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**THE RBC VIEW OF PAKISTAN: A DECLARATION OF
STYLIZED FACTS AND ESSENTIAL MODELS**

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The RBC View of Pakistan: A Declaration of Stylized Facts and Essential Models¹

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Abstract

In this paper, we establish the nature of short-run fluctuations of the Pakistani economy over the period of 1981-2010. There have been significant changes in the nature of the Pakistani economy over the last few decades. Therefore, we focus our detailed analysis on the last few decades where it seems more appropriate to investigate the nature and causes of business cycles in Pakistan. Furthermore, we evaluate the performance of a typical RBC and an augmented RBC model with an exogenous FDI shock in explaining cyclical fluctuations experienced by the Pakistani economy. We find that a simple RBC model performs poorly in terms of matching relevant second-order moments of short run fluctuations as depicted by the data. However, augmented RBC model performs better compared to the simple RBC model.

Keywords: Business Cycles, Emerging Economies, FDI,

JEL Classification: .

Non-Technical Summary

Varieties of dynamic stochastic general equilibrium (DSGE) models have become popular tools to understand conjuncture and also study optimal fiscal and monetary policies in most developed countries. The most basic and the first DSGE model is the real business cycle (RBC) model and it emphasizes the role of technological change as the main driver for short term fluctuations. The popularity of DSGE models in developed countries however has not translated into them being utilized more widely in developing economy realm.

Though in recent decade some progress has been made on business cycles for developing economies and is mainly focused on South American countries Garcia-Cicco et al (2010), Aguiar & Gopinath (2007). However, there are not many business cycle models for South Asian countries. To the best of author's knowledge, only work for India has been undertaken for both rigorously establishing the business cycle facts of the economy as well as empirically evaluating a RBC model.

This project is the first of its kind to explore short-run fluctuations of the Pakistani economy as well as checking the ability of the RBC model setup to match regularities of key macroeconomic variables -namely output, consumption and investment. Pakistan is an interesting country to put through the RBC lens for it has been subject to continuous regime changes and shares borders with a mélange of volatile, autocratic and democratic neighbors. Our investigation says that:

- The Pakistan economy has been stuck at a per capita real growth of about 2.5% for the last three decades- we call this the 'Pakistani growth rate.'
- Pakistan has always been an agrarian economy, in terms of employment, from its independence till now even after more than 60 years. However, the sectoral emphasis in terms of 'output' has shifted away from agriculture to services while industry has stagnated.
- The long run ratios of the economy such as investment to output, private consumption to output, government consumption to output, exports to output and imports to output all display volatility over time instead of having a stable trend.

- Looking further into disaggregated components of investment, we find that share of private investment in aggregate investment has taken over the share of public investment over the last couple of decades.
- This decline in share of public investment, in particular over the last decade, has been accompanied by an increase in foreign direct investment to the economy.
- The government consumption to output ratio is also very volatile and seems to follow a cyclical pattern. It has been particularly more volatile over the last decade.
- For the period from 1981-2010, de-trended output is significantly positively correlated with de-trended imports, de-trended aggregate investment, de-trended government consumption, de-trended private consumption, de-trended exports and de-trended FDI.
- The contemporaneous correlation of de-trended output is strongest with de-trended imports, de-trended aggregate investment and de-trended FDI. This implies that imports, investment and FDI play an important role to explain conjuncture.
- The co-movement between de-trended output and other de-trended macro variables differ significantly by decade.
- De-trended exports is the only clear leading indicator of de-trended output.
- All de-trended macroeconomic variables are more volatile than de-trended output for the period 1981-2010.
- The de-trended government consumption and de-trended imports are around five times as volatile as de-trended output. De-trended exports are about three and half times as volatile as output. Lastly, de-trended FDI is the most volatile variable.
- The de-trended aggregate investment, private consumption, government consumption, exports, imports and FDI are all significantly positively correlated with de-trended output. This pro-cyclicality is strongest for de-trended imports, aggregate investment and FDI.

- Technology shock together with an investment shock in the form of FDI explain better the Pakistani economic conjecture than a simple RBC model.

1 Introduction

What are the stylized facts of the Pakistani economy? Has the nature of the economy shifted with significant changes in the social and political landscape of the country? What is the current nature of the Pakistani economy (especially the last few decades)? What drives short run fluctuations in the economy? Is the Pakistani economy driven by technology shocks as advocated by the well known Real Business Cycles (RBC) literature or is it driven by external factors?

Answers to these basic questions are long overdue and in this paper we attempt to answer some of these questions in detail. In this paper, we attempt to establish the nature and structure of Pakistani economy over the last 5 decades, focusing mainly on the period of 1981-2010. After the initial look into the salient features of the economy, we discuss some ‘stylized facts’ for the Pakistani economy. In addition, we also uncover that these ‘stylized facts’ of the economy have been changing over the last few decades.

Pakistan is an interesting country to study due to its rich history. The original state of Pakistan was established on 14 August 1947 in the eastern and north-western regions of modern day India. Following civil unrest in its eastern province and a war with India, the eastern region, now known as Bangladesh, obtained independence in 1971-leaving the north-western region to become modern day’s Pakistan. This followed five years of political turmoil with a military coup in 1977 ending in 1988, and yet another ‘soft’ military regime starting in 1999 that ended in 2007. One would imagine that the flip-flopping of regimes which started in late 1970’s would make Pakistan a prime candidate for Aguiar and Gopinath’s (2007) shocks-to-trend approach.

Indeed, here is the first remarkable stylized fact about Pakistan. The Pakistani economy is stuck at a per capita real growth of about 2.5%-we call this the ‘Pakistani’ growth rate and it holds between 1981-2010 and the decades within.¹

The above fact led us to focus on the period of 1981-2010 for an in-depth analysis of short-run fluctuations. By short run fluctuations, we mean that the empirical analysis in this paper uses the conventional frequencies for isolating data from its

¹In contrast, both India and Bangladesh have achieved higher real per capita growth rates especially in the last two decades.

trend to study business cycles. In addition, we also break down the data decade by decade to get a clear picture of changing nature of the cyclical fluctuations in the economy.

Furthermore, we evaluate the fit of the Pakistani macroeconomic data with a simple RBC model during the 1981-2010 period. We find that a simple RBC model does a good job of matching some of the relevant moments from the data. However, it fails to account for the increased relative volatility of consumption and investment as shown by the data.

In order to further improve the fit, we introduce an augmented RBC model, with an exogenous foreign direct investment (FDI) shock as the main innovation. We propose this modification as FDI is found to be most volatile, strongly correlated with detrended output and has been rising when the share of public investment has been falling. We find that the augmented RBC model performs better than the simple RBC model for some of the moments but more importantly delivers the increased relative volatility of consumption and investment. However, the augmented model does poorly on the absolute magnitude of volatilities across the board.

These findings add to the expanding literature on the business cycles in developing countries. Existing real business cycles models for developing countries (Garcia-Cicco et al (2010), Aguiar & Gopinath (2007)) have been mainly focused on South American countries. Aguiar & Gopinath (2007) in their paper suggested that business cycles for developing economies can be explained by a simple RBC model with the addition of a trend stationary productivity shock on top of the traditional exogenous technology shock. However, recent work by Garcia-Cicco et al (2010) challenges the main result of the earlier work of Aguiar & Gopinath (2007) and reports that for both Chile and Argentina a simple RBC model with a trend stationary technology shock fails to explain their respective business cycles.

It is plain that not all emerging economies are the same and therefore it should be no surprise that the nature and behaviour of their economies differ from each other. These economic differences between developing countries should be addressed rigorously to completely understand the business cycles of any particular developing country. This line of reasoning has recently become quite popular, as the last few years has seen a rapid increase in the literature related to country-specific business cycles models for different developing countries. This paper contributes to this specific

strand of literature.

Unfortunately, even with the recent surge in the literature on business cycles in developing economies, there are not many business cycle models for South Asian countries. To the best of author's knowledge, only work for India has been undertaken for both rigorously establishing the business cycle facts of the economy as well as evaluating the fit of a typical RBC model to explain the 'stylized facts' of a South Asian economy see for example Ghate, Pandey, Patnaik (2012). This project is the first of its kind to explore short-run fluctuations of the Pakistani economy as well as checking the ability of the RBC model setup in matching the empirical moments for the relevant macroeconomic variables (Output, Consumption & Investment).

The main reason for the dearth of economic research relating to business cycles of developing countries is the lack of availability of relevant time series data at appropriate frequency. Therefore, anyone interested in studying the cyclical fluctuations of a country like Pakistan has to deal with data availability and consistency issues even for the most basic macroeconomic variables.

There is no quarterly data available for the relevant macroeconomic series of Pakistan. The main variables of interest such as output, private consumption, government expenditures, investment, exports and imports² are only available at an annual frequency. However, even with annual data there are still issues of consistency from different sources. In this scenario, it is hard to even establish the 'stylized facts' of the economy. In this paper, we mainly use 'annual' data from Pakistani sources such as the Pakistan Bureau of Statistics, the Government of Pakistan and the State Bank of Pakistan. However, for some of the analysis related to business cycles we also use data from IMF's International Financial Statistics database.

In addition, on the calibration front there are no agreed upon values for even the most basic parameters such as the discount rate β or depreciation rate δ . There is also a severe lack of understanding and knowledge regarding the micro-foundations of the economy. This is partially being addressed by the national surveys being conducted by the State Bank of Pakistan in the labour, product and credit markets.

The rest of the paper is organized as follows. Section 2 presents some structural facts of the economy as well as the basic 'stylized facts' of economic fluctuations' in Pakistan. Because this is the first exercise of its kind, the reader is warned that

²Import and Export dataset is the exception where data is available at a monthly frequency.

Section 2 is long. Readers wishing to skip details and a framework for explaining some results are invited to proceed to Section 2.5 for a summary of the various stylized facts. Section 3 presents our model. Section 4 presents the calibration, the impulse response functions and evaluates the model's ability to capture the basic features of the data. The last section concludes.

2 Some Relevant Empirical Facts of Pakistani Economy

In this section, we present some structural facts on the Pakistani economy in general and how the nature of the economy has shifted over the last 50 years. After discussing the structure of the economy, we then take a look at HP filtered series of relevant variables to establish some 'stylized facts' of short run economic fluctuations in Pakistan.

2.1 Some Basic Structural Facts

Pakistan has been an agrarian economy from the start of its existence. The agriculture sector accounted for more than 1/2 of total output in 1950. However, over time the share of agriculture in total production has been steadily declining and was a little more than 1/5 of the total Output in 2010.

On the other hand, Figure 1 shows that the decline in the share of agriculture in production has been accompanied by an increase in the share of the services sector in total output. The share of the services sector in production has increased to more than 1/2 of overall production in the economy and has been for most part of the last few decades. Furthermore, starting from a low base, the share of industry in production has increased almost 3 folds since 1950. The industrial sector now accounts for a little less than 3/10 of total output. However, the industrial sector has stagnated over the last three decades.

Figure 1, points out that the nature of the Pakistani economy has shifted from being agrarian to more service oriented. However, limiting attention only to sectoral shares of output can be misleading for fully understanding the changing nature and evolution of the Pakistani economy.

Even though the contribution of the agriculture sector has declined significantly on the production side, the lower two panels of Figure 1 point out that in real terms the Pakistani economy is still very much agrarian, as almost 1/2 of all employed persons in the country are still working in the agriculture sector.

This is puzzling as there is a difference in the pattern of sectoral breakdown of employment and production. It is true that the sectoral share of employed persons has moved in the same direction as the sectoral share of output for both the agriculture and services sector. On the other hand, the share of employed persons working in the industrial sector has continuously remained more or less the same over the last few decades.

This puzzle can be addressed in several ways.

First, a possible increase in labour productivity of the services and industry sector, the latter to a smaller extent, as well as a possible productivity slowdown in the agriculture sector. In other words, almost 1/2 of Pakistani labour force (employed persons in agriculture) is currently producing around 1/5 of its output.

Second, the presence of a large informal sector, not part of our current analysis, can not be ignored. The informal sector remains a big part of the agricultural sector and available statistics undermine its dynamics and size.

Another possible explanation for the persistence of employed persons in the agriculture sector can be the supportive policies of government. Presently, the government announces the support/indicative price for the top 2 crops (wheat and sugarcane) to incentivize the agrarian economy. The government also used to actively intervene in the rice and cotton crops until 1995/6.³ Such government interventions distort households' labour supply decisions in favour of the agriculture sector and contribute to sustained employment in this sector.

An allied explanation is the failure of the industrial sector to generate employment due to ill-thought labour market reforms. For example Ahmed et al. (Forthcoming) observe a declining real-wages in the industrial sector due to the market wages being pegged to the minimum-wage levels while the latter are only being revised irregularly over time.

³Wasti (2012), Table 2.12 (a)

2.2 Long-run Facts

After looking at some structural facts of the economy, let's consider long run behavior of macroeconomic variables. In the business cycle literature, it is the usual practice to look at the long-run ratios of consumption to output, investment to output ratio, government consumption to output as well as exports and imports to output ratio before studying the short term fluctuations.

Aggregate Investment & Other Disaggregated Components of Aggregate Investment

Let's first consider the long run behaviour of investment over the last few decades. Investment is widely considered the main driver of long run growth of any economy. In Figure 2A(i), we can see that investment, captured by fixed capital formation, to output ratio has been moving between 0.10 to 0.22 over the last five decades. However, for the period from 1981-2010 the ratio has fluctuated between a high of 0.21 in 2008 and a low of around 0.14 in 1999 and 2010 respectively. Another interesting observation from the second panel of Figure 2A is that the volatility of the investment to output ratio has become much more pronounced over the last 10-15 years.

Due to the previously mentioned importance of investment, the rest of the panels of Figure 2A(i) help us in taking a detailed look at different components of aggregate investment in order to better understand the source of volatility in the investment to output ratio. The rest of the graphs in Figure 2A(i) are presented as a share of given component of total investment in total investment (e.g share of private investment in total investment). This helps us in clearly identifying the relevant patterns in different disaggregated components of investment.

After breaking down aggregate investment into public and private investment we can see that the share of public investment in total investment⁴ has been declining from late 80's until very recently. Furthermore, from 1990 onwards the share of private investment has overtaken the share of public investment and was more than twice the size of public investment (0.72 vs 0.28). This shift over the last 20 years or so can be

⁴The share of public investment in GDP has also declined over the last 20 years. The share of public investment of GDP has been reduced from a high of 0.10 in 1987 to a low of 0.04 in 2010.

a result of changes in the financial sector due to financial sector reforms starting in late 80's and early 90's. Finally, this significant gap between the share of private and public sector investment seems to be stabilizing over the last few years.

