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MONETARY POLICY COORDINATION REVISITED IN A TWO-BLOC DSGE MODEL

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Monetary Policy Coordination Revisited in a Two-Bloc DSGE Model*

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Abstract

We reassess the gains from monetary policy coordination within the confines of the canonical NOEM in the light of three issues. First, the literature uses a number of cooperative and non-cooperative equilibrium concepts that do not always clearly distinguish commitment and discretionary outcomes, and in some cases adopts inappropriate concepts. Second, our analysis is welfare based. Moreover, as with much of this literature, we adopt a linear-quadratic approximation of the actual non-linear non-quadratic stochastic optimization problem facing the monetary policymakers. Our second objective then is to re-assess welfare gains using an accurate approximation for such a problem, a feature that for the most part is lacking in previous studies. Finally, we examine the issue where the monetary authority is restricted to rules that are *operational* in two senses: first, the zero lower bound constraint is imposed on the optimal rule and second, we study simple Taylor-type commitment rules that unlike fully optimal rules are easily monitored by the public.

JEL Classification: E52, E37, E58

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1 Introduction

Following the seminal contribution of Obstfeld and Rogoff (1996), chapter 10, New Keynesian open economy DSGE modelling, the 'New Open Economy Macroeconomics', has been a highly active area.[1] Obstfeld and Rogoff developed a non-stochastic, perfect foresight two-country general equilibrium model with first flexible prices, and then price-rigidity. This model formed the basis for the emergence of a wave of New Open Economy stochastic general equilibrium models that have been used to examine the potential welfare gains from monetary policy coordination. The earlier of these studies were based on a very basic New Open Economy Model (NOEM) that assumed perfect financial markets, complete exchange rate pass-through, the absence of a traded sector, wage flexibility and other features that kept the analysis reasonably tractable (though ultimately the reliance on numerical simulations to quantify the gains from cooperation still remained).[2]

The more recent papers have seen a reassessment of these gains using more empirical and more developed DSGE models incorporating various persistence mechanisms, incomplete financial markets, incomplete exchange rate pass-through and a non-traded sector.[3]In the words of Canzoneri *et al.* (2005), "what is yet to come" is the reassessment of welfare gains from coordination using empirical DSGE models that embody these features.

This paper does not attempt this ambitious goal. Our aim instead is to remain more or less within the confines of the canonical NOEM and to examine the no gains result in the light of three issues. First, the literature uses a number of cooperative and non-cooperative equilibrium concepts that do not always clearly distinguish commitment and discretionary outcomes, and in some cases adopts inappropriate concepts. Second, our analysis is welfare based. Moreover, as with much of this literature, we adopt a linear-quadratic approximation of the actual non-linear non-quadratic stochastic optimization problem facing the monetary policymakers. Our second objective then is to re-assess welfare gains using an accurate approximation for such a problem, a feature that for the most part is lacking in previous studies. Finally, we examine the issue where the monetary authority is restricted to rules that are *operational* in two senses: first, the zero lower bound

[1]See also Obstfeld and Rogoff (2000) and a survey by Lane (2001).

[2]See, for example, Benigno and Benigno (2001), Obstfeld and Rogoff (2002) Clarida *et al.* (2002) and Pappa (2004a).

[3]See, in particular, Sutherland (2004), Batini *et al.* (2005), Liu and Pappa (2005), Coenen *et al.* (2007).

constraint is imposed on the optimal rule and second, we study simple Taylor-type rules that unlike fully optimal rules are easily monitored by the public.

2 The Model

In this section set out a standard model, similar to Pappa (2004b).[4] In her model there is no non-traded sector, complete financial markets, only PCP setting, flexible wages, no government spending, no habit, no indexation and only productivity shocks. Her model allows for home bias and a non-unitary elasticity of substitution in the choice of domestic and imported goods. We generalize to include external habit in consumption, government spending, preference shocks, an oil shock and price indexing. Details of the model are as follows.

2.1 Households

There are ν households in the 'home' bloc and ν^* households in the 'foreign' bloc. A representative household r in the home bloc maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[\frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1-\sigma} + U_{M,t} \frac{\left(\frac{M_t(r)}{P_t}\right)^{1-\varphi}}{1-\varphi} - U_{L,t} \frac{L_t(r)^{1+\phi}}{1+\phi} + u(G_t) \right] \tag{1}$$

where E_t is the expectations operator indicating expectations formed at time t, β is the household's discount factor, $U_{C,t}$, $U_{M,t}$ and $U_{L,t}$ are preference shocks $C_t(r)$ is an index of consumption, $L_t(r)$ are hours worked, $H_{C,t}$ represents the habit in consumption, or desire not to differ too much from other households, and we choose $H_{C,t} = hC_{t-1}$, where $C_t = \frac{1}{\nu} \sum_{r=1}^{\nu} C_t(r)$ is the average consumption index, $h \in [0, 1)$. When $h = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the intertemporal elasticity of substitution)[5]. $M_t(r)$ are end-of-period nominal money balances and G_t is exogenous per capita real government spending assumed to be exclusively on non-traded domestic output. An analogous symmetric intertemporal utility is defined for the 'foreign' representative household and the corresponding variables (such as consumption) are denoted by $C_t^*(r)$, etc.

[4]Clarida *et al.* (2002) is a special case of her model. At various places in the paper we indicate how their model and results can be obtained.

[5]When $h \neq 0$, σ is merely an index of the curvature of the utility function.

The representative household r must obey a budget constraint:

$$
\begin{aligned}
P_t C_t(r) + E_t[Q_{t,t+1} D_{t+1}(r)] + M_t(r) &= (1-T_t) W_t(r) L_t(r) + D_t(r) + M_{t-1}(r) \\
&+ (1-T_t)\Gamma_t(r) + TR_t \qquad (2)
\end{aligned}
$$

where P_t is a Dixit-Stiglitz CPI price index defined in (4) below, $D_{t+1}(r)$ is a random variable denoting the payoff of the portfolio purchased at time t and $Q_{t,t+1}$, the stochastic discount factor, is the period-t price of an asset that pays one unit of domestic currency in a particular state of period $t+1$ divided by the probability of an occurrence of that state given information available in period t. $W_t(r)$ is the wage rate, T_t the income tax rate and $\Gamma_t(r)$ are dividends from ownership of firms.[6] Finally TR_t are lump-sum transfers to households by the government net of lump-sum taxes

Assume the existence of nominal one-period riskless bonds denominated in domestic currency with nominal interest rate I_t over the interval $[t, t+1]$. Then arbitrage considerations imply that $E_t[Q_{t,t+1}] = \frac{1}{1+I_t}$. In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand for each consumer's labor supplied by ν identical households is given by

$$
L_t(r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} L_t \qquad (3)
$$

where $W_t = \left[\frac{1}{\nu}\sum_{r=1}^{\nu} W_t(r)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ and $L_t = \left[\frac{1}{\nu}\sum_{r=1}^{\nu} L_t(r)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ are the average wage index and average employment respectively.[7]

Let the number of differentiated goods produced in the home and foreign blocs be n_H and n_F respectively. Each good is produced by a single firm and we assume that the the ratio of households to firms are the same in each bloc, i.e., $\frac{\nu}{n} = \frac{\nu^*}{n^*}$. It follows that n and n^* (or ν and ν^*) are measures of *size*. Then the per capita consumption index in the home bloc is given by

$$
\begin{aligned}
C_t(r) &= \left[\mathrm{w}^{\frac{1}{\mu}} C_{H,t}(r)^{\frac{\mu-1}{\mu}} + (1-\mathrm{w})^{\frac{1}{\mu}} C_{F,t}(r)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} ;\ \mu \neq 1 \\
&= \mathrm{w}^{-\mathrm{w}}(1-\mathrm{w})^{-(1-\mathrm{w})} C_{H,t}(r)^{\mathrm{w}} C_{F,t}(r)^{1-\mathrm{w}} ;\ \mu = 1 \qquad (4)
\end{aligned}
$$

[6] The tax rate T_t can be interpreted as a total tax wedge (see Levine *et al.* (2006)).

[7] Note that if we normalize $\nu = 1$ then as is more customary in the literature we can write $W_t \simeq \left[\int_0^1 W_t(r)^{1-\eta} dr\right]^{\frac{1}{1-\eta}}$. However here we allow for different sized blocs with the foreign number of households $\nu^* \neq \nu$.

where μ is the elasticity of substitution between home and foreign traded goods,

$$C_{H,t}(r) = \left[\left(\frac{1}{n_H}\right)^{\frac{1}{\zeta}} \sum_{f=1}^{n_H} C_{H,t}(f,r)^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \tag{5}$$

$$C_{F,t}(r) = \left[\left(\frac{1}{n_F}\right)^{\frac{1}{\zeta}} \sum_{f=1}^{n_F} C_{F,t}(f,r)^{(\zeta-1)/\zeta)}\right]^{\zeta/(\zeta-1)} \tag{6}$$

$C_{H,t}(f,r)$ and $C_{F,t}(f,r)$ denote the home consumption of traded variety f produced in blocs H and F respectively, ζ is the elasticities of substitution between varieties in each bloc (note that we impose equality between blocs for this traded elasticity, i.e., $\zeta^* = \zeta$), and

$$\mathrm{w} = \frac{n_H \omega}{n_H \omega + n_F(1-\omega)} \tag{7}$$

In (7) $\omega \in [\frac{1}{2}, 1]$ is a parameter that captures the degree of 'bias' in the home bloc. If $\omega = 1$ we have autarky, while the lower extreme of $\omega = \frac{1}{2}$ gives us the case of no home bias (perfect integration). If blocs are of equal size then $n_H = n_F$, $\mathrm{w} = \omega$ and consumption only favours home consumption if there is home bias.[8] In the absence of home bias $\mathrm{w} = \frac{n_H}{n_H+n_F}$, $\mathrm{w} + \mathrm{w}^* = 1$ and domestic/foreign consumption decisions depend only on relative size.

If $P_{H,t}(f)$, $P_{F,t}(f)$ are the prices in domestic currency of the good produced by firm f in the relevant bloc, then the optimal intra-temporal decisions are given by standard results:

$$C_{H,t}(r,f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\zeta} C_{H,t}(r)\,; \;\; C_{F,t}(r,f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\zeta_T} C_{F,t}(r) \tag{8}$$

$$C_{H,t}(r) = \mathrm{w}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\mu} C_t(r)\,; \;\; C_{F,t}(r) = (1-\mathrm{w})\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t(r) \tag{9}$$

where aggregate price indices for domestic and foreign consumption bundles of traded goods are given by, respectively,

$$P_{H,t} = \left[\frac{1}{n_H}\sum_{f=1}^{n_H} P_{H,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{10}$$

$$P_{F,t} = \left[\frac{1}{n_F}\sum_{f=1}^{n_F} P_{F,t}(f)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{11}$$

[8] The case $\mu \to 1$ without home bias is studied in Clarida *et al.* (2002). The effect of home bias in open economies is also examined in Corsetti *et al.* (2002) and De Fiore and Liu (2002).

and the aggregate price indices P_t, P_t^* are given by

$$\begin{aligned} P_t &= U_{OIL,t}\left[\mathrm{w}(P_{H,t})^{1-\mu} + (1-\mathrm{w})(P_{F,t})^{1-\mu}\right]^{\frac{1}{1-\mu}} \; ; \; \mu \neq 1 \\ &= U_{OIL,t}(P_{H,t})^{\mathrm{w}}(P_{F,t})^{1-\mathrm{w}}\; ; \qquad \mu = 1 \qquad (12) \\ P_t^* &= U_{OIL,t}\left[\mathrm{w}^*(P_{F,t}^*)^{1-\mu^*} + (1-\mathrm{w}^*)(P_{H,t}^*)^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} \\ &= U_{OIL,t}(P_{F,t}^*)^{\mathrm{w}^*}(P_{H,t}^*)^{1-\mathrm{w}^*}\; ; \qquad \mu^* = 1 \qquad (13) \end{aligned}$$

where $U_{OIL,t}$ is an oil price shock. Aggregate nominal consumption is then given by

$$P_t C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \qquad (14)$$

It now follows that relative CPI prices $\frac{S_t P_t^*}{P_t}$, the 'real exchange rate', and the terms of trade, defined as the domestic currency relative price of imports to exports, $\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$, are related by the relationship

$$\begin{aligned} RER_t \equiv \frac{S_t P_t^*}{P_t} &= \frac{\left[\mathrm{w}^*(\mathcal{T}_t)^{1-\mu^*} + (1-\mathrm{w}^*)^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}}}{\left[\mathrm{w} + (1-\mathrm{w})\mathcal{T}_t^{1-\mu}\right]^{\frac{1}{1-\mu}}}\; ; \;\; \mu \neq 1, \mu^* \neq 1 \\ &= \mathcal{T}_t^{\mathrm{w}+\mathrm{w}^*-1}\; ; \;\; \mu = \mu^* = 1, \qquad (15) \end{aligned}$$

Thus if $\mu = \mu^*$, then the law of one price applies to the CPI price indices iff $\mathrm{w}^* = 1 - \mathrm{w}$. The latter condition holds if there is no home bias. If there is home bias, the real exchange rate, $\frac{S_t P_t^*}{P_t}$, rises (a depreciation) as the terms of trade, $\mathcal{T}$, rises (a depreciation).[9]

Now consider the consumption, money demand and labour supply decisions of the representative household. We first consider the case of flexible wages. Then maximizing (1) subject to (2) and (3), treating habit as exogenous, and imposing symmetry on households (so that $C_t(r) = C_t$, etc) yields standard results:

$$\begin{aligned} Q_{t,t+1} &= \beta \frac{MU_{t+1}^C}{MU_t^C}\frac{P_t}{P_{t+1}} \qquad (16) \\ MU_t^M &= MU_t^C\left[\frac{I_t}{1+I_t}\right] \qquad (17) \\ \frac{W_t(1-T_t)}{P_t} &= -\frac{1}{\left(1-\frac{1}{\eta}\right)}\frac{MU_t^L}{MU_t^C} \equiv \frac{1}{\left(1-\frac{1}{\eta}\right)}MRS_t \qquad (18) \end{aligned}$$

[9]Clarida *et al.* (2002) assumed no bias and $\mu = \mu^* = 1$ in which case $RER_t = 1$ (law of one price for CPI indices).

where MU_t^C, MU_t^M and $-MU_t^L$ are the marginal utility of consumption, money holdings and the marginal disutility of work respectively. Taking expectations of (16) we arrive at the following familiar Keynes-Ramsey rule:

$$1 = \beta(1 + I_t)E_t \left[\frac{MU_{t+1}^C}{MU_t^C} \frac{P_t}{P_{t+1}} \right] \tag{19}$$

In (17), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. Given the central bank's setting of the latter and ignoring seignorage in the government budget constraint, (17) is completely recursive to the rest of the system describing our macro-model and will be ignored in the rest of the paper. In (18) the real disposable wage is proportional to the marginal rate of substitution between consumption and leisure, $-\frac{MU_t^L}{MU_t^C}$, this constant of proportionality reflecting the market power of households that arises from their monopolistic supply of a differentiated factor input with elasticity η.

