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AGGREGATE STABILITY AND BALANCED-BUDGET RULES

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Aggregate Stability and Balanced-Budget Rules*

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Abstract

It has been shown that under perfect competition and a Cobb-Douglas production function, a basic real business cycle model may exhibit indeterminacy and sunspots fluctuations when income tax rates are determined by a balanced-budget rule. This paper introduces in an otherwise standard real business cycle model a more general and data coherent class of production functions, namely a constant elasticity of substitution production function. We show that the degree of substitutability between production factors is a key ingredient to understand the (de)stabilising properties of a balanced-budget rule. Furthermore, when we calibrate the model consistently with the empirical evidence, i.e. the elasticity of substitutions between labour and capital below unity, balanced-budget rules deliver determinacy for a broad range of OECD countries.

Keywords: Constant Elasticity of Substitution; Balanced-Budget Rules; Indeterminacy; Business Cycles.

JEL Classification: E32; E62.

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1 Introduction

A recurring debate in American politics regards the possibility of inserting in the U.S. constitution the requirements that federal government operates under a balanced budget rule (BBR). Indeed, most states have constitutional or statutory limitations restricting their ability to run deficits in the state's general fund. Balanced-budget limitations may be either prospective (beginning-of-the-year) requirements or retrospective (end-of-the-year) requirements. In Europe, the Maastricht Treaty required that EMU countries had a deficit below 3% imposing *de facto* a political restriction on the possibility for governments to deviate *"too far"* from a balanced budget fiscal policy.

In economic literature there is no shortage of discussion on the BBR. Both traditional business cycle literature (see among others Lucas and Stokey, 1983, King et al., 1988, Eggertsson 2008) and political economy literature (see among others Alesina and Perotti 1996, Besley and Smart 2007, Azzimonti et al. 2010) study under different perspectives the normative properties of adopting a BBR.

On the other hand, a different strand of literature focuses on the stabilising properties of a BBR on equilibrium determinacy. The main contribution to this can be found in Schmitt-Grohè and Uribe (1997). The authors find that in a standard neoclassical growth model isomorphic to Hansen (1985) or King et al. (1988), imposing on the fiscal authority a balanced-budget requirement may induce self-fulfilling expectations, hence indeterminacy. The intuition for this result goes as follows: under a BBR, when agents expect higher tax rates in the future, for a given level of capital stock, hours worked and therefore the rental rate of capital will be lower (the marginal product of capital is decreasing in the capital/labour ratio). The decrease in expected return on capital lowers current labour supply via its effect on the marginal utility of income, leading to a decline in current production. Given that the tax rate is an increasing function of income, the government is obliged, following a BBR to increase the tax rate today. This countercyclical tax policy will help fulfil agents' initial expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations. Furthermore, as Schmitt-Grohè and Uribe show, using an empirical based calibration, BBR would generate indeterminacy in the group of G7 countries if they were to adopt it.

This paper extends Schmitt-Grohè and Uribe's model by considering a more general class of production function, namely the constant elasticity of substitution (CES) production functions as in Arrow et al. (1961), which nests the traditional Cobb-Douglas (CD) as a particular case. We justify the introduction of this production technology into an otherwise basic real business cycle workhorse in two ways. Firstly, from a purely theoretical point of view, by simply varying a single model's parameter, namely the elasticity of substitution between production factors, the CES can be used to treat the production inputs, i.e. labour and capital, both as gross complements (elasticity of substitution below one) or gross substitutes (elasticity of substitution above one). Secondly, from an empirical point of view, recent studies, i.e. Klump et al. (2006, 2008), Chirinko (2008), León-Ledesma et al. (2010), reject the CD specification in favour of CES production function in which labour and capital are gross complements.

We obtain two sets of results. First, we show analytically that the degree of substitutability between production factors is a key ingredient to understand the (de)stabilising properties of a BBR. Second, when we calibrate the model in order to match the empirical evidence, i.e. elasticity of substitution below unity, the instability problems that affect the Schmitt-Grohè and Uribe model disappear for a broad range of OECD countries.

The main intuition for this result goes as follows. When the elasticity of substitution between capital and labour is below unity, production factors are gross complements. This causes labour hours to be more tightly coupled to the stock of capital. Consequently, equilibrium hours worked can respond less freely to belief shocks, avoiding generating the type of endogenous fluctuations which characterise the CD case.

From this, the aim of the paper is not only to contribute to the debate about the (de)stabilising properties of a BBR (see among others Guo and Harrison 2004, Giannitsarou, 2007, Linnemann, 2008, Anagnostopoulos and Giannitsarou, 2010) but also to develop an interest in the use of CES production technology in the analysis of economic policy issues.

The rest of the paper is organised as follows. Section 2 presents the model with only labour tax and the determinacy analysis. Section 3 adds to the benchmark model capital taxation and discusses the policy implications of introducing a BBR in a set of OECD countries. Section 4 concludes.

2 The Model

In this section we derive analytically the main results of the paper. In order to do this we analyse a continuous-time one sector real business model which consists of households, firms and government. Government purchases are constant and the only source of government revenues is a labour income tax. The initial stock of public debt is zero and the government is subject to a balanced-budget requirement. The government budget constraint is given by $G = \tau_t^h H_t w_t$, where G indicates government purchases of goods, τ_t^h denotes labour tax rate, w_t the pretax wage, and H_t hours worked. Firms hire labour and rent capital in a perfectly competitive manner. We generalise the standard Cobb-Douglas (CD) production function by employing a normalised version of the Constant Elasticity of Substitution (CES) production function. This represents the only difference between the model presented here and the one in Schmitt-Grohè and Uribe (1997).

The economy is populated by a unit measure of identical infinitely-lived households. Each household starts in period 0 with a positive stock of capital, K_0 and chooses path for consumption, C_t, hours and capital, so to maximise the present value of its lifetime utility where disutility of labour is linear, i.e. infinite Frisch elasticity:

$$\max \int_0^\infty e^{-\rho t} \left(\log C_t - AH_t\right) dt \tag{1}$$

subject to $K_t \geq 0$ and to the standard budget constraint:

$$\dot{K}_t = \left(\mu_t - \delta\right) K_t + (1 - \tau_t^h) w_t H_t - C_t \tag{2}$$

where $\rho \in (0, 1)$ is the subjective discount factor, $A \in [0, +\infty)$ is a standard utility parameter, μ_t denotes the rental rate of capital, $\delta \in (0, 1)$ is the depreciation rate. A dot above a variable denotes the time derivative, so $\dot{K}_t$ are net investment. The first order conditions associated with this problem are:

$$AC_t = (1 - \tau_t^h) w_t \tag{3}$$

$$\dot{C}_t = (\mu_t - \delta - \rho) C_t \tag{4}$$

The first equation states that the slope of the indifference curve of the representative households equates to the slope of the after tax real wage. The second equation is the consumption Euler equation. Alongside those we have the following transversality condition:

$$\lim_{t \to \infty} e^{-\rho t} \frac{K_t}{C_t} = 0 \tag{5}$$

Government revenues are given by a tax on labour income, government purchases are constant and the fiscal authority follows a balanced budget rule of the type:

$$G = \tau_t^h w_t H_t \tag{6}$$

where G denotes government purchases of goods.

