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SPECTRUM PROPERTY RIGHTS VERSUS A COMMONS MODEL: EXPLOITATION OF MESH NETWORKS

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Abstract

This paper combines models and ideas from radio-engineering literature and economics to address the need for regulation of spectrum allocation in a commons scenario. It discusses under what conditions a laissez-faire policy towards spectrum usage would engender the inefficiencies of a spectrum commons allocation regime; to overcome such potential inefficiency, centralised allocation or a formal market for spectrum (with well-defined property rights) is required.

JEL Classification: L10, L50, L96

Keywords: mesh networks, spectrum allocation, spectrum commons model.

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Contents

1 Introduction

A central broad theme in the literature on the economics of radio spectrum addresses the need for the regulation of spectrum allocation, and whether a laissez-faire policy towards spectrum usage would engender the inefficiencies of a traditional commons outcome. The concept of the spectrum commons and some of the associated problems have been well documented, an example of this discussion is provided by Webb and Cave (2003) who ask where we should draw the line between laissez-faire and regulation.

The so-called tragedy of the commons arises because of the interaction between capacity constraints and unfettered use of this scarce capacity. In the context of radio spectrum, Shannon s Law (Shannon (1948)) has traditionally been taken to place an upper bound on spectrum capacity and, thus, imply a potential commons problem for laissez-faire policies. In fact, more recent engineering literature has extended Shannon s seminal work and models now exist where, in principle, the capacity constraint can be relaxed. This may appear when mobile receivers act simultaneously as transmitters (as relays); as in the case of some mesh networks.

From an economic viewpoint, these results can be related to the fact that consumers can simultaneously produce demand-side and supply-side externalities. Thus, the more consumers use a service, i.e. with increasing number of users or active transceiving nodes, the overall throughput within the network increases.

This paper shows how such models can be combined with economics to investigate the potential for a tragedy (or a triumph) of the commons under laissez-faire policy framework. Our approach provides an interesting illustration of the research philosophy outlined above: precise modelling helps identify key information that needs to be known before a decision can be made of where to draw the line between property rights and commons policies towards spectrum. For example, the distribution of users may be important in producing an efficient mesh network, or the propensity for delay may matter, or perhaps one can strategically populate hot-spots to remove potential congestion. Our work may also have more immediate policy implications to the extent that (currently popular) wireless LANs, in terms of overall throughput can be regarded as are inferior to mesh networks and that there is economic benefit from setting aside spectrum for experimenting with a commons approach.

2 Spectrum Property Rights Versus a Commons Model

Benkler (2002) describes a 'traditional model' of communication based on the *lone, stupid receiver* (LSR). Here 'lone' refers to the fact that all signals are send directly from the sender to the receiver and 'stupid' to the inability of the receiver to distinguish between electromagnetic radiation of the same frequency and power. The LSR can then send and receive intelligible messages only if the spectrum management regime restricts individuals to radiate at a particular frequency, power, location and time-frame.[1]

He then argues that two main reasons why the traditional model is no longer applicable. The first is that the dramatic *decrease in the cost of computation* means that receivers are able to distinguish signals of the same frequency and power by using computationally intensive (but no longer expensive) encoding and decoding techniques. The second reason is the existence of *cooperation or diversity gains* between users of a particular *network* where a network is a group of users who are both senders and receivers of signals situated at nodes communicating with each other over wireless channels without any centralized control of traffic flow.

A crucial issue for the viability of a commons model is the relationship between *throughput*, defined as the average amount of information (bits) per second that can

[1] Our model of spectrum prices and channel trading in Section 3 and 4 respectively is, in the Benkler sense, a traditional model in that we assume that operators needs to transmit at different frequencies to avoid destructive interference and each purchases a licence to use a particular channel in a particular time frame. By implication the LSR does not employ sophisticated and computationally intensive interference mitigation schemes.

be transmitted by every sender node to its chosen destination node, and the number of nodes. This will involve a scheduling policy with a possible *delay*, T, and relaying. The precise definition of feasible throughput is as follows:

Definition: Feasible Throughput. Let n be the number of nodes. Then $\lambda(n)$ bit/sec for every node is feasible if there is a scheduling policy such that by operating the network in a multi-hop fashion and buffering at intermediate nodes when awaiting transmission there is a time $T < \infty$ such that in every interval $[tT, (t+1)T]$, $t = 1, 2, \cdots$, every node can send $T\lambda(n)$ bits to its chosen destination node.[2]

In the following section we survey work by Gupta and Kumar (2000) and Grossglauser and Tse (2002) that models the throughput $\lambda(n)$ using mesh technnologies. Since locations are random we need to formulate throughput in probability terms as probability limits. Then we examine the implication of these results for the existence of coordination gains and for the possible existence of a 'tragedy of the commons'.