2.2 Producers

In the domestic sector, each good differentiated good f is produced by a single firm f using only differentiated labour with another constant returns CES technology:

$$Y_t(f) = A_t \left[\left(\frac{1}{\nu}\right)^{\frac{1}{\eta}} \sum_{r=1}^{\nu} L_{i,t}(f,r)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \equiv A_t L_t(f) \tag{20}$$

where $L_t(f,r)$ is the labour input of type r by firm f and A_t is an exogenous shock capturing shifts to trend total factor productivity in this sector. Minimizing costs $\sum_{f=1}^{\nu} W_t(r)L_t(f,r)$ gives the demand for each household's labour by firm f as

$$L_t(f,r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} L_t(f) \tag{21}$$

and aggregating over firms leads to the demand for labor as shown in (3).[10]

In a equilibrium of equal households, all wages adjust to the same level W_t. For later analysis it is useful to define the real marginal cost (MC) as the wage relative to domestic

[10] Note that in a *symmetric equilibrium* of identical firms and households, total demand for labour of type r by firms is $L_t(r) = \sum_{f=1}^{n_H} L_t(f,r)$. Hence $L_t = \sum_{f=1}^{n_H} L_t(f) = \sum_{r=1}^{n_H} L_t(r)$, $n_H L_t(f) = \nu L_t(r)$. Such a symmetric equilibrium applies to the flexi-price case of our model, but *not* to the sticky-price case where, at each point in time, some firms are locked into price and wage contracts, but others are re-optimizing these contracts.

producer price. Using (18) this can be written as

$$\text{MC}_t \equiv \frac{W_t}{A_t P_{H,t}} = \frac{\eta U_{L,t}}{(\eta-1)(1-T_t)A_t} L_t^{\phi}(C_t - H_{C,t})^{\sigma}\left(\frac{P_t}{P_{H,t}}\right) \tag{22}$$

Turning to price-setting we assume that there is a probability of $1-\xi_H$ at each period that the price of each good f is set optimally to $\hat{P}_{H,t}(f)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation.[11] With indexation parameter $\gamma_H \geq 0$, this implies that successive prices with no re-optimization are given by $\hat{P}_{H,t}(f)$, $\hat{P}_{H,t}(f)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H}$, $\hat{P}_{H,t}(f)\left(\frac{P_{H,t+1}}{P_{H,t-1}}\right)^{\gamma_H}$, ... For each producer f the objective is at time t to choose $\hat{P}_{H,t}(f)$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{t+k}(f)\left[\hat{P}_{H,t}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - P_{H,t+k}\text{MC}_{t+k}\right] \tag{23}$$

where $Q_{t,t+k}$ is the discount factor over the interval $[t, t+k]$, subject to a common[12] downward sloping demand from domestic consumers and foreign importers of elasticity ζ as in (8). The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{T,t+k}(f)\left[\hat{P}_{H,t}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - \frac{\zeta}{(\zeta-1)} P_{H,t+k}\text{MC}_{t+k}\right] = 0 \tag{24}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{H,t+1}^{1-\zeta} = \xi_H\left(P_{H,t}\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H}\right)^{1-\zeta} + (1-\xi_H)(\hat{P}_{H,t+1}(f))^{1-\zeta} \tag{25}$$

The first-order condition (24) is cumbersome to manipulate. However it is possible to express this price-setting rule in terms of *difference equations* that are far easier to manipulate. To do this first note that

$$Y_{t+k}(j) = \left(\frac{\hat{P}_{H,t}}{P_{H,t+k}}\right)^{-\zeta} Y_{t+k} \tag{26}$$

and multiplying both sides of (24) by $(\frac{\hat{P}_{H,t}}{P_{H,t}})^{\zeta} MU_t^C$ and in addition noting that $P_{H,t+k}/P_{H,t} = \Pi_{H,t+k} ... \Pi_{H,t+1}$, the firms' staggered price setting can be succinctly described by

$$Q_{H,t} = \Lambda_t / H_t \tag{27}$$

[11] Thus we can interpret $\frac{1}{1-\xi_H}$ as the average duration for which prices are left unchanged.

[12] Recall that we have imposed a symmetry condition $\zeta = \zeta^*$ at this point; i.e., the elasticity of substitution between differentiated goods produced in any one bloc is the same for consumers in both blocs.

where we have defined variables $\Pi_{H,t}$, $\tilde{\Pi}_{H,t}$, $Q_{H,t}$, H_t and Λ_t by

$$\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}} \tag{28}$$

$$\tilde{\Pi}_{H,t} \equiv \frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\gamma_H}} \tag{29}$$

$$Q_{H,t} \equiv \frac{\hat{P}_{H,t}}{P_{H,t}} \tag{30}$$

$$H_t - \xi\beta E_t[\tilde{\Pi}_{H,t+1}^{\zeta-1} H_{t+1}] = Y_t \, MU_t^C \tag{31}$$

$$\Lambda_t - \xi\beta E_t[\tilde{\Pi}_{H,t+1}^{\zeta} \Lambda_{t+1}] = \frac{Y_t \, MC_t \, MU_t^C}{(1-1/\zeta)} = \frac{Y_t \frac{W_t}{A_t P_t} MU_t^C}{(1-1/\zeta)} \tag{32}$$

and from (25) aggregate inflation is given by

$$1 = \xi_H \tilde{\Pi}_{H,t}^{\zeta-1} + (1-\xi_H) Q_{H,t}^{1-\zeta} \tag{33}$$

This completes the supply-side in the home bloc. Analogous results hold for the foreign bloc.

2.3 Price Dispersion

The impact of price dispersion arises from labour input being the same for each individual, but dependent on demand for each good:

$$L_t = \int_0^1 L_t(j)dj = \frac{Y_t}{A_t} \int_0^1 \frac{Y_t(j)}{Y_t} dj = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\zeta} dj \tag{34}$$

Now define price dispersion $\Delta_t = \int_0^1 (\frac{P_{H,t}(j)}{P_{H,t}})^{-\zeta} dj \geq 1$. Equality is reached only when prices are flexible and therefore the same, as all firms are identical except in their timing of price changes. Now we can write down aggregate output as

$$Y_t = \frac{A_t L_t}{\Delta_t} \leq A_t L_t \tag{35}$$

which clearly highlights the output distortion caused by price dispersion $\Delta_t \geq 1$.

Price dispersion is linked to inflation as follows. Assuming as before that the number of firms is large we obtain the following dynamic relationship:

$$\Delta_t = \xi \tilde{\Pi}_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi) Q_{H,t}^{-\zeta} \tag{36}$$

2.4 The Equilibrium

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the home consumer good we obtain

$$Y_t = \frac{A_t L_t}{\Delta_t} = C_{H,t} + \frac{\nu^*}{\nu} C^*_{H,t} + G_t \tag{37}$$

$$\tag{38}$$

Fiscal policy is rudimentary: a balanced government budget constraint[13]

$$P_{H,t}G_t + TI_t = T_t(P_{H,t}C_{H,t} + \frac{\nu^*}{\nu} S_t P^*_{H,t} C^*_{H,t} + P_{H,t}G_t) \equiv T_t \, GDP_t \tag{39}$$

where GDP_t is nominal GDP, completes the model. As in Coenen *et al.* (2005) we further assume that changes in government spending are financed exclusively by changes in lump-sum taxes with the tax rates T_t, held constant at its steady-state value.

From (16) and its foreign counterpart we have

$$Q_{t,t+1} = \beta \frac{MU^C_{t+1}}{MU^C_t} \frac{P_t}{P_{t+1}} = \beta \frac{MU^{C*}_{t+1}}{MU^{C*}_t} \frac{P^*_t S_t}{P^*_{t+1} S_{t+1}} \tag{40}$$

Let $z_t = \frac{S_t P^*_t}{P_t} \frac{MU^C_t}{MU^{C*}_t}$. Then assuming identical holdings of initial wealth in the two blocs, (40) implies that $z_{t+1} = z_t = z_0$ where initial relative consumption in prices denominated in the home currency reflects different initial wealth in the two blocs. Therefore[14]

$$\frac{MU^C_t}{MU^{C*}_t} = \frac{z_0 P_t}{S_t P^*_t} = \frac{z_0}{RER_t} \tag{41}$$

Given nominal interest rates I_t, I^*_t the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition. Then the equilibrium is defined at $t = 0$ as stochastic sequences C_t, $C_{H,t}$, $C_{F,t}$, $P_{H,t}$, $P_{F,t}$, P_t, M_t, W_t, $Y_{H,t}$, L_t, , L_t, $\hat{P}_{H,t}$, 16 foreign counterparts C^*_t, etc, RER_t, and $\mathcal{T}_t$, given past price indices and exogenous processes $U_{C,t}$, $U_{M,t}$, $U_{L,t}$, A_t, TR_t, G_t and foreign counterparts.

[13] In this cashless economy, we ignore seignorage and consistent with this we later ignore the utility from money balances in the household welfare function.

[14] (41) is the risk-sharing condition for consumption, because it equates marginal rate of substitution to relative price, as would be obtained if utility were being jointly maximized by a social planner (see Sutherland (2002)). Note that (79) and (41) together imply the stochastic UIP condition (see Benigno and Benigno (2001)).

2.5 Steady State

A deterministic zero-inflation steady state, denoted by variables without the time subscripts, $U_{C,t} = U_{OIL,t} = 1$ and $U_{L,t} = \kappa$ is given by

$$C_H = \mathrm{w}\left(\frac{P_H}{P}\right)^{-\mu} C \tag{42}$$

$$C_F = (1-\mathrm{w})\left(\frac{P_F}{P}\right)^{-\mu} C \tag{43}$$

$$\begin{aligned} P &= \left[\mathrm{w}P_H^{1-\mu} + (1-\mathrm{w})P_F^{1-\mu}\right]^{\frac{1}{1-\mu}} \; ; \; \mu \neq 1 \\ &= P_H^{\mathrm{w}} P_F^{1-\mathrm{w}} \; ; \; \mu = 1 \end{aligned} \tag{44}$$

$$\frac{W(1-T)}{P} = \frac{\kappa L^{\phi}((1-h)C)^{\sigma}}{1-\frac{1}{\eta}} \tag{45}$$

$$1 = \beta(1+I) \tag{46}$$

$$\Delta = 1 \tag{47}$$

$$Y = AL \tag{48}$$

$$P_H = \hat{P}_H = \frac{W}{A\left(1-\frac{1}{\zeta_T}\right)} \tag{49}$$

$$P_F = SP_F^* \tag{50}$$

$$Y = C_H + \frac{\nu^*}{\nu} C_H^* + G \tag{51}$$

$$T = \frac{P_H G + TR}{GDP} = \frac{P_H G + TR}{PC + P_H G} \tag{52}$$

plus the 11 foreign counterparts and

$$\mathcal{T} = \frac{P_F}{P_T} \tag{53}$$

$$\begin{aligned} RER &= \frac{SP^*}{P} = \frac{\left[\mathrm{w}^*\mathcal{T}^{1-\mu^*} + 1 - \mathrm{w}^*\right]^{\frac{1}{1-\mu^*}}}{\left[\mathrm{w} + (1-\mathrm{w})\mathcal{T}^{1-\mu}\right]^{\frac{1}{1-\mu}}} \; ; \; \mu \neq 1, \mu^* \neq 1 \\ &= \mathcal{T}^{\mathrm{w}+\mathrm{w}^*-1} \; ; \; \mu = \mu^* = 1 \end{aligned} \tag{54}$$

$$\frac{C(1-h)}{C^*(1-h^*)} = \left(\frac{RER_t}{z_0}\right)^{\frac{1}{\sigma}} \tag{55}$$

We now have gives 25 equations to determine the steady state of 27 endogenous variables: C, C_H, C_F, P, W, L, I, Δ, Y, $P_H = \hat{P}_H$, P_F, T, 12 foreign counterparts C^* etc, $\mathcal{T}$, S, and RER given G, TR and z_0.

To pin down price levels we need to re-introduce money equate money demand and its foreign counterpart with exogenously set money supplies in the two blocs, which then gives us a determinate steady state of the model. It is convenient to assume that money supplies in our steady state are set so as to result in $S = 1$ and dispense with the money demand equations. Furthermore, as is standard in general equilibrium models, we choose units of output appropriately so that steady-state prices of the two goods in their own currencies are unity; i.e, $P_H = P_F^* = 1$. With this normalization and the fact that the law of one price holds in the steady state, we have that $P = P_F = \mathcal{T} = RER = 1$. Similarly for the foreign bloc $P^* = P_H^* = 1$. Thus in *the steady state we can normalize all prices at unity*, an extremely convenient property when it comes to the linearization.

2.5.1 The Inefficiency of the Steady State

In our model there are three sources of inefficiency: the tax wedge, labour and output market power, and external habit. Later in the LQ approximation of the policymakers' optimization problems these inefficiencies in the steady state play a prominent role. In a symmetric model of two identical economies the steady state trade balance is zero and we can appeal to results from Choudhary and Levine (2006). Then the zero-inflation steady state output in the market economy above and the steady state of the social optimum are given respectively by

$$Y^{\phi+\sigma} = \frac{A^{1+\phi}(1-T)(1-\frac{1}{\zeta})(1-\frac{1}{\eta})}{\kappa(1-h)^{\sigma}(1-g_y)^{\sigma}} \tag{56}$$

$$\hat{Y}^{\phi+\sigma} = \frac{A^{1+\phi}(1-\beta h)}{\kappa(1-h)^{\sigma}(1-g_y)^{\sigma}} \tag{57}$$

It follows that we can measure the net effect of distortions by

$$\Phi_y \equiv \left(1 - \frac{Y}{\hat{Y}}\right) = 1 - \frac{(1-T)\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right)}{(1-h\beta)} \geq \text{or} \leq 0 \tag{58}$$

In the case where there is no habit persistence ($h = 0$), then $\Phi_y > 0$. Then tax distortions and market power in the output and labour markets, captured by the elasticities $\eta \in (0, \infty)$ and $\zeta \in (0, \infty)$ respectively, drive the market equilibrium output below the efficient level. If $h = T = 0$ and $\eta = \zeta = \infty$, tax distortions and market power both disappear, $\Phi_y = 0$ and the steady state market equilibrium is efficient. But if $h > 0$, this leads to the possibility that $\Phi_y < 0$ and then the market equilibrium output is actually *above* the efficient level (see Choudhary and Levine (2006)). Then the household's consumption-leisure choose leads to excessive levels of work effort and consumption and insufficient leisure relative to the social optimum.

