33) and (31) we have

$$F_E(\rho) = \left( \frac{\rho T'(r - E(Eq'' + q'))}{\rho(Eq'T'' + T') + rT''} - Eq' \right) \quad (45)$$

$$G_\lambda(\rho) = \frac{rT' \left[ 2 + \frac{T''}{T'}(r + Eq') \right]}{\rho(Eq'T'' + T') + rT''} \quad (46)$$

$$\Theta(\rho) = \frac{rE[-q'(2T' + rT'' + Eq'T'') - Eq''T']}{\rho(Eq'T'' + T') + rT''} \quad (47)$$

We are interested in the sign of  $\frac{d\mu_1}{d\rho}$ . If it is positive, then increasing risk-aversion pushes  $\mu_1$  towards zero and towards pure hysteresis. If the derivative is negative, it has the opposite effect and risk-aversion actually improves the stability of the dynamic system. Differentiating (42) we have

$$\frac{d\mu_1}{d\rho} = \frac{1}{2} \left( \frac{dF_E}{d\rho} + \frac{dG_\lambda}{d\rho} \right) \left( 1 - \frac{(F_E + G_\lambda)}{\sqrt{(F_E + G_\lambda)^2 - 4\Theta}} \right) + \frac{2}{\sqrt{(F_E + G_\lambda)^2 - 4\Theta}} \frac{d\Theta}{d\rho} \quad (48)$$

Differentiating (32) we have that

$$\frac{d\Theta}{d\rho} = (Eq'T'' + T') \frac{rE[q'(2T' + rT'' + Eq'T'') + Eq''T']}{(\rho(Eq'T'' + T') + rT'')^2} > 0 \quad (49)$$

and so the last term in (48) is positive. Since  $F_E + G_\lambda < \sqrt{(F_E + G_\lambda)^2 - 4\Theta}$  the first term has the sign of  $\frac{dF_E}{d\rho} + \frac{dG_\lambda}{d\rho}$ . Consider first derivatives in the vicinity of  $\rho = 0$ . Differentiating (45) and (46), at  $\rho = 0$  we find that

$$\frac{d(F_E + G_\lambda)}{d\rho} = -Eq' - \frac{1}{r(T'')^2} [(Eq'T'' + T')(2T' + T''Eq') + T''T'E(Eq'' + q')] \quad (50)$$

which is negative. It follows that in the vicinity of  $\rho = 0$ , the effect of increasing risk aversion on the degree of stability of the macro-economy is ambiguous. However we can establish an unambiguous result for higher values of  $\rho$ , the case where  $r \ll \rho\bar{q}$ ,  $\bar{q}$  being the quit rate in the steady-state. Then from the calibration that follows we can establish that  $\frac{\Theta}{(F_E + G_\lambda)^2} \ll 1$  and  $-F_E \gg G_\lambda$ . Then expanding (42) as a binomial series we have

$$\mu_1 = \frac{F_E + G_\lambda - |F_E + G_\lambda| \left[ 1 - \frac{\Theta}{(F_E + G_\lambda)^2} \right]^{\frac{1}{2}}}{2} \approx F_E + G_\lambda + \frac{\Theta}{|F_E + G_\lambda|} \approx F_E \quad (51)$$