The representative firm produces output Y_t, hires labour at a rate w_t and rents capital at a rate μ_t according to a CES production function of the type:

$$Y_t = B \left[\alpha \left(K_t\right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(H_t\right)^{\frac{\sigma - 1}{\sigma}}\right]^{\frac{\sigma}{\sigma - 1}} \tag{7}$$

where $\alpha \in (0, 1)$. Note that when $\sigma = 1$ the CES collapses to the CD case, when $\sigma \to 0$ it collapses to the Leontief case where capital and labour are perfect complements, while when $\sigma \to \infty$, capital and labour become perfect substitutes.

Let F_H and F_K denote the first derivatives of the production function with respect to labour and capital respectively. The representative firm maximises the stream of profits as:

$$\max \int_0^\infty e^{-\rho t} \left(Y_t - w_t H_t - \mu_t K_t\right) dt \tag{8}$$

subject to the production function as defined in (7). The first order conditions for this problem are:

$$F_{H,t} = w_t = (1-\alpha)B\left(\frac{Y_t}{H_t}\right)^{\frac{1}{\sigma}} \tag{9}$$

$$F_{K,t} = \mu_t = \alpha B\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} \tag{10}$$

These two equations state that the marginal products of the production inputs equate to their corresponding prices. Following Klump and de LaGrandville (2000a, b), Guo and Lansing (2010) and Cantore et al. (2010), we "normalise" the standard CES production function so that all steady-state allocations and factor income shares are held constant as the input substitution elasticity is changed. Normalisation removes the problem that arises from the fact that labour and capital are measured in different units. Under CD, normalisation plays no role since, due to its multiplicative form, differences in units are absorbed by the scaling constant. The CES function, by contrast, is highly non-linear, and so, unless correctly normalised, out of its two key parameters - the distribution parameter α and the substitution elasticity σ - only the latter is deep. The former turns out to be affected by the size of the substitution elasticity and factor income shares. Accordingly if one is interested in model sensitivity with respect to production parameters (as here), normalisation is essential to have interpretable comparisons. Given the aim of the paper we normalise the CES with a CD production function.

In order to complete the description of the model we need to define the aggregate resource constraint for the economy as:

$$Y_t = C_t + G + \dot{K}_t + \delta K_t \tag{11}$$

Steady State

The adopted normalisation implies that our model shares the same steady state as Schmitt-Grohè and Uribe (1997). Given the distortive nature of fiscal policy, in the steady state there is a Laffer curve-type of relation between tax rates and tax revenues. From the steady state relations of the endogenous variables, it is easy to show that government revenues are zero when tax rates are either zero or one and positive in between. Provided that, government revenues are a continuum function of the tax rate, there must be a τ^{h*} that maximises G. In turn τ^{h*} can be found as the solution of $\frac{\partial G}{\partial \tau^h} = 0$ as:

$$G\frac{(1-\bar{\alpha})\left(\tau^h\right)^2 - 2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^h + \left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)}{\tau^h\left(1-\tau^h\right)\left(1-\tau^h\left(1-\bar{\alpha}\right)-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)} = 0$$

where $\bar{\alpha}$ is the income share of capital (i.e. the usual CD parameter) obtained via the normalisation procedure outlined in the Appendix, and $\bar{\alpha}\frac{\delta}{\delta+\rho}$ is independent of τ^h. Note that for

$\tau^h \in (0,1)$, G and the denominator are always positive.[1] So τ^{h*} corresponds with the zeros of the polynomial:

$$(1-\bar{\alpha})\left(\tau^h\right)^2 - 2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^h + \left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right) \tag{12}$$

When $\tau^h = 0$ the above expression takes the value of $1-\bar{\alpha}\frac{\delta}{\delta+\rho} > 0$, while when $\tau^h = 1$ it takes the value of $\bar{\alpha}\left(\frac{\delta}{\delta+\rho}-1\right) < 0$. Hence one of the zeros happens when $\tau^h \in (0,1)$ and the other when $\tau^h > 1$. Therefore there is a unique maximum of government revenues, τ^{h*} between 0 and 1.

Determinacy Analysis

We log-linearise the structural equations around a normalised steady state (Appendix A). As previously discussed, we follow the normalisation procedure presented by Guo and Lansing (2010) and Cantore et al. (2010). This allows us to use the CD production function as a steady state benchmark. After some straightforward manipulations we can rewrite the model as one involving just two dynamic variables, namely consumption and capital accumulation (see Appendix B for details on the log-linearisation). The model can be represented as:

$$\begin{bmatrix} \dot{c}_t \\ \dot{k}_t \end{bmatrix} = \begin{bmatrix} -(\delta+\rho)\left(\frac{(1-\bar{\alpha})\left(1-\tau^h\right)}{\bar{\alpha}-\sigma\tau^h}\right) & -(\delta+\rho)\frac{1}{\sigma}\left(1+\bar{\alpha}\frac{1-\sigma\tau^h}{\sigma\tau^h-\bar{\alpha}}\right) \\ \frac{Y}{K}\sigma(1-\bar{\alpha})\frac{\tau^h-1}{\bar{\alpha}-\sigma\tau^h}-\frac{C}{K} & \frac{Y}{K}\bar{\alpha}\left(\frac{-\sigma\tau^h+1}{\bar{\alpha}-\sigma\tau^h}\right)-\delta \end{bmatrix} \begin{bmatrix} c_t \\ k_t \end{bmatrix} \tag{13}$$

where $\bar{\alpha}$ is the long run capital income share, i.e. the usual CD parameter, a lower case variable identifies its log-linearised value and a variable without time index identifies its steady state value. Let J be the matrix of this linear system. Since (13) contains one predetermined, k_t, and one non-predetermined variable, c_t, the perfect-foresight equilibrium will be determinate if, and only if, the two eigenvalues of J have different signs. Being the determinant of J the product of its eigenvalues, the system is determinate if and only if the determinant of J is negative. However, if the determinant is positive and the trace (being the sum of the eigenvalues of J) is negative, i.e. both eigenvalues of J are negative, the perfect-foresight equilibrium will be indeterminate. Finally, if both the determinant and the trace of J are positive, i.e. both eigenvalues of J are positive, the perfect-foresight equilibrium will be unstable. Determinant and trace of J are respectively

$$Det(J) = \frac{\delta(\rho+\delta)(1-\bar{\alpha})}{\frac{\delta\alpha}{\delta+\rho}(\sigma\tau-\bar{\alpha})}\left((1-\bar{\alpha})\left(\tau^h\right)^2 - 2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^h + \left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\right) \tag{14}$$