2.6 Linearized Model

Linearizing about the steady state set out in section 2.5 we obtain the following state-space representation. All variables are expressed in deviation form[15]

$$
\begin{aligned}
a_{t+1} &= \rho_a a_t + \epsilon_{a,t+1} \\
a^*_{t+1} &= \rho^*_a a^*_t + \epsilon^*_{a,t+1} \\
u_{C,t+1} &= \rho_C u_{C,t} + \epsilon_{C,t+1} \\
u^*_{C,t+1} &= \rho^*_C u^*_{C,t} + \epsilon^*_{C,t+1} \\
g_{t+1} &= \rho_g g_t + \epsilon_{g,t+1} \\
g^*_{t+1} &= \rho^*_g g^*_t + \epsilon^*_{g,t+1} \\
e_{t+1} &= \rho_e e_t + \epsilon_{e,t+1} \\
e^*_{t+1} &= \rho^*_e e^*_t + \epsilon^*_{e,t+1} \\
\mathrm{oil}_{t+1} &= \rho_o \mathrm{oil}_t + \epsilon_{o,t+1} \\
E_t mu^C_{t+1} &= mu^C_t - (i_t - E_t \pi_{t+1}) \\
E_t mu^{C*}_{t+1} &= mu^{C*}_t - (i^*_t - E_t \pi^*_{t+1}) \\
\beta E_t \pi_{H,t+1} &= \pi_{H,t} - \gamma_H \pi_{H,t-1} - \lambda_H mc_t - e_t \\
\beta E_t \pi^*_{F,t+1} &= \pi^*_{F,t} - \gamma^*_F \pi_{F,t-1} - \lambda^*_F mc^*_t - e^*_t
\end{aligned}
$$

Non-state-space variables, o_t, are given by

$$
\begin{aligned}
mrs_t &= mu^L_t - mu^C_t \\
mrs^*_t &= mu^{L*}_t - mu^{C*}_t
\end{aligned}
$$

[15]That is, for a typical variable X_t, $x_t = \frac{X_t - X}{X} \simeq \log\left(\frac{X_t}{X}\right)$ where X is the baseline steady state. For variables expressing a rate of change over time, π_t and i_t, $x_t = X_t - X$. Since steady-state inflation is zero π_t is the actual inflation rate, but $i_t = I_t - I$. Since $\frac{X_t - X}{X} \simeq \log\left(\frac{X_t}{X}\right)$ a log-linearization leads to practically the same linear form of the dynamic model, but, as we will see, to quite different quadratic approximations of the loss function.

$\pi_{H,t}$, $\pi^*_{F,t}$	producer price inflation over interval $[t-1,t]$
$\pi_{F,t}$, $\pi^*_{H,t}$	imported price inflation over interval $[t-1,t]$
π_t, π^*_t	CPI inflation over interval $[t-1,t]$
i_t, i^*_t	nominal interest rate over interval $[t,t+1]$
mc_t, mc^*_t	marginal cost
mu^C_t, mu^{C*}_t	marginal utility of consumption
mu^L_t, mu^{L*}_t	marginal utility of labour supply
c_t, c^*_t	consumption
y_t, y^*_t	output
l_t, l^*_t	employment
a_t, a^*_t	total factor productivity shock
g_t, g^*_t	government spending shock
$u_{C,t}$, $u^*_{C,t}$, $u_{L,t}$, $u^*_{L,t}$	preference shocks
e_t, e^*_t	mark-up shock
oil_t	oil shock
rer_t	real exchange rate
$\tau = -\tau^*$	terms of trade
β	discount parameter
γ_H, γ^*_F	indexation parameter
h, h^*	habit parameters
$\lambda_H = \frac{(1-\beta\xi_H)(1-\xi_H)}{\xi_H}$	Phillips Curve Parameter in H Country
$\lambda^*_F = \frac{(1-\beta\xi^*_F)(1-\xi^*_F)}{\xi^*_F}$	Phillips Curve Parameter in F Country
$1-\xi_H$, $1-\xi^*_F$	probability of a price re-optimization
σ, σ^*	risk-aversion parameter
ϕ, ϕ^*	disutility of labour supply parameter
$\text{w} - \frac{1}{2}$, $\text{w}^* - \frac{1}{2}$	degree of home bias

Table 1: Summary of Notation (Variables in Deviation Form)

$$
\begin{aligned}
\frac{\sigma}{(1-h)}c_t &= -mu_t^C + u_{C,t} + \frac{h\sigma}{(1-h)}c_{t-1} \\
\frac{\sigma}{(1-h)}c_t^* &= -mu_t^{C*} + u_{C,t}^* + \frac{h^*\sigma^*}{(1-h^*)}c_{t-1}^* \\
mu_t^L &= \phi l_t + u_{L,t} \\
mu_t^{L*} &= \phi^* l_t^* + u_{L,t}^* \\
mc_t &\equiv mrs_t - a_t + p_t - p_{H,t} \\
mc_t^* &\equiv mrs_t^* - a_t^* + p_t^* - p_{F,t}^* \\
c_{H,t} &= c_t - \mu(p_{H,t} - p_t) = c_t + \mu(1-\mathrm{w})\tau_t \\
c_{H,t}^* &= c_t^* - \mu^*(p_{H,t}^* - p_t^*) = c_t^* + \mathrm{w}^*\mu^*\tau_t \\
c_{F,t} &= c_t - \mu(p_{F,t} - p_t) = c_t - \mu\mathrm{w}\tau_t \\
c_{F,t}^* &= c_t^* - \mu^*(p_{F,t}^* - p_t^*) = c_t^* - \mu^*(1-\mathrm{w}^*)\tau_t \\
y_t &= \alpha_H c_{H,t} + \alpha_F c_{H,t}^* + (1-\alpha_H-\alpha_F)g_t \\
y_t^* &= \alpha_F^* c_{F,t}^* + \alpha_H^* c_{F,t} + (1-\alpha_F^*-\alpha_H^*)g_t^* \\
l_t &= y_t - a_t \\
l_t^* &= y_t^* - a_t^*
\end{aligned}
$$

$$
\begin{aligned}
\pi_t &\equiv p_t - p_{t-1} = \mathrm{w}\pi_{H,t} + (1-\mathrm{w})\pi_{F,t} + \mathrm{oil}_t \\
\pi_t^* &= \mathrm{w}^*\pi_{F,t}^* + (1-\mathrm{w}^*)\pi_{H,t}^* + \mathrm{oil}_t \\
\pi_{H,t}^* &\equiv p_{H,t}^* - p_{H,t-1}^* = \pi_{H,t} + \pi_t^* - \pi_t - \Delta rer_t \\
\pi_{F,t} &\equiv p_{F,t} - p_{F,t-1} = \Delta rer_t + \pi_t - \pi_t^* + \pi_{F,t}^* \\
E_t\pi_{t+1} &= \mathrm{w}E_t\pi_{H,t+1} + (1-\mathrm{w})E_t\pi_{F,t+1} + \rho_o\mathrm{oil}_t \\
E_t\pi_{t+1}^* &= \mathrm{w}^*E_t\pi_{F,t+1}^* + (1-\mathrm{w}^*)E_t\pi_{H,t+1}^* + \rho_o\mathrm{oil}_t \\
E_t\pi_{H,t+1}^* &= E_t\pi_{H,t+1} + E_t\pi_{t+1}^* - E_t\pi_{t+1} - (E_t rer_{t+1} - rer_t) \\
E_t\pi_{F,t+1} &= E_t rer_{t+1} - rer_t + E_t\pi_{t+1} - E_t\pi_{t+1}^* + E_t\pi_{F,t+1}^* \\
E_t rer_{t+1} &= E_t mu_{t+1}^{C*} - E_t mu_{t+1}^C \\
rer_t &= mu_t^{C*} - mu_t^C \\
(\mathrm{w}^* + \mathrm{w} - 1)\tau_t &= rer_t
\end{aligned}
$$

The flexi-price zero expected inflation economy and output gap are given by

$$
\begin{aligned}
E_t\widehat{mu}^C_{t+1} &= \widehat{mu}^C_t - \hat{r}_t \qquad (\text{ determines } \hat{r}_t) \\
E_t\widehat{mu}^{C*}_{t+1} &= \widehat{mu}^{C*}_t - \hat{r}^*_t \qquad (\text{ determines } \hat{r}^*_t) \\
\widehat{mc}_t &= 0 = \widehat{wr}_t - a_t - (1-w)\hat{\tau}_t \qquad (\text{ determines } \widehat{wr}_t) \\
\widehat{mc}^*_t &= 0 = \widehat{wr}^*_t - a^*_t + (1-w^*)\hat{\tau}_t \qquad (\text{ determines } \widehat{wr}^*_t) \\
\widehat{mrs}_t &= \widehat{wr}_t = \widehat{mu}^L_t - \widehat{mu}^C_t \qquad (\text{ determines } \widehat{mu}^C_t) \\
\widehat{mrs}^*_t &= \widehat{wr}^*_t = \widehat{mu}^{L*}_t - \widehat{mu}^{C*}_t \qquad (\text{ determines } \widehat{mu}^{C*}_t) \\
\frac{\sigma}{(1-h)}\hat{c}_t &= -\widehat{mu}^C_t + u_{C,t} + \frac{h\sigma}{(1-h)}\hat{c}_{t-1} \\
\frac{\sigma}{(1-h)}\hat{c}^*_t &= -\widehat{mu}^{C*}_t + u^*_{C,t} + \frac{h^*\sigma^*}{(1-h^*)}\hat{c}^*_{t-1} \\
\widehat{mu}^L_t &= \phi\hat{l}_t + u_{L,t} + u_{C,t} \\
\widehat{mu}^{L*}_t &= \phi^*\hat{l}^*_t + u^*_{L,t} + u^*_{C,t} \\
\widehat{rer}_t &= \widehat{mu}^{C*}_t - \widehat{mu}^C_t \\
\hat{l}_t &= \hat{y}_t - a_t \\
\hat{l}^*_t &= \hat{y}^*_t - a^*_t
\end{aligned}
$$

$$
\begin{aligned}
\hat{y}_t &= \alpha_H\hat{c}_t + \alpha_F\hat{c}^*_t + [\alpha_H\mu(1-\mathrm{w}) + \alpha_F\mu^*\mathrm{w}^*]\hat{\tau}_t \\
\hat{y}^*_t &= \alpha^*_F\hat{c}^*_t + \alpha^*_H\hat{c}_t - [\alpha^*_F\mu^*(1-\mathrm{w}^*) + \alpha^*_H\mu w]\hat{\tau}_t \\
(\mathrm{w}^* + \mathrm{w} - 1)\hat{\tau}_t &= \widehat{rer}_t \\
\mathrm{ogap}_t &= \hat{y}_t - y_t \\
\mathrm{ogap}^*_t &= \hat{y}^*_t - y^*_t
\end{aligned}
$$

The whole model can now be written in the required state space form as

$$
\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t\mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{o}_t + C \begin{bmatrix} i_t \\ i^*_t \end{bmatrix} + D\epsilon_{t+1} \tag{59}
$$

$$
F\mathsf{o}_\mathsf{t} = H \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{60}
$$

where $\mathsf{z}_t = [\text{shocks } a_t, a^*_t \text{ etc}, \pi_{H,t-1},\ \pi^*_{F,t-1}]$ is a vector of predetermined variables and $\mathsf{x}_t = [mu^C_t, mu^{C*}_t, \pi_{H,t}, \pi_{F,t}]$ is a vector of non-predetermined or 'jump' variables.

3 Equilibrium Concepts

3.1 Cooperation, Non-Cooperation, Commitment and Discretion

Optimal policy can be formulated independently by each monetary authority. However In addition to the time-inconsistency problem there is a second classical problem first raised by Hamada (1976): in an open economy, rules designed for the single economy may perform badly in a world Nash equilibrium when all countries pursue similar optimal policies. In the open economy the optimal monetary policy requires all policymakers to cooperate, maximizing an agreed global welfare, and to be able to commit not just with respect to each other but collectively with respect to the private sector too. These considerations lead to a number of possible equilibria depending on whether policymakers cooperate and can commit to the private sector and whether they can commit with respect to each other (i.e., can cooperate).

Consider symmetrical equilibria in the sense that all authorities can either commit or not with respect to the private sector. In the absence of any commitment mechanism for players all authorities must independently pursue discretionary policies (non-cooperation with discretion (ND)). If authorities can cooperate (i.e., can commit to each other) and can commit with respect to the private sector, then the socially optimal policy with respect to an agreed global objective function can be achieved (cooperation with commitment to the private sector, CC). The remaining possible equilibria are those where (for some reason) authorities can commit to each other but not to the private sector (cooperation with discretion, CD) or vice versa, they can commit to the private sector but not to each other (non-cooperation with commitment to the private sector, NC). Table 6 summarizes these four possibilities.

	Commitment	Discretion
Cooperation	CC	CD
Non-cooperation	NC	ND

Table 2. Possible Equilibria

For linear-quadratic dynamic games, these equilibria are formulated in Levine and Currie (1987a), Levine and Currie (1987b), Currie and Levine (1993), Currie *et al.* (1996)) and summarized in Appendix A. *General* procedures, not specific to any one model, for their calculation and software for their computation have been developed (see Kemball-Cook *et al.* (1995).) In a two-bloc model the *potential gains from commitment in the absence of coordination* can be quantified by comparing the welfare in equilibria NC and ND. These 'gains' can be negative: as in Levine and Currie (1987b), for an ad hoc 'Old Keynesian' model *commitment without coordination may be counterproductive*. Similarly one can assess the *potential gains from coordination in the absence of commitment* by comparing equilibria CD and ND and revisit the possibility of *counterproductive cooperation* found by Rogoff (1985).

To realize the full potential gain from monetary policy coordination between the two blocs requires a combination of commitment and coordination; i.e., equilibrium CC and this can be be quantified by comparing CC with the non-cooperative alternatives, NC or ND. The first wave of the new Keynesian open economy models that revisited this old issue in the literature cited above suggested that these gains are not substantial compare with the gains from stabilization. Referring to table 6, Clarida *et al.* (2002) compare CD and ND and show there exists gains from CC if and only if $\sigma \neq 1$. Pappa (2004a) and Benigno and Benigno (2001) compare CC and NC. Pappa (2004a) shows gains are small and Benigno and Benigno (2001) show that CC can be sustained as an NC equilibrium by delegation to a central bank with an appropriate loss function. Finally Currie and Levine (1993) compare CC and ND, but using an ad hoc model and utility function.