This shows that Pakistani economy is moving towards an economy where the government is moving towards liberalization. This can attributed partly to bad-luck and partly to strategy. Bad-luck because Pakistan has faced multiple balance-of-payments crisis. In particular, Pakistan has participated in 13 different IMF programs over the last three decades. Its tax to GDP ratio in 2011 was under 10%; one the of lowest in the region for an economy if its size. These considerably restricted government's focus and means forcing it towards a 'less' managed economy. Strategy because following 1991 financial sector reforms, the private sector was expected to take the lead for investments. And to some extent this did happen as we discuss later.

Another interesting observation from Figure 2A(ii) about the investment dynamics of Pakistan is the fact that starting from last decade the decline in share of public investment in overall investment has been accompanied by the increase in the share of Foreign Direct Investment (FDI) in overall investment. Not surprisingly, this surge in FDI in Pakistan is strongest for the most part of the last decade (2001-2008) where there was a global boom in FDI in developing countries in general and South Asian countries in particular. However, just like for other countries FDI inflows to Pakistan have dried up since the great recession and financial crisis starting in 2008. The share of FDI in total investment has declined by almost 50% in the last three years.

For Pakistan, the share of FDI in total investment reached a peak value of 0.17 in 2007 before declining in the last few years. This further emphasizes the importance of foreign investment in complementing domestic private investment of the country which in turn is mainly responsible for the economic growth. In order to further evaluate this bold claim, we look closely at the link between domestic and foreign investment in the next sub-section.

Private Consumption, Government Consumption, Exports & Imports

After investigating the long run behaviour of aggregate investment and its various disaggregated components, we turn our attention to long run behaviour of other macroeconomic variables. In the last two panels of Figure 2B(i), we present the long run behaviour of private consumption in Pakistan. The ratio of private consumption

to output has fluctuated considerably between 0.68 and 0.84 during the period of 1960-2010. Hence, it is clear that private consumption has always been the largest component of aggregate output for our economy.

However, it is important to note that the significant role of private consumption from the data comes with a caveat. The data for private consumption in Pakistan is not collected or gathered but instead private consumption is computed as a residual from the income identity equation. Furthermore, Malik (2011) using Pakistani data found that private consumption data from national accounts is significantly different from the consumption data gathered from household surveys.

The long run ratio of private consumption to output is volatile and it shows some cyclical behavior. Over the period of 1981-2010, the share of private consumption in output declined initially reaching the lowest value of 0.68 in 1991. However, since reaching the lowest value of 0.68 the share has had an increasing trend with a value of 0.82 in 2010.

After analyzing the long-run behavior of private consumption next up is the long-run behavior of government consumption. The ratio of government consumption to output also exhibits significant volatility with values ranging between 0.08 to 0.17 (see Figure 2B(ii)). In particular, the last decade shows a number of episodes of upward and downward movement in the ratio of government consumption to output. The share of government consumption in output is also exhibiting cyclical patterns rather than taking a stable long run value.

Moving on to trade related variables of exports and imports, we consider their long run behavior over the last five decades. In Figure 2B(iii), there is a significant jump upwards in both exports to output and imports to output ratio in early to mid 70's. The value of exports to output ratio almost doubled between 1972 and 1973, similarly the value of imports to output ratio also increased significantly between 1972 and 1973. Over the last three decades (1981-2010), the share of export in output has fluctuated between 0.10 and 0.17. On the other hand, imports to output ratio has been between 0.14 and 0.24.

The behavior of exports to output and imports to output ratio over the last three decades is in line with other evidence suggesting that Pakistan over the last 30 years has started behaving more and more like a small open economy. This is further supported by the fact that more than 1/3 of output was due to trade (exports +

imports) on average over the period from 1981-2010.

Before moving on to other empirical evidence, it is important to establish the linkages between different macroeconomic variables and business cycle features. Let's revisit the claim made earlier that the economy has started behaving differently over the last three decades as opposed to the earlier period. Looking at Table 2, the differences in magnitude of these long run ratios as measured by mean, median, volatility as measured by coefficient of variation, and another measure of volatility (std.dev/median) are obvious.

These differences are less pronounced for the magnitude of some long run ratios. The absolute value of different long run ratios are similar for the two periods of 1960-1980 and 1981-2010 except for the trade related ratios of exports to output and imports to output. The value of both trade variables has increased significantly as a share of overall output. For the exports to output ratio the average value was 0.09 for the earlier period and 0.14 for the latter period. Similarly, the imports to output ratio on average was 0.15 for the period spanning 1960-1980 and the average was 0.20 for the period between 1981-2010. On the other hand, the average of private consumption to output ratio has decreased slightly from 0.79 to 0.75 and investment to output ratio has increased from 0.15 to 0.17 respectively. Finally, the value of government consumption to output on average has remained almost same throughout the last five decades at around 0.11.

The main findings from Table 2 are the significant changes in volatility measure of coefficient of variation between the two periods for almost all of the variables. There is a significant decrease in the volatility of both exports to output ratio and imports to output ratio from the period of 1960-1980 to the period of 1981-2010. The coefficient of variation for both investment to output ratio and FDI to output ratio have also reduced significantly for the latter period as compared to the former period. Interestingly, government consumption to output ratio is the only long run ratio with a significantly increased volatility during the 1981-2010 period as oppose to the earlier period of 1960-1980. The volatility of private consumption to output ratio as measured by the coefficient of variation is very similar for the two periods being compared. However, as mentioned before, any analysis involving private consumption should be interpreted cautiously due to the residual nature of the private consumption data.

After the brief comparison of the two periods of 1960-1980 and 1981-2010 based on the magnitude and volatility of long run ratios and our previous findings about the structure of the economy, we will now focus on the period from 1981-2010 for the rest of the empirical analysis. As explained earlier this period is important as it followed Pakistan-Bangladesh break up.

2.3 Correlations & Dynamic Correlations

In order to better understand the relationship between different macroeconomic variables, we look at contemporaneous correlations and dynamic correlations in this section. In particular, we want to empirically establish the linkages between output and aggregate investment, FDI, private consumption, government consumption, exports and imports. This exercise will help identify the macroeconomic variables to focus on in order to develop a better understanding of the economy and to develop a relevant model of business cycle fluctuations of Pakistan. For this part of the paper, we consider HP filtered data for the period of 1981-2010 and decade by decade as well.

Aggregate Investment & Other Disaggregated Components of Aggregate Investment

To capture cyclical aspects using de-trended data, we first look at investment related variables, as investment and private consumption are both significantly correlated with output according to Table 3B.

In order to de-trend different macroeconomic series, we use HP filter with $\lambda = 100$, which is the value normally used in the literature for annual data. This value of λ is appropriate for Pakistan see Choudhary, Hanif and Iqbal (Forthcoming).

The Table 3A describes the linkages between de-trended private investment, de-trended all public investment, de-trended public investment, de-trended government investment, de-trended private domestic investment, de-trended foreign direct investment and de-trended output.

Over the period of 1981-2010, aggregate investment, private investment, all public investment, public investment, government investment and foreign direct investment are all significantly positively correlated with output.

The Table 3A also shows that aggregate investment is significantly positively correlated with all the disaggregate components of aggregate investment. However, it is most strongly positively correlated with private investment. Interestingly, private investment is strongly positively correlated with private domestic investment, government investment and FDI.

However, looking at the data by decades we see different results for different decades. The relationship of investment with its disaggregated components is different for the three periods of 1981-1990, 1991-2000 & 2001-2010. This once again points towards the continuously evolving nature of economic fluctuations in Pakistan over the last few decades.

However, since the early 1990's private investment has become the major driving force behind aggregate investment in Pakistan. This is further supported by Table 3A, where correlations are significantly positive for each of the last two decades for de-trended aggregate investment and de-trended output as well as between de-trended private investment and de-trended output for the last decade.

Looking at the period of 1981-1990, we find some strange observations, such as strong negative correlation between private investment and output as well as between private domestic investment and output. During this period, aggregate investment was significantly positively correlated with both all public investment and public investment. Also, de-trended government investment was strongly negatively correlated with both private and private domestic investment. This behaviour of aggregate investment and its different components is in line with our earlier explanations regarding military rule and nationalization of investment during this period.

During the last two decades, we find more typical behaviour of aggregate investment and its disaggregated components. For both decades, de-trended output is positively correlated with aggregate investment as well as with all public investment, public investment, government investment and foreign direct investment.

In addition, for the last decade de-trended output is significantly positively correlated with aggregate investment, private investment, private domestic investment and foreign direct investment. The significant relationship between private investment and aggregate investment is also supported by the correlations reported in Table 3A for the last decade. Finally, the strong link between private investment and aggre-

gate investment as well as output is due to both the domestic⁵ component of private investment as well as the foreign⁶ one.

Another important result from Table 3A is that foreign direct investment has steadily gained importance over time in explaining short-run fluctuations of aggregate investment as well as the output in particular over the last two decades. The contemporaneous correlation of FDI with output has increased significantly from 0.07 in the 80's to 0.73 for the last two decades. This increased significance of foreign investment is further supported by the correlation coefficient of 0.92 between aggregate investment and FDI during the last decade.

After the analysis based on contemporaneous correlations, we turn our attention to dynamic correlations in order to better understand the lead-lag relationship between different de-trended variables of interest. This exercise should be ideally done at a quarterly or monthly frequency, but due to unavailability of relevant macroeconomic series at appropriate frequency, we look at dynamic correlations at an annual frequency.

The dynamic correlations of different de-trended aggregate and disaggregated macro variables with de-trended output are presented in Figures 3 & 4. Figure 3 presents the dynamic correlations for HP filtered series of output with HP filtered series of different disaggregated components of investment. The data is annual from 1981 to 2010, and the Figure 3 plots the correlation between de-trended real GDP, Y_t and I_{t+j} against j , where I represents either disaggregated component of aggregate investment or de-trended disaggregated component of aggregate investment. In order to further evaluate the changing nature of the economy, each panel also has the relevant dynamic correlations by the decade as well.

In Figure 3, the second panel is showing the dynamic correlation for HP filtered real GDP with HP filtered real aggregate investment. The de-trended aggregate investment is positively correlated with de-trended output for both lags and both leads for the covered period of 1981-2010. However, it is interesting to note that the dynamic correlations differ from one decade to another. During the last decade, de-trended aggregate investment is positively correlated with de-trended output for the first lag as well as for both leads.

⁵private domestic investment

⁶foreign direct investment

The behavior of de-trended private investment as measured in terms of its dynamic correlation with de-trended output is similar to the behavior of de-trended aggregate investment as shown in Figure 3. For the last decade, de-trended private investment is significantly positively correlated with the de-trended output at both first lag and first lead. For the earlier two decades, it is clear that de-trended private investment was a lagging indicator of de-trended output. However, for the last decade de-trended private investment seems to be both leading and lagging the de-trended output.

The de-trended all public investment is positively correlated with de-trended output for the last two decades at the first lead as well as for the whole period of 30 years. Therefore, de-trended all public investment usually lags de-trended output for the period from 1981-2010. However, only for the 80's, de-trended all public investment is a leading indicator of de-trended output.

On the other hand, de-trended public investment is positively correlated with de-trended output for the whole period of 30 years as well as for all three decades at the first lead. Therefore, de-trended public investment lags de-trended output during the time period covered in our analysis.

In Figure 3, we can see that de-trended government investment is positively correlated with de-trended output at the first lag for the first two decades as well as for the whole period from 1981-2010. Furthermore, for both 80's and 90's de-trended government investment is negatively correlated with de-trended output for both leads. However, during the last decade de-trended government investment is positively correlated with de-trended output for both leads and negatively correlated for both lags. Therefore, de-trended government investment was a leading indicator of de-trended output for the first two decades and a lagging indicator for the last decade.

The evidence presented so far in this section has repeatedly pointed towards the importance of private investment in explaining economic behaviour of Pakistan over the last few decades. Therefore, in order to disentangle the importance of domestic and foreign component of private investment for short-run economic fluctuations we look at the dynamic correlations of de-trended private domestic investment and de-trended foreign direct investment with de-trended output respectively.

The de-trended private domestic investment is positively correlated with de-trended output for the whole period as well as for 80's and the last decade for the first lead. However, it is also positively correlated with output at the first lag for the last decade

and at both lags for the whole period of 1981-2010. This point towards increasing importance of fluctuations in private domestic investment for the fluctuations in output.

Finally, de-trended foreign direct investment is positively correlated with de-trended output at the first lead for the all three decades as well as for the complete period from 1981 to 2010. However, more importantly for the whole period as well as the last decade fluctuations in FDI is strongly positively correlated with fluctuations in output at the first lag as well. This implies that fluctuations in FDI not only follow fluctuations in output but can also lead to fluctuations in the output. Therefore, the role of foreign direct investment in driving Pakistani economy during the last decade as well as for the complete period of our analysis is important for de-trended variables as well.

The short run fluctuations in output are driven by short-run fluctuations in aggregate and private investment. Furthermore, the short run fluctuations in FDI and private domestic investment are strongly correlated with short run fluctuations of output specially for the last decade.

Private Consumption, Government Consumption, Exports & Imports

After detailed discussion of contemporaneous correlations and dynamic correlations of de-trended aggregate investment and de-trended output as well as de-trended components of aggregate investment with de-trended output, we turn our attention to other macroeconomic variables such as private consumption, government consumption, exports and imports.

The Table 3B describes the linkages between different de-trended components of output such as de-trended private consumption, de-trended government consumption, de-trended exports and de-trended imports with de-trended output.

Over the period of 1981-2010, de-trended private consumption, de-trended government consumption, de-trended exports and de-trended imports are all strongly positively correlated with the de-trended output. The Table 3B also shows that de-trended imports is most strongly positively correlated with de-trended output. Interestingly, both de-trended private consumption and de-trended government consumption are positively correlated with de-trended imports. However, government consumption is also significantly positively correlated with both aggregate invest-

ment and FDI. The de-trended exports is only significantly positively correlated with de-trended output. Finally, de-trended imports are positively correlated with all other de-trended components of output except for de-trended exports.

However, looking at the data by decades we see different results for different decades.

The relationship of de-trended output and different de-trended components of output is significantly different for the period of 1981-1990 compared to the other two decades. For this period only, de-trended government consumption is significantly positively correlated with the de-trended output. The de-trended aggregate investment is strongly negatively correlated with de-trended private consumption and positively correlated with de-trended exports. However, for this period de-trended exports is significantly negatively correlated with de-trended private consumption. Finally, de-trended imports is positively correlated with de-trended private consumption for this period.

