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ENDOGENOUS GROWTH AND CONSUMPTION AGGREGATION

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Endogenous Growth and Consumption Aggregation

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Abstract

In this paper general CES consumption preferences are introduced into an endogenous growth model à la Bernard, Eaton, Jensen, and Kortum (2003) and Eaton and Kortum (2001). This is in contrast to the more generally used assumption of logarithmic preferences. The paper shows that the CES preference structure does not alter the expected profits from engaging in R&D and therefore the growth path. This is proof that the analytically more convenient logarithmic preferences do not sacrifice generality. It is argued that the driving force behind this result is the common assumption of undirected research.

Keywords: CES Preferences, Endogenous Growth, Research

JEL classification: O30, O40

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1 Introduction

In the literature on growth through creative destruction, the incentives to engage in R&D are key in determining the long-run growth rate of an economy. Assumptions about product competition in turn affect these incentives. For example, both in Aghion and Howitt (1992) and Grossman and Helpman (1991), allowing for competition from lower quality products reduces the incentive to engage in R&D and hence the growth rate. In the present paper, competition between different varieties is introduced, by employing the general CES preference form introduced by Dixit and Stiglitz (1977). The state-of-the-art good in sector j faces competition both from lower quality products in the same sector as well as products from other sectors $j' \neq j$. The model considered here builds on the static model introduced by Bernard, Eaton, Jensen, and Kortum (2003) (BEJK), who use the general CES form in a static trade context. The dynamic aspect of creative destruction in the present paper builds on the dynamic trade model of Eaton and Kortum (2001). In contrast to these models, I consider a closed economy and focus on how the CES preferences affect the growth rate of the economy. I show that although competition is increased, the results of the model with general CES preferences are identical to those with logarithmic preferences. Although the added competition effectively lowers the expected price that can be charged for any good j, it is shown that the expected *profits* from innovating are identical under both types of preferences. The basic intuition behind this result stems from the fact that under undirected research, an innovator will expect to charge a price for his variety that is equal to the general price index in the economy. If the price for any good j is identical to the general price index, the exact value of the elasticity of substitution is irrelevant for expenditure shares. Therefore, *ceteris paribus*, expected profits from R&D are not affected by the elasticity of substitution between goods if research is undirected. Consequently, neither are research effort, nor ultimately, growth.

2 Consumers

Consumption is over a continuum of goods (x), indexed by $j \in [0,1]$. Time is continuous and at each t, a mass L_t of agents exists. Following BEJK, individuals maximize the following instantaneous utility function,

$$U_{it} = \left[\int_0^1 x_{it}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where σ is the elasticity of substitution between different types of goods, subject to their budget constraint at t,

$$\int_0^1 p_t(j) x_{it}(j) \leq Y_{it}, \tag{2}$$

where $p_t(j)$ is the price of good j at t and Y_{it} is income of agent i. Subscript i is omitted whenever a variable is the same for all individuals. The elasticity of intertemporal substitution is unity and the interest rate is equal to the discount rate, which is assumed constant.

The first order conditions of this problem are

$$\left[\frac{x_{it}(j)}{x_{it}(j')} \right]^{1/\sigma} = \frac{p_t(j')}{p_t(j)} \qquad \forall j \neq j'. \tag{3}$$

For the expected value of R&D, it will be useful to calculate the expenditure share of good j at t, which is

$$p_t(j) \cdot x_{it}(j) = Y_{it} \left[\frac{p_t(j)}{p_t} \right]^{1-\sigma}. \tag{4}$$

where $p_t = [\int_0^1 p_t(j)^{1-\sigma}\, dj]^{1/(1-\sigma)}$ is the ideal price index for this economy.

3 Supply Side

The supply side of the economy is modeled as a one-country version of BEJK. For proofs, the reader is referred to the original article. Labor is the only input in production and differently to BEJK, the wage is the numeraire. Technology determines how efficiently labor can be transformed into goods. Only the highest efficiency level (the state-of-the-art) will be in use. However, it is useful to keep track also of the second-highest efficiency for producing good j. Denoting $z_{1t}(j)$ as the highest efficiency level at t and $z_{2t}(j)$ as the second highest, the lowest cost of producing good j is

$$C_{1t}(j) = \frac{1}{z_1(j)}. \tag{5}$$

Growth will be driven through increasing efficiency of production. Firms compete in prices. Competition from other varieties j' implies a profit-maximising markup for good j of $\bar{m} = \sigma/(\sigma - 1)$ for $\sigma > 1$[1]. The additional competition from the second-lowest cost producer of j constrains the state-of-the-art producer to charge the lowest price between the marginal costs of the second-lowest producer, and $\bar{m}C_{1t}(j)$:

$$p_t(j) = min\{C_{2t}(j), \bar{m}C_{1t}(j)\}. \tag{6}$$

At each t, a measure R_t of individuals are engaged in research (as opposed to working in the production sector). To each researcher, ideas arrive as a Poisson

[1]For $\sigma \leq 1$, I assume, following BEJK, that $\bar{m} = \infty$.

process with parameter α[2]. Each idea is the realization of two random variables: the good j to which it pertains, and the efficiency $q(j)$ with which j can be produced. While the type of good is drawn from the uniform distribution over $[0, 1]$, it is assumed that the efficiency $q(j)$ is drawn from the Pareto distribution, $H(q) = 1 - q^{-\theta}$, which is the same for all sectors[3].