3.2 Welfare-Based Versus Real World Equilibria

In most of the recent literature the policymaker pursues the welfare-based objective based on the underlying utility function of the household. Then the gains from coordination are calculated as the increase in welfare. However Svensson (2003) proposes a totally different equilibrium concept: central banks in reality adopt a loss function of the form

$$\sum_{t=0}^{\infty} \beta^t \left[o_t^2 + w_\pi \pi_t^2 + w_i i_t^2 \right] \tag{61}$$

for the home bloc, where $o_t = y_t - \hat{y}_t$ is the output gap.

What we observe is not (61) but reaction functions of various possible forms:

$$i_t = \rho i_{t-1} + \theta_\pi \pi_{H,t} + \theta_y ogap_t \tag{62}$$

$$i_t = \rho i_{t-1} + \theta_\pi \pi_{H,t} + \theta_y y_t \tag{63}$$

$$i_t = \rho i_{t-1} + \theta_\pi \pi_{H,t} + \theta_y \Delta y_t \tag{64}$$

Then armed with estimates ρ, θ_π and θ_y we can reverse-engineer the implied coefficients w_π and w_i from the non-cooperative Nash equilibrium in such rules. This is computationally expensive but possible, at least for the simple model in this section. Having obtained w_π and w_i the corresponding cooperative equilibrium can be calculate. Finally the welfare gains can be evaluated using the welfare-based utility function.

4 LQ Approximation

This section sets out the two forms of linear-quadratic approximation of the non-linear stochastic optimization problems that characterize the welfare-based equilibria concepts set out in the previous section. We distinguish between an equilibrium of social planners and Ramsey planners.

4.1 Social Planners

Without cooperation the home policymaker be her a social or Ramsey planners at time $t = 0$ maximizes an expected loss function

$$\Omega_0 = E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{U_{L,t}}{1+\phi} L_t^{1+\phi} \right] \equiv E_0 \sum_{t=0}^{\infty} \beta^t \hat{W}_t \tag{65}$$

subject to the *resource constraint*

$$\begin{aligned} Y_t = \frac{A_t L_t}{\Delta_t} &= C_{H,t} + C_{H,t}^* + G_t = \mathrm{w} \left(\frac{P_{H,t}}{P_t} \right)^{-\mu} C_t + (1 - \mathrm{w}^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu^*} C_t^* \\ &= U_{OIL,t} \left(\left[\mathrm{w} + (1-\mathrm{w}) \mathcal{T}_t^{1-\mu} \right]^{\frac{\mu}{1-\mu}} C_t + \left[\mathrm{w}^* \mathcal{T}_t^{1-\mu} + (1-\mathrm{w}^*) \right]^{\frac{\mu^*}{1-\mu^*}} C_t^* \right) + G_t \\ & \quad (\mu \neq 1, \mu^* \neq 1) \end{aligned} \tag{66}$$

and the *risk-sharing condition*

$$\frac{U_{C,t}(C_t^* - hC_{t-1}^*)^{\sigma^*}}{U_{C,t}^*(C_t - hC_{t-1})^{\sigma}} = RER_t = \frac{\left[\mathrm{w}^* (\mathcal{T}_t)^{1-\mu^*} + (1-\mathrm{w}^*)^{1-\mu^*} \right]^{\frac{1}{1-\mu^*}}}{\left[\mathrm{w} + (1-\mathrm{w}) \mathcal{T}_t^{1-\mu} \right]^{\frac{1}{1-\mu}}} \quad (\mu \neq 1, \mu^* \neq 1) \tag{67}$$

Since staggered price-setting is absent, $\Delta_t = 1$ and these are the only constraints facing the home social planner. In a *non-cooperative social planner's equilibrium*, given the foreign social planner's allocation C_t^*, L_t^*, the terms of trade are pinned down by the risk-sharing condition leaving the home planner able to choose C_t, L_t to maximize (65) subject to the resource constraint (66). For the foreign bloc we define an analogue of (65) denoted by Ω_0^*, its resource constraint

$$\begin{aligned} Y_t^* = \frac{A_t^* L_t^*}{\Delta_t^*} &= C_{F,t}^* + C_{F,t} + G_t^* = \mathrm{w}^* \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\mu^*} C_t^* + (1-\mathrm{w}) \left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t \\ &= U_{OIL,t} \left(\left[\mathrm{w}^* + (1-\mathrm{w}^*)\mathcal{T}_t^{\mu^*-1}\right]^{\frac{\mu^*}{1-\mu^*}} C_t^* + \left[\mathrm{w}\mathcal{T}_t^{\mu-1} + (1-\mathrm{w})\right]^{\frac{\mu}{1-\mu}} C_t \right) + G_t^* \\ &\quad (\mu \neq 1, \mu^* \neq 1) \end{aligned} \tag{68}$$

with $\Delta_t^* = 1$.

In the *cooperative social planners equilibrium* the two policymakers jointly maximize some agreed linear combination $\alpha\Omega_0 + (1-\alpha)\Omega_0^*$, $\alpha \in [0,1]$, with respect to C_t, L_t and C_t^*, L_t^* subject to (66), (68) and (67).

We restrict our results to the case of a symmetric equilibrium of identical economies with a unitary elasticity $\mu = \mu^* = 1$. Then for the non-cooperative and cooperative steady states we have, respectively

$$(\hat{Y}^{NC})^{\phi+\sigma} = \frac{A^{1+\phi}(1-\beta h)\mathrm{w}\left(2(1-\mathrm{w})(1-\beta h) + \frac{2\mathrm{w}-1}{\sigma}(1-h)\right)}{\kappa(1-h)^\sigma(1-g_y)^\sigma\left(4\mathrm{w}(1-\mathrm{w})(1-\beta h) + \frac{(2\mathrm{w}-1)^2}{\sigma}(1-h)\right)} \tag{69}$$

$$(\hat{Y}^{C})^{\phi+\sigma} = \frac{A^{1+\phi}(1-\beta h)}{\kappa(1-h)^\sigma(1-g_y)^\sigma} \tag{70}$$

For later use we require the Taylor series second-order expansions, about these two steady states, of the single-period loss functions, W_t and W_t^* in the non-cooperative equilibrium, and $W_t^C \equiv W_t + W_t^*$ for the symmetric cooperative equilibrium. These are given respectively by

$$\begin{aligned}
\hat{W}_t^{NC} &= w_c(c_t - hc_{t-1})^2 + w_y y_t^2 + w_{ya} y_t a_t + w_\tau^{NC} \tau_t^2 + w_{c\tau}^{NC} c_t \tau_t + w_{h\tau}^{NC}(c_t - hc_{t-1})\tau_t \\
&+ w_{g\tau}^{NC} \tau_t g_t + w_{cc}^{NC}(c_t - hc_{t-1}) u_{C,t} + w_{oc}^{NC} oil_t c_t + w_{ya}^{NC} y_t a_t \\
&+ w_{yl}^{NC} y_t(u_{C,t} + u_{L,t}) + w_{hC*}^{NC}(c_t - hc_{t-1}) u_{C,t}^* + w_{\tau C*}^{NC} \tau_t u_{C,t}^* + w_{oy}^{NC} oil_t y_t + w_{o\tau}^{NC} oil_t \tau_t \\
&+ (t.i.p)^{NC} + \text{third order terms} \qquad (71)\\
\hat{W}_t^{C} &= w_c((c_t - hc_{t-1})^2 + (c_t^* - hc_{t-1}^*)^2) + w_y(y_t^2 + (y_t^*)^2) + w_{ya}(y_t a_t + y_t a_t^*) + w_\tau^C \tau_t^2 \\
&+ w_{cc}^C(u_{C,t}(c_t - hc_{t-1}) + u_{C,t}^*(c_t^* - hc_{t-1}^*)) + w_{oc}^C oil_t(c_t + c_t^*) \\
&+ w_{ya}^C(y_t a_t + y_t^* a_t^*) + w_{yl}^C(y_t(u_{C,t} + u_{L,t}) + y_t^*(u_{C,t}^* + u_{L,t}^*)) \\
&+ (t.i.p)^C + \text{third order terms} \qquad (72)
\end{aligned}$$

where weights w_y etc are derived in Appendix B. Note that cooperative quadratic form is not a simple sum of the non-cooperative forms; in particular the contribution of the terms of trade and shock processes are different in these two cases.

4.2 Ramsey Planners

Ramsey planners in both non-cooperative and cooperative games have the same objectives as their social planing counterpart, but without the ability to plan consumption and labour supply paths. Instead they face a decentralized economy given by resource constraints, the market-sharing condition plus the price-setting behaviour of firms and the households' Euler equations. Gathering up previous results the former, for the home bloc, are given by

$$Q_{H,t} = \Lambda_t / H_t \qquad (73)$$

$$H_t - \xi\beta E_t[\tilde{\Pi}_{H,t+1}^{\zeta-1} H_{t+1}] = \frac{P_t Y_t}{P_{H,t}} MU_t^C \qquad (74)$$

$$\Lambda_t - \xi\beta E_t[\tilde{\Pi}_{H,t+1}^{\zeta} \Lambda_{t+1}] = \frac{Y_t \, MC_t \, MU_t^C}{(1 - 1/\zeta)} = \frac{Y_t \frac{W_t}{A_t P_{H,t}} MU_t^C}{(1 - 1/\zeta)} \qquad (75)$$

where $\Pi_{H,t}$, $\tilde{\Pi}_{H,t}$ and $Q_{H,t}$ are defined by

$$\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}} \qquad (76)$$

$$\tilde{\Pi}_{H,t} \equiv \frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\gamma_H}} \qquad (77)$$

$$1 = \xi_H \tilde{\Pi}_{H,t}^{\zeta-1} + (1 - \xi_H) Q_{H,t}^{1-\zeta} \qquad (78)$$

and the Euler equation is

$$1 = \beta(1+I_t)E_t\left[\frac{(C_{t+1}-hC_t)^{-\sigma}}{(C_t-hC_{t-1})^{-\sigma^*}}\frac{1}{(1+\Pi_{t+1})}\right] \tag{79}$$

Analogous results apply to the foreign bloc.

Much of the literature (for example, Clarida *et al.* (2002), Pappa (2004a)) now assumes that the Ramsey planners can use tax instruments T_t and T_t^* to bring the zero-inflation steady state of the decentralized economies in line with the social optima. For the non-cooperative and cooperative games this require respectively, tax wedges

$$T^{NC} = 1 - \frac{(1-h\beta)\mathrm{w}\left(2(1-\mathrm{w})(1-\beta h)+\frac{2\mathrm{w}-1}{\sigma}(1-h)\right)}{4\mathrm{w}(1-\mathrm{w})(1-\beta h)+\frac{(2\mathrm{w}-1)^2}{\sigma}(1-h))\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right)} \tag{80}$$

$$T^{C} = 1 - \frac{(1-h\beta)}{\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right)} \tag{81}$$

Then $\Phi_y = 0$ and the zero-inflations, zero-trade-balance steady state of section 2.6 is efficient. Note first, that $T^{NC} > T^C$: in a Nash equilibrium of social planners, the incentive to improve the terms of trade by restricting output leads each planner to choose a higher distortionary tax wedge. Second, as w increases from $\mathrm{w} = \frac{1}{2}$, for the case of no bias, to $\mathrm{w} = 1$ for two closed economies then T^{NC} falls from $T^{NC} = \frac{1}{2}(1+T^C)$ to $T^{NC} = T^C$.

We have calibrated our symmetric two-bloc model to US data and in particular chosen 15% and 20% mark-ups in the product and labour markets respectively. This gives $\zeta = 7.674$ and $\eta = 5$. With the habit and discount factor calibrated at $h = 0.5$ and $\beta = 0.99$ (both on a quarterly basis), it follows that the optimal cooperative tax wedge is $T^C = 0.274$ and the non-cooperative rate can be as high as $T^{NC} = 0.637$. We have chosen the calibrated value $\mathrm{w} = 0.75$ for which $T^{NC} = 0.533$. Interestingly, these tax rates compares with a total tax wedge (consisting of taxes on consumption and income plus social security contribution) of $T = 0.373$ for the US and $T = 0.641$ for the euro area, reported in Coenen *et al.* (2007).

The nature of the game is a two-stage process which we refer to as the *two-stage Ramsey game*. At stage 1 tax wedges are chosen so as to bring the steady state of the decentralized economy in line with the socially optimal allocation. In a non-cooperative game, each social planner's choice of consumption and leisure is a best response to the choice of the other; i.e., a Nash equilibrium in the individual blocs' social optima. In the quadratic approximation (71), terms independent of policy (t.i.p), involve outcomes in the other bloc and are t.i.p only for this particular game.

In the second stage the monetary instruments are used to achieve, as far as possible, the outcome of the first stage, but now there is staggered price-setting, inflation and costs of inflation from price dispersion. From Appendix B, the Ramsey loss functions can now be shown to take the approximate quadratic form

$$W_t^{NC} = \hat{W}_t + w_\pi(\pi_{H,t} - \gamma_H \pi_{H,t-1})^2 \tag{82}$$

$$(W_t^*)^{NC} = (\hat{W}_t^*)^{NC} + w_\pi^*(\pi_{F,t}^* - \gamma_F^* \pi_{F,t-1}^*)^2 \tag{83}$$

$$W_t^C = \hat{W}_t^C + w_\pi(\pi_{H,t} - \gamma_H \pi_{H,t-1})^2 + w_\pi^*(\pi_{F,t}^* - \gamma_F^* \pi_{F,t-1}^*)^2 \tag{84}$$

where w_π and w_π^* are defined in that Appendix.

The Nash equilibrium at this stage depends on the monetary instrument; these could be inflation targets with nominal interest rates subsequently chosen to exactly achieve these targets; or they can be the nominal interest rates themselves. The Nash equilibria in nominal interest rates can be open-loop with the authorities responding to each other's interest rate paths. More appropriate in a stochastic environment with commitment are closed-loop Nash equilibria with each authority choosing their best response to each other's feedback commitment rule.