During the 1990's, all different de-trended components of output are strongly positively correlated with de-trended output. The de-trended private consumption has a contemporaneous correlation of 0.85 with de-trended output. The de-trended private consumption itself is positively correlated with de-trended aggregate investment with a correlation coefficient of 0.56. The de-trended government consumption is significantly positively correlated with de-trended FDI, de-trended exports and de-trended imports. However, the correlation of de-trended government consumption is strongest with de-trended FDI. Both de-trended exports and de-trended imports are positively correlated with de-trended output as well as all other de-trended components of output.

During the period of 2001-2010, de-trended output is positively correlated with all de-trended components of output and with significant correlations with de-trended aggregate investment, de-trended FDI and de-trended imports. These significant correlations of de-trended output with different de-trended components of output implies that short-run economic fluctuations in the last decade was not driven only by investment or consumption or trade but it was a combination of all.

The most interesting finding for this period is the significant positive correlation of de-trended imports with de-trended aggregate investment, de-trended FDI & de-trended government consumption. These findings combined with the importance of

de-trended FDI in driving de-trended output as discussed previously once again emphasizes the importance of external shocks for the medium-run economic fluctuations in Pakistan.

The dynamic correlations of different de-trended components of output with de-trended output are presented in Figure 4.

Figure 4 presents the correlations for de-trended series of output with different components of output. The data is annual from 1981 to 2010, and the figure plots the correlation between real GDP, Y_t and PC_{t+j} , GC_{t+j} , EXP_{t+j} , IMP_{t+j} against j , where PC, GC, EXP and IMP are private consumption, government consumption, exports and imports respectively.

In order to establish the changing nature of the economy, each panel also has the relevant dynamic correlation by decade as well.

In Figure 4, the first panel is showing the dynamic correlation of de-trended output with de-trended private consumption. The de-trended private consumption is positively correlated with de-trended output for the period from 1981-2010 for both leads as well as for the first lag. However, it is interesting to note that dynamic correlations between private consumption and output differ considerably decade by decade.

During the 80's, de-trended private consumption is negatively correlated with de-trended output for both lags and the first lead. This implies that for the period of 1981-1990 de-trended private consumption is neither a leading nor a lagging factor for the de-trended output.

For the last decade, de-trended private consumption is positively correlated with de-trended output at both first lag and first lead as well. This means that fluctuations in private consumption are both impacted by fluctuations in output as well as impacting output fluctuations. The positive correlation between de-trended private consumption and de-trended output is stronger for the first lag as compared to the first lead for the last decade.

Therefore, the relationship between private consumption and output has been evolving over time and the increased importance of private consumption for the economy has been highlighted by this simple analysis.

The de-trended government consumption is positively correlated with de-trended output for the first lead for all three decades as well as for the whole period of 1981-2010. In addition, for all three decades de-trended government consumption and

de-trended output is negatively correlated for both lags. For the complete period of analysis, there is a positive correlation between de-trended government consumption and de-trended output at the first lag as well. However, the positive correlation is stronger between de-trended government consumption and de-trended output for the first lead even for the whole period. Therefore, de-trended government consumption is clearly a lagging indicator for de-trended output for the three decades considered in our analysis. However, it can be both leading and lagging indicator for the period of 1981-2010.

Now turning to trade related variables, we first look at the behaviour of exports. For the period of 1981-2010, there is only slight positive correlation between de-trended exports and de-trended output for the first lead but a strong positive correlation for both lags. Therefore, de-trended exports are clearly a leading indicator of de-trended output for the period of 1981-2010.

However, looking at dynamic correlations of de-trended exports and de-trended output decade by decade paints a different picture. During the 80's & 90's de-trended exports has almost no correlation with de-trended output at first lag and positively correlated at the first lead. This implies that fluctuations in exports followed fluctuations in output during the first two decades.

On the other hand, for the last decade we find the exact opposite of what happened during the first two decades. In the period from 2001-2010, de-trended exports is significantly positively correlated with de-trended output at both lags and negatively correlated for both leads. This implies that fluctuations in exports were followed by fluctuations in output. In other words, an increase in de-trended exports led to an increase in de-trended output during this period. Therefore, de-trended exports was a leading indicator of de-trended output for the last decade. This further lend support to the importance of external factors for economic fluctuations in Pakistan.

Finally, fluctuations in de-trended imports has mainly been a result of fluctuations in de-trended output. For the period of 1981-2010, de-trended imports has been positively correlated with de-trended output for both lags and both leads. However, the behaviour of de-trended imports and de-trended output is markedly different for all three decades. For the 80's, there seems to be no relationship between the two variables neither at lags nor at leads. In the second decade of interest, de-trended imports are a lagging indicator of de-trended output. Finally, during the last decade

de-trended imports was positively correlated with de-trended output at both first lead and first lag. However, the positive correlation between de-trended output and de-trended imports is much stronger for the first lead compared to the first lag. Therefore, even for the last decade we can't claim de-trended imports to be a leading variable for de-trended output. At best, de-trended imports are both a lagging and leading indicator of de-trended output for the last decade.

The empirical results discussed in this section once again highlight the importance of external factors such as fluctuations in exports as a leading cause or indicator of fluctuations in aggregate output for a small open economy like Pakistan. In particular, as pointed out earlier all of these results are found to be much more stronger for the period starting from 1990's and much stronger for the last decade.

2.4 Not So 'Stylized Facts' of Pakistani Business Cycles

After a detailed discussion of different structural and empirical facts of Pakistani economy over the period of 1981-2010, now we can finally introduce and discuss the so called stylized facts of the Pakistani business cycles for our main period of interest as well as for each of the three decades that we have analysed throughout the empirical section. The business cycles are usually characterized by **volatility, relative volatility, co-movement and persistence** of different macroeconomic variables of a given economy. Before continuing with our analysis, it is important to mention that for a developing economy like Pakistan 'stylized facts' are not very well known and there is no clear consensus on what are actually the 'stylized facts' of the economy. Data inconsistencies are one of reasons for this lack of consensus as well as the continuously changing nature of economic fluctuations in Pakistan. Therefore, in this section we present our not so 'stylized facts' for the business cycles of Pakistan for de-trended series of the relevant macroeconomic variables. Furthermore, we only discuss the 'stylized facts' for the last 30 years and the relevant statistics for each decade are reported in Table 4 in the appendix.

The second moments for de-trended output, aggregate investment, private consumption, government consumption, exports, imports and FDI are reported in Table 4 of the appendix. The business cycles are usually characterized by volatility, relative volatility, co-movement and persistence of different de-trended macroeconomic

variables of a given economy. Therefore, we investigate these second order moments closely for the Pakistani economy for the period of 1982-2010 in order to get a better understanding of Pakistani business cycles over the last three decades.

The de-trended output for Pakistan over the last 30 years is more volatile than most developed economies⁷. The magnitude of volatility of de-trended real output is very similar for the whole period of 30 years as well as for the last two decades. However, rather than discussing the absolute volatility of different macroeconomic variables for all the periods, we will focus our attention on relative volatility with respect to real output of these variables. The relative volatility is measured as a ratio of the standard deviation of a given de-trended macroeconomic variable to the standard deviation of de-trended output.

For the period of 1981-2010, all the de-trended macroeconomic variables are more volatile than de-trended output. The de-trended aggregate investment is about four and half times as volatile as de-trended real output. Private consumption is clearly the least volatile among all other components of output. Private consumption is less than one and half time as volatile as output. On the other hand, government consumption is the second most volatile variable after FDI. The de-trended government consumption and de-trended imports are around 5 times as volatile as de-trended output. The de-trended exports is around three and a half times as volatile as output. Finally, foreign direct investment is the most volatile variable relative to output. The relative volatility of FDI to output is 17.6.

The nature of economic fluctuations in Pakistan has been continuously evolving. Therefore, one should be aware of what has happened over time in the economy as well as focus on the main features of the last few years to come up with an appropriate business cycles model. During the period 2001-2010, relative volatility of aggregate investment, government consumption, imports and FDI is higher from the magnitudes of relative volatilities of these variables for the whole period. On the other hand, relative volatility of private consumption and exports with respect to output has been consistently declining over time and reaching their lowest values in the last decade. This decline in relative volatility indicates possible similarities between Pakistani business cycles as well as business cycles of developed economies in recent times. In particular, the decline in relative volatility of private consumption

⁷reference here

with respect to output can be a sign that the Pakistani economy is moving towards behaving like a developed economy as far as the second order moments are concerned.

The co-movement of de-trended output with de-trended macro variables helps us in classifying them as pro-cyclical, a-cyclical or counter-cyclical variables with respect to output. During 1981-2010, de-trended aggregate investment, private consumption, government consumption, exports, imports and FDI all are significantly positively correlated with de-trended output. This strong pro-cyclical behavior is strongest for imports, aggregate investment, and FDI. The de-trended private consumption, government consumption and exports are also but to a lesser degree significantly positively correlated with output.

This trend still holds for the last decade. All the de-trended variables including private consumption, government consumption and exports are positively correlated with de-trended output. However, de-trended aggregate investment is clearly most significantly positively correlated with de-trended output. The de-trended imports and de-trended FDI are the other two significantly pro-cyclical variables. The behavior of different de-trended macro variables over the last three decades in general and for the last few years in particular consistently highlight the importance of aggregate investment and imports in explaining fluctuations in output for Pakistani economy over the last few decades and possibly in the future.

Finally, we consider the persistence of relevant de-trended macro variables for the period 1981-2010 as well as by decade. All the de-trended variables exhibit some persistence. However, output, aggregate investment, FDI and imports are the most persistent out of all the variables. The persistence level has stayed the same for output and aggregate investment and has been increasing for some of the other de-trended variables. In particular, the autocorrelation of exports and FDI has both increased for the last decade.

2.5 Summary of Stylized Facts by Category

In order to focus on the main empirical findings for Pakistan over the last few decades, lets recall the main points of this Section:

2.5.1 Structural facts and long-run ratios for the period 1960-2010 and decades in-between

- The Pakistan economy is stuck at a per capita real growth of about 2.5%-we call this 'Pakistani' growth rate.
- Pakistan has always been an agrarian economy, in terms of employment, from its independence till now even after more than 60 years.
- Having said that, over the sixty year period, employment shares in relation to total employment have been steady for industry, declined for agriculture but expanded for the services sector.
- The sectoral emphasis in terms of output has shifted away from agriculture to services while industry has stagnated.
- The long run ratios of the economy such as investment to output, private consumption to output, government consumption to output, exports to output and imports to output all display volatility over time instead of having a stable trend.
- The volatility of the investment to output ratio has become more pronounced over the last two decades.
- Looking further into disaggregated components of investment, we find that share of private investment in aggregate investment has taken over the share of public investment over the last couple of decades.
- This decline in share of public investment, in particular over the last decade, has been accompanied by an increase in foreign direct investment to the economy.
- The long run ratio of private consumption to output is volatile and displays cyclical behaviour. Private consumption has always been the largest component of output in Pakistan. However, it is important to keep in mind that private consumption data in Pakistan is computed as a residual.

- The government consumption to output ratio is also very volatile and seems to follow a cyclical pattern. It has been particularly more volatile over the last decade.
- Both trade related long run ratios, namely exports to output and imports to output show a significant jump in the early 70's.
- The magnitude of both the exports to output and imports to output ratio increased significantly in early 70's.
- Trade has accounted for more than one third of output over the last three decades. This lends credence to treating Pakistan as a small open economy.

2.5.2 Correlations and Dynamic Correlations for the period 1981-2010

- For the period from 1981-2010, de-trended output is significantly positively correlated with de-trended imports, de-trended aggregate investment, de-trended government consumption, de-trended private consumption, de-trended exports and de-trended FDI.
- The contemporaneous correlation of de-trended output is strongest with de-trended imports, de-trended aggregate investment and de-trended FDI.
- The co-movement between de-trended output and other de-trended macro variables differ significantly by decade.
- During the 80's only de-trended government consumption is significantly positively correlated with de-trended output. Furthermore, all other de-trended variables have almost no correlation with de-trended output during the first decade in our analysis.
- On the other hand, there is a positive correlation between de-trended output and all de-trended macroeconomic variables for the 90's. The positive correlation is statistically significant for de-trended private consumption, de-trended exports, de-trended FDI, de-trended imports and de-trended government consumption.
- During 90's, de-trended private consumption and de-trended exports are the most strongly correlation macro variables with de-trended output.

- For the last decade, once again there is a positive correlation between de-trended output and all de-trended macro variables of interest.
- However, the positive correlation is significant only for aggregate investment, imports and FDI.
- Most of the de-trended macroeconomic variables are lagging behind de-trended output or for most of them, for the last decade are both lagging and leading de-trended output. Therefore, dynamic correlation analysis points out that de-trended exports is the only clear leading indicator of de-trended output.
- The leading relationship of de-trended exports with de-trended output is strongest for the last decade.

2.5.3 The stylized facts of business cycles in Pakistan for 1981-2010

- All de-trended macroeconomic variables are more volatile than de-trended output for the period 1981-2010.
- The de-trended aggregate investment is four and a half times as volatile as de-trended output. The de-trended private consumption is the least volatile variable, with it being less than one and half times as volatile as de-trended output.
- The de-trended government consumption and de-trended imports are around five times as volatile as de-trended output. De-trended exports are about three and half times as volatile as output. Lastly, de-trended FDI is the most volatile variable.
- During the last decade, relative volatility of aggregate investment, government consumption, imports and FDI is higher from the values for the whole period.
- The de-trended aggregate investment, private consumption, government consumption, exports, imports and FDI are all significantly positively correlated with de-trended output.
- This pro-cyclicality of variables is strongest for de-trended imports, aggregate investment and FDI.

- The de-trended output is significantly positively correlated with de-trended aggregate investment, de-trended imports and de-trended FDI for the last decade.
- The de-trended macroeconomic variables that show persistence during the period of 1981-2010 are output, aggregate investment and FDI. For the last decade, de-trended exports and de-trended imports also show persistence in addition to de-trended output, aggregate investment and FDI.