[1]To see why the denominator is positive, note that $1-\tau^h(1-\bar{\alpha})-\bar{\alpha}\frac{\delta}{\delta+\rho} > 1-(1-\bar{\alpha})-\alpha\frac{\delta}{\delta+\rho}$. The latter can be written as $\bar{\alpha}\left(1-\frac{\delta}{\delta+\rho}\right)$ which is positive.

$$Trace\,(J) = \frac{(\rho+\delta)\left(1-\sigma\tau^h\right) - (\rho+\delta)\left(\left(1-\tau^h\right)(1-\bar{\alpha})\right)}{\bar{\alpha}-\sigma\tau^h} - \delta \tag{15}$$

The next two propositions describe the possible equilibrium outcomes.

Proposition 1 *Necessary and sufficient conditions for equilibrium determinacy are:*

$$\begin{aligned} if\ \sigma &\in [\bar{\alpha}, +\infty) \Longrightarrow \tau^h \in [0, \min\left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right)) \cup \tau^h \in (\max\left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right), 1] \\ else\ \sigma &\in [0, \bar{\alpha}) \Longrightarrow \tau^h \in [0, \tau^{h*}) \end{aligned}$$

Proof. See Appendix C. ■

Proposition 2 *Let define* $\varepsilon_1 = \frac{\bar{\alpha}\rho}{\rho\sigma - (1-\bar{\alpha})(\delta+\rho)}$. *When the system is not determinate:*

$a)$ $If\ \sigma \in \left[0, 1+\frac{\delta(1-\bar{\alpha})}{\rho}\right] \cap \frac{\bar{\alpha}}{\sigma} < \tau^{h*}$, *the system is never unstable. It is indeterminate if* $\tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right)$

$b)$ $If\ \sigma \in \left[0, 1+\frac{\delta(1-\bar{\alpha})}{\rho}\right] \cap \frac{\bar{\alpha}}{\sigma} > \tau^{h*}$, *the system is never indeterminate.*
It is unstable if $\tau^h \in \left(\tau^{h*}, \min\left(\frac{\bar{\alpha}}{\sigma}, 1\right)\right)$.

$c)$ $If\ \sigma \in \left(1+\frac{\delta(1-\bar{\alpha})}{\rho}, +\infty\right) \cap \varepsilon_1 > \tau^{h*}$, *the system is never unstable.*
It is indeterminate if $\tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right)$

$d)$ $If\ \sigma \in \left(1+\frac{\delta(1-\bar{\alpha})}{\rho}, +\infty\right) \cap \varepsilon_1 < \tau^{h*}$, *the system is indeterminate if* $\tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \varepsilon_1\right)$
and unstable if $\tau^h \in \left(\varepsilon_1, \tau^{h*}\right)$

Proof. See Appendix C. ■

Proposition 1 states the main result of the paper. Under a BBR, the elasticity of substitution between production factors changes markedly the determinacy properties of the model. A few points are noteworthy here. First, the value of τ^{h*} is not function of σ. This is a direct consequence of the normalisation procedure, i.e. at steady state the Laffer curve is independent of σ. Second, the threshold tax rate which may cause model's instability is not only a function, as in Schmitt-Grohè and Uribe (1997), of the steady state share of capital to output, i.e. $\bar{\alpha}$, but also and in a non-trivial way of the elasticity of substitution between input factors, i.e. σ. Increasing the complementarity between factors, i.e. lowering σ, increases the upper bound on the labour tax rate, $\frac{\bar{\alpha}}{\sigma}$ which ensures determinacy, while the opposite is true when σ increases.[2] Furthermore it is interesting to note that for particularly low values of σ, the upper bound on the tax rate is represented by τ^{h*}, this happens if $\frac{\bar{\alpha}}{\sigma} > \tau^{h*}$.

[2]Note that our results perfectly nest SGU (1997) when $\sigma = 1$.

The second set of results is aimed to classify the equilibrium outcomes when the model displays some sort of sunspot fluctuations, i.e. either stationary or not. Unlike the CD specification we are able to induce instability in the system when income labour tax is the only source of government revenue.

Intuitions: a Closer Look at the Labour Market and the Laffer Curve

The equilibrium conditions obtained can be explained by the interactions between the effects of the elasticity of substitutions between factors on the *"equilibrium labour demand schedule"* (LDS henceforth), and the shape of the Laffer curve. As in the Schmitt-Grohè and Uribe model, our LDS may slope upwards because increases in aggregate hours worked are accompanied by decreases in the tax rate. However in our model the elasticity of substitution between factors greatly modifies the LDS's slope, in turn affecting markedly the equilibrium outcomes. To see why let us write the after-tax labour demand function (in log deviation from steady state) as:

$$\widetilde{w}_t = \frac{\tau^h - \bar{\alpha}/\sigma}{1-\tau^h} h_t + \frac{\bar{\alpha}/\sigma}{1-\tau^h} k_t \tag{16}$$

where $\widetilde{w}_t \equiv \hat{w}_t - \frac{\tau^h}{1-\tau^h}\hat{\tau}_t^h$ denotes the log deviation of the after-tax wage rate from the steady state. Note that if $\tau^h < \frac{\bar{\alpha}}{\sigma}$ the slope of the LDS is negative while it is positive in the opposite case, i.e. $\tau^h > \frac{\bar{\alpha}}{\sigma}$. It is worth stressing that the elasticity of substitution parameter implies that the higher the complementarity (substituability) between production factors, the higher (lower) is the steady state tax rate threshold that flips the sign of the LDS's slope. Moreover, the change in sign in the LDS's slope can take place for tax rates higher than τ^{h*}, i.e. on the decreasing side of the Laffer curve, or may not occur at all, i.e. if $\sigma \leq \bar{\alpha}$.