3 Feasible Throughput in a Network

3.1 Shannon's Theorem and the Tramsmission Constraint

We start with the basic theorem due to Shannon (1948) (theorem 17, page 43) and others relating the *maximum* capacity (C), bandwidth (B), power of the signal (P) and background (white) noise (N) *for a single receiver* as

$$C = B \log_2 \left[\frac{P + N}{N} \right] \tag{1}$$

This means that in order to transmit at a rate of R bits per sec, the signal-to-noise ratio, $\frac{P}{N}$, must satisfy the transmission constraint $R < B \log_2 \left[\frac{P+N}{N} \right]$ or

$$\frac{P}{N} > 2^{\frac{R}{B}} - 1 = \beta \equiv \beta(R/B)\,; \quad \beta'(\cdot) > 0 \tag{2}$$

[2]This definition implies that nodes are 'semi-intelligent', processing and routing data.

In (2), β is the minimum signal-to-noise ratio (or signal-to-interference ratio, SIR) that is required for successful communication by a single receiver. $\beta(\cdot)$ is an increasing function of $\frac{R}{B}$ so that for a given rate of transmission, the minimum SIR falls as more bandwidth is provided.

Now consider a network of n nodes lying in the disk of unit area of radius $\frac{1}{\sqrt{\pi}}$. Each node transmits using a common radio channel of bandwidth B. The location of the ith node at time t is $X_i(t)$. Nodes are randomly located and this randomness is crucial for the results on throughput. Nodes can either be fixed, as in Gupta and Kumar (2000); or mobile as in Grossglauser and Tse (2002). For the case of fixed node positions, $\{X_i\}$ are identically and independently distributed and uniformly distributed over the disk of unit area. For example if the centre of the disk is at $(0,0)$ then coordinates (x_i, y_i) for node i are chosen with x_i and y_i independently and uniformly distributed over $(-1,1)$. For the case of mobile nodes the location of the ith node at time t is given by $X_i(t)$ where the process $\{X_i(t)\}$ is a stationary and ergodic uniform[3] distribution on the disk.

At time t, each of the n nodes is either a *source* or a *destination* node. Let $P_i(t)$ be the *transmit power* of node i such that the received power at node j is $P_i(t)\gamma_{ij}(t)$. Let N_0 be the background noise. Then applying (2) to nodes i and j, the condition for i to transmit to j at a rate R at bandwidth B is

$$\frac{P_i(t)\gamma_{ij}(t)}{N_0 + \sum_{k\neq i} P_k(t)\gamma_{kj}(t)} > \beta\left(R/B\right) \tag{3}$$

A standard result (see Gupta and Kumar (2000) and Grossglauser and Tse (2002)) is that signal power decays with distance r as $r^{-\alpha}$ where $\alpha > 2$ is usually assumed. Thus we have

$$\gamma_{ij} = \frac{1}{\mid X_i(t) - X_j(t) \mid^{\alpha}} \tag{4}$$

[3]That is, the probability density function is constant over time and these densities can be calculated by time averaging, over t, rather than than ensemble averaging, over i. Of course, it is possible that this approach to the distribution of nodes is not applicable in the case of mobile radio. In particular, the distribution of mobile terminals 'follows' population distributions and major trunk routes. Further research is suggested to examine these claims in detail.

A further consideration is the existence of *processing gain* $L > 1$ which has the effect of making the signal received by node j from i appear more powerful owing to the use of encoding. This has the effect of reducing the interference from transmissions by other users so that (3) becomes

$$\frac{P_i(t)\gamma_{ij}(t)}{N_0 + \frac{1}{L}\sum_{k\neq i} P_k(t)\gamma_{kj}(t)} > \beta\,(R/B) \tag{5}$$

Processing gain L increases with the bandwidth of the common channel used by the network. Assume for simplicity that all nodes transmit at the same power. Then Putting $N_0 = P_i(t) = P(t)$ and combining (4) and (5) it can be seen that transmission from i to j will be unsuccessful if there is another node X_k simultaneously transmitting within a distance.

$$\mid X_k - X_j \mid \leq (\beta/L)^{\frac{1}{\alpha}} \mid X_i - X_j \mid \tag{6}$$

That is, there cannot be another sender in a disk proportional to the transmission distance, $\mid X_i - X_j \mid$ by a factor $(\beta/L)^{\frac{1}{\alpha}}$. For a given transmission rate R, this factor decreases easing the constraint if the bandwidth increases, reducing the minimum SIR β required, or if α increases, thus increasing the rate at which the channel gain γ_{ij} falls over distance, or if the processing gain L increases. Figure 1 illustrates this constraint.