3 Essential Models

After discussing in detail different structural and so called stylized facts of the Pakistani economy over the last few decades, we finally turn our attention to an appropriate economic model that can help us explain the aggregate fluctuations of the economy. Our empirical evidence has repeatedly pointed towards the importance of aggregate investment, FDI and imports in explaining aggregate fluctuations in output over the last 30 years and in particular for the last decade. However, in this paper we start from the simplest of RBC model first and then introduce and augmented version of our simple real business cycle model. The idea is to see how far these simplest of models can take us in explaining recent aggregate fluctuations of Pakistani economy.

3.1 A Simple Real Business Cycles Model

In order to evaluate the ability of a simple real business cycles model to capture cyclical fluctuations of Pakistani economy, we consider the most basic RBC model which is easily available in most macro books. Our model is a closed economy representative agent model with exogenous technology shock. The representative agent in our simple RBC model tries to maximize the following separable utility function over an infinite time horizon.

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[\ln C_t + \psi \frac{(1 - N_t)^{1-\sigma}}{1 - \sigma} \right] \quad (1)$$

where E_t is the expectation at time t , β is the discount rate, C_t is the consumption of our representative agent at time t , N_t represents the number of hours spent working, $\frac{-1}{\sigma}$ is the frisch elasticity of leisure and ψ is the parameter explaining the utility gained

by our representative agent through leisure. This utility function is consistent with balance growth.

The representative agent tries to maximize the infinite stream of utilities given that in each period he/she faces the following budget constraint. The equation below is actually the typical aggregate resource constraint for a closed economy without government.

$$C_t + I_t = Y_t \tag{2}$$

where I_t is the aggregate investment in the economy at time t and Y_t is aggregate output.

The production in this economy follows a simple cobb-douglas function. The output in the economy depends on physical capital, labor and technology according to the following equation.

$$Y_t = K_t^{1-\alpha} (A_t N_t)^\alpha \tag{3}$$

Y_t is the output, K_t represents physical capital, A_t denotes technology and N_t is labor. $(1 - \alpha)$ is the share of capital in production.

The physical capital in this economy gets accumulated according to following capital accumulation equation:

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{4}$$

where δ is the depreciation rate of capital.

Finally, we model technology by using total factor productivity as a proxy for technological progress. Technology is exogenous as is typical in RBC literature and it follows an AR-1 process.

$$\ln(A_{t+1}) = \rho_A \ln(A_t) + \epsilon_{A,t} \tag{5}$$

3.2 Augmented Real Business Cycles Model

The augmented RBC model is similar to the typical closed economy RBC model presented earlier. Indeed, the augmented model also has a representative agent that tries to maximize the infinite stream of discounted utilities given the budget constraint. The main innovation is the introduction of an exogenous FDI shock and the division of aggregate investment into domestic and foreign components in our capital stock

accumulation Eq. (6). This model is isomorphic to RBC model with investment-specific technology shock as studied by Greenwood, Hercowitz and Huffman (1988) and Greenwood, Hercowitz and Krusell (1997). The main difference between their model and ours is that they use the relative price of investment as the exogenous shock to aggregate investment and we use FDI. However, the propagation mechanism in both models work in a very similar manner

To be precise, there are now two types of investment in our closed economy. I_t denotes domestic investment which is our typical aggregate investment that is usually incorporated in these closed economy RBC models. On the other hand, I_t^* is the foreign investment which we are interpreting as foreign direct investment:

$$K_{t+1} = (1 - \delta)K_t + I_t(I_t^*) \quad (6)$$

The reason we introduce FDI in our augmented RBC model is the abundant empirical evidence presented in the previous section supporting the significant role of FDI and trade variables in driving the business cycle fluctuations of the Pakistani economy over the last few decades and in particular for the last decade.

Our specification in above equation implies that there is strong complementarity between foreign and domestic investments in Pakistan. This is an important point as we also tried more general alternative specifications, such as CES, which produced an empirical fit inferior to our model. Therefore, we need the assumption of complementarity between the two types of investment to get our model to capture key facts from the data. However, another possibility is that both domestic and foreign investments respond to an underlying common shock process, such a investment-specific technology shock, changes in tax or regulatory structure, business confidence, stability, political change etc., that drives investments decisions.

Naturally, we should have considered a small open economy model in order to properly model the FDI channel as well as the external sector. However, the purpose of this paper is to mainly establish some structural and stylized facts of Pakistani economy and evaluate the performance of simple RBC closed economy models in replicating the relevant moments of Pakistani economy. The idea is to see how far we can get in terms of matching the stylized facts of the economy even with these simple models.

Therefore, we model this foreign component of investment as simply an exogenous

shock consistent with how a typical exogenous technology shock is incorporated in these models.

$$\ln(I_{t+1}^*) = \rho_{I^*} \ln(I_t^*) + \epsilon_{I^*,t} \quad (7)$$

where ρ_{I^*} is the persistence of the exogenous fdi shock and $\epsilon_{I^*,t}$ is the standard error associated with the shock.

Finally, technology shock is the same as before,

$$\ln(A_{t+1}) = \rho_A \ln(A_t) + \epsilon_{A,t} \quad (8)$$

where ρ_A and $\epsilon_{A,t}$ are the persistence and standard error of the technology shock respectively.

4 Calibration & Results

In this section, we first discuss the calibration of different parameters of the model. It is important to have a good understanding of rationale behind picking different parameter values in order to properly evaluate the fit of the model. After calibration, we compare second order moments obtained from simulations of our two models and their empirical counterpart. Finally, we take a brief look at the impulse response functions from both simple and augmented RBC model for both exogenous shocks to technology and FDI.

4.1 Calibration

Due to data limitations all parameters in our model are calibrated for annual frequency. There are 10 parameters in total with 6 structural and 4 shock related parameters. Structural parameters can be categorized into utility and production function related parameters. We have generally adopted two approaches in terms of calibrating parameters for our models. Some of the parameters, for which estimation remained an issue due to lack of reliable and detailed data, are picked from existing RBC/DSGE literature for developing and developed countries. Some of the parameters with available data, have been calibrated using partial estimation/computation approach.

First of all, we discuss parameters related to household utility. The value of discount factor β used in the existing literature ranges from 0.925 to 0.99 for annual frequency for developing countries. We decided to use a value of 0.95 as it falls in the middle of that range as well as being the widely accepted value for RBC/DSGE models of annual frequency for developed countries like the US. Furthermore, this calibrated value of β is also consistent with long run behavior of Pakistani economy.

Ψ reflects household's preference for leisure and a value of 2.80 for this parameter is taken from DiCecio and Nelson (2007). Coefficient of labour supply in utility function ϕ is fixed at 1.5 following Fagan and Messina (2009). This value is consistent with the posterior mean reported by Smets and Wouter (2007).

α and δ are the main parameters related to production. To calibrate the share of capital in production α , we took a value of 0.50 which is quite close to the average of capital shares of other developing countries as reported by Liu (2008). Depreciation rate δ has been set at 0.10 which is within the range of values used in the literature for annual DSGE models for developed and developing countries such as $\delta = 0.1255$ as used by Garcia, et al. (2006). In addition, balance sheet analysis of joint stock companies listed at the Karachi Stock Exchange reveals that the overall depreciation rate has been close to 10 percent. Therefore, in order to be consistent with both the existing literature and empirical evidence from Pakistani firms we use the value of 0.10.

The two exogenous shock processes for technology and FDI are estimated using the method of King and Rebelo (2000). Following our estimation we set persistence ρ_A and standard deviation of technology shock σ_A to 0.90 and 0.02 respectively. Similarly ρ_{I^*} and σ_{I^*} are fixed at 0.59 and 0.34 respectively. The data for these estimations of shock related parameters has been acquired from Federal Bureau of Statistics (FBS) and State Bank of Pakistan (SBP). The details of estimation of shock parameters are discussed in the appendix at the end of the paper.

4.2 Results

To evaluate the performance of two models discussed in this paper, we will compare steady-state ratios from the models with their empirical counterpart. Furthermore, second order moments such as standard deviation, contemporaneous correlation with

output and autocorrelation of different variables will also be evaluated from our models and their fit with the empirical evidence presented earlier.

In our closed economy models, the two relevant long-run ratios are private consumption to output ratio and aggregate investment to output ratio. Furthermore, output in this case is defined as the combination of private consumption and aggregate investment. The Table in the appendix shows that our model respectively give values of 0.67 and 0.33 for private consumption to output and aggregate investment to output ratio. The values obtained from our models are similar to the values obtained from the data. However, our models overshoots the value of investment to output ratio and undershoots the private consumption to output ratio.

The Table in the appendix show different second order moments from the two models as well as their empirical counterpart for the period from 1981-2010 and the last decade respectively. We had earlier reported the second order moments for output, aggregate investment, private consumption, government consumption, exports, imports and FDI from annual data. However, since our models are closed economy models we will only compare the second order moments for output, private consumption, aggregate investment and FDI from our models and their empirical counterparts.

The basic RBC model with only technology shock underestimates the absolute and relative volatility of both private consumption and aggregate investment. In particular, the relative volatility of aggregate investment and private consumption are 2.28 and 0.45 respectively according to the RBC model. However, according to Pakistani annual economic data these should be around in the range of 4.50 to 6.32 for aggregate investment and between 0.89 and 1.31 for private consumption.

On the other hand, the augmented RBC model does relatively better for both private consumption and aggregate investment in terms of their absolute volatility. The relative volatility of private consumption with respect to output is 1.04 and relative volatility of aggregate investment is 2.80. The augmented model still underestimates the relative volatility of aggregate investment but it is higher than the simple RBC model. The improvement in relative volatility is significant for private consumption as the relative volatility more than doubles from the simple model version and also lies within the reported empirical range.

In terms of contemporaneous correlation of macro variables with output, simple

RBC model overshoots the correlation for both aggregate investment as well as private consumption. The Table in appendix shows that augmented model does a better job of matching the correlations of both aggregate investment with output and private consumption with output. In particular, correlation of aggregate investment and output for the period 1981-2010 was 0.72 from the data. According to the RBC model this correlation was 0.98 and 0.74 according to the augmented model. Therefore, augmented RBC model outperforms the basic RBC model in terms of matching correlation moments.

Finally, in terms of autocorrelation moments both models perform equally well. Both models do a good job of matching the autocorrelation coefficient of output. However, both models underestimate the autocorrelation of aggregate investment and overestimate the persistence of private consumption. This can be due to the continually changing dynamics of private consumption and aggregate investment and possibly due to the residual nature of computing private consumption in Pakistan.

Overall, the augmented RBC model with technology and FDI shocks outperforms the simple RBC model with only technology shock in terms of matching the empirical second order moments such as volatility, contemporaneous correlation and autocorrelation.

4.3 Impulse Response Functions

After considering the steady state ratios and second order moments for both models and their empirical counterparts, next we analyze the impulse response functions (IRF) generated in response to the respective exogenous shocks for the two models. The simple RBC model has the typical exogenous technology shock. On the other hand, augmented RBC model also has an exogenous FDI shock in addition to the exogenous technology shock.

Figure 7 shows that, in a simple RBC model a positive technology shock leads to a rise in investment, output and consumption. As a result of a positive technology shock investment rises the most followed by output and consumption. These impulse responses are in line with impulse response functions of a typical RBC model.

The Figure 8 shows the impulse response function in response to both technology and FDI shocks for the augmented RBC model. The impulse response functions

for investment, consumption and output in response to a technology shock for the augmented model are almost identical to the IRFs from the simple RBC model.

Interestingly, in response to a positive FDI shock in the augmented RBC model investment, consumption and output all rise immediately. As expected, the magnitude of the rise is largest for investment as FDI shock operates directly through the investment channel. After a few periods, investment falls below the steady state level before reverting back to original pre-shock levels. The impulse response of aggregate investment tapers off after a few periods, in line with the smaller persistence of the exogenous FDI shock. The behaviour of impulse response functions for consumption and output is very similar to their response to an exogenous technology shock. The only difference is their magnitude and persistence is adjusted according to the parameters of the FDI shock. Also, the FDI shocks gets propagated to output indirectly through aggregate investment. This is different from the case of technology shock as technology directly impacts the output. The IRF of consumption displays a hump shape as is already well documented in the literature.

5 Conclusion

In this paper, we establish some basic empirical facts of the ‘documented’ Pakistani economy over the last few decades. In addition, we also conduct detailed analysis using contemporaneous and dynamic correlations to identify the relevant macroeconomic linkages that can explain short run fluctuations in output. This paper also discovers some stylized facts regarding business cycle fluctuations of Pakistani economy in particular over the last few decades.

Furthermore, we evaluate the performance of a typical RBC and an augmented RBC model with an exogenous FDI shock in explaining cyclical fluctuations experienced by the Pakistani economy. We find that augmented RBC model performs better compared to the simple RBC model in terms of matching long-run ratios as well as second order moments of Pakistani economy.

To conclude, although we have discussed in detail the evolution of the economy and where it stands now, it strikes us that the only ‘stable’ stylized fact is the instability that exists in key economic ratios and relationships. This is especially true for the last three decades. What is more of a concern is that this instability has increased over the

last decade. Furthermore, this paper is a small step in understanding and modeling the short term economic fluctuations of a small open economy like Pakistan. However, this paper shows that even a simple model with an exogenous external sector shock can go far in explaining business cycles of the country for the last 30 years and in particular for the last decade.

Shock process $(\rho_A, \rho_{I^*}, \sigma_A, \sigma_{I^*})$

TFP series is obtained by using residuals of estimated neo-classical production function through following regression:

$$\ln Y_t = \alpha \ln K_t + (1 - \alpha) \ln L_t + \ln A_t$$

To estimate ρ_A , we estimate the following equation:

$$\ln A_t = c + \rho_A \ln A_{t-1} + u_t^A$$

σ_A is calculated using residuals of above equation.

Owing to the unavailability of actual data, capital stock series has to be calculated using interpolation methods. There are different ways to calculate capital stock series and parameters of technology shock process are sensitive to variations in capital stock series. Using different series, we get a range of estimates for ρ_A 0.85-0.95 and σ_A 0.0095-0.025. From these ranges, we choose values of 0.9 and 0.02 for ρ_A and σ_A respectively.

To obtain ρ_{I^*} and σ_{I^*} , we estimate the following equation:

$$\ln I_t^* = c + \rho_{I^*} \ln I_{t-1}^* + \mu_t^{I^*}$$

Using log of real per capita FDI, estimation yields values of 0.59 for ρ_{I^*} . Standard deviation of residuals from above regression yields estimate of σ_{I^*} that is 0.34.