From *Proposition 1* it is easy to see that whenever the LDS's slope is positive (negative) and $\tau^h \in [0, \tau^{h*})$ the system is indeterminate (determinate). The intuition for this goes as follows: under a BBR, when agents expect higher tax rates in the future, for a given level of capital stock, hours worked and therefore the rental rate of capital will be lower (the marginal product of capital is decreasing in the capital/labour ratio). In turn, since consumers want to invest less in the future, this leads to a reduction of current labour demand and to a decline in current production. Given that the tax rate is an increasing function of income (the model is on the increasing side of the Laffer curve), the government is obliged, following a BBR, to increase the tax rate today. This countercyclical tax policy will help to fulfil agents' initial expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations. These effects are consistent with a positive slope in the LDS. As σ decreases below unity, production factors become gross complements. This causes the demand of labour to be more tightly coupled to the stock of capital which is predetermined. Consequently, equilibrium hours worked can respond less freely to belief shocks, i.e. the lower σ the higher the steady state tax rates required to flip the sign of the LDS and hence to induce a reduction of the equilibrium hours worked, in turn making endogenous fluctuations more difficult to occur. These effects are reversed when production factors are gross substitutes, i.e. $\sigma > 1$. Furthermore, whenever the model is not determinate and $\tau^h < \tau^{h*}$, sunspot fluctuations are stationary. This is because an increase in *current* taxation generates a further decrease in the equilibrium hours worked. This decrease pushes fiscal policy to implement a further increase in current taxation. As a result, (for a given expected increase in the future tax rate) the tax rate in period 0 is larger in absolute value than the tax rate in period t' for any $t' > 0$, so that the sequence of tax rates converges to the steady state, i.e. indeterminacy.

Furthermore, when the steady state tax rate is greater than τ^{h*}, i.e. on the decreasing side of the Laffer curve, a positive slope in the LDS is a necessary condition for determinacy. This is because while expectations of higher future tax rates generate a decrease in labour demand, fiscal policy needs to cut taxes in order to balance its budget, in turn contrasting the initial expectations of higher tax rates.

We are also able to induce instability in the model with only labour income tax. This occurs whenever the slope of the LDS is negative and steady state taxes are higher than the peak of the Laffer curve. The intuition for this result is the following. When the complementarity between factors is so strong that $\frac{\bar{\alpha}}{\sigma} > \tau^{h*}$, and agents expect higher future taxes, equilibrium hours worked increase. Because the system is in the decreasing side of the Laffer curve fiscal policy needs to raise taxation in order to balance its budget. While higher taxation today helps to self-fulfil agents' expectations, it decreases current labour demand putting thus downward pressure on taxes. As a result, (for a given expected increase in the future tax rate) tax rates in period 0 increase less in absolute value than tax rates in period t' for any $t' > 0$ so that the sequence of tax rates have an explosive path, i.e. instability.

Finally, note that we can induce instability even when production factors are gross substitute and the tax rate is smaller than the peak of the Laffer curve. This is possibly due to the perverse effect of the high degree of substitutability and the positive slope of the LDS. However, given that, as discussed below, this occurs for empirically implausible[3] values of σ, this result goes beyond the aim of the present exercise. Hence we leave future research to explore this conjecture in more detail.

A Numerical Example

In order to give a primal flavour of how the elasticity of substitution changes quantitatively the determinacy analysis, we present a numerical example. The model is calibrated to annual frequency. The parameters' values are, with the obvious exception of σ, the same as in Schmitt-Grohé and Uribe (1997) and are $\rho = 0.04$ for the annual interest rate, $\delta = 0.1$ for the capital depreciation rate, the utility parameter A is calibrated to 1 and the CD capital income share in output is $\bar{\alpha} = 0.3$.

[3] For standard parametrisation $1 + \frac{\delta(1-\bar{\alpha})}{\rho} = 2.75$. This value is outside the plausible estimates of σ.

In order to have a comprehensive picture on how the elasticity parameter modifies the (de)stabilising properties of a BBR, we allow σ to vary between 0.2 and 1.8.

Results of this exercise are presented in Figure (1). Each subplot presents the values of labour tax rates that induce determinacy (white area), indeterminacy (black area) and instability (red area). As discussed in detail before, reducing the value of σ reduces the indeterminacy area, in turn expanding the possibility that for a given tax rate, a BBR delivers determinacy. Finally, when $\bar{\alpha}/\sigma > \tau^{h*}$, a BBR may induce instability for steady state labour tax rates greater than the peak of the Laffer curve (bottom right subplot).

Note that for empirically plausible values of the elasticity of substitutions, i.e. $\sigma \in [0.4 - 0.6]$, the labour tax rates that may induce indeterminacy are greater than the one reported by empirical studies[4], i.e. $\tau^h \in [0.25 - 0.47]$.

3 Policy Implications

This section discusses the policy implications of introducing a CES production function under balanced-budget fiscal policy. To this end we add capital income taxation as an additional source of government revenue.[5] In this case, government budget constraint takes the form:

$$G = \tau_t^h w_t H_t + (\mu_t - \delta) \tau_t^k K_t \tag{17}$$

where τ_t^k is the tax rate and the term $-\tau_t^k \delta K_t$ represents the depreciation allowance. In this analysis we consider the case where government purchases are fixed and τ^h and τ^k vary in the same proportion to balance the budget, i.e. $\hat{\tau}_t^h = \hat{\tau}_t^k = \hat{\tau}_t$.

The determinacy analysis is represented in figure (2). To be consistent with the labour income tax case, we let the elasticity of substitution parameter σ vary between 0.2 and 1.8. The solid line represents the points where $\tau^h = \tau^k$ and it corresponds to the case of an income tax regime with depreciation allowance. As before, each subplot presents the values of labour and capital tax rates that induce determinacy (white area), indeterminacy (black area) and instability (red area). The central graph represents the CD case discussed by Schmitt-Grohè and Uribe (1997), i.e. $\sigma = 1$. As in Schmitt-Grohè and Uribe (1997) and Guo and Harrison (2004), the presence of an endogenous labour tax rate is the key destabilising ingredient of the model. If labour taxes are endogenous, as the capital tax rate increases (either as a steady state value of an endogenous τ^k or as being fixed), the range of indeterminacy with respect to labour taxes becomes larger. In particular, indeterminacy arises for increasingly smaller labour tax rates. However, as described in details above, decreasing (increasing) the elasticity of substitution between factors decreases (increases) the potential destabilising effects of endogenous labour tax rate, in turn reducing

[4] Christiano (1988) reportes estimates of of the labour income tax between 0.25 and 0.43, while Mendoza et al. reportes estimates between 0.27 and 0.47.

[5] Given the aim of the paper we ignore consumption taxes which are a significant source of revenues in the European countries. See Giannitsrou (2007) for a detailed discussion on this theme.

(augmenting) both the possibility of indeterminacy and instability for a given mix of capital-labour taxation.

Next, we aim to include some historical perspective on the (de)stabilising properties of a BBR. To this end we use the estimated tax rates taken from Mendoza (1994) for five countries (US, UK, Japan, Canada and Germany) and we perform two types of exercises. In the first one we use the estimated labour and capital tax rates for 1996 as in Schmitt-Grohé and Uribe (1997) and in the second one we extend the analysis to the the period $1965 - 1996$.

The estimated labour and capital tax rates for 1996 are summarized in table 1.