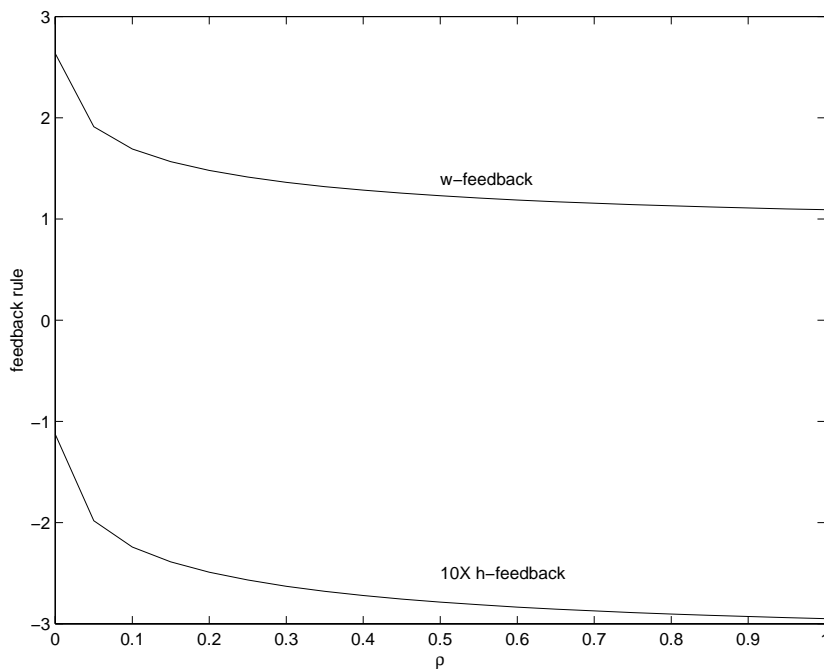


Figure 2: **Feedback Rules as  $\rho$  changes**

From (45) for  $r \ll \rho\bar{q}$ ,  $\bar{q}$ ,  $r \ll E(Eq'' + q')$  and it follows that  $\frac{dF_E}{d\rho} < 0$ . Hence we have established the proposition:

**Proposition 4**

**For  $r \ll \rho\bar{q}$  an increase in risk aversion increases the degree of stability of the economy.**

The intuition for this result can best be seen by examining the decision variables of the firm, trajectories for the wage rate and the hiring rate,  $\{w(t), h(t)\}$  in terms of a *policy feed-back rule* that responds to the current state of employment in the firm and the value of the trained worker. In terms of aggregate employment (assuming all firms are identical), in the vicinity of the steady state  $[\bar{w}, \bar{h}]$  the policy rule of the representative firm is given by

$$\begin{bmatrix} w(t) - \bar{w} \\ h(t) - \bar{h} \end{bmatrix} = \begin{bmatrix} w_E & w_\lambda \\ h_E & h_\lambda \end{bmatrix} \begin{bmatrix} E(t) - \bar{E} \\ \lambda(t) - \bar{\lambda} \end{bmatrix} \quad (52)$$

Now consider the firm is adjusting to the steady state along a trajectory where  $e(t) > \bar{e}$ . For  $r \ll \rho\bar{q}$  we have seen that  $\mu_1 \approx F_E$  and hence from (37)  $\lambda(t) \approx \bar{\lambda}$ .

Then the policy rule takes the approximate form

$$\begin{bmatrix} w - \bar{w} \\ h - \bar{h} \end{bmatrix} = \begin{bmatrix} w_E \\ h_E \end{bmatrix} (e(t) - \bar{e}) = \begin{bmatrix} [(Eq'' + q')T' + Eq'T''h_E] \\ h_E \end{bmatrix} (E(t) - \bar{E}) \quad (53)$$

substituting for  $w_E$  from (30). Now compare a firm with relatively low and high degrees of risk aversion. In the latter case  $h_E$  is small so the firm responds to being above the equilibrium employment level by raising the wage, but only changes the hiring rate slightly. In the more risk averse case, for  $r < E(Eq'' + q')$  (easily satisfied in our calibration),  $h_E < 0$  and grows in absolute size as  $\rho$  increases, so the wage is raised *by less* than in the less risk-averse case, quits are therefore *higher*, and now the firm responds by *lowering* the hiring rate. As a result of these differences employment returns faster to its steady state for the risk averse firm. Clearly adjustment for  $e(t) < \bar{e}$  is symmetrical: now the more risk averse firm hires more and engineers less quits than the less risk averse risk neutral firm. In short, *firms are self-stabilizing in the face of shocks, and the more risk averse firm will stabilize employment by more than the less risk averse firm.* Figure 2 plots the exact feedback coefficients  $w_E - M_{22}^{-1}M_{21}w_\lambda$  of the wage on employment (the ‘w-feedback’) and the corresponding ‘h-feedback’  $h_E - M_{22}^{-1}M_{21}h_\lambda$  computed without the approximation used in the analysis, for parameter values discussed in next subsection. The same features revealed in the analysis carry over to the exact solution: the positive w-feedback on employment falls and the negative h-feedback rises in absolute value, increasing the self-stabilization of the firm as risk aversion increases.