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Appendix A

Table 1

Average GDP Share

Decade	Agriculture	Non-Agricultural
1950's	48.0	52.0
1960's	40.4	59.6
1970's	33.7	66.3
1980's	27.6	72.4
1990's	25.4	74.6
2000's	22.6	77.8

Average Share of Agriculture Sector

Decade	GDP Share	Employment Share
1950's	48.0	
1960's	40.4	56.3
1970's	33.7	56.2
1980's	27.6	51.5
1990's	25.4	47.5
2000's	22.6	43.8

Average Employment Share

Decade	Agriculture	Non-Agricultural
1960's	56.3	43.7
1970's	56.2	43.8
1980's	51.5	48.5
1990's	47.5	52.8
2000's	43.8	56.2

Table 2

LONG RUN RATIOS

1960-2010						
	PC/Y	GC/Y	GDFCF/Y	FDI/Y	EXP/Y	IMP/Y
Min	0.68	0.08	0.11	0.00	0.06	0.09
Max	0.84	0.17	0.21	0.04	0.17	0.24
Average	0.77	0.11	0.16	0.01	0.12	0.18
Std.dev	0.04	0.02	0.02	0.01	0.04	0.04
Median	0.76	0.11	0.17	0.01	0.13	0.18
Std.dev/Average	5.2%	17.9%	14.4%	119.5%	28.9%	21.8%
Std.dev/Median	5.2%	18.3%	14.2%	156.0%	27.4%	21.0%

1960-1980						
	PC/Y	GC/Y	GDFCF/Y	FDI/Y	EXP/Y	IMP/Y
Min	0.72	0.10	0.11	0.00	0.06	0.09
Max	0.84	0.15	0.21	0.01	0.15	0.23
Average	0.79	0.11	0.15	0.00	0.09	0.15
Std.dev	0.03	0.01	0.03	0.00	0.03	0.04
Median	0.79	0.11	0.15	0.00	0.08	0.14
Std.dev/Average	4.1%	9.7%	18.3%	132.6%	28.2%	26.2%
Std.dev/Median	4.1%	9.9%	18.5%	267.2%	32.5%	27.6%

Table 2

LONG RUN RATIOS

1981-2010						
	PC/Y	GC/Y	GDFCF/Y	FDI/Y	EXP/Y	IMP/Y
Min	0.68	0.08	0.14	0.00	0.10	0.15
Max	0.83	0.17	0.21	0.04	0.17	0.24
Average	0.75	0.11	0.17	0.01	0.14	0.20
Std.dev	0.04	0.02	0.02	0.01	0.02	0.03
Median	0.75	0.12	0.17	0.01	0.15	0.20
Std.dev/Average	5.1%	21.6%	10.2%	92.3%	13.5%	13.0%
Std.dev/Median	5.1%	21.1%	10.2%	133.9%	13.3%	12.7%

% Change between (1960-1980) and (1981-2010)						
	PC/Y	GC/Y	GDFCF/Y	FDI/Y	EXP/Y	IMP/Y
Min	-4.9%	-19.7%	30.7%	-275.9%	63.9%	69.2%
Max	-1.6%	14.4%	-2.6%	231.9%	17.0%	2.5%
Average	-4.6%	4.1%	10.1%	309.9%	62.6%	29.4%
Std.dev	18.4%	133.4%	-38.3%	185.3%	-21.9%	-35.7%
Median	-5.2%	9.8%	11.8%	469.5%	90.7%	40.2%
Std.dev/Average	24.0%	124.1%	-44.0%	-30.4%	-52.0%	-50.3%
Std.dev/Median	24.9%	112.5%	-44.8%	-49.9%	-59.0%	-54.2%

Table 3A

HP FILTER DATA

1981-2010

	Output	Investment	Private Investment	All Public Investment	Public Investment	Government Investment	Priavte Domestic Investment	Foreign Direct Investment
Output	1.00							
Investment	0.72***	1.00						
Private Investment	0.53***	0.92***	1.00					
All Public Investment	0.71***	0.76***	0.45**	1.00				
Public Investment	0.41**	0.52***	0.23	0.86***	1.00			
Government Investment	0.81***	0.76***	0.59***	0.73***	0.29	1.00		
Priavte Domestic Investment	0.30	0.76***	0.93***	0.23	0.08	0.37**	1.00	
Foreign Direct Investment	0.71***	0.68***	0.58***	0.55***	0.25	0.73***	0.28	1.00

1981-1990

	Output	Investment	Private Investment	All Public Investment	Public Investment	Government Investment	Priavte Domestic Investment	Foreign Direct Investment
Output	1.00							
Investment	-0.06	1.00						
Private Investment	-0.58*	0.30	1.00					
All Public Investment	0.43	0.68**	-0.49	1.00				
Public Investment	0.27	0.79***	-0.18	0.88***	1.00			
Government Investment	0.44	0.15	-0.72**	0.67**	0.23	1.00		
Priavte Domestic Investment	-0.55	0.23	0.95***	0.51	-0.15	-0.82***	1.00	
Foreign Direct Investment	0.07	0.15	-0.20	0.26	-0.03	0.60*	-0.49	1.00

Table 3A

HP FILTER DATA

1991-2000

	Output	Investment	Private Investment	All Public Investment	Public Investment	Government Investment	Private Domestic Investment	Foreign Direct Investment
Output	1.00							
Investment	0.53	1.00						
Private Investment	0.05	0.72**	1.00					
All Public Investment	0.70**	0.69**	0.00	1.00				
Public Investment	0.61*	0.69**	0.04	0.97***	1.00			
Government Investment	0.66**	0.39	-0.14	0.69**	0.48	1.00		
Private Domestic Investment	-0.28	0.48	0.89***	-0.23	-0.16	-0.37	1.00	
Foreign Direct Investment	0.73**	0.37	0.09	0.44	0.37	0.47	-0.37	1.00

2001-2010

	Output	Investment	Private Investment	All Public Investment	Public Investment	Government Investment	Private Domestic Investment	Foreign Direct Investment
Output	1.00							
Investment	0.87***	1.00						
Private Investment	0.84***	0.97***	1.00					
All Public investment	0.52	0.73**	0.61*	1.00				
Public Investment	0.46	0.60*	0.40	0.89***	1.00			
Government Investment	0.34	0.54	0.62*	0.64**	0.22	1.00		
Private Domestic Investment	0.86***	0.92***	0.98***	0.52	0.32	0.59*	1.00	
Foreign Direct Investment	0.73**	0.92***	0.96***	0.57*	0.33	0.66**	0.89***	1.00

Table 3B

HP FILTER DATA

1981-2010

	Output	Investment	Foreign Direct Investment	Private Consumption	Government Consumptions	Exports	Imports
Output	1.00						
Investment	0.72***	1.00					
Foreign Direct Investment	0.71***	0.68***	1.00				
Private Consumption	0.60***	0.23	0.31*	1.00			
Government Consumptions	0.61***	0.58***	0.66***	0.12	1.00		
Exports	0.43**	0.29	0.21	-0.11	0.24	1.00	
Imports	0.76***	0.78***	0.66***	0.60***	0.67***	0.30	1.00

1981-1990

	Output	Investment	Foreign Direct Investment	Private Consumption	Government Consumptions	Exports	Imports
Output	1.00						
Investment	-0.06	1.00					
Foreign Direct Investment	0.07	0.15	1.00				
Private Consumption	0.15	-0.68**	-0.11	1.00			
Government Consumptions	0.66**	-0.03	-0.18	-0.19	1.00		
Exports	0.00	0.56*	-0.35	-0.78***	0.26	1.00	
Imports	0.01	-0.40	-0.52	0.68**	0.11	-0.27	1.00

Table 3B

HP FILTER DATA

1991-2000

	Output	Investment	Foreign Direct Investment	Private Consumption	Government Consumptions	Exports	Imports
Output	1.00						
Investment	0.53	1.00					
Foreign Direct Investment	0.73**	0.37	1.00				
Private Consumption	0.85***	0.56*	0.60*	1.00			
Government Consumptions	0.58*	0.02	0.86***	0.53	1.00		
Exports	0.78***	0.08	0.80***	0.60*	0.84***	1.00	
Imports	0.67**	0.53	0.76**	0.83***	0.75**	0.66**	1.00

2001-2010

	Output	Investment	Foreign Direct Investment	Private Consumption	Government Consumptions	Exports	Imports
Output	1.00						
Investment	0.87***	1.00					
Foreign Direct Investment	0.73**	0.92***	1.00				
Private Consumption	0.54	0.28	-0.01	1.00			
Government Consumptions	0.50	0.76**	0.74**	-0.22	1.00		
Exports	0.41	0.16	0.28	0.06	0.02	1.00	
Imports	0.79***	0.92***	0.78***	0.40	0.73**	0.17	1.00

Table 4

Relevant Moments of HP Filtered Macroeconomic Variables (IFS)

Volatility of Macro Variables				
	1981-2010	1981-1990	1991-2000	2001-2010
Real Output	0.0219	0.0070	0.0222	0.0233
Real Investment	0.0983	0.0185	0.0635	0.1475
Real FDI	0.3849	0.3091	0.3151	0.4353
Real Private Consumption	0.0287	0.0242	0.0291	0.0208
Real Government Consumption	0.1203	0.0509	0.0715	0.1560
Real Exports	0.0776	0.0835	0.0617	0.0621
Real Imports	0.1110	0.0431	0.1019	0.1329

Relative Volatility of Macro Variables				
	1981-2010	1981-1990	1991-2000	2001-2010
Real Output	1.00	1.00	1.00	1.00
Real Investment	4.50	2.63	2.85	6.32
Real FDI	17.61	44.05	14.17	18.66
Real Private Consumption	1.31	3.44	1.31	0.89
Real Government Consumption	5.50	7.26	3.22	6.69
Real Exports	3.55	11.90	2.78	2.66
Real Imports	5.08	6.14	4.58	5.69

Table 4

Relevant Moments of HP Filtered Macroeconomic Variables (IFS)

Contemporaneous Correlation of Macro Variables with Output				
	1981-2010	1981-1990	1991-2000	2001-2010
Real Output	1.00	1.00	1.00	1.00
Real Investment	0.72***	-0.06	0.53	0.87***
Real FDI	0.71***	0.07	0.73**	0.73**
Real Private Consumption	0.60***	0.15	0.85***	0.54
Real Government Consumption	0.61***	0.66**	0.58*	0.50
Real Exports	0.43**	0.00	0.78***	0.41
Real Imports	0.76***	0.01	0.67**	0.79***

Autocorrelation of Macro Variables				
	1981-2010	1981-1990	1991-2000	2001-2010
Real Output	0.58	-0.23	0.23	0.59
Real Investment	0.62	-0.22	0.09	0.61
Real FDI	0.40	-0.26	0.25	0.55
Real Private Consumption	0.37	0.17	0.00	0.10
Real Government Consumption	0.33	-0.13	0.37	-0.09
Real Exports	0.30	-0.24	0.20	0.48
Real Imports	0.51	0.11	0.21	0.36

Table 5

Comparison of Relevant Moments of HP Filtered Macroeconomic Variables

Volatility of Macro Variables				
	1981-2010	2001-2010	RBC Model	Augmented RBC Model
Real Output	0.0219	0.0233	0.0142	0.0305
Real Investment	0.0983	0.1475	0.0324	0.0853
Real Private Consumption	0.0287	0.0208	0.0063	0.0317
Real FDI	0.3849	0.4353		0.3161

Relative Volatility of Macro Variables				
	1981-2010	2001-2010	RBC Model	Augmented RBC Model
Real Output	1.00	1.00	1.00	1.00
Real Investment	4.50	6.32	2.28	2.80
Real Private Consumption	1.31	0.89	0.44	1.04
Real FDI	17.61	18.66		10.36

Contemporaneous Correlation of Macro Variables with Output				
	1981-2010	2001-2010	RBC Model	Augmented RBC Model
Real Output	1.00	1.00	1.00	1.00
Real Investment	0.72***	0.87***	0.98	0.74
Real Private Consumption	0.60***	0.54	0.88	0.47
Real FDI	0.71***	0.73**		0.73

Autocorrelation of Macro Variables				
	1981-2010	2001-2010	RBC Model	Augmented RBC Model
Real Output	0.58	0.59	0.50	0.54
Real Investment	0.62	0.61	0.45	0.31
Real Private Consumption	0.37	0.10	0.69	0.76
Real FDI	0.40	0.55		0.30

Figure 1



Figure 2A (i)

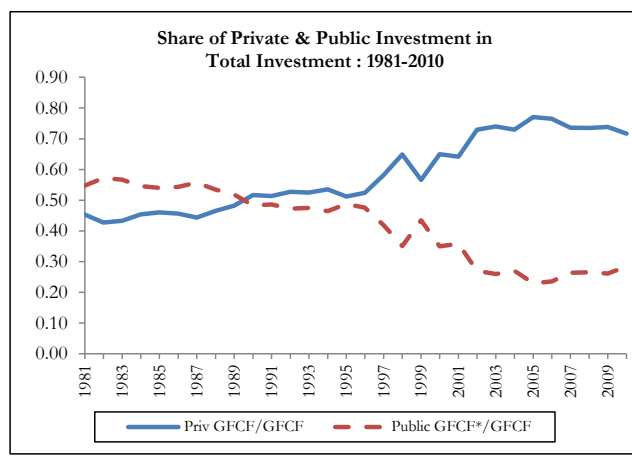
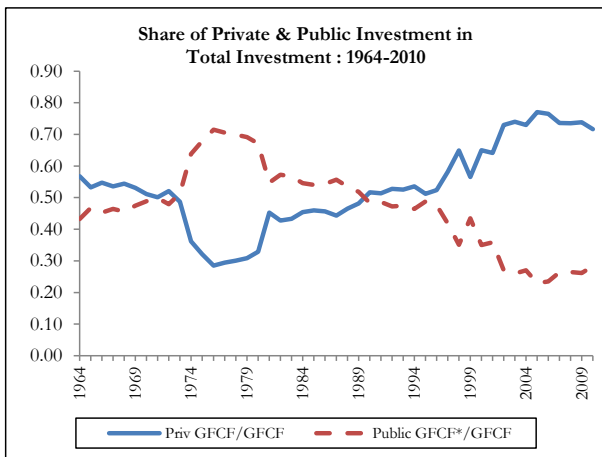
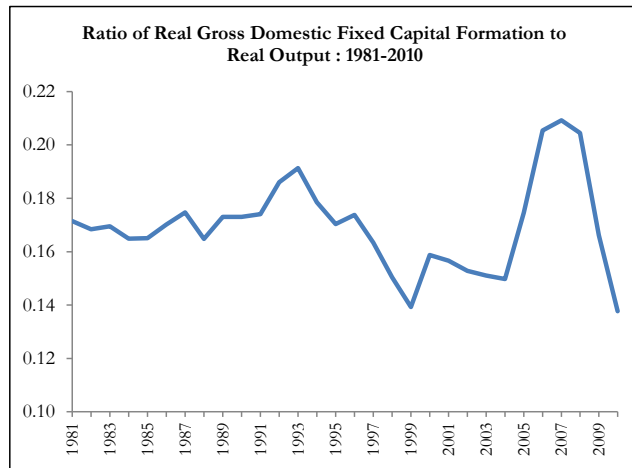
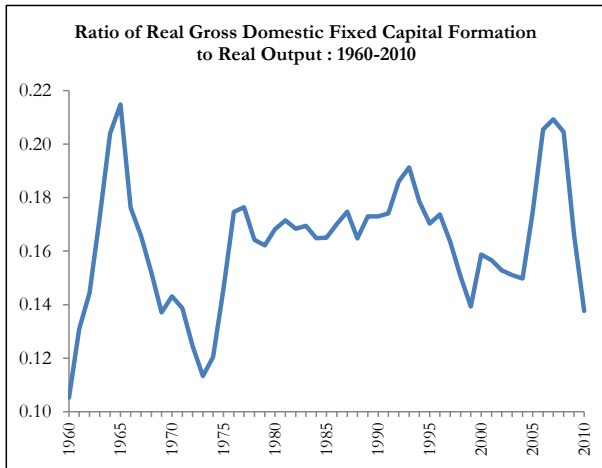