Since we are interested in the gains from monetary policy coordination with independent central banks we need to consider the Ramsey planner as a monetary authority with only monetary but not fiscal instruments available. We refer to this as a *single-stage Ramsey game*. Then tax wedges are given in the monetary policy game. Under what circumstances are the quadratic approximations (82) and (84) then appropriate for Ramsey games? If the fiscal authorities have set their tax wedges close to T^{NC} then (82) is a good 'small distortions' approximation. But then the tax wedge is far higher than that for cooperation and (84) is *not* a good 'small distortions' approximation for that game. A similar argument holds if the fiscal authorities have set their tax wedges close to T^C. *In short,* (82) *and* (84) *cannot both be good 'small distortions' approximations if the tax wedge is given to the monetary authority.*

Given the game we are interested in we now consider two Ramsey planners choosing monetary instruments to maximize household welfare in an environment consisting of a decentralized economy with possibly large distortions in the zero-inflation steady state..

We show in Levine *et al.* (2007), the procedure for achieving an accurate LQ approximation for each equilibrium concept is as follows[16]:

1. Define the optimization problem for the Ramsey planner. For the cooperative this is a standard problem. For non-cooperative games we need to define the appropriate equilibrium concept. Our ultimate aim is to obtain an accurate quadratic approximation of welfare for the state-space representation of the game, (59) and (60). Since interest-rates are given in this representation, we choose an open-loop Nash equilibrium in interest rate paths.

2. Set out the deterministic non-linear form of each Ramsey problem, to maximize the representative agents utility subject to non-linear dynamic constraints.

3. Write down the single Lagrangian for the cooperative problem, and the Lagrangians for the two blocs for the non-cooperative problem. Associated with each Lagrangian is a Hamiltonian consisting of the utility and a sum of *all* appropriately expressed constraints for the decentralized economy time multipliers.

4. Calculate the first order conditions. We do not require the initial conditions for an optimum since we ultimately only need the steady-state about which we are approximating.

5. Calculate the steady state of the first-order conditions. The terminal condition implied by this procedure is that the system converges to this steady state.

6. Calculate a second-order Taylor series approximation, about the steady state, of the Hamiltonian associated with the Lagrangian or Lagrangians in 2.

7. Calculate a first-order Taylor series approximation, about the steady state, of the first-order conditions and the original constraints.

8. Use 4. to eliminate the steady-state Lagrangian multipliers in 5. By appropriate elimination both the Hamiltonian and the constraints can be expressed in minimal form.

[16]MATLAB software to implement this procedure is in preparation and will be available on request from the authors.

This then gives us the accurate LQ approximation of the original non-linear optimization problem in the form of a minimal linear state-space representation of the constraints and a quadratic form of the utility expressed in terms of the states. The quadratic form of the utility function obtained for the cooperative Ramsey planners is then appropriate for games CC and CD irrespective of the monetary instrument; that obtained for the non-cooperative Ramsey planners is appropriate for games NC and ND, but only where interest rates are the instruments.

We have now set out two quite distinct procedures for obtaining a LQ state-space representation for different equilibria. In the two-stage Ramsey games the planner has access to a fiscal instrument and uses monetary policy to minimize a quadratic approximation of the loss function that the social planner would choose. In the single-stage Ramsey game the monetary authority must deal with an economy that is distorted in the steady state and minimizes a quadratic approximation of loss function appropriate for decentralized economy with the nominal interest rate as the instrument. It should be emphazized that these l*ead to quite different non-cooperative equilibria concepts* even in the case where all labour market, output market, external habit and distortionary taxes disappear.

5 The Zero Lower Bound Constraint

We can impose an interest rate ZLB in a straightforward way by modifying the LQ optimization problemsi. As in Woodford (2003), chapter 6, this is implemented by modifying the home bloc welfare loss function to

$$\Omega_t = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t' Q \mathsf{y}_t + w_i i_t^2] \tag{85}$$

with an analogous adjustment for the foreign bloc. As explained in Levine *et al.* (2006), the policymaker's optimization problem is to choose an unconditional distribution for i_t (i.e., the steady-state variance) shifted to the right about a new non-zero steady-state inflation rate and a higher nominal interest rate, such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_i *for each of our policy rules* so that $z_0(p)\sigma_i < R$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $R = \frac{1}{\beta} - 1 + \bar{\pi}$ is the steady-state nominal interest rate, σ_i is the unconditional variance and $\bar{\pi}$ is the new steady-state inflation rate. Given σ_i the steady-state positive inflation rate that will ensure $i_t \geq 0$ with probability $1-p$ is given by

$$\bar{\pi} = \max[z_0(p)\sigma_i - \left(\frac{1}{\beta} - 1\right) \times 100, 0] \tag{86}$$

6 Gains from Coordination and Commitment

We now turn to numerical results. We assume symmetrical blocs with the calibration set out in Appendix C. Only two-stage Ramsey games, with tax wedges chosen to eliminate the distortions in the zero-inflation steady state, are considered.

6.1 Results with no ZLB Constraint

First let us ignore the ZLB constraint. Table 3 presents results for this case. The simple commitment rules feed back on current domestic (GDP price deflator) inflation in each bloc and take the form

$$i_t = \rho i_{t-1} + \theta_\pi \pi_{H,t} \tag{87}$$

$$i_t^* = \rho^* i_{t-1} + \theta_\pi^* \pi_{F,t}^* \tag{88}$$

SIMCC is the coordinated rule whilst SIMNC is chosen in a Nash game between the countries. c_e is the percentage consumption permanent equivalent gain from CC compared with each alternative given by $c_e = \frac{\Omega_0 - \Omega_0^{CC}}{(1-h)} \times 10^{-2}$. $\text{var}(i_t)$ is the steady-state conditional variance of the nominal interest rate. The welfare loss functions are given by (82)-(84) for the non-cooperative and cooperative games with additional terms penalizing interest rate variabilities, as in (85). In these simulations we have set the penalty on the interest rate variability at $w_i = 1.5$.[17] Figure 1 compares the impulse responses for the regimes CC, CD, ND and SIMCC following a 1% negative shock to the productivity parameter A_t at $t = 0$.

[17]For $w_i < 1.5$ we found that our iterative procedure for the non-cooperative equilibrium with discretion, NCD, did not converge.

Regime	Rule	Ω_0	$c_e(\%)$	var(i_t)	$\bar{\pi}$	prob ZLB
CC	complex	11.20	0	1.54	0	0.21
CD	not applicable	17.16	0.12	5.00	0	0.33
ND	not applicable	16.62	0.11	5.60	0	0.34
SIMCC	$(\rho, \theta_\pi) = (1.00, 3.32)$	11.64	0.008	1.87	0	0.23
SIMNC	$(\rho, \theta_\pi) = (1.00, 4.17)$	11.71	0.010	2.19	0	0.25

Table 3. Gains from Coordination and Commitment: no ZLB Constraint.[18]

A number of features stand out from these results. First, comparing the cooperative regime with commitment, CC, with the cooperative regime with discretion we see there are small, but not insignificant gains from commitment of 0.1% permanent increase in consumption about the steady state. But comparing cooperation and non-cooperation under discretion the gains are actually negative (but very small); i.e., ND dominates CD or, in other words, *cooperation without commitment can be counterproductive*. This result was first found by Rogoff (1985), but here the cooperative loss arises purely from the stabilization problem.

Comparing the optimized Taylor rule under cooperative and non-cooperation, SIMCC and SIMNC, we see the gains are very small, of the order of 0.002% permanent increase in consumption.. However the conclusion that cooperative brings small benefits needs to be qualified by a consideration of the steady state variances. These are all high especially with the discretionary regimes CD and ND with a high probability of hitting the ZLB on the interest rate. This indicates that the rules we have designed are not operational. The next subsection addresses this shortcoming.

6.2 Imposing the ZLB Constraint

In our linear-quadratic framework we can write the intertemporal expected welfare loss at time $t = 0$ as the sum of stochastic and deterministic components, $\Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0$. By increasing w_i we can lower σ_i thereby decreasing $\bar{\pi}$, given by (86), and reducing the

[18] In order to compare cooperative and non-cooperative outcomes, although the latter equilibria are calculated using the single-period loss function W_t^{NC}, given by (82), the value for the expected welfare per country, Ω_0, reported in the table then uses W_t^C in (84).

deterministic component. But this welfare improvement in the deterministic component of the welfare loss comes about at the expense of increasing the stochastic component. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes the ZLB constraint, $i_t \geq 0$ with probability $1 - p$.

Table 4 shows the results of this optimization procedure under commitment using the loss functions given by (82)-(84). We choose $p = 0.025$. Given w_i, denote the expected inter-temporal loss (stochastic plus deterministic components) at time $t = 0$ by $\Omega_0(w_i)$. This includes a term penalizing the variance of the interest rate which does not contribute to utility loss as such, but rather represents the interest rate lower bound constraint. *Actual* utility, found by subtracting the interest rate term, is given by $\Omega_0(0)$. The steady state inflation rate, $\bar{\pi}$, that will ensure the lower bound is reached only with probability $p = 0.025$ is computed using (86). Given $\bar{\pi}$, we can then evaluate the deterministic component of the welfare loss, $\bar{\Omega}_0$. Since in the new steady state the real interest rate is unchanged, the steady state involving real variables is also unchanged, so from (82)-(84) we can write[19]

$$\bar{\Omega}_0(0) = w_\pi (1 - \gamma_H)^2 \bar{\pi}^2 \tag{89}$$

for the home bloc with an analogous result for the foreign bloc.

From the table we see that the optimal way of imposing the ZLB constraint is to shift the distribution to the right by choosing a small steady state inflation rate in both countries of 0.10% per quarter and to reduce the variance of the nominal interest rate to $\sigma_i^2 = \sigma_i^{*\,2} = 0.30$ by choosing a weight $w_i = w_i^*$. Figures 2 and 3 illustrate this optimization procedure.

[19]Both the ex-ante optimal and the optimal time-consistent deterministic welfare loss that guide the economy from a zero-inflation steady state to $\pi = \bar{\pi}$ differ from $\bar{\Omega}_0(0)$ (but not by much because the steady state contribution by far outweighs the transitional contribution).

w_i	var(i_t)	$\tilde{\Omega}_0(w_i)$	$\tilde{\Omega}_0(0)$	$\bar{\pi}$	$\bar{\Omega}_0(0)$	$\Omega_0(0)$
1	1.77	10.82	9.94	1.60	57.67	67.61
5	0.85	13.17	11.07	0.80	14.37	25.44
10	0.51	14.79	12.25	0.40	3.59	15.84
15	0.36	15.85	13.19	0.17	0.64	13.83
16	0.34	16.02	13.36	0.13	0.39	13.75
17	**0.32**	**16.18**	**13.52**	**0.10**	**0.22**	**13.74**
18	0.30	16.33	13.67	0.04	0.10	13.77
20	0.27	16.61	13.96	0.01	0.00	13.96

Table 4. Optimal Commitment with a Nominal Interest Rate ZLB.

6.2.1 Cooperation with Discretion (CD)

Turning to the regime cooperation with discretion (CD) we follow the same procedure as for CC to arrive at the optimal choice of $\sigma_i^2 = \sigma_i^{*2} = 0.30$ and $w_i = w_i^*$ that achieves the ZLB constraint. Figures 4 and 5 show the result. As before, to achieve the ZLB constraint requires a non-zero steady state inflation, but now under discretion it is far higher than under commitment. Whereas under commitment the trade-off between a high steady-state inflation rate and a smaller stochastic welfare loss can be exploited to drastically reduce the ultimate loss, this is not the case under discretion and highlights an important difference between stabilization policy under commitment and discretion. For the latter we see that the *steady-state inflation – stochastic welfare loss trade-off is far less favourable*. As the weight on interest rate variability increases beyond $w_i = w_i^* = 9$, both the unconditional variance of the interest rate, and the steady-state inflation rate needed to reduce the probability of hitting the ZLB to $p = 0.025$ increase. Now the optimal choice of the weight is $w_i = w_i^* = 7$ with a optimal steady state inflation rate at $\bar{\pi} = 2.76\%$. This is a somewhat counterintuitive result that can be explained in general by the fact that under discretion, a policymaker lacks the leverage to manage the economy she would enjoy under commitment. More specifically, the constraint on using the interest rate, captured by increasing the weight w_i beyond a certain point, simply results in a more volatile economy including interest rate volatility.

6.2.2 Non-Cooperation with Discretion

Turning to non-cooperation with discretion, the home country policymaker now chooses a weight w_i and a steady-state inflation rate $\bar{\pi}$ to achieve its ZLB constraint. In a Nash game the corresponding choice of w_i^* and $\bar{\pi}$ by the foreign policymaker is taken as given. This leads to a reaction function in (w_i, w_i^*) space shown in figure 6. If policymakers were to cooperate just in the choice of (w_i, w_i^*) it turns out that they would choose $w_i = w_i^* = 8.1$ and achieve welfare outcomes in the subsequent non-cooperative and discretionary setting of interest rates of $\Omega_0 = \Omega_0^* = 207.75$. However in the full non-cooperative game each policymaker would try for rules that are more aggressive than the other with a lower weight on interest rate variability. The intention given the setting of the other country is to manipulate the exchange rate in the face of shocks. However, this is a beggar-thy-neighbour strategy and in equilibrium each ends up with a sub-optimally low choice of the weights at the intersection of the reaction functions at $w_i = w_i^* = 7.6$ with $\Omega_0 = \Omega_0^* = 208.29$. At this equilibrium $\bar{\pi} = \bar{\pi}^* = 2.82$ and $\sigma_i^2 = \sigma_i^{*\,2} = 3.82$. The outcomes under the three regimes CC, CD and ND are summarized below in table 5. With the ZLB constraint imposed we now observe that with discretion there are small but not insignificant *gains* from cooperation of $c_e = 0.06\%$, compared with counter-productive coordination found previously.

6.2.3 Current Inflation Commitment Rules

The results for the current inflation commitment rule wirh commitment (SIMCC) are shown in figures 7 and 8 and summarized in the table below. The main feature is that with the ZLB constraint, the costs of simplicity rise substantially from $c_e = 0.008\%$ to $c_e = 0.07\%$. The optimal choice of weight is $w_i = w_i^* = 29$ with a steady state inflation rate $\bar{\pi} = 0.20\%$. For SIMNC the interest rate variance rises a little with a corresponding increase in the steady state inflation rate to $\bar{\pi} = 0.27\%$. Gains from cooperation with current inflation commitment rules rise from $c_e = 0.002\%$ without a ZLB to $c_e = 0.005\%$ with a ZLB, a more than doubling of a rather small effect.