The technological frontier consists of the states-of-the-art across all sectors. As efficiencies are draws from the Pareto distribution, the state-of-the-art efficiencies, $z(j)$, that enter the technological frontier can also be treated as realizations of a random variable, Z, with a given distribution F. BEJK show that the joint distribution of Z_1 and Z_2 is of a generalized Fréchet form:

$$F(z_1, z_2) = Pr[Z_1 \leq z_1, Z_2 \leq z_2] = \left[1 + T_t(z_2^{-\theta} - z_1^{-\theta})\right] e^{-T_t z_2^{-\theta}} \tag{7}$$

for $0 \leq z_2 \leq z_1$, drawn independently across goods j.

It is obvious from (5) that the efficiency level is the only variable aspect determining the marginal costs of production. Consequently, (7) will govern the distribution of the minimum costs for producing j. Considering both C_1 and C_2, BEJK show that their joint distribution is given by

$$G(c_1, c_2) = Pr[C_1 \leq c_1, C_2 \leq c_2] = 1 - e^{-T_t c_1^{\theta}} - T_t c_1^{\theta} e^{-T_t c_2^{\theta}} \tag{8}$$

for $c_1 \leq c_2$. The markup charged for good j, $M(j) = p(j)/C_1(j)$, is the realization of a random variable M which is drawn from a Pareto distribution truncated at the monopoly markup

[2] α can be seen as the efficiency of research.

[3] $\theta > 1$ governs the variation in efficiencies of production. The higher θ, the lower the variability in efficiencies across sectors.

$$H(m) = Pr[M \leq m] = \begin{cases} 1 - m^{-\theta} & 1 \leq m < \bar{m} \\ 1 & m \geq \bar{m} \end{cases}$$

and the price index is given by

$$p_t = \gamma T_t^{-1/\theta}, \tag{9}$$

where

$$\gamma = \left[\frac{1 + \theta - \sigma + (\sigma - 1)\bar{m}^{-\theta}}{1 + \theta - \sigma} \Gamma \left(\frac{1 + 2\theta - \sigma}{\theta} \right) \right]^{1/(1-\sigma)},$$

and $\Gamma(\cdot)$ denotes the gamma function.

4 Innovation

Let m' denote the inventive step of a new idea. It is distributed as $H(m') = 1 - m'^{-\theta}$. Also, let $G_1(c_1) = Pr[C_1 \leq c_1] = 1 - e^{-T_t c_1^\theta}$ denote the marginal distribution of the lowest cost. The probability that an idea q will be the most efficient is given by $1 - G(1/q)$, and the probability that q undercuts the lowest cost by a factor $m' \geq 1$ is $1 - G(m'/q)$. Integrating over the Pareto distribution of q gives the probability $b(m')$ that an idea will have an inventive step of at least m':

$$b(m') = \int_1^\infty [1 - G(m'/q)] dH(q) \approx \frac{1}{T_t (m')^\theta} \tag{10}$$

and setting $m' = 1$ gives the probability $b_t(1) = 1/T_t$ of idea q surpassing the previous state-of-the-art at all. This implies that the probability of any state of the art idea z still being the state-of-the-art by time $s > t$ is $b_s(1)/b_t(1) = T_t/T_s$ (see BEJK).

Researchers who have a state-of-the-art idea can patent it and sell the patent to a firm producing consumption goods for the present expected value of the idea. Let Y_t denote total expenditure at date t, $Y_t = \int_0^{L_t} y_{it} di$, and let $p_t(j)c_t(j) = \int_0^{L_t} p_t(j)c_{it}(j)di$ denote total expenditure on good j at t. Then the expected profit flows from engaging in R&D are

$$\begin{aligned}
\Pi_t &= \int_0^1 p_t(j)c_t(j) - \frac{1}{z(j)}c_t(j)dj \\
&= \int_0^1 p_t(j)c_t(j) - p_t(j)c_t(j)m^{-1}dj \\
&= \int_0^1 Y_t \left[\frac{p_t(j)}{p_t}\right]^{1-\sigma} [1 - m(j)^{-1}]dj \\
&= \frac{Y_t}{p_t^{1-\sigma}} \int_1^\infty E[p_t^{1-\sigma}|M' = m'](1 - m'^{-1})dH(m')
\end{aligned} \tag{11}$$

where the last line changes the variable of integration from j to the distribution of the inventive step, $H(m')$ and $E[p_t^{1-\sigma}|M' = m']$ is the expected price that can be charged given inventive step m'. This profit flow differs from the profit flows used in, e.g., Eaton and Kortum (2001) and Kortum (1997) in that the price of variety j relative to the price index and $\bar{m}$ matter. Solving (11) under the assumption that $\sigma < 1 + \theta$[4] leads to