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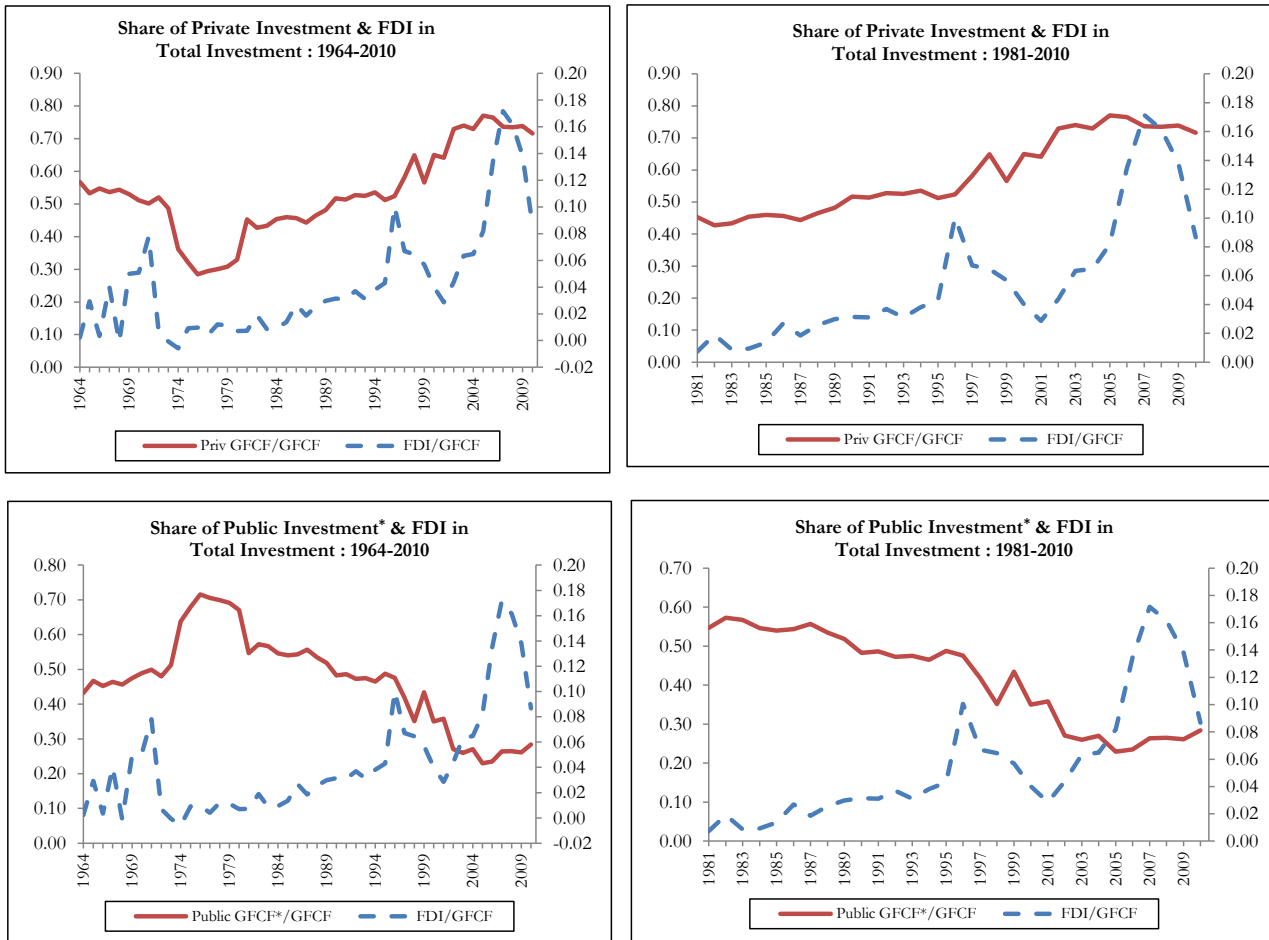


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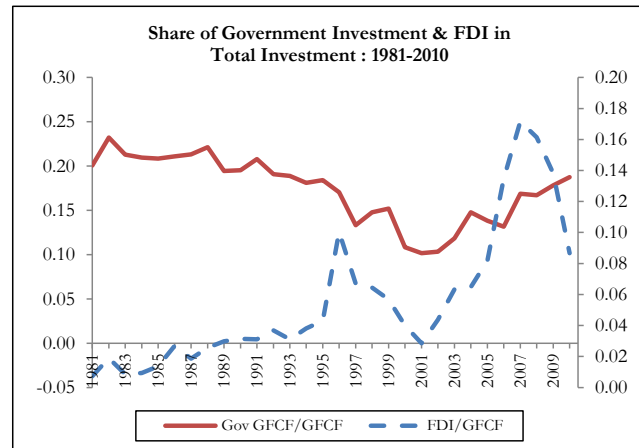
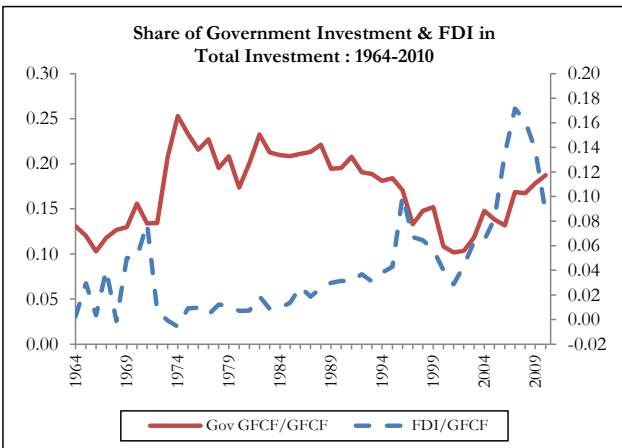
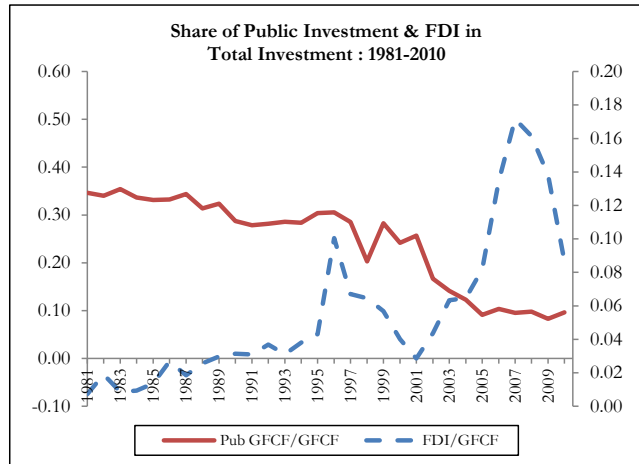
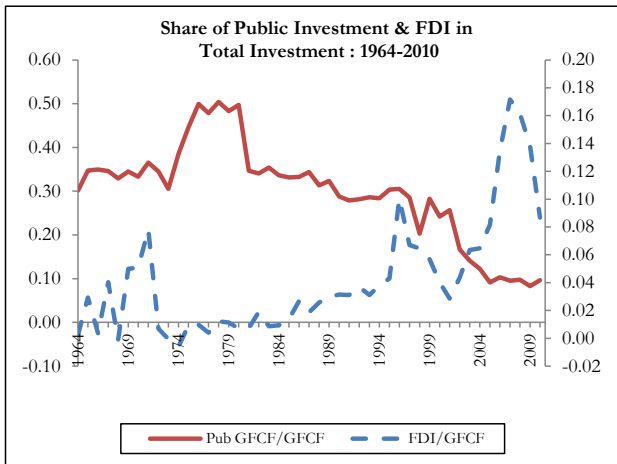


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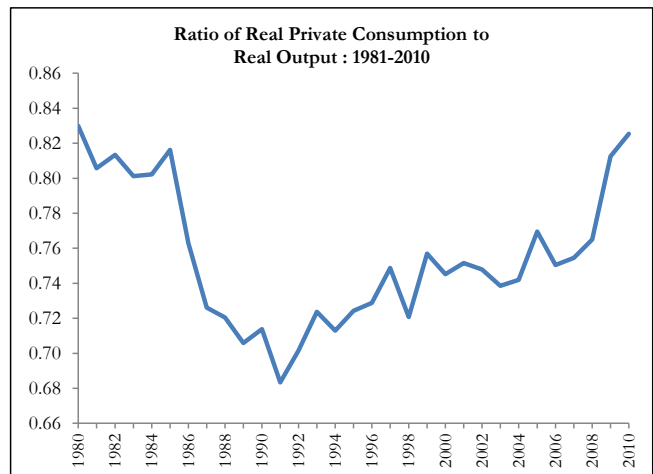
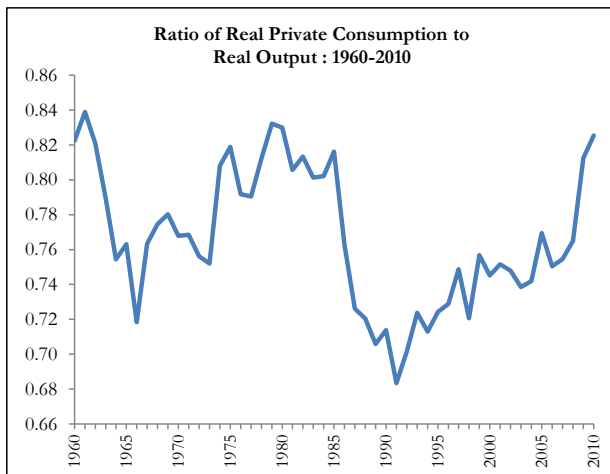
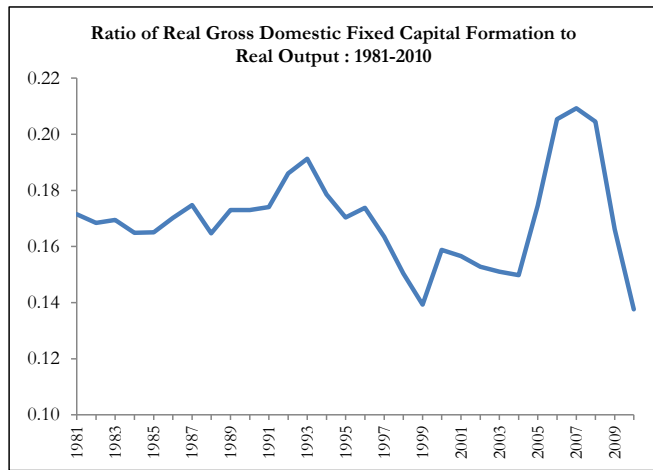
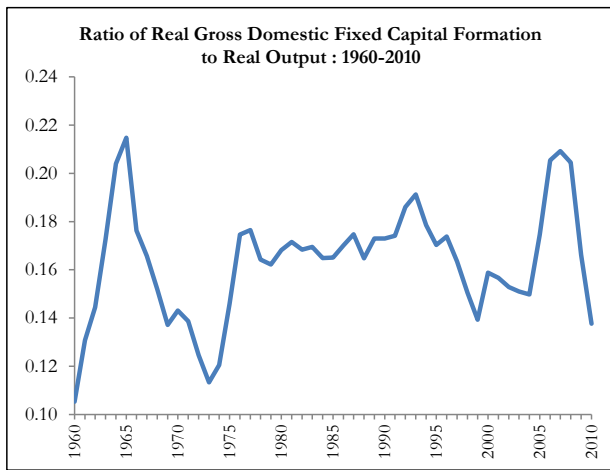


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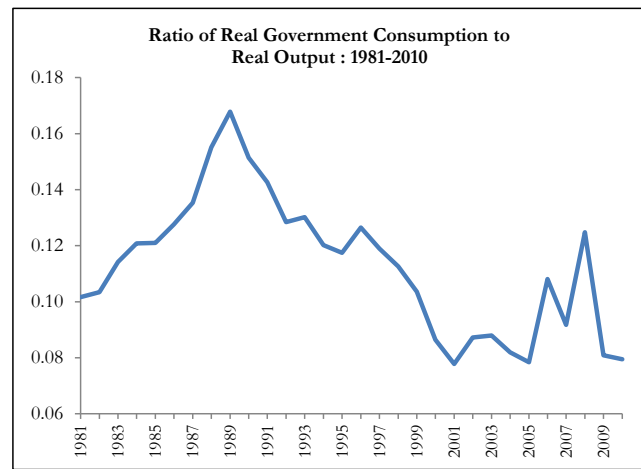
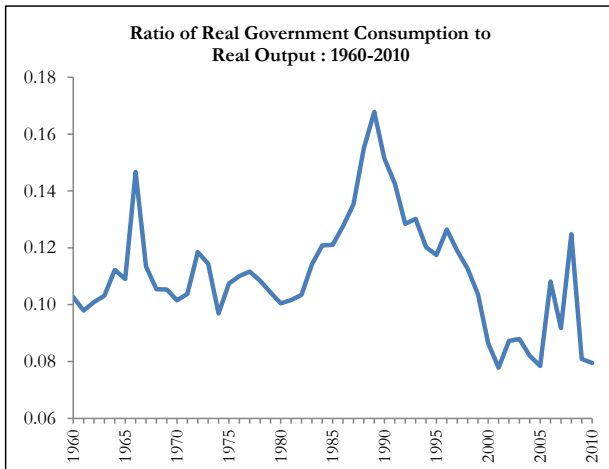
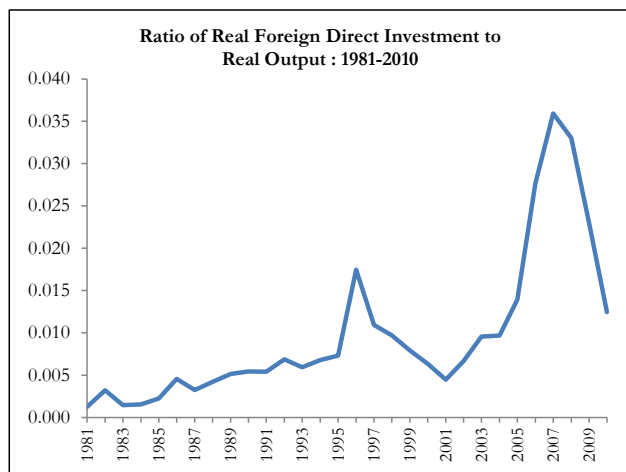
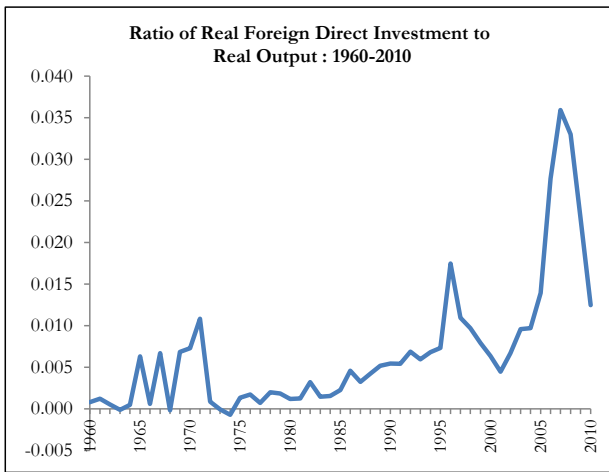


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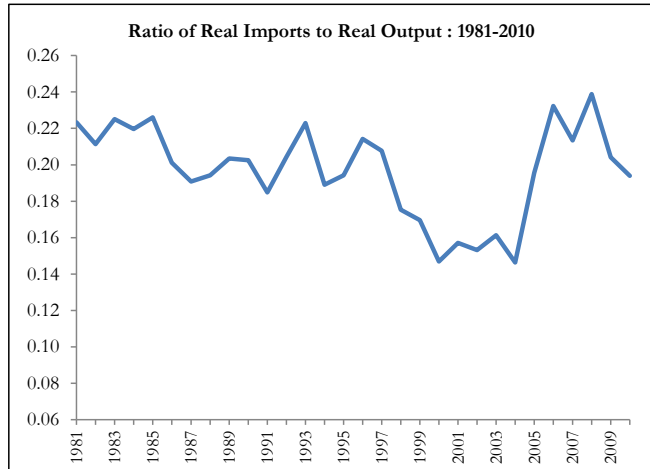
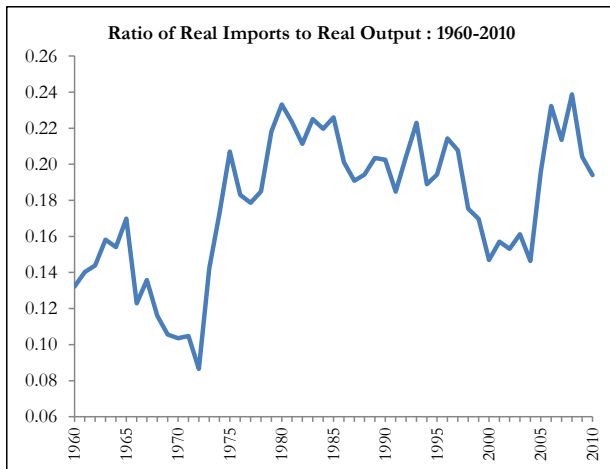
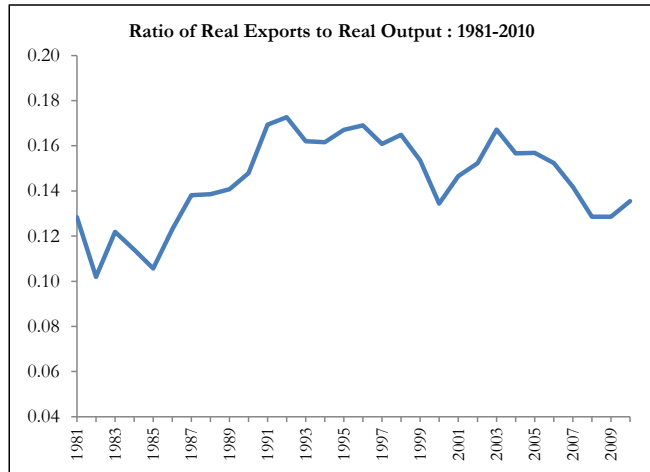
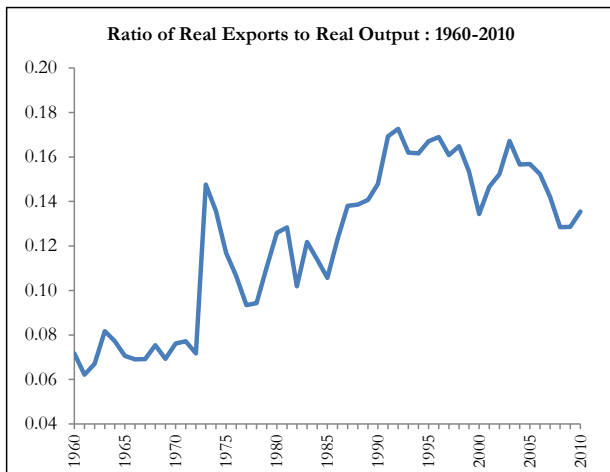


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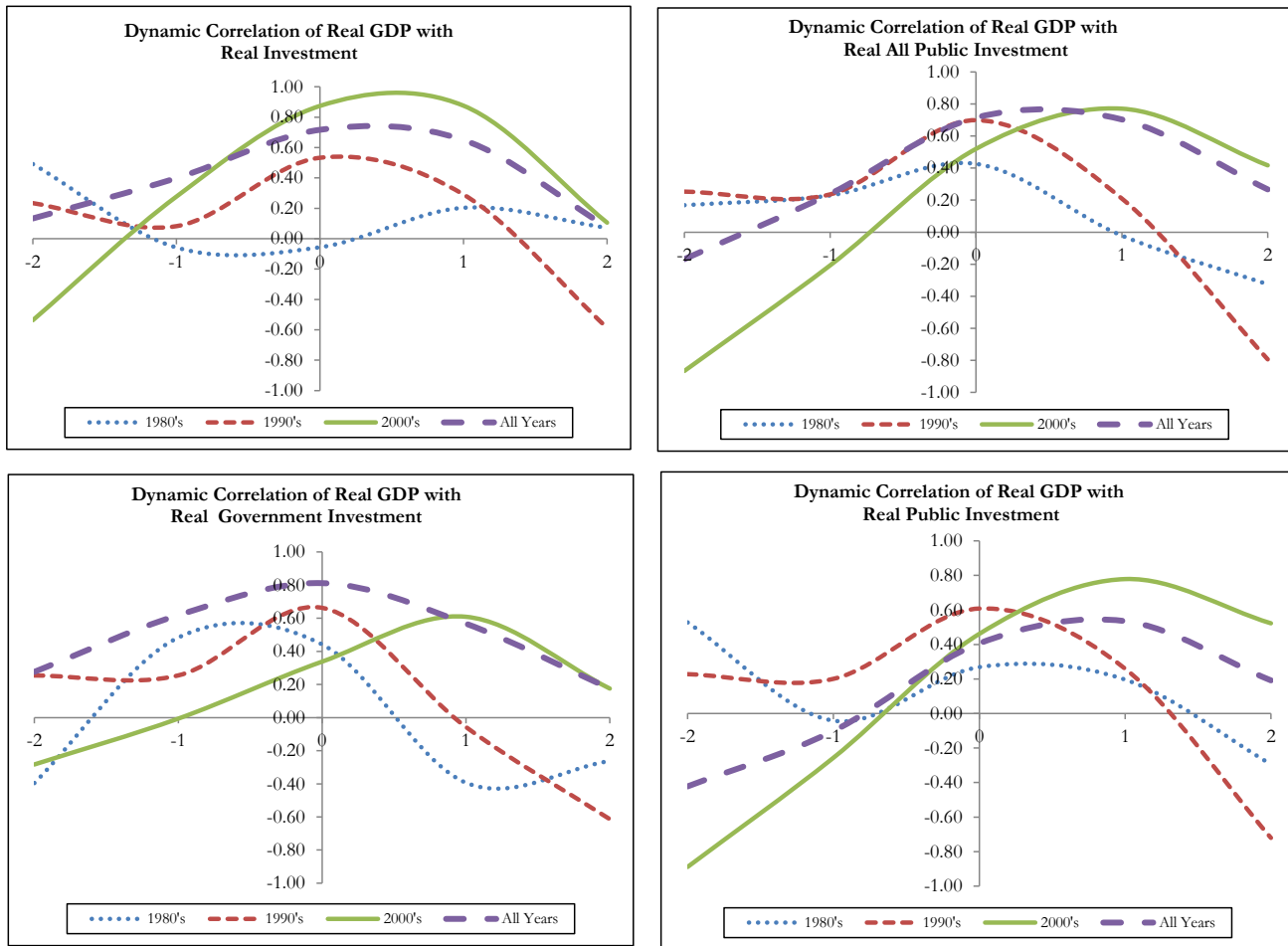


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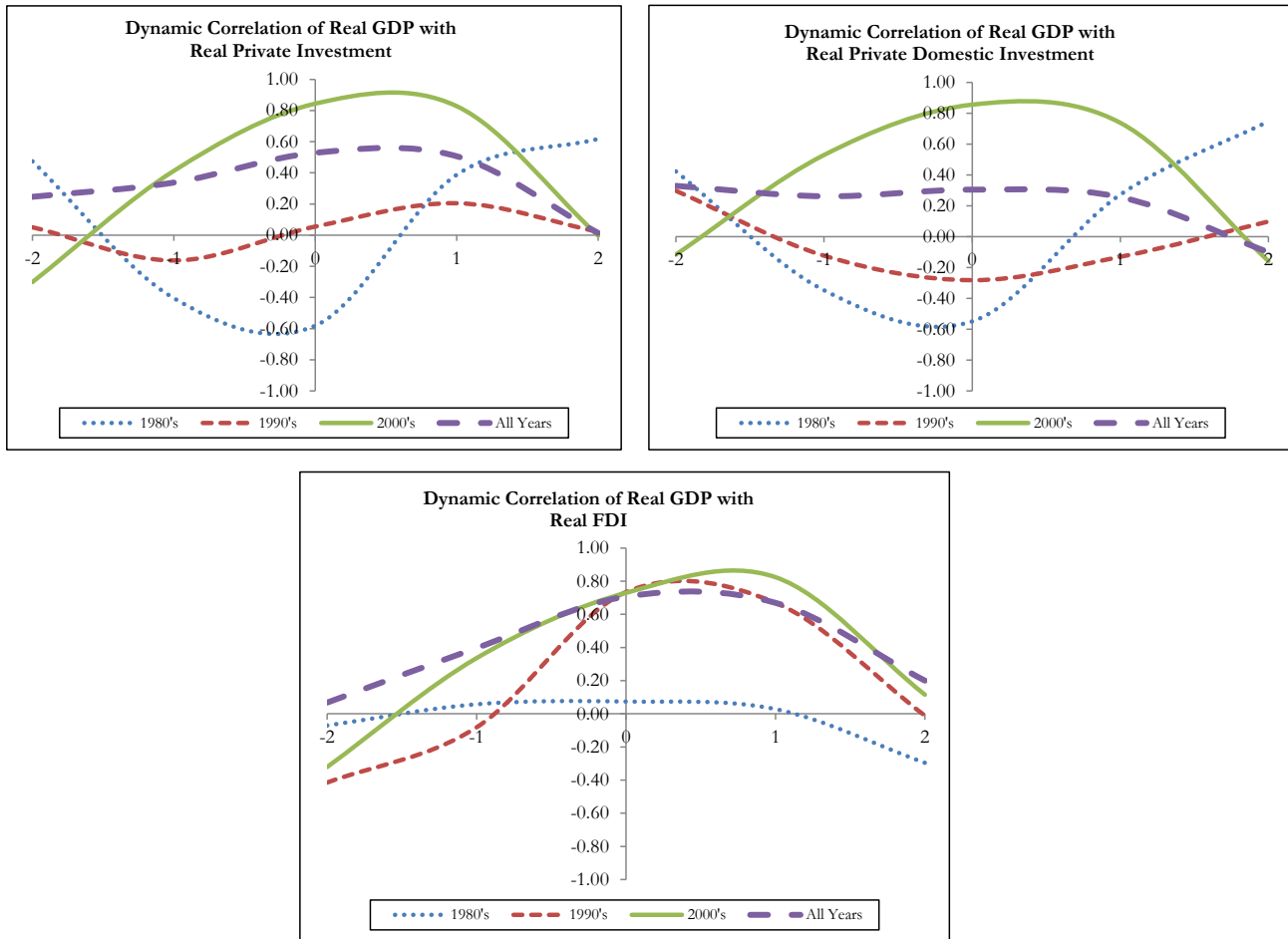


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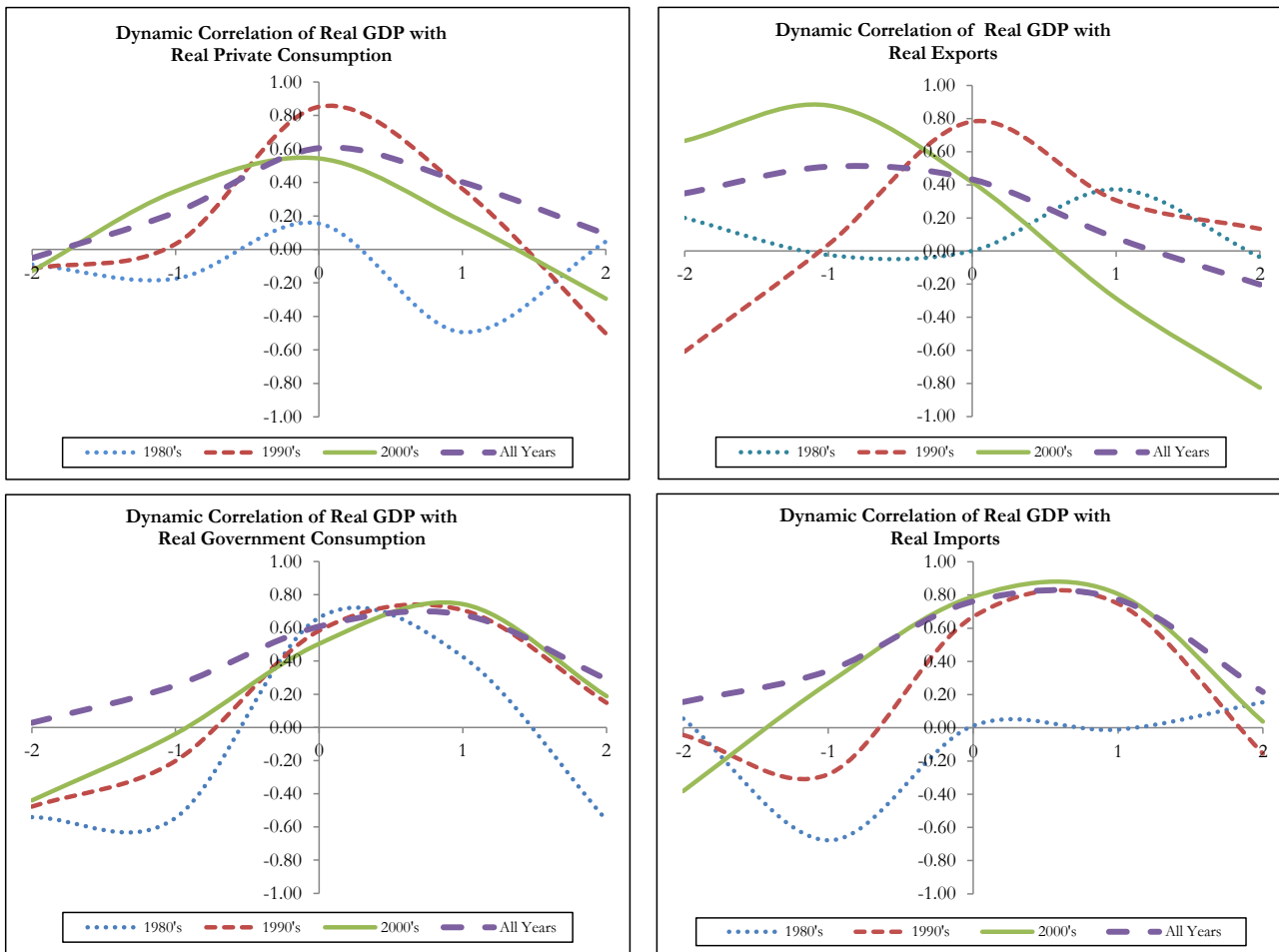


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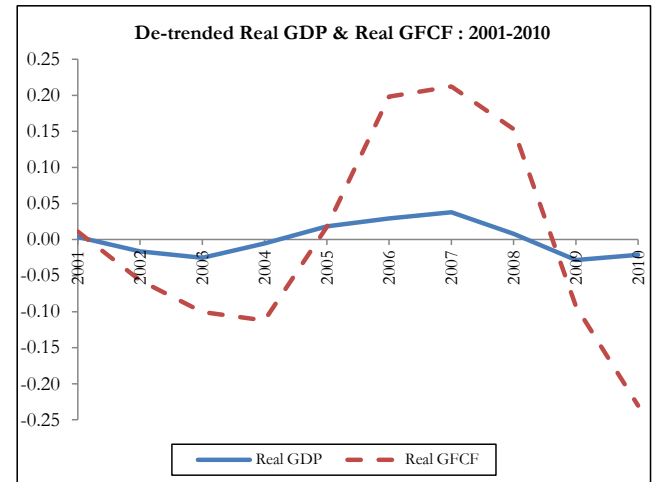
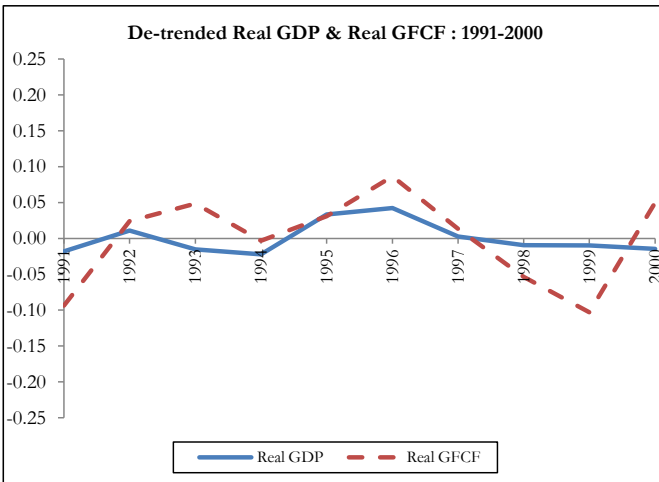
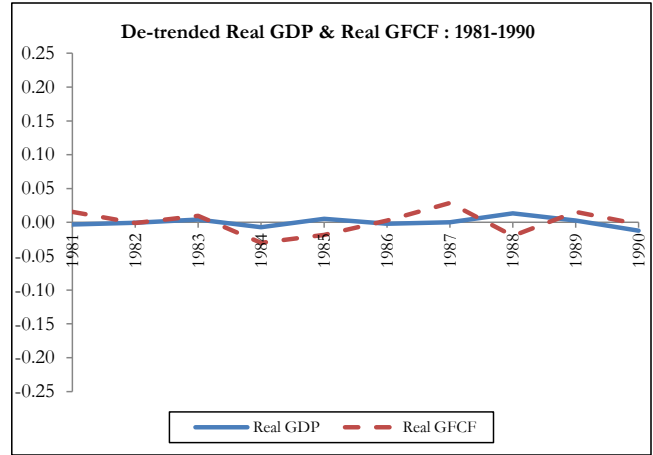
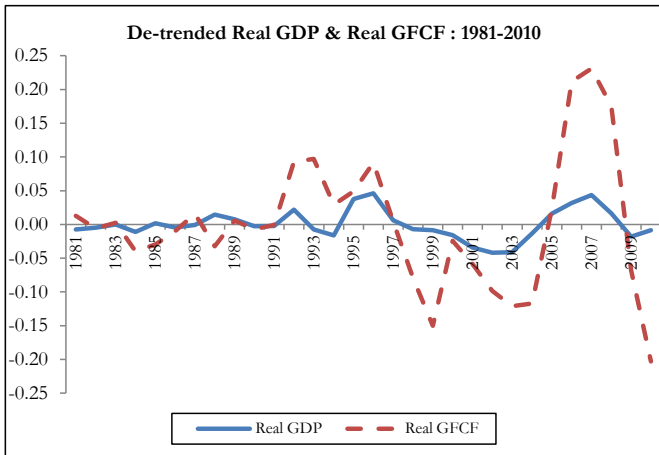


Figure 5 (ii)

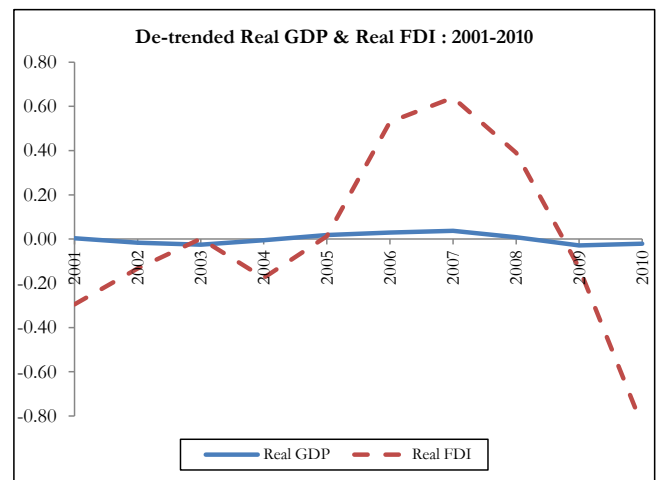
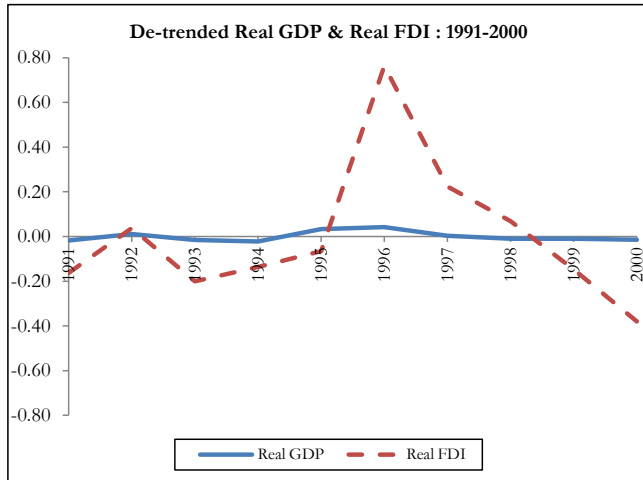
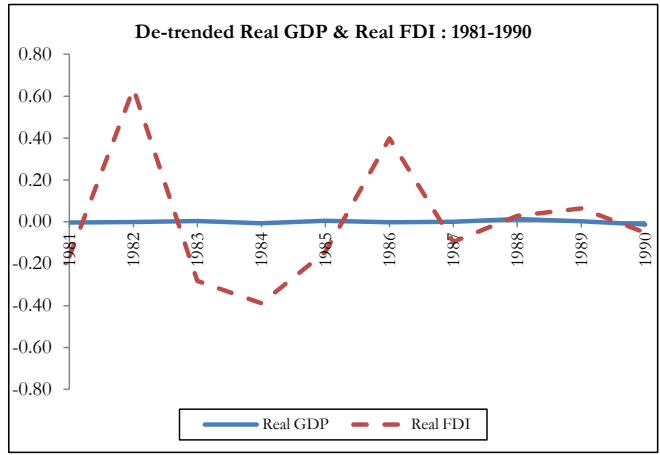
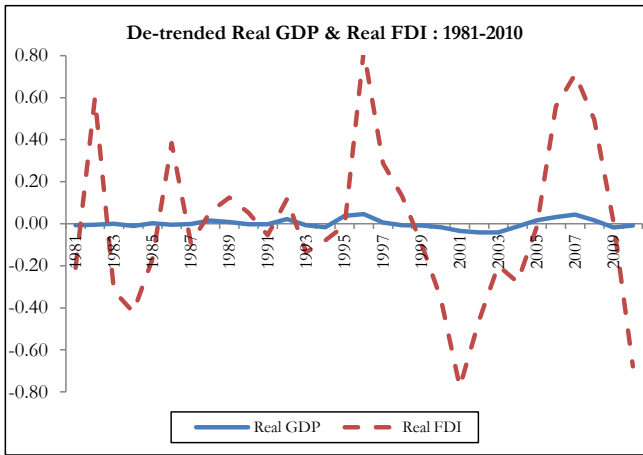


Figure 5 (iii)

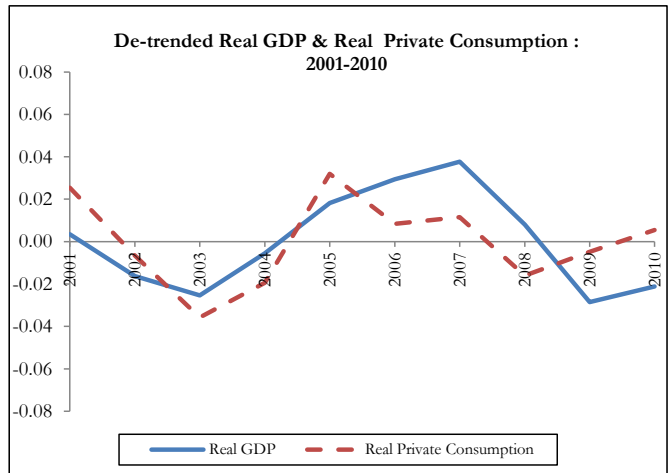
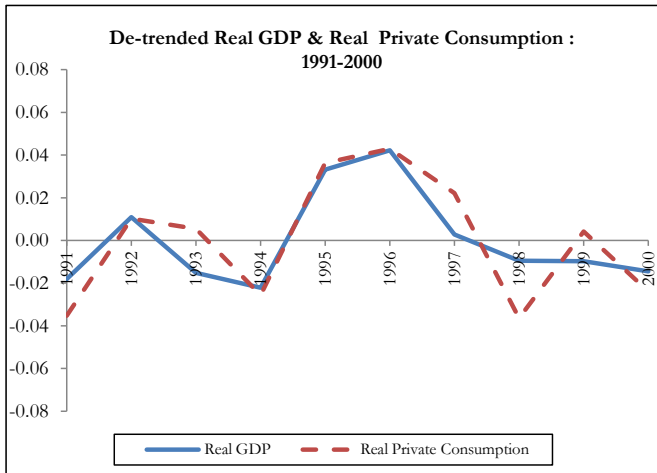
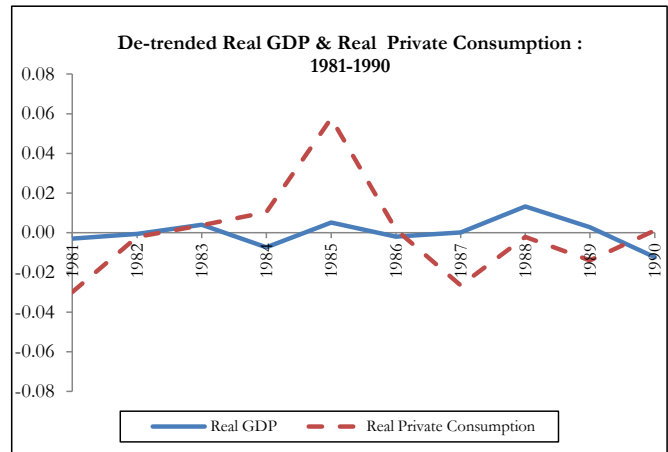