Regime	Rule	Ω_0	$c_e(\%)$	$\text{var}(i_t)$	$\bar{\pi}(\%)$	prob ZLB
CC	complex	13.74	0	0.32	0.10	0.025
CD	not applicable	205.47	3.83	3.75	2.76	0.025
ND	not applicable	208.29	3.89	3.82	2.82	0.025
SIMCC	$(\rho, \theta_\pi) = (1.000, 0.574)$	17.11	0.0674	0.382	0.20	0.025
SIMNC	$(\rho, \theta_\pi) = (1.000, 0.622)$	17.35	0.0722	0.418	0.27	0.025

Table 5. Gains from Coordination and Commitment with a ZLB Constraint.

6.2.4 Summary

The results of all five regimes with a ZLB are summarized in table 5 which should be compared with table 3 without a zero lower bound. The main result that emerges is that gains from commitment and cooperation rise substantially with ZLB considerations, though the cooperative gains with commitment rules are still small. The cost of simplicity in pursuing a simple current inflation rule, rather than its fully optimal counterpart also rise substantially with a ZLB constraint imposed, but the gains still remain. Gains from cooperation where policymakers cannot commit to the private sector are significant with ZLB considerations.

7 Conclusions

The main conclusion from our numerical results is that studies that concludes gains from monetary policy coordination, but ignore the ZLB constraint on nominal interest rates, may be misleading. We have shown that with ZLB considerations cooperative gains increase substantially, and in discretionary equilibria the gain is significant.

This result was obtained using a rudimentary New Keynesian model with a unitary elasticity of substitution between home and imported goods, complete financial markets, complete exchange rate pass-through and no non-traded sector. Relaxing these assumptions is known to increase gains from monetary policy. Future research will pursue this path in a two-bloc model estimate by Bayesian methods. We also intend to examine the case of large distortions in one-stage Ramsey games, examine the policy regime NC and pursue the implications of 'real world equilibria'.

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A The Policy Rules

Consider first the deterministic problem. Substituting out for outputs, the state-space representation is:

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t \tag{A.1}$$

where z_t is an $(n-m)\times 1$ vector of predetermined variables including non-stationary processes, z_0 is given, $w_t = [i_t, i^*_t]^T$ is a vector of policy variables, x_t is an $m \times 1$ vector of non-predetermined variables and $x^e_{t+1,t}$ denotes rational (model consistent) expectations of x_{t+1} formed at time t. Then $x^e_{t+1,t} = x_{t+1}$ and letting $y^T_t = [z_t, x_t]^T$, (A.1) becomes

$$y_{t+1} = Ay_t + Bw_t \tag{A.2}$$

Define target variables s_t by

$$s_t = My_t + Hw_t \tag{A.3}$$

and the policymakers' loss function under cooperation at time t by

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty}\lambda^t[s_{t+i}^T Q_1 s_{t+i} + w_{t+i}^T Q_2 w_{t+i}] \tag{A.4}$$

which we rewrite as

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty}\lambda^t[y_{t+i}^T Q y_{t+i} Q y_{t+i} + 2y_{t+i}^T U w_{t+i} + w_{t+i}^T R w_{t+i}] \tag{A.5}$$

where $Q = M^T Q_1 M$, $U = M^T Q_1 H$, $R = Q_2 + H^T Q_1 H$, Q_1 and Q_2 are symmetric and non-negative definite R is required to be positive definite and $\lambda \in (0,1)$ is discount factor. The procedures for evaluating the three policy rules are outlined in the rest of this appendix (or Currie and Levine (1993) for a more detailed treatment).

A.1 The Optimal Policy: Cooperation with Commitment (CC)

Consider the policy-maker's *ex-ante* optimal policy at $t = 0$. This is found by minimizing Ω_0 given by (A.5) subject to (A.2) and (A.3) and given z_0. We proceed by defining the Hamiltonian

$$H_t(y_t, y_{t+1}, \mu_{t+1}) = \frac{1}{2}\lambda^t(y_t^T Q y_t + 2y_t^T U w_t + w_t^T R w_t) + \mu_{t+1}(Ay_t + Bw_t - y_{t+1}) \tag{A.6}$$

where μ_t is a row vector of costate variables. By standard Lagrange multiplier theory we minimize

$$L_0(y_0, y_1, \ldots, w_0, w_1, \ldots, \mu_1, \mu_2, \ldots) = \sum_{t=0}^{\infty} H_t \tag{A.7}$$

with respect to the arguments of L_0 (except z_0 which is given). Then at the optimum, $L_0 = \Omega_0$.

Redefining a new costate vector $p_t = \lambda^{-1}\mu_t^T$, the first-order conditions lead to

$$w_t = -R^{-1}(\lambda B^T p_{t+1} + U^T y_t) \tag{A.8}$$

$$\lambda A^T p_{t+1} - p_t = -(Qy_t + Uw_t) \tag{A.9}$$

Substituting (A.8) into (A.2)) we arrive at the following system under control

$$\begin{bmatrix} I & \lambda B R^{-1} B^T \\ 0 & \lambda(A^T - UR^{-1}B^T) \end{bmatrix} \begin{bmatrix} y_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} A - BR^{-1}U^T & 0 \\ -(Q - UR^{-1}U^T & I \end{bmatrix} \begin{bmatrix} y_t \\ p_t \end{bmatrix} \tag{A.10}$$

To complete the solution we require $2n$ boundary conditions for (A.10). Specifying z_0 gives us $n-m$ of these conditions. The remaining condition is the 'transversality condition'

$$\lim_{t\to\infty} \mu_t^T = \lim_{t\to\infty} \lambda^t p_t = 0 \tag{A.11}$$

and the initial condition

$$p_{20} = 0 \tag{A.12}$$

where $p_t^T = \left[\begin{array}{cc} p_{1t}^T & p_{2t}^T \end{array} \right]$ is partitioned so that p_{1t} is of dimension $(n-m) \times 1$. Equation (A.3), (A.8), (A.10) together with the $2n$ boundary conditions constitute the system under optimal control.

Solving the system under control leads to the following rule

$$w_t = -F \left[\begin{array}{cc} I & 0 \\ -N_{21} & -N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{A.13}$$

$$\left[\begin{array}{c} z_{t+1} \\ p_{2t+1} \end{array} \right] = \left[\begin{array}{cc} I & 0 \\ S_{21} & S_{22} \end{array} \right] G \left[\begin{array}{cc} I & 0 \\ -N_{21} & -N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{A.14}$$

$$N = \left[\begin{array}{cc} S_{11} - S_{12}S_{22}^{-1}S_{21} & S_{12}S_{22}^{-1} \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{array} \right] = \left[\begin{array}{cc} N_{11} & N_{12} \\ N_{21} & N_{22} \end{array} \right] \tag{A.15}$$

$$x_t = -\left[\begin{array}{cc} N_{21} & N_{22} \end{array} \right] \left[\begin{array}{c} z_t \\ p_{2t} \end{array} \right] \tag{A.16}$$

where $F = -(R + B^T SB)^{-1}(B^T SA + U^T), G = A - BF$ and

$$S = \left[\begin{array}{cc} S_{11} & S_{12} \\ S_{21} & S_{22} \end{array} \right] \tag{A.17}$$

partitioned so that S_{11} is $(n-m) \times (n-m)$ and S_{22} is $m \times m$ is the solution to the steady-state Ricatti equation

$$S = Q - UF - F^T U^T + F^T RF + \lambda(A - BF)^T S(A - BF) \tag{A.18}$$

The cost-to-go for the optimal policy (OP) at time t is

$$\Omega_t^{OP} = -\frac{1}{2}(\mathrm{tr}(N_{11}Z_t) + \mathrm{tr}(N_{22}p_{2t}p_{2t}^T)) \tag{A.19}$$

where $Z_t = z_t z_t^T$. To achieve optimality the policy-maker sets $p_{20} = 0$ at time $t = 0$. At time $t > 0$ there exists a gain from reneging by resetting $p_{2t} = 0$. It can be shown that $N_{22} < 0$, so the incentive to renege exists at all points along the trajectory of the optimal policy. This is the time-inconsistency problem.

A.2 Optimized Simple Rules (SIMCC and SIMNC)

We now consider simple sub-optimal rules of the form

$$w_t = Dy_t = D \left[\begin{array}{c} z_t \\ x_t \end{array} \right] \tag{A.20}$$

where D is constrained to be sparse in some specified way. Rule (A.20) can be quite general. By augmenting the state vector in an appropriate way it can represent a PID (proportional-integral-derivative)controller (though the paper is restricted to a simple proportional controller only).

First consider the design of cooperative simple rules. Substituting (A.20) into (A.5) gives

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty} \lambda_t y_{t+i}^T P_{t+i} y_{t+i} \tag{A.21}$$

where $P = Q + UD + D^T U^T + D^T RD$. The system under control (A.1), with w_t given by (A.20), has a rational expectations solution with $x_t = -Nz_t$ where $N = N(D)$. Hence

$$y_t^T P y_t = z_t^T T z_t \tag{A.22}$$

where $T = P_{11} - N^T P_{21} - P_{12}N + N^T P_{22}N$, P is partitioned as for S in (A.17) onwards and

$$z_{t+1} = (G_{11} - G_{12}N)z_t \tag{A.23}$$

where $G = A + BD$ is partitioned as for P. Solving (A.23) we have

$$z_t = (G_{11} - G_{12}N)^t z_0 \tag{A.24}$$

Hence from (A.25), (A.22) and (A.24) we may write at time t

$$\Omega_t^{SIM} = \frac{1}{2} z_t^T V z_t = \frac{1}{2}\mathrm{tr}(VZ_t) \tag{A.25}$$

where $Z_t = z_t z_t^T$ and V satisfies the *Lyapunov* equation

$$V = T + H^T V H \tag{A.26}$$

where $H = G_{11} - G_{12}N$. At time $t = 0$ the optimized simple rule is then found by minimizing Ω_0 given by (A.25) with respect to the non-zero elements of D given z_0 using a standard numerical technique. An important feature of the result is that unlike the previous solution the optimal value of D is not independent of z_0. That is to say

$$D = D(z_0)$$

For the non-cooperative case, in a closed-loop Nash equilibrium we assume each policymaker chooses rules $w_t = Dy_t$ and $w_t^* = D^* y_t$ independently taking the rule of the other bloc as given. The equilibrium is then computed by iterating between the two countries until the solutions converge.

A.3 The Stochastic Case

Consider the stochastic generalization of (A.1)

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t + \begin{bmatrix} u_t \\ 0 \end{bmatrix} \tag{A.27}$$

where u_t is an $n \times 1$ vector of white noise disturbances independently distributed with $\text{cov}(u_t) = \Sigma$. Then, it can be shown that certainty equivalence applies to all the policy rules apart from the simple rules (see Currie and Levine (1993)). The expected loss at time t is as before with quadratic terms of the form $z_t^T X z_t = \text{tr}(X z_t, Z_t^T)$ replaced with

$$E_t\left(\text{tr}\left[X\left(z_t z_t^T + \sum_{i=1}^{\infty} \lambda^t u_{t+i} u_{t+i}^T\right)\right]\right) = \text{tr}\left[X\left(z_t^T z_t + \frac{\lambda}{1-\lambda}\Sigma\right)\right] \tag{A.28}$$

where E_t is the expectations operator with expectations formed at time t.

Thus for the optimal policy with commitment (A.19) becomes in the stochastic case

$$\Omega_t^{OP} = -\frac{1}{2}\text{tr}\left(N_{11}\left(Z_t + \frac{\lambda}{1-\lambda}\Sigma\right) + N_{22} p_{2t} p_{2t}^T\right) \tag{A.29}$$

For the simple rule, generalizing (A.25)

$$\Omega_t^{SIM} = -\frac{1}{2}\text{tr}\left(V\left(Z_t + \frac{\lambda}{1-\lambda}\Sigma\right)\right) \tag{A.30}$$

The optimized cooperative simple rule is found at time $t = 0$ by minimizing Ω_0^{SIM} given by (A.30). Now we find that

$$D^* = D^*\left(z_0 + \frac{\lambda}{1-\lambda}\Sigma\right) \tag{A.31}$$

or, in other words, the optimized rule depends both on the initial displacement z_0 and on the covariance matrix of disturbances Σ. The non-cooperative rule for the stochastic case follows as before.

A.4 Non-Cooperation with Commitment (NC)

In Liu and Pappa (2005) a NC regime with commitment is used which is *open-loop* in character. For stochastic environments a *closed-loop* equilibrium is more appropriate. Suppose country 1 assumes $w_t^* = 0$ (no control) or some other initial rule and calculates an optimal rule with reputation. Following the analysis in the CC regime this will take the form

$$w_t = D \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix} \tag{A.32}$$

with

$$p_{2t+1} = H_{21} z_t + H_{22} p_{2t} \tag{A.33}$$

and $p_{20} = 0$. Now consider the response of country 2 which faces a system under control of the form

$$\begin{bmatrix} z_{t+1} \\ p_{2t+1} \\ x^e_{t+1,t} \end{bmatrix} = \begin{bmatrix} A_{11} + E_{11} & E_{12} & A_{12} \\ H_{21} & H_{21} & 0 \\ A_{21} + E_{21} & E_{22} & A_{22} \end{bmatrix} \begin{bmatrix} z_t \\ p_{2t} \\ x_t \end{bmatrix} + B_2 w^*_t \tag{A.34}$$

where $E = B_1 D$ is partitioned as for A. The optimal responses will now be a rule of the form

$$w^*_t = D^* \begin{bmatrix} z_t \\ p_{2t} \\ p^*_{2t} \end{bmatrix} \tag{A.35}$$

$$p^*_{2t+1} = C^*_{21} \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix} + C_{22} p^*_{2t} \tag{A.36}$$

and $p^*_{20} = 0$.

Now replace the initial rule of country 2 with (A.35) and recalculate w_t for country 1. The new optimal response will be of the form

$$w_t = D \begin{bmatrix} z_t \\ p_{2t} \\ p^*_{2t} \\ \hat{p_{2t}} \end{bmatrix} \tag{A.37}$$

p_{2t} is then up-dated with $\hat{p_{2t}}$ and country 2 responds in a similar way. Iterating in this fashion we arrive at stationary rules of the form

$$w_t = D \begin{bmatrix} z_t \\ p_{2t} \\ p^*_{2t} \end{bmatrix} : w^*_t = D^* \begin{bmatrix} z_t \\ p_{2t} \\ p^*_{2t} \end{bmatrix} \tag{A.38}$$

provided the algorithm converges. The expected welfare losses in equilibrium are then given by an expression analogous to (A.19) for both countries.