### 3.3 Numerical Illustration and Quantification

To illustrate these results and attempt to quantify their the degree to which risk averse firms self-stabilize employment, we turn to numerical computations. We assume the following functional forms:

$$q(E) = q_0 E^\beta; \beta > 1 \quad (54)$$

$$T(h) = \frac{1}{2}th^2 \quad (55)$$

This leaves parameters  $\Lambda$ ,  $q_0$ ,  $\beta$  and  $t$ . The interest rate  $r$  is exogenous and we set its value to a plausible average real interest rate. By choice of units we can normalize  $\Lambda = 1$  so that a fully employed labour force produces one unit of output.  $\bar{E}$  is given by  $1 - \bar{U}$  where  $\bar{U}$  is the natural rate of unemployment for which econometric estimates are available. To calibrate the remaining three parameters we use the steady state relationships given by (17) to (19) and econometric estimates for the unemployment rate, and data for the quit rate  $q$ . We need one more item of data to complete the calibration. Data on training costs  $T(h)E$  is available, but leads to implausibly low degrees of persistence. We therefore appeal to estimates of the latter to pin down the final parameter.

First we note that from (18)  $w(E) = Eq'(E)$  and with our chosen functional forms we have

$$w'(E) = 2\beta^2 q_0^2 t E^{2\beta-1} > 0 \quad (56)$$

$$w''(E) = 2\beta^2(2\beta - 1)q_0^2 t E^{2\beta-2} > 0 \quad (57)$$

Thus the 'wage curve',  $w(E)$ , has the familiar convex shape.

Now suppose we use data on training costs (as a proportion of total output)  $\bar{T}\bar{E} = \overline{TC}$ , say, and on quit rate  $\bar{q}$ . Then we have that  $\frac{1}{2}t\bar{q}^2\bar{E} = \overline{TC}$  and hence  $t$  is calibrated as:

$$t = \frac{2\overline{TC}}{\bar{E}\bar{q}^2} \quad (58)$$

Given  $t$  and exogenous  $r$ , from (19) and  $\bar{w} = \beta t \bar{q}^2$  we can solve for  $\beta$  as

$$\beta = \frac{1 - \frac{\overline{TC}}{\bar{E}} - t\bar{q}r}{t\bar{q}^2} \quad (59)$$

Finally  $q_0$  is now calibrated as  $q_0 = \frac{\bar{q}}{\bar{E}^\beta}$ , though in fact this parameter is not required for the stability analysis. We have completed the calibration of parameters  $\beta$  and  $t$ . For the computations of  $\mu_1$  we then use

$$T' = t\bar{q}; \quad T'' = t \quad (60)$$

$$q' = \frac{\beta\bar{q}}{\bar{E}}; \quad q'' = \frac{\beta(\beta - 1)\bar{q}}{\bar{E}^2} \quad (61)$$

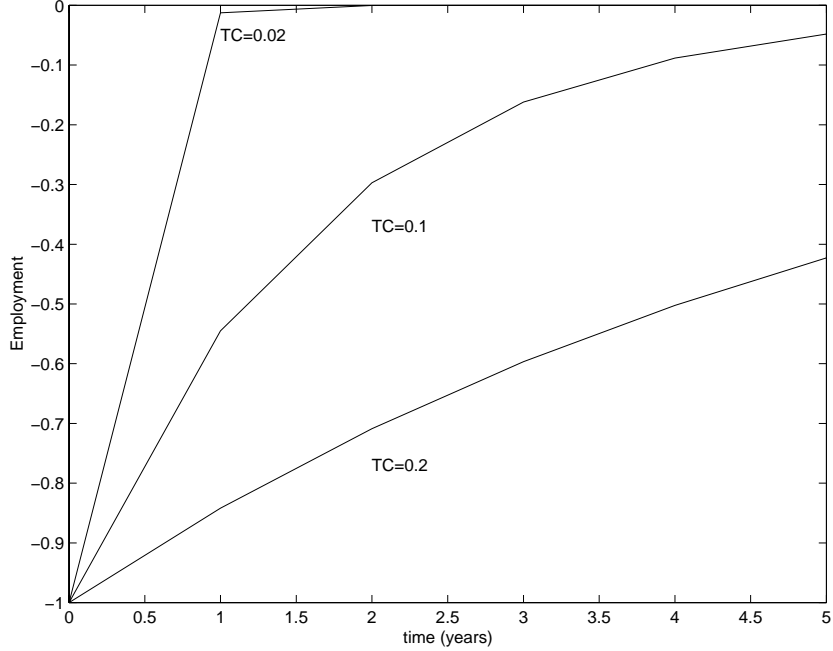


Figure 3: **Calibration of Training Costs (TC)**

Then substituting into (45), (46) and (46) we have

$$F_E = \frac{\rho\bar{q}(r - \bar{q}\beta^2)}{(\rho\bar{q}(\beta + 1) + r)} - \beta\bar{q} \quad (62)$$

$$G_\lambda = \frac{r[(2 + \beta)\bar{q} + r]}{(\rho\bar{q}(\beta + 1) + r)} \quad (63)$$

$$\Theta = - \frac{\bar{q}\beta r[(2 + \beta)\bar{q} + r]}{(\rho\bar{q}(\beta + 1) + r)} = -\bar{q}\beta G_\lambda \quad (64)$$