[4]Eaton and Kortum (2002) have estimated different values for θ for 19 OECD countries in the 1990s. The lowest estimate for θ they found was 3.60 and the highest 12.86, making this restriction empirically plausible.

$$
\begin{aligned}
\Pi_t &= \frac{Y_t}{p_t^{1-\sigma}} \left[\int_1^{\bar{m}} E[p_t^{1-\sigma} | M' = m'](1 - m'^{-1}) \theta m'^{-(\theta+1)} dm' \right. \\
&\quad \left. + \int_{\bar{m}}^{\infty} E[p_t^{1-\sigma} | M' = m'](1 - \bar{m}^{-1}) \theta m'^{-(\theta+1)} dm' \right] \\
&= \frac{Y_t}{p_t^{1-\sigma}} \left[\int_1^{\bar{m}} E[C_2^{1-\sigma}] \theta m'^{-(1+\theta)} (1 - m'^{-1}) dm' \right. \\
&\quad \left. + \int_{\bar{m}}^{\infty} E[(\bar{m} C_2 / m')^{1-\sigma}] \theta m'^{-(1+\theta)} (1 - \bar{m}^{-1}) dm' \right] \qquad (12) \\
&= \frac{Y_t}{p_t^{1-\sigma}} \cdot E[C_2^{1-\sigma}] \theta \left[\int_1^{\bar{m}} m'^{-(1+\theta)} dm' - \int_1^{\bar{m}} m'^{-(2+\theta)} dm' \right. \\
&\quad \left. + \int_{\bar{m}}^{\infty} \bar{m}^{1-\sigma} m'^{-(\theta+2-\sigma)} dm' - \int_{\bar{m}}^{\infty} m'^{-(\theta+2-\sigma)} \bar{m}^{-\sigma} dm' \right] \\
&= \frac{Y_t}{p_t^{1-\sigma}} \cdot E[C_2^{1-\sigma}] \theta \left[\frac{1}{\theta(1+\theta)} + \frac{(\sigma-1)\bar{m}^{-\theta}}{\theta(1+\theta-\sigma)} - \frac{\sigma \bar{m}^{-(1+\theta)}}{(1+\theta)(1+\theta-\sigma)} \right]
\end{aligned}
$$

Using the definition of the price index from (9) and the fact that $E[C_2^{1-\sigma}] = \Phi^{-\frac{1-\sigma}{\theta}} \Gamma(\frac{1-\sigma+2\theta}{\theta})$ (from BEJK), equation (12) simplifies to

$$
\begin{aligned}
\Pi_t &= Y_t \frac{1}{\theta + (\sigma-1)(\bar{m}^{-\theta} - 1)} \left[\frac{\theta - (\sigma-1)}{1+\theta} + (\sigma-1)\bar{m}^{-\theta} - \frac{\theta}{1+\theta} \sigma \bar{m}^{-(1+\theta)} \right] \\
&= Y_t \frac{1}{\theta + (\sigma-1)(\bar{m}^{-\theta} - 1)} \left[\frac{\theta + (\sigma-1)(\bar{m}^{-\theta} - 1)}{1+\theta} + \frac{\theta \sigma \bar{m}^{-\theta}(1 - \bar{m}^{-1} - 1/\sigma)}{1+\theta} \right] \\
&= Y_t \frac{1}{1+\theta}
\end{aligned}
\qquad (13)
$$

where the penultimate to the last line uses the fact that $\bar{m} = \frac{\sigma}{\sigma-1}$. This is exactly the same expected profit flow as under logarithmic preferences, despite the increased competition. This result is due to undirected research: any innovator expects an inventive step m' such that he will charge a price equal to the price index of the economy. In this case the exact value of the elasticity of substitution between varieties is irrelevant.

As expected profit flows under the general Dixit-Stiglitz preferences are identical to the logarithmic case, the equilibrium R&D intensity will turn out to be identical to a one-country version of Eaton and Kortum (2001). Indeed, following their derivations, it can be shown that total output at time t in the above model is

$$Y_t = \frac{(1+\theta)(1-\beta)}{\theta} L_t,$$

where β is the fraction of the population who choose to engage in R&D at t, which is constant over time. Thus, total nominal output is growing at rate n, due to population growth. It can be shown that the general price index grows at a rate of $-n/\theta$, i.e., real output per capita grows at rate n/θ.

5 Conclusions

The present paper combines a general CES preference structure, as found in Bernard, Eaton, Jensen, and Kortum (2003), with a model of growth through innovation, as presented in Eaton and Kortum (2001). These Dixit-Stiglitz preferences introduce competition across varieties into the growth model, something that is not usually included in the endogenous growth literature. The present paper shows that expected profits from innovating are unaffected by this increased competition if research is undirected. Consequently, research effort and growth are also unaffected. It is concluded that the use of logarithmic preferences does not sacrifice generality as far as the induced amount of R&D under undirected research is concerned.

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