Information can be transmitted either directly from a source to its destination or can be relayed through one or more other nodes. Assume that all nodes transmit at the same power P. In the absence of cooperation transmissions must be direct. With cooperation the network members can appoint a coordinator or *scheduler* who chooses an optimal *relay policy* consisting of a choice of transmitting nodes and their relay path to the required destination. Now consider separately fixed and mobile nodes.

3.2 Fixed Nodes

For fixed nodes long-range direct transmissions between many pairs is infeasible due to excessive interference. Then in the absence of coordination, the only way the

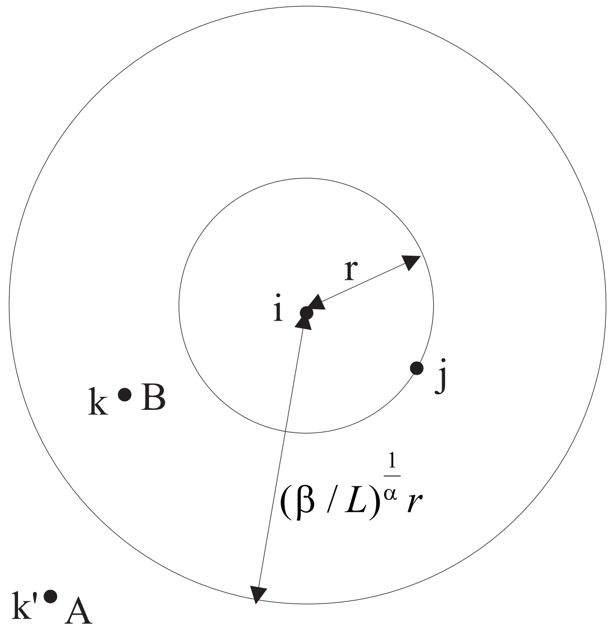

Figure 1: The Transmission Constraint. Successful $i - j$ transmission if nearest other node is at A, but unsuccessful transmission if at B

network can function is introduce more radio channels. With coordination and a common channel, most communication has to occur between nearest neighbours at distances of order $\frac{1}{\sqrt{n}}$ with each message being relayed by the scheduler through many intermediate nodes. Then Gupta and Kumar (2000) show:

Result 1. **For fixed nodes with relaying, there exists a constant c independent of n and N_0 and decreasing in β such that**

$$\lim_{n \to \infty} \Pr \left\{ \lambda(n) = \frac{cR}{\sqrt{n \log n}} \text{ is feasible} \right\} = 1 \tag{7}$$

In other words, $\sqrt{n \log n}\lambda(n)$ converges in probability to cR as $n \to \infty$. We shall loosely refer to this as $\text{plim}\lambda(n) = \frac{cR}{\sqrt{n \log n}}$.

3.3 Mobile Nodes

Grossglauser and Tse (2002) consider the case of mobile nodes where the process $\{X_i(\cdot)\}$ are i.i.d., stationary and ergodic. Without relaying, the number of concurrent transmissions over long distances is still interference limited, but with mobility any two nodes can be expected to be close to each other from time to time. Then Grossglauser and Tse (2002) obtain the result

Result 2. **For mobile nodes without relaying, there exists a constant c' independent of n and N_0 and decreasing in β such that**

$$\lim_{n\to\infty} \Pr\left\{\lambda(n) = c' n^{-\frac{1}{1+\frac{\alpha}{2}}} R \text{ is feasible}\right\} = 1 \tag{8}$$

Now allow mobility and coordination. Then with relaying a result of remarkable importance is obtained:

Result 3. **For mobile nodes with relaying, there exists a constant c'' independent of n and N_0 and decreasing in β such that**

$$\lim_{n\to\infty} \Pr\left\{\lambda(n) = c'' R \text{ is feasible}\right\} = 1 \tag{9}$$

Thus throughput converges in probability to a non-zero positive constant as $n \to \infty$. Thus in the limiting large numbers case, adding nodes does not actually reduce anyone's capacity to use the system; every consumer brings with her an additional input that eliminates the scarcity of bandwidth.