From these expressions it is clear that if  $\rho\bar{q} > r$ , then  $\frac{\Theta}{(F_E + G_\lambda)^2} \ll 1$ , justifying the approximations made in the analysis of the previous section.

We can now compute  $\mu_1$  using these functional forms and parameters. Figure 3 shows the employment response to a unit negative shock (i.e.,  $E(0) - \bar{E}) = -1$ ) about the steady state for empirical data  $\bar{q} = 0.1$ ,  $\bar{U} = 0.05$ ,  $\bar{TC} = 0.02$  and  $r = 0.025$  and compares the response with those with higher values of  $TC$ . Clearly using data that suggests  $TC = 0.02$  leads to an implausibly quick responses: the economy returns to equilibrium within a year! A 50% recovery within 3 years fits in with the stylized facts and from our graphs this suggests the  $TC$  should be between

0.1 and 0.2. Our results suggest that much of this cost of training does not appear as measured investments and counts as the ‘intangible investment’ highlighted in Parente and Prescott (2000). Their estimate of total intangible investment could be as much as 50% of GDP for the US. Of this sum an amount 15% for training is therefore reasonable.

With these parameter values, increasing risk aversion then increases the absolute size of the negative eigenvalue (figure 4) making the macro-economy *more* stable, and *decreases* unemployment persistence following a negative shock to employment (figure 5). The policy rules for the risk-neutral and risk averse firms are contrasted in 6. For the risk averse case wages are higher, quits are therefore lower and more hiring takes place confirming the increased self-stabilization of the firm following a negative employment shock. Another way of quantifying the degree of hysteresis is to evaluate the asymptotic variance of employment subject to a white noise shock of variance  $\sigma^2$ . This is given by  $\frac{\sigma^2}{-\mu_1}$ . Then the effect of increasing  $\rho$  from 0 to 1 is to decrease the variance of employment by a factor  $\frac{\mu_1(0)}{\mu_1(1)}$  which turns out to be about  $\frac{2}{3}$  in our example.

## 4 Conclusion

The question we have examined is what explains the delays in the speed of adjustment of unemployment rate towards its steady-state levels. We have shown that risk-aversion behaviour in firms tends to speed up the adjustment process towards the steady-state employment levels as the firms attempt to minimise fluctuations in profits. One clear policy implication stands out: if firms are risk averse there is *less* need for macroeconomic policy stabilization through monetary and fiscal policy. Our calibrated model suggests however that risk averse behaviour removes at most one-third of employment variation, therefore still leaving a substantial role for government. The topic of how the institutional structure of the economy and the degree of risk-aversion in firms are related, and how these may differ across countries to cause different adjustment speeds, awaits further work.

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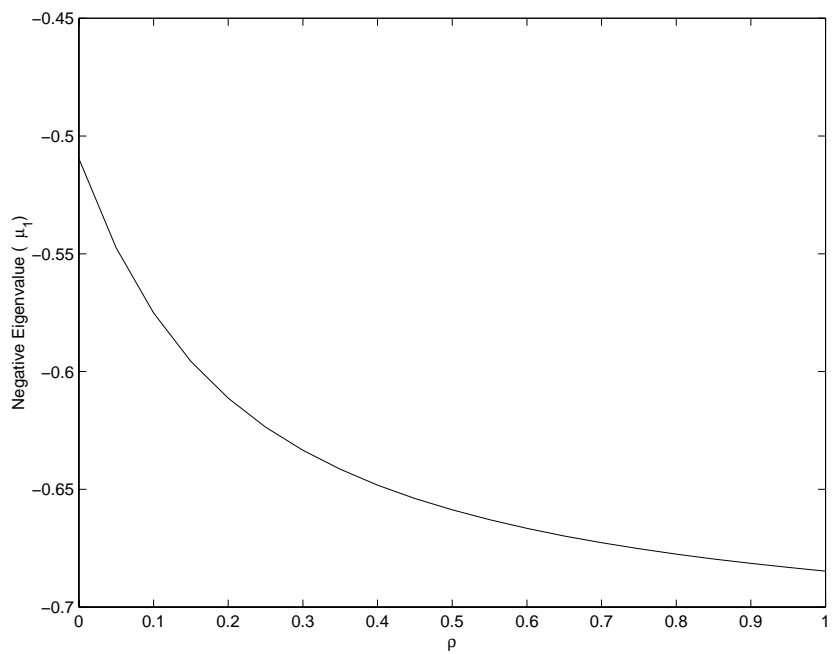