Table 1 - 1996 Estimated Tax Rates

	Capital Tax	Labour Tax
United States	0.3962	0.2773
United Kingdom	0.4717	0.2440
Japan	0.4261	0.2743
Canada	0.5066	0.3263
Germany	0.2391	0.4238

Table 2 shows how the determinacy analysis changes for the considered countries when a BBR is coupled with an empirical consistent CES production function. Our analysis points out that the US, UK, Japan and Canada are in the determinate area while Germany is in the determinate area when $\sigma < 0.57$, a value that is very close to the upper level of the estimated range of the elasticity of substitution between input factors.

Table 2 - Stability Properties

	$\sigma = 1$	$\sigma \in [0.4 - 0.6]$
United States	Indeterminate	Determinate
United Kingdom	Indeterminate	Determinate
Japan	Indeterminate	Determinate
Canada	Indeterminate	Determinate
Germany	Indeterminate	Determinate if $\sigma < 0.57$

Figures 3, 4, 5, 6 and 7 show the results of the second policy exercise. In each figure we have three graphs. Graph (a) shows how the level of labour and capital rate moves along the sample. Graph (b) is a scatter-plot of a country's labour and capital tax rates. This graph complements figure (2) as it provides a graphical view of the "position" of a single country in the determinacy figure. Ultimately, graph (c) shows the threshold upper value of σ required for a BBR in a particular country, in a particular year (given its tax scheme) to induce determinacy.

As can be seen from graphs (a), labour and capital taxes increase in almost all the countries. This trend is more evident in the US, Japan, Canada and Germany while in UK there is a slight trend in the labour taxes but capital tax rates do not exibit such a behaviour. Moreover in all countries, with the exception of Germany, capital tax rates are higher than labour tax rates.

The consequences for determinacy of the different mix of tax rates are shown in graph (c). This leads us to some considerations. Firstly, the threshold values of σ decrease over time. In Japan, Canada and Germany this trend is more marked than elsewhere. For the UK we record a slight decrease in the determinacy threshold from 1965 to 1996 but it is not as pronounced as in the previous countries and it does not exhibit a clear trend. These results are determined by the growth of tax rates over the sample, i.e. the higher the increase, the faster the decrease in the threshold value os σ.

Secondly when the whole sample is considered and with empirically plausible values of the elasticity of substitutions, i.e. $\sigma \in [0.4 - 0.6]$, a BBR may lead, *ceteris paribus,* to determinacy, in the US, the UK, Japan and Canada. As before, only Germany displays some indeterminacy problems but for values of σ that are very close to the the upper level of the estimated range. At the base of these results lies the destabilising role of the endogenous labour income tax. *Ceteris paribus*, for a given level of elasticity of substitution between factors, the likelihood of being in a determinate area increases when capital tax rates are higher than labour tax rates (figure 2).

4 Conclusions

The aim of this paper is to study how the introduction of a general class of production function, namely a CES, which is able to match the empirical evidence on the substitutability between labour and capital, changes the local determinacy analysis of a neoclassical economy where fiscal policy follows a balanced budget rule.

We obtain two sets of results. Firstly, we show analytically that the degree of substitutability between production factors is a key ingredient in understanding the (de)stabilising properties of a BBR. Secondly, when we calibrate the model in order to match the empirical evidence, i.e. elasticity of substitution below unity, the instability problems that affect the Schmitt-Grohè and Uribe model disappear for a broad range of OECD countries.

From this, the aim of the paper is not only to contribute to the large debate about the (de)stabilising properties of a BBR (see among others Guo and Harrison 2004, Giannaritsarou, 2007, Linnemann, 2008) but also to develop an interest in the use of CES production technology in the analysis of economic policy issues.

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Appendix A - The Normalisation Procedure

As stated before we are using a normalised CES production function. The meaning of the normalisation is that the family of CES production function need a common benchmark point. Since the elasticity of substitution is defined as a point of elasticity we need to fix the benchmark values for the level of production, factor inputs and for the marginal rate of substitution or equivalently for the per-capita production, capital deepening and factor income share.

Therefore we need to recalibrate the parameters B and α each time that the elasticity of substitution σ is varied to have the factor shares and the steady state allocations constant. In particular, when σ is varied, we want to maintain the value of B equal to its value of the Cobb-Douglas output steady-state case and the value of the parameter α is set to maintain at each point the steady state capital income share, therefore its value must be equal to 0.7.

For our analysis we use as reference point the normalised quantities for the CD case with $\sigma = 1$ and $B = 1$.

In order to achieve that result the parameters used takes the form:

$$\alpha = \frac{\bar{\alpha}}{\bar{\alpha} + (1-\bar{\alpha})\left(\frac{K^{CD}}{H^{CD}}\right)^{\psi}} \tag{18}$$

$$B = \frac{Y^{CD}}{\left[\alpha\left(K^{CD}\right)^{\psi} + (1-\alpha)\left(H^{CD}\right)^{\psi}\right]^{1/\psi}} \tag{19}$$

where K^{CD} and H^{CD} are the steady state values of capital and labour of a Cobb-Douglas production function.

Appendix B - The Log-Linearised System

In order to study the local determinacy we log-linearise the structural equations around the non-stochastic steady state. Here we present the log-linearise system. A lower case variable identifies $c_t = \log\left(\frac{C_t}{C}\right)$. $\hat{\tau}_t^h$ identifies the log-linearised labour tax rate. $\hat{w}_t$ is the log-linearisation real wage and $\hat{\mu}_t$ the log-linearised real interest rate.

$$c_t = \frac{-\tau^h}{1-\tau^h}\hat{\tau}_t^h + \hat{w}_t \tag{20}$$

$$0 = \hat{\tau}_t^h + \hat{w}_t + h_t \tag{21}$$

$$\dot{k}_t = \frac{Y}{K}y_t - \delta k_t + -\frac{C}{K}c_t \tag{22}$$

$$\dot{c}_t = \mu\hat{\mu}_t \tag{23}$$

$$\hat{\mu}_t = \frac{1}{\sigma}\left(y_t - k_t\right) \tag{24}$$

$$\hat{w}_t = \frac{1}{\sigma}\left(y_t - h_t\right) \tag{25}$$

$$y_t = \alpha\left(B\frac{K}{Y}\right)^{\psi} k_t + (1-\alpha)\left(B\frac{H}{Y}\right)^{\psi} h_t \tag{26}$$

where $\psi = \frac{\sigma-1}{\sigma}$. If we substitute (18) and (19) into the previous expression we obtain:

$$y_t = \overline{\alpha}k_t + (1-\overline{\alpha})h_t$$

Appendix C - The Determinacy Analysis

The model can be reduced to one involving only two dynamic variables, namely, consumption (non-predetermined) and capital (predetermined), as:

$$\begin{bmatrix} \dot{c}_t \\ \dot{k}_t \end{bmatrix} = J \begin{bmatrix} c_t \\ k_t \end{bmatrix} \tag{27}$$

where:

$$J = \begin{bmatrix} \mu\frac{\bar{\alpha}+\tau^h-\bar{\alpha}\tau^h-1}{\bar{\alpha}-\sigma\tau^h} & -\mu\frac{1}{\sigma}\left(1+\bar{\alpha}\frac{-\sigma\tau^h+1}{\sigma\tau^h-\bar{\alpha}}\right) \\ \frac{Y}{K}\sigma\left(1-\bar{\alpha}\right)\frac{\tau^h-1}{\bar{\alpha}-\sigma\tau^h} - \frac{C}{K} & \frac{Y}{K}\bar{\alpha}\frac{-\sigma\tau^h+1}{\bar{\alpha}-\sigma\tau^h} - \delta \end{bmatrix}$$