Table 1 summarises these three results. It is the entry for coordination across mobile nodes that perhaps contains the most economic interest. Effectively, this says that every consumer of 'mobile' services (however defined) is also a producer of these services since s/he can be used to transmit information between other parties. Thus, as well as the demand-side externalities traditionally associated with communications systems, we have identified coincident supply-side externalities as well. These externalities may be a feature of extended network settings influencing the operation of the given network (e.g. see Cave *et al.* (2002)).

	Cooperation (Relaying)	Non-Cooperation
Fixed Nodes	plim $\lambda(n) = \frac{cR}{\sqrt{n \log n}}$	nodes require separate channels
Mobile Nodes	plim $\lambda(n) = c''R$	plim $\lambda(n) = c' n^{-\frac{1}{1+\frac{\alpha}{2}}} R$

Table 1: Cooperation Gains between Network Users

4 Cooperation gains: Tragedy of the Commons or a Triumph of Cooperation Gains?

Given feasible throughput $\lambda(n)$ as a function of the number of users n, the possibility of a tragedy of the commons can now be investigated. Let p be the value of transmitting each bit/sec. Let κ be the total cost of establishing each radio link in the network. Then the in terms of probability limits (dropping the prefix 'plim') the value of total traffic is given by

$$V(n) = pn\lambda(n) - \kappa n \tag{10}$$

In a *free-entry equilibrium* users will join the network until the value per user is

$$\frac{V(n)}{n} = p\lambda(n) - \kappa = 0 \tag{11}$$

If $\lambda(n)$ is decreasing in n then this has a solution at $n = n^e$, say.

The *social optimum* by contrast is found from

$$\frac{dV(n)}{dn} = p\left[n\frac{d\lambda(n)}{dn} + \lambda(n)\right] = \kappa \tag{12}$$

If $\lambda(n)$ is decreasing in n and $\lambda''(n) < 0$, then the social optimum $n = n^*$ exists and $n^* < n^e$; i.e., the free-entry equilibrium results in too many users in the network compared with the social optimum and there is a tragedy of the commons as illustrated in Figure 2. This is the case if nodes are fixed with relaying and mobile without relaying. Then there is a role for a spectrum regulator to issue spectrum licences to transmit on the common bandwidth to only $n^* < n^e$ users.

To take an example suppose that nodes are mobile but there is no relaying so that in the probability limit $\lambda(n) = c'n^{-\frac{1}{1+\frac{\alpha}{2}}}R$. Then using (11) and (12) we can derive

$$n^e = \left[\frac{pc'R}{\kappa}\right]^{1+\frac{\alpha}{2}} \tag{13}$$

$$n^* = \left[\frac{pc'R}{\left(1+\frac{2}{\alpha}\right)\kappa}\right]^{1+\frac{\alpha}{2}} \tag{14}$$

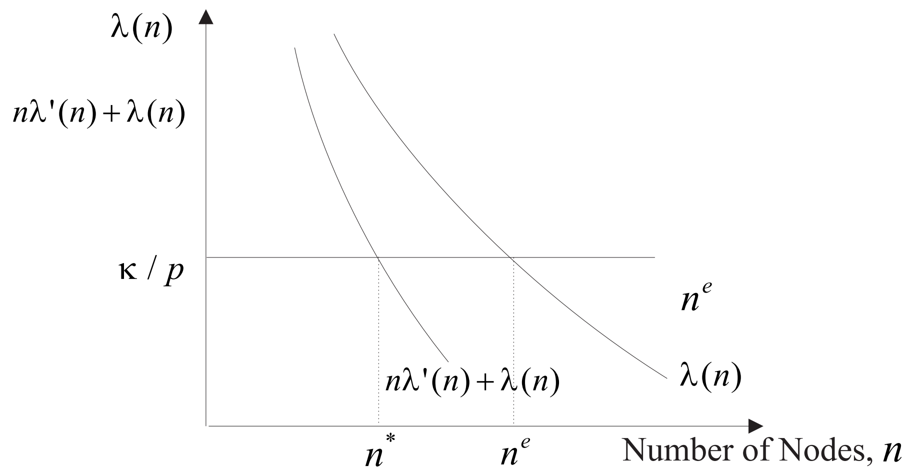

Figure 2: The Tragedy of the Commons

and hence

$$\frac{n^*}{n^e} = \frac{1}{\left(1+\frac{2}{\alpha}\right)^{1+\frac{\alpha}{2}}} \tag{15}$$

For $\alpha = 2$ then $\frac{n^*}{n^e} = \frac{1}{4}$ which rises as α increases until as $\alpha \to \infty$ the free-entry equilibrium converges to the social optimum. The reason for this is that as α increases, interference becomes more localised and less of an externality. In the limit as α becomes infinite, this externality disappears altogether.

However if nodes are randomly mobile as in Grossglauser and Tse (2002), $\lambda(n) = c''$ in the probability limit. Users will keep joining the network indefinitely without any loss of throughput. This is achieved through relaying and enhanced cooperation, so the tragedy of the commons is transformed into a triumph of network cooperation and the role for the regulator disappears.

It should be stressed that this is a theoretical result based on a restricted set of assumptions of the model, particularly regarding the distribution of the mobile nodes. Furthermore, this triumphal outcome is limited to the cases where either one technology (e.g. mesh) is deployed or there is a common air interface to which all technologies subscribe. Perhaps more crucially it assumes that the transmission with delay imposes no loss in value to the user. As the number of nodes increases, more and more messages are queued in transit at the relay nodes. Even though the throughput as defined at the beginning of this section does not degrade, the mean response time of the system does. This may not pose problems for non-real-time services such as email, but it would not be a suitable solution for any service requiring bi-directional real-time communication.[4] Further, while the overall throughput will increase with the number of users, the increase of useful data throughput (i.e. non-redundant data) has yet to be investigated. All these caveats suggest areas for future research.

5 Policy Implications and Conclusions

Webb and Cave (2003) have suggested a basis for determining how much radio spectrum should be made available for unlicensed applications, which in the context of UK legislation means applications specifically exempted from licensing on an individual basis. The paper points to the extensive debate taking place in the U.S surrounding some key new technologies and whether spectrum can be used as a common resource.

We concur with Webb and Cave's conclusion that there is no consensus view from these or other related papers that the regulator may draw on. However, by providing a rigorous analysis for spectrum commons we offer some conclusions on this matter. Webb and Cave suggest that a commons approach works where there is little probability of congestion. However it is still the case that with most technologies, even those that make use of advanced frequency hopping and avoidance algorithms, congestion will reduce capacity particularly where the technologies operate in a mixed commons environment. Using the results of Gupta and Kumar (2000) and Grossglauser and Tse (2002) we have shown that a single mesh technology solution in a mobile environment can, under certain assumptions about the randomness of mobile users, transform the tragedy into a triumph of commons. This is a startling conclusion whose robustness with respect to the modelling assumptions

[4]We are grateful to Robert Leese for pointing this out.

is one important area for future research.

Turning to policy implications and opportunities, the most difficult three issues for the regulator to address are spectrum congestion, bandwidth for high data rate services and overcoming the rural telecommunications divide. What is needed is a radically new approach if these three are to be addressed simultaneously. A model based on an internet with mobility, mesh technology, a common air interface and adaptive power control has some merit. To use an analogy: if what inhibits the growth of broadband demand is the cost of deploying base stations then it would seem sensible to use spare capacity within the mobile terminals to 'fill in' by off-loading traffic in a co-operative manner. The twin challenges for the regulator are to stimulate the early development of this vision and to accommodate it in the right spectrum space.

This paper is part of a general research agenda that addresses policy questions in the area of spectrum allocation by the combination of models from radio engineering and economics. Our approach complements this literature by offering rigorous modelling of the issues identified, whilst advancing it by the multidisciplinary approach adopted.

A valuable result of the formal modelling of spectrum commons is that it allows us to see the circumstances under which results do (not) hold. Accordingly, this allows us to think about the factors that influence where one should draw the line between different policy options. The work we have drawn on (Grossglauser and Tse (2002)) makes several important assumptions whose relaxation is needed before one can use them as a basis for policy towards any form of spectrum commons. Thus, the supply-side externality generated by additional, mobile, consumers will generally be limited by the distribution of population and typical patterns of activity. Clearly, it is important to test the robustness of such results to distributional assumptions; and system simulations to investigate this are required. Also the questions of what might happen in the model when there are heterogeneous technologies and when issues like delivery delay needs to be considered for the different traffic types. These also require further research before one can confidently claim any particular triumph for commons spectrum models.

More practical questions are also raised by our results. If, in a suitably developed model of mesh technologies, the commons problem is less sever, what sorts of technologies should be encouraged to take advantage of this? Wireless LANs, for example, can be seen as a limited multi-hop technology but are they the most suitable for harnessing the supply-side externalities identified?

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