Figure 4: **The Negative Eigenvalue as  $\rho$  changes**

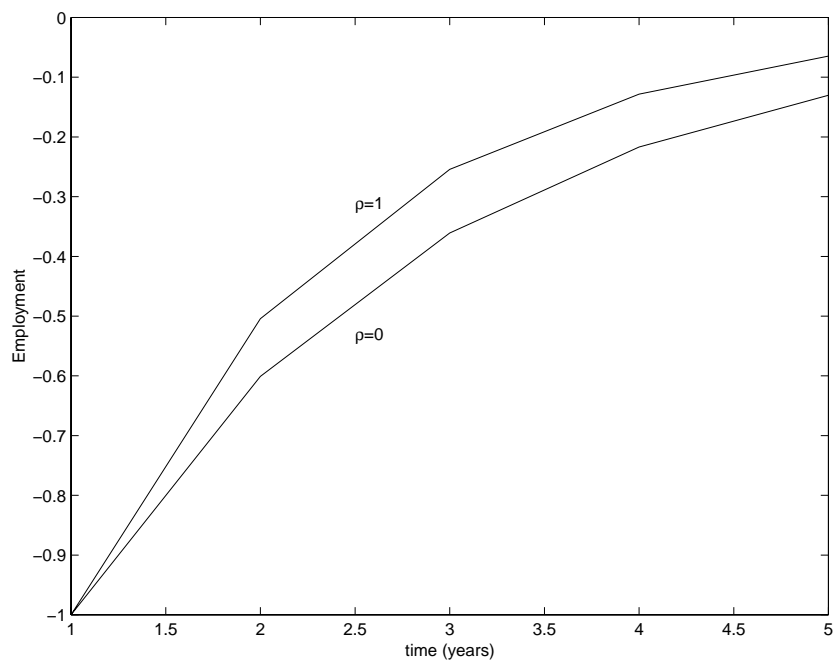


Figure 5: **Employment Trajectories as  $\rho$  changes**

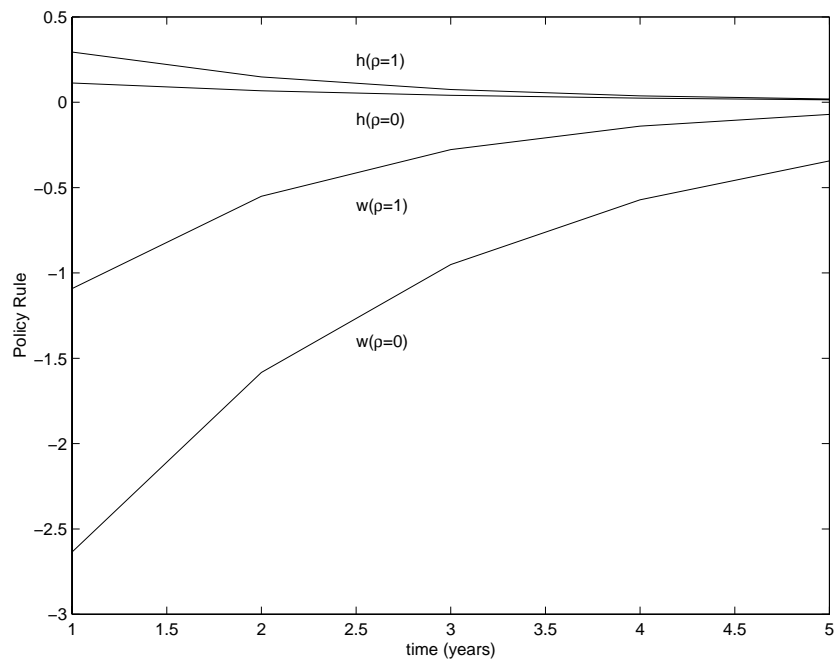


Figure 6: Policy Trajectories as  $\rho$  changes