The system is determinate if and only if:

$$Det\left(J\right) < 0$$

While the system displays indeterminacy if and only if:

$$Det\left(J\right) > 0 \ \cap Trace\left(J\right) < 0 \tag{28}$$

i.e. both eigenvalues of J are negative, and it is unstable if and only if:

$$Det\left(J\right) > 0 \cap Trace\left(J\right) > 0$$

i.e. both eigenvalues of J are positive.

Proof of Propositions 1 and 2

Determinant and trace of J are respectively:

$$Det\left(J\right) = \frac{\delta\left(\rho+\delta\right)\left(1-\bar{\alpha}\right)}{\frac{\delta\alpha}{\delta+\rho}\left(\sigma\tau^h-\bar{\alpha}\right)}\left(\left(1-\bar{\alpha}\right)\left(\tau^h\right)^2 - 2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^h + \left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\right)$$

$$Trace\left(J\right)=\frac{\left(\rho+\delta\right)\left(1-\sigma\tau^{h}\right)-\left(\rho+\delta\right)\left(\left(1-\tau^{h}\right)\left(1-\bar{\alpha}\right)\right)}{\bar{\alpha}-\sigma\tau^{h}}-\delta$$

Sign of the Determinant

The sign of the determinant is the product of the sign of $\frac{\delta(\rho+\delta)(1-\bar{\alpha})}{\frac{\delta\alpha}{\delta+\rho}(\sigma\tau-\bar{\alpha})}$ and $(1-\bar{\alpha})\left(\tau^{h}\right)^{2}-2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^{h}+\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)$.

Part 1: Sign of $\frac{\delta(\rho+\delta)(1-\bar{\alpha})}{\frac{\delta\alpha}{\delta+\rho}(\sigma\tau-\bar{\alpha})}$:

$$\frac{\delta\left(\rho+\delta\right)\left(1-\bar{\alpha}\right)}{\frac{\delta\alpha}{\delta+\rho}\left(\sigma\tau-\bar{\alpha}\right)}>0 \text{ if and only if } \tau^{h}>\frac{\bar{\alpha}}{\sigma}$$

Part2: Sign of $(1-\bar{\alpha})\left(\tau^{h}\right)^{2}-2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^{h}+\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)$:

See Steady State section in the main text, i.e:

$$(1-\bar{\alpha})\left(\tau^{h}\right)^{2}-2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^{h}+\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)>0 \text{ if and only if } \tau^{h}\in[0,\tau^{h*})$$

Furthermore note that if $\tau^{h}=0.5$, $(1-\bar{\alpha})\left(\tau^{h}\right)^{2}-2\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)\tau^{h}+\left(1-\bar{\alpha}\frac{\delta}{\delta+\rho}\right)>0$. Putting Part 1 and part 2 together one obtains that the determinant is positive:

$$\begin{aligned} if\ \tau^{h*} &> \frac{\bar{\alpha}}{\sigma} \Longrightarrow \tau^{h}\in(\frac{\bar{\alpha}}{\sigma},\tau^{h*}) \\ else\ \tau^{h*} &< \frac{\bar{\alpha}}{\sigma} \Longrightarrow \tau\in(\tau^{h*},\min(\frac{\bar{\alpha}}{\sigma},1)) \end{aligned}$$

Sign of the Trace

The trace of J is:

$$Trace\left(J\right)=\frac{\left(\rho+\delta\right)\left(1-\sigma\tau^{h}\right)-\left(\rho+\delta\right)\left(\left(1-\tau^{h}\right)\left(1-\bar{\alpha}\right)\right)}{\bar{\alpha}-\sigma\tau^{h}}-\delta$$

It is easy to show that the conditions for $Trace\left(J\right)<0$ are:

$$\begin{aligned} if\ \bar{\alpha} &< \sigma\leq 1+\frac{\delta\left(1-\bar{\alpha}\right)}{\rho} \rightarrow \quad \tau^{h}>\frac{\bar{\alpha}}{\sigma} \\ elseif\ \sigma &> 1+\frac{\delta\left(1-\bar{\alpha}\right)}{\rho}\rightarrow\frac{\bar{\alpha}}{\sigma}<\tau^{h}<\frac{\bar{\alpha}\rho}{\rho\sigma-\left(1-\bar{\alpha}\right)\left(\delta+\rho\right)}=\varepsilon_{1} \\ elseif\ \sigma &< \bar{\alpha}\rightarrow\ Trace\left(J\right)>0\ \forall\ \tau^{h}\in[0,1] \end{aligned}$$

Result 1: Determinacy

The system as defined in (27) is determinate if and only if $Det\,(J) < 0$, hence:

$$\begin{aligned} if\ \sigma &\in [\bar{\alpha}, +\infty) \Longrightarrow \tau^h \in [0, \min\left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right)) \cup \tau^h \in (\max\left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right), 1] \\ else\ \sigma &\in [0, \bar{\alpha}) \Longrightarrow \tau^h \in [0, \tau^{h*}) \end{aligned}$$

Result 2: Indeterminacy-Instability

When $Det\,(J) > 0$, directly from the sign of the trace of J we can easily find the following equilibrium conditions:

$$\begin{aligned} a)\ If\ \sigma &\in \left[0, 1 + \frac{\delta\,(1-\bar{\alpha})}{\rho}\right] \cap \frac{\bar{\alpha}}{\sigma} < \tau^{h*},\ \text{the system is never unstable. It is indeterminate if } \tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right) \\ b)\ If\ \sigma &\in \left[0, 1 + \frac{\delta\,(1-\bar{\alpha})}{\rho}\right] \cap \frac{\bar{\alpha}}{\sigma} > \tau^{h*},\ \text{the system is never indeterminate.} \\ &\quad \text{It is unstable if } \tau^h \in \left(\tau^{h*}, \min\left(\frac{\bar{\alpha}}{\sigma}, 1\right)\right). \\ c)\ If\ \sigma &\in \left(1 + \frac{\delta\,(1-\bar{\alpha})}{\rho}, +\infty\right) \cap \varepsilon_1 > \tau^{h*},\ \text{the system is never unstable.} \\ &\quad \text{It is indeterminate if } \tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \tau^{h*}\right) \\ d)\ If\ \sigma &\in \left(1 + \frac{\delta\,(1-\bar{\alpha})}{\rho}, +\infty\right) \cap \varepsilon_1 < \tau^{h*},\ \text{the system is indeterminate if } \tau^h \in \left(\frac{\bar{\alpha}}{\sigma}, \varepsilon_1\right) \\ &\quad \text{and unstable if } \tau^h \in \left(\varepsilon_1, \tau^{h*}\right) \end{aligned}$$