A.5 Cooperation with Discretion (CD)

As for CC we only give the outline solution. This is given by the iterative scheme

$$J_t = -(A_{22} + N_{t+1} A_{21})^{-1} (N_{t+1} A_{11} + A_{21}) \tag{A.39}$$

$$K_t = -(A_{22} + N_{t+1} A_{21})^{-1} (N_{t+1} B^1 + B^2) \tag{A.40}$$

$$N_t = -J_t + K_t F_t \tag{A.41}$$

$$F_t = (\overline{R}_t + \lambda \overline{B}_t^T S_{t+1} \overline{B}_t)(\overline{U}_t + \lambda \overline{B}_t^T S_{t+1} \overline{A}_t) \tag{A.42}$$

$$\overline{Q}_t = Q_{11} + J_t^T Q_{21} + Q_{12} J_t + J_t^T Q_{22} J_t \tag{A.43}$$

$$\overline{U}_t = U^1 + Q_{12} K_t + J_t^T U^2 + J_t^T Q_{22} J_t \tag{A.44}$$

$$\overline{R}_t = R + K_t^T Q_{22} K_t + U^{2t} K_t + K_t^T U^2 \tag{A.45}$$

$$S_t = \overline{Q}_t - \overline{U}_t F_t - F_t^T \overline{U}_t^T + F_t^T \overline{R}_t F_t + \lambda(\overline{A}_t - \overline{B}_t F)^T S_{t+1}(\overline{A}_t - \overline{B}_t F_t) \tag{A.46}$$

where, to ease the notational burden, the subscript c has been dropped in C_c, U_c and R_c. If these converge to stationary values $J, K, N, F, \overline{Q}, \overline{U}, \overline{R}$ and S then the solution is given by

$$w_t = -Fz_t \tag{A.47}$$

$$x_t = -Nz_t \tag{A.48}$$

where

$$z_{t+1} = [A_{11} + A_{12}J - (B^1 + A_{12}K)F]z_t + u_t \tag{A.49}$$

with z_0 given. The expected welfare loss from time t onwards is given by

$$\Omega_0 = \frac{1}{2}\mathrm{tr}(S(z_t z_t^T + \Sigma/(1-\lambda))) \tag{A.50}$$

A.6 Non-cooperative Equilibria with Discretion (ND)

Regime ND is a Nash equilibrium found by iterating between the two policy-makers together and the private sector in a Cournot-like adjustment process. For the case of two countries acting independently we now have three players. There are a number of ways in which the iteration may now proceed. The method we chose is to pass from country 1 to the private sector to country 2 to the private sector and so on. Then given initial values for D, D^* and N, provided the iteration converges, we arrive at the ND equilibrium.

B Quadratic Approximation of Utility Function

We use a small distortions approximation to the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\kappa}{1+\phi}\left(\frac{Y_t}{A_t}\right)^{1+\phi} \Delta_t^{1+\phi} \right] \tag{B.1}$$

where we use the resource constraints rather than full set of constraints that involve price setting. For the purposes of this paper we set the elasticity μ between home and foreign goods to 1, and also assume that home bias w is the same in each bloc. Inflation does not enter any of the resource constraints, and it is easy to show that the second order approximation involving inflation stems directly from (B.1), and is given by $\frac{1}{2}w_\pi(\pi_{H,t} - \gamma_{H,t}\pi_{H,t-1})^2$ where $w_\pi = \frac{\xi\zeta}{(1-\beta\xi)(1-\xi)}$ in (82) of the main text. The remaining terms of the quadratic approximation are then obtained as the second-order expansion to the stationary point of the Lagrangian involving (B.1) and the resource constraints:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \quad & \Bigg[\quad \frac{(C_t - Z_t)^{1-\sigma}}{1-\sigma} - \frac{\kappa}{1+\phi}\left(\frac{Y_t}{A_t}\right)^{1+\phi} + \lambda_{1t}(T_t^{\frac{2w-1}{\sigma}}(C_t^* - Z_t^*)U_{C,t}^{*\frac{-1}{\sigma}} - (C_t - Z_t)U_{C,t}^{\frac{-1}{\sigma}}) \\ & + \quad \lambda_{2t}(Z_t - hC_{t-1}) + \lambda_{3t}(Y_t - U_{oil,t}(wT_t^{1-w}C_t + (1-w)T_t^w C_t^*) - G_t) && \text{(B.2)} \\ & + \quad \lambda_{4t}(Y_t^* - U_{oil,t}((1-w)T_t^{-w}C_t + wT_t^{w-1}C_t^*) - G_t) + \lambda_{5t}(Z_t^* - hC_{t-1}^*) \Bigg] && \text{(B.3)} \end{aligned}$$

In the first order conditions we set $G = g_y Y$, and obtain the equilibrium value for the Nash solution as in the main text. Other relevant steady state values for the second order approximation are

$$C = (1-g_y)Y \quad \lambda_3 = \kappa\frac{Y^\phi}{A^{1+\phi}} \quad \lambda_1 = \lambda_3\frac{2(1-w)}{2(1-\beta h)(1-w) + \frac{2w-1}{\sigma}(1-h)}$$
$$\lambda_4 = \lambda_3\frac{(1-w)(2w(1-\beta h) - \frac{(2w-1)}{\sigma}(1-h))}{w(2(1-\beta h)(1-w) + \frac{2w-1}{\sigma}(1-h))} \tag{B.4}$$

The second-order approximation (apart from the inflation contribution described above) is then given by

$$\begin{aligned}
\hat{W}_t^{NC} &= -\frac{\sigma}{2}C^{1-\sigma}(1-h)^{-1-\sigma}(c_t - hc_{t-1})^2 - \frac{\kappa\phi}{2}\left(\frac{Y}{A}\right)^{1+\phi} y_t^2 + \kappa(1+\phi)\left(\frac{Y}{A}\right)^{1+\phi} y_t a_t \\
&- \frac{C}{2}\left(\frac{(2w-1)^2}{\sigma^2}(1-h)\lambda_1 + 2w\frac{2w-1}{\sigma}(1-h)\lambda_1 + (\lambda_3+\lambda_4)w(1-w)\right)\tau_t^2 \\
&+ C(\lambda_3-\lambda_4)w(2w-1)c_t\tau_t + C\frac{2w-1}{\sigma}\lambda_1(c_t - hc_{t-1})\tau_t - C(\lambda_3-\lambda_4)w\tau_t y_t \\
&- \frac{\lambda_1}{2w(1-w)}(y_t oil_t + \frac{(2w-1)^2}{\sigma}(1-h)oil_t c_t + 2w(1-w)oil_t\tau_t - g_y oil_t g_t) \\
&+ C(\lambda_3-\lambda_4)wg_y\tau_t + \frac{\lambda_1(1-h)}{\sigma}C\tau_t(u_{C,t}^* - u_{C,t}) + (\frac{\lambda_3+\lambda_4}{1-\beta h} + \frac{\lambda_1}{\sigma})(c_t - hc_{t-1})u_{C,t} \\
&- \kappa\left(\frac{Y}{A}\right)^{1+\phi} y_t(u_{C,t} + u_{L,t})
\end{aligned} \tag{B.5}$$

For the cooperative case the problem is much simpler; the welfare function of course includes welfare from both home and foreign blocs, and the risk-sharing condition is redundant. We then obtain

$$\begin{aligned}
\hat{W}_t^{C} &= -\frac{\sigma}{2}C^{1-\sigma}(1-h)^{-1-\sigma}[(c_t - hc_{t-1})^2 +](c_t^* - hc_{t-1}^*)^2] - \frac{\kappa\phi}{2}\left(\frac{Y}{A}\right)^{1+\phi}(y_t^2 + y_t^{*2}) \\
&+ \kappa(1+\phi)\left(\frac{Y}{A}\right)^{1+\phi}(y_t a_t + y_t^* a_t^*) - C^{1-\sigma}(1-h)^{-\sigma}[(c_t - hc_{t-1})u_{C,t} + (c_t^* - hc_{t-1}^*)u_{C,t}^*] \\
&- \frac{Y^\phi C}{A^{1+\phi}}oil_t(c_t + c_t^*) - \frac{Y^{1+\phi}}{A^{1+\phi}}[y_t(u_{C,t} + u_{L,t}) + y_t(u_{C,t}^* + u_{L,t}^*)] - w(1-w)\frac{Y^\phi C}{A^{1+\phi}}\tau_t^2
\end{aligned} \tag{B.6}$$

Finally the steady state expressions for can be used to eliminate the unobservable parameter κ and obtain the quadratic expressions (71) and (72) in the main text.

C Calibration

Our empirical US-Euro model will be estimated and calibrated without imposing symmetry on any parameters with the following exceptions. As we have noted $\zeta^* = \zeta$. In addition, in order to formulate optimal cooperative policies we need to impose a common discount factor $\beta = \beta^*$. This also ensures that the zero-inflation steady state sees a common nominal interest rate in the two blocs

The following ***fundamental parameters*** need to be estimated or calibrated:

1. **Disturbances** , ρ_a, ρ_a^*, ρ_g, ρ_g^*, ρ_e, ρ_e^* and corresponding standard deviations.
2. **Preference Parameters** h, h^*, σ, σ^*, $\beta = \beta^*$, ϕ, ϕ^*, μ, μ^*,
(Note: unidentified $\zeta^* = \zeta$, are only needed later for the welfare analysis; ω, ω^* are derived below are not really needed.)
3. **Bloc Size** n, n^*, ν, ν^* (Note $n^* = 1 - n$, $\nu^* = 1 - \nu$ and $\frac{n^*}{n} = \frac{\nu^*}{\nu}$, so we only need to calibrate n, the relative population size of the H-bloc.)
4. **Pricing** γ_H, ξ_H, γ_F^*, ξ_F^*
5. **Labour Market Elasticities of Substitution η, η^*.**

The remaining parameters to estimate are *the bias parameters* ω and ω^*. In principle these can be treated as any other parameter. However we adopt an alternative procedure is to use trade data so as to equate

$$\frac{P_F C_F}{P_T C_T} = (1 - \mathrm{w}) = \text{ import share of traded consumption in H bloc} \tag{C.7}$$

$$\frac{P_H^* C_H^*}{P_T^* C_T^*} = (1 - \mathrm{w}^*) = \text{import share of traded consumption in F bloc} \tag{C.8}$$

which calibrate w and w*. We also need to calibrate: $\frac{C}{Y}$, $\frac{C^*}{Y^*}$, and $\frac{Y}{Y^*}$.

We now have the following ***derived parameters*** as functions of estimated or calibrated parameters

- $\alpha_H = \mathrm{w}\frac{C}{Y}$
- $\alpha_F = (1 - \mathrm{w}^*)\frac{n^* C^*}{nY} = (1 - \mathrm{w}^*)\frac{n^* C^*}{nY^*}\frac{Y^*}{Y}$
- $\alpha_F^* = \mathrm{w}^*\frac{C^*}{Y^*}$
- $\alpha_H^* = (1 - \mathrm{w})\frac{nC}{n^* Y^*} = (1 - \mathrm{w})\frac{nC}{n^* Y}\frac{Y}{Y^*}$

  Note if $TB = 0$ in the steady state (which we have assumed in the linearization above), then $Y = C + TB = C$ and $Y^* = C^*$, so $\frac{C}{Y} = \frac{C^*}{Y^*} = 1$ in the expressions for α_H, etc.

- $\lambda_H = \frac{(1-\beta\xi_H)(1-\xi_H)}{\xi_H}$

- $\lambda_F^* = \frac{(1-\beta\xi_F^*)(1-\xi_F^*)}{\xi_F^*}$

- $\omega = \frac{\text{w}\frac{n^*}{n}}{1-\text{w}\left(1-\frac{n^*}{n}\right)}$=w if $n = n^*$

- $\omega^* = \frac{\text{w}^*\frac{n}{n^*}}{1-\text{w}^*\left(1-\frac{n}{n^*}\right)}$=w* if $n = n^*$

From the linearization it is apparent that parameters $\beta = \beta^*$, $\zeta = \zeta^*$, η and η^* are not identified and are therefore calibrated along with w, w* and the relative bloc sizes.

In the results reported in the present version of the paper we assume symmetric blocs with $n = n^*$ and all parameters identical. The oil shock is common, the technology shocks can be correlated in principle, but not in these results. Other shocks are independent. $\rho_i = 0.7$ for all shocks $i = a, g$ etc, all standard deviations are 1%. There is no government sector ($c_y = 1$). $\phi = 1.7$, $\mu = 1$, $\xi_H = \xi_F = 2/3$ corresponding to a 3-quarter Calvo price contract, $\zeta = 7.674$ corresponding to a 15% mark-up, $h = 0.5$, $\sigma = 2$, $\beta = 0.99$ and there is no indexation. Import shares are assumed to be 25% so $w = w^* = 0.75$.

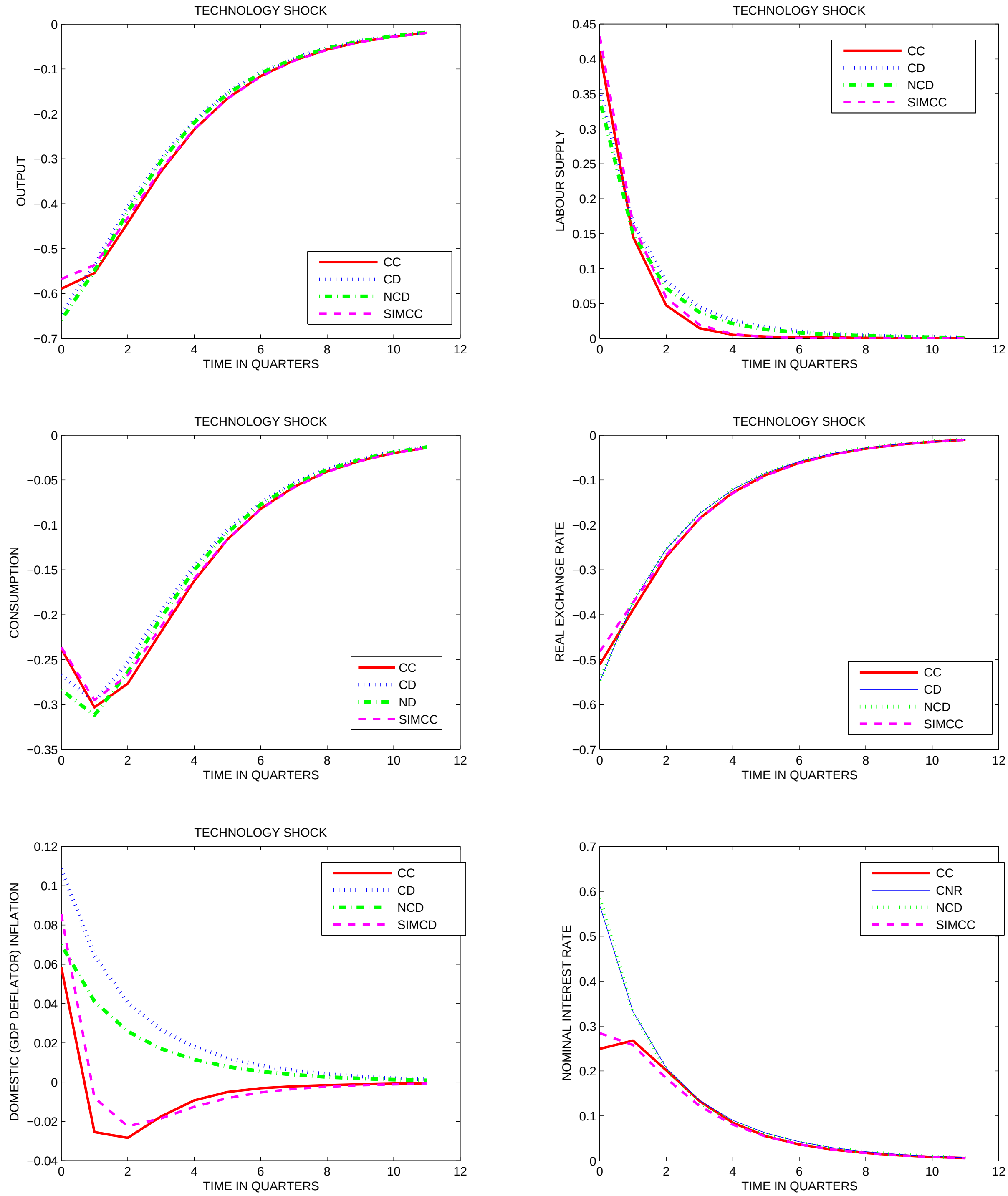

Figure 1: **Responses to a Negative Technology Shock: No ZLB Constraint Imposed.**

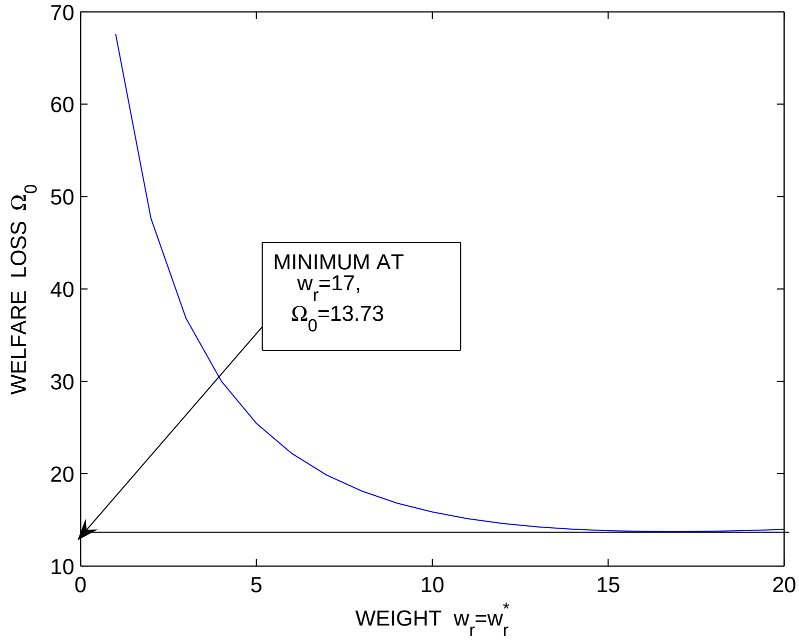

Figure 2: **Cooperation with Commitment: Imposing the ZLB**

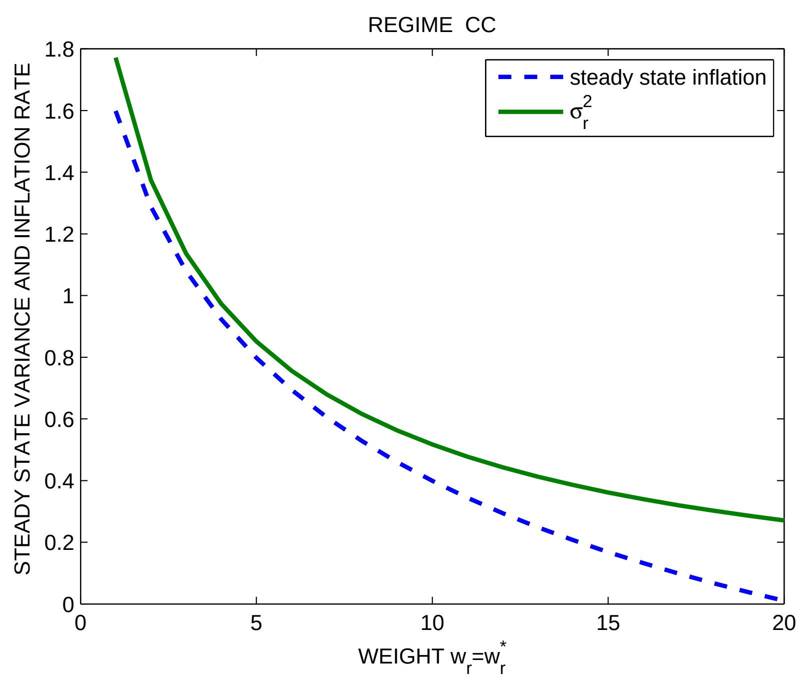

Figure 3: **Cooperation with Commitment: Imposing the ZLB**

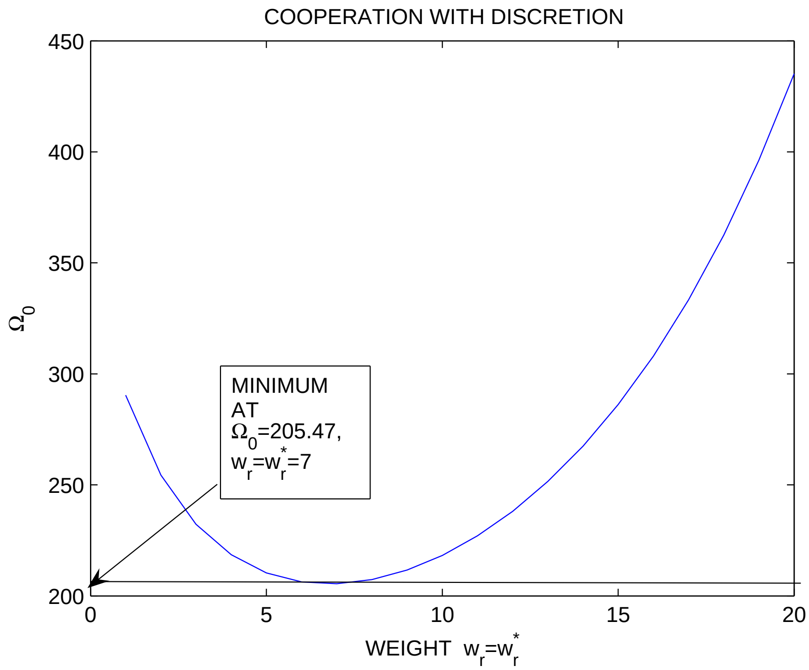

Figure 4: **Cooperation with Discretion: Imposing the ZLB**

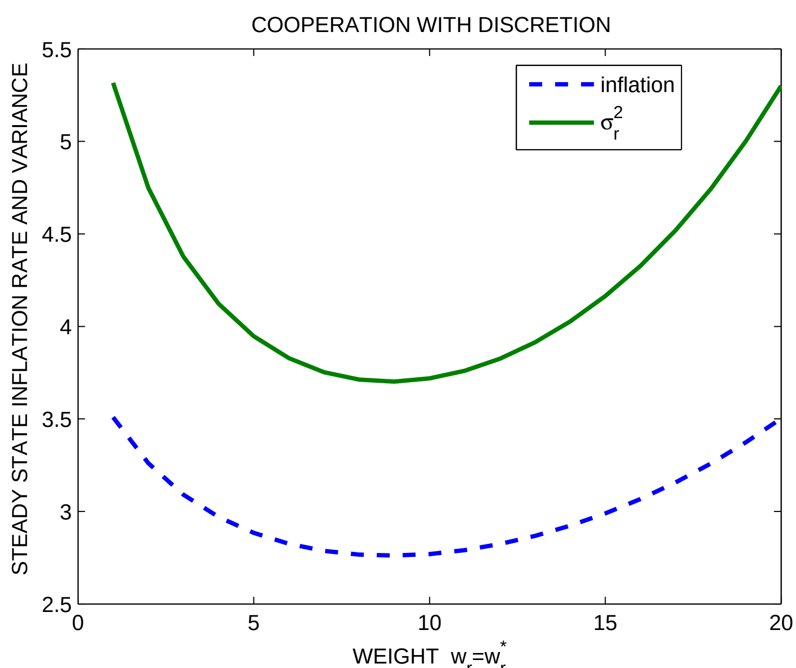

Figure 5: **Cooperation with Discretion: Imposing the ZLB**

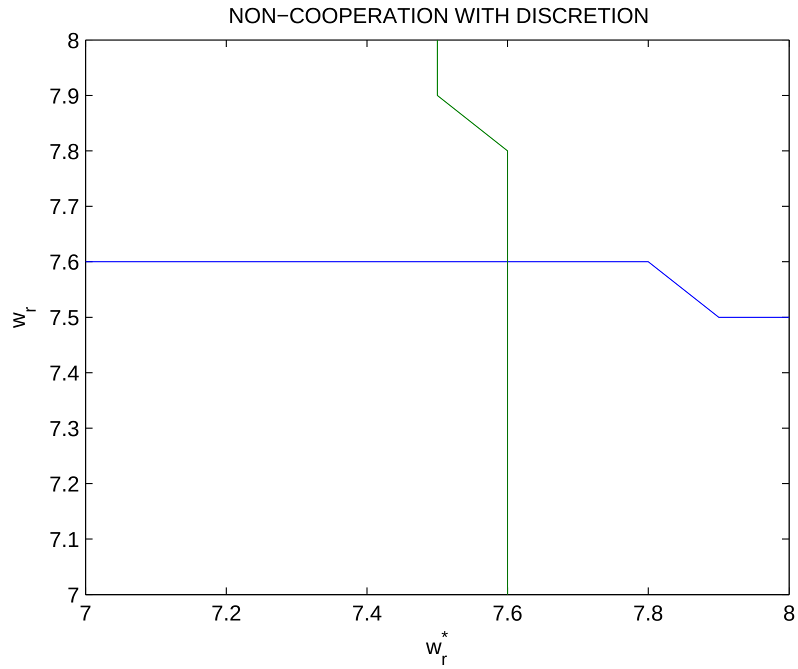

Figure 6: **Non-Cooperation with Discretion: Imposing the ZLB**

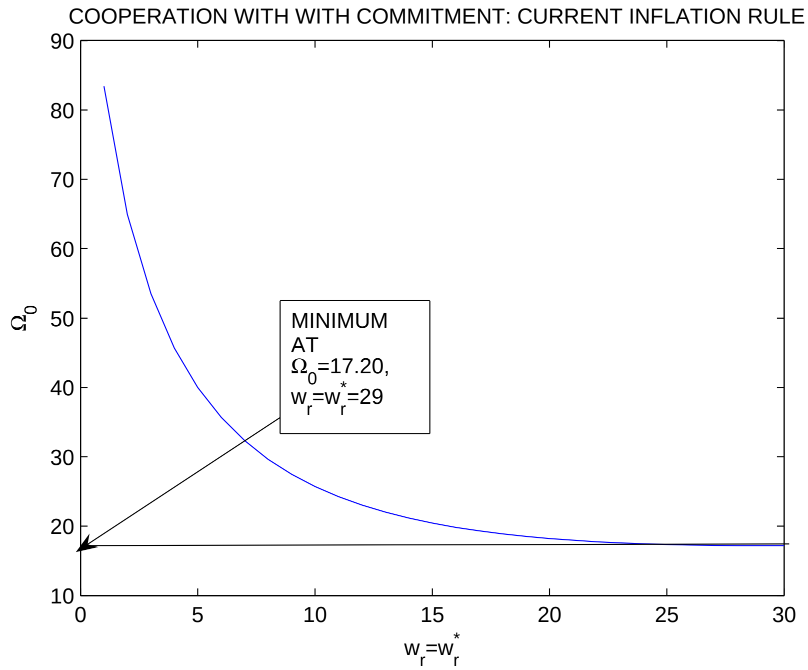

Figure 7: **Cooperation with a Commitment Current Inflation Rule: Imposing the ZLB**

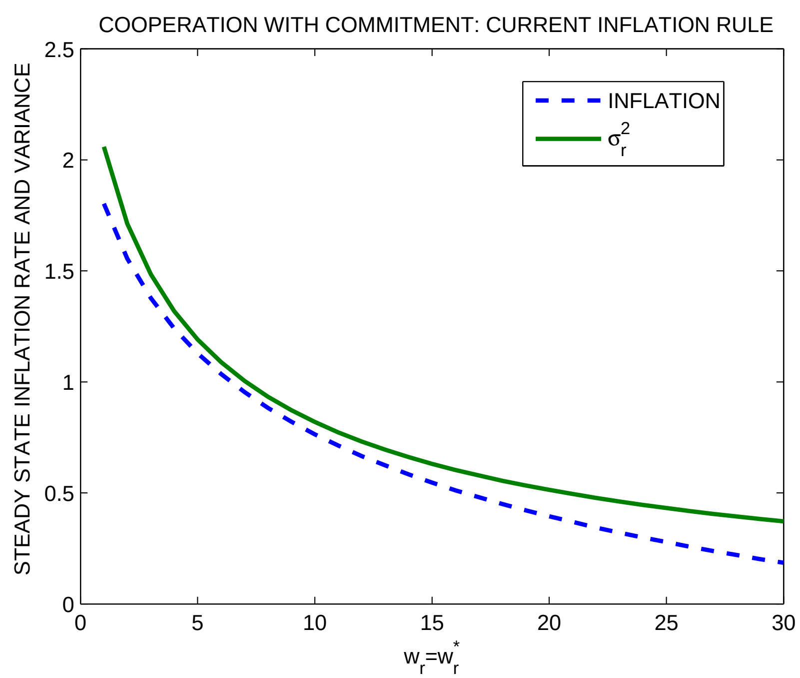

Figure 8: **Cooperation with a Commitment Current Inflation Rule: Imposing the ZLB**