Figures

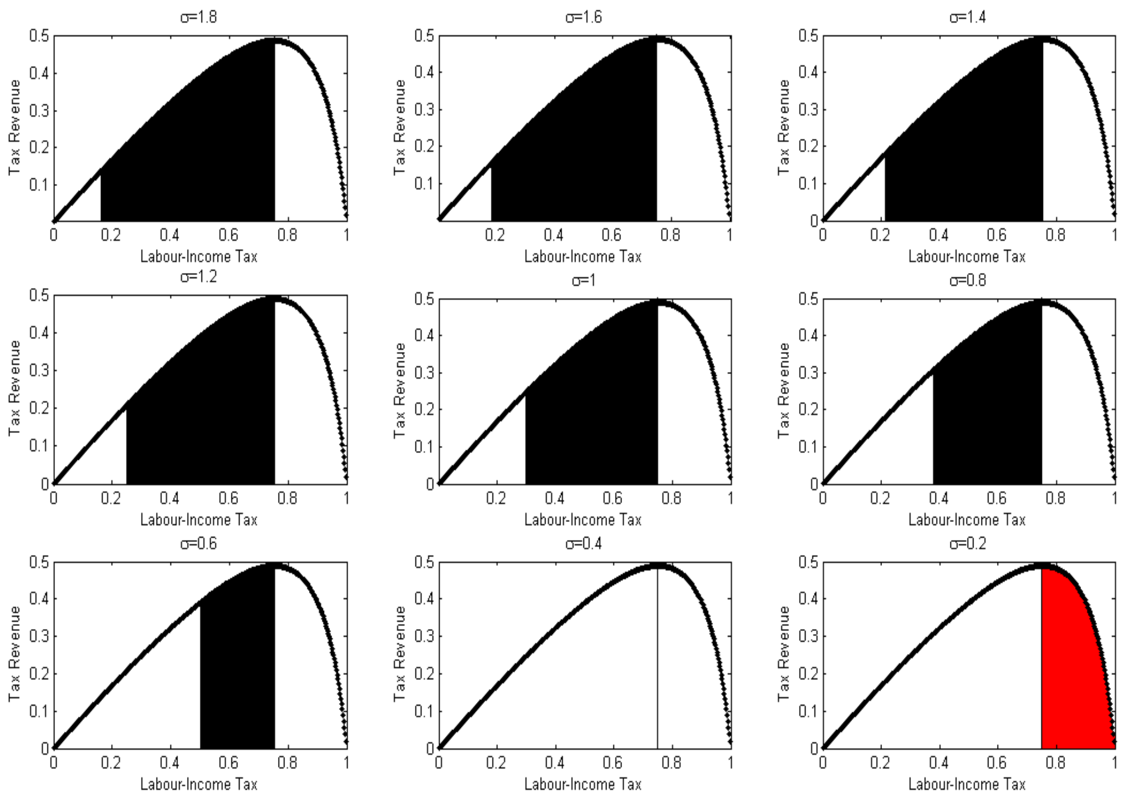

Figure 1 - Determinacy analysis with labour tax only. White area, determinacy. Black area, indeterminacy. Red area, instability.

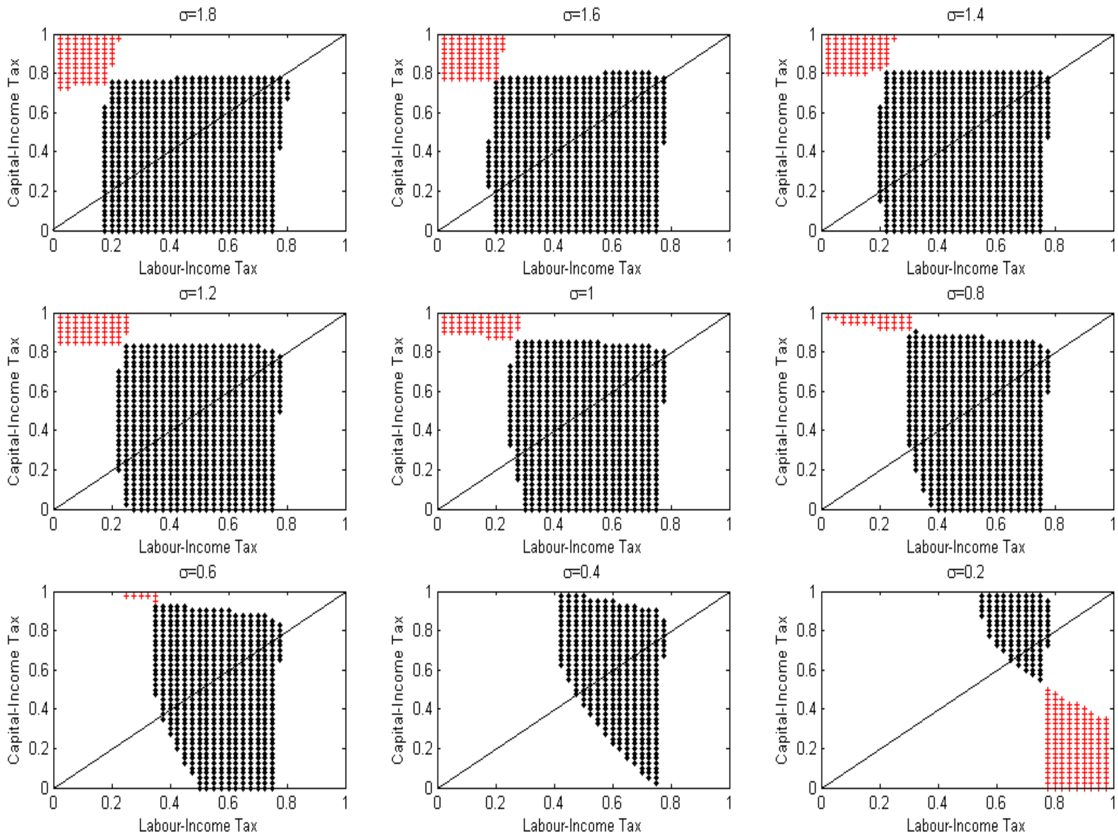

Figure 2 - Determinacy analysis with labour and capital taxes. White area, determinacy. Black dots, indeterminacy. Red dots, instability. The pairs (τ^h, τ^k) for which $\tau^h = \tau^k$ (the solid line) correspond to the case of an income tax regime with depreciation allowance.

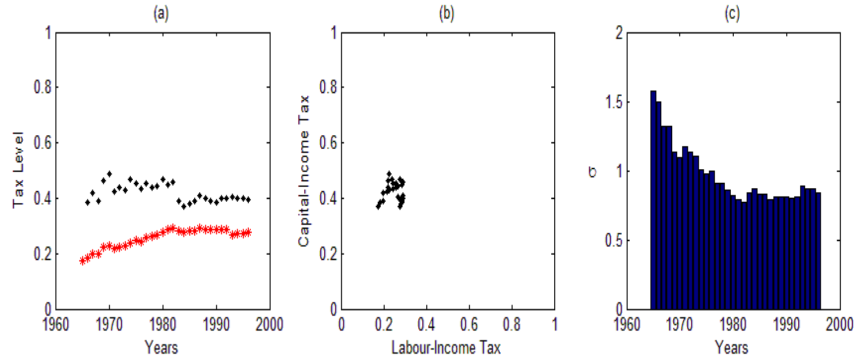

Figure 3 - United States - Graph (a): capital (black dots) and labour (red stars) tax rates trend. - Graph (b) capital and labour tax rates scatterplot - Graph (c) determinancy level for a given year.

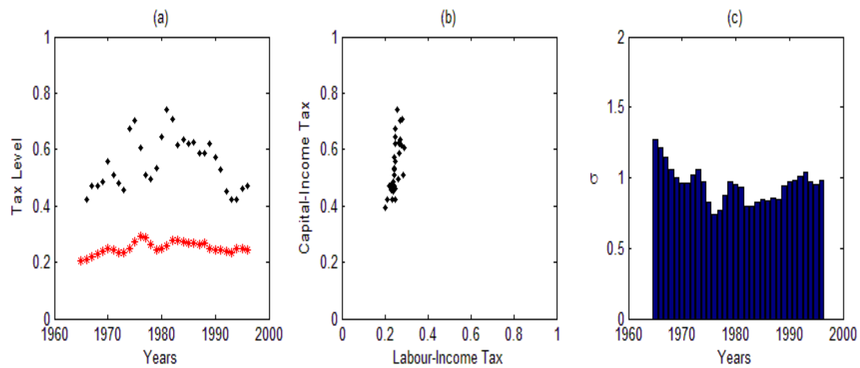

Figure 4 - United Kingdom - - Graph (a): capital (black dots) and labour (red stars) tax rates trend. - Graph (b) capital and labour tax rates scatterplot - Graph (c) determinancy level for a given year.

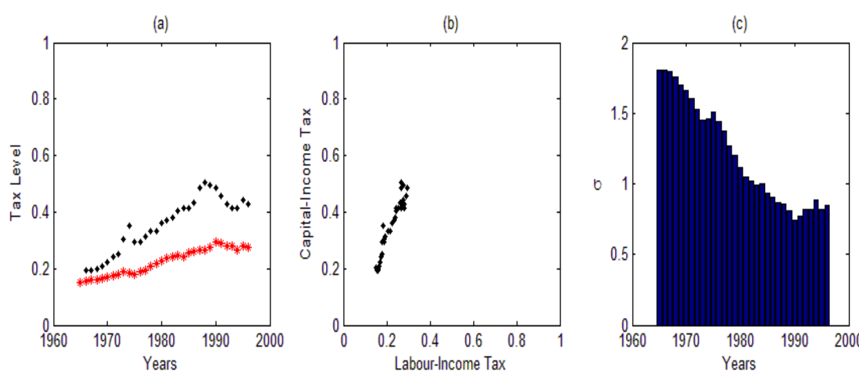

Figure 5 - Japan - Graph (a): capital (black dots) and labour (red stars) tax rates trend. - Graph (b) capital and labour tax rates scatterplot - Graph (c) determinancy level for a given year.

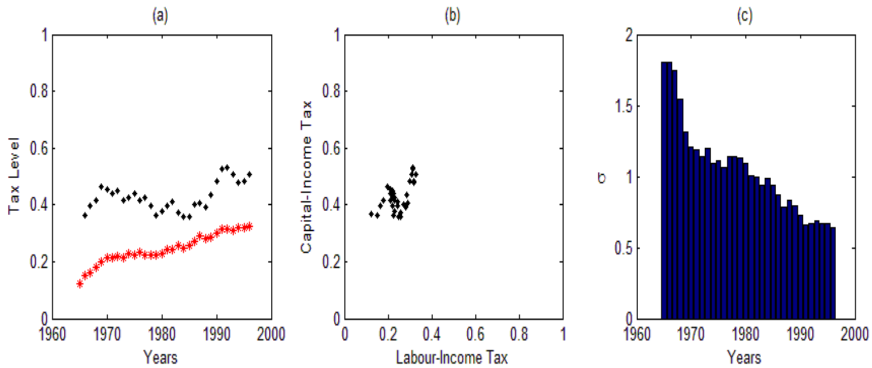

Figure 6 - Canada - Graph (a): capital (black dots) and labour (red stars) tax rates trend. - Graph (b) capital and labour tax rates scatterplot - Graph (c) determinancy level for a given year.

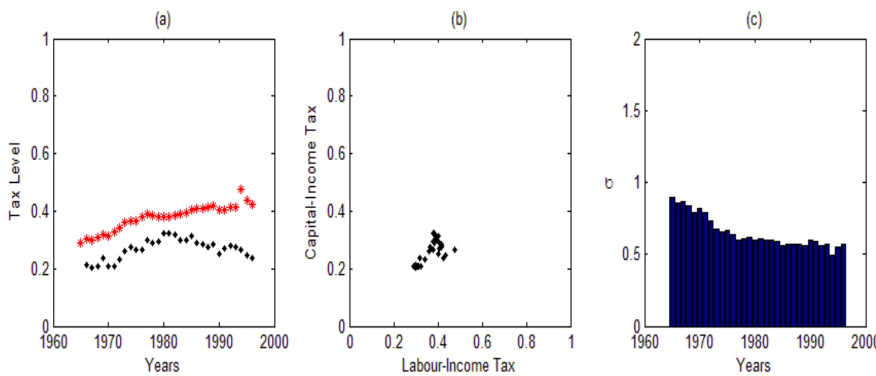

Figure 7 - Germany - Graph (a): capital (black dots) and labour (red stars) tax rates trend. - Graph (b) capital and labour tax rates scatterplot - Graph (c) determinancy level for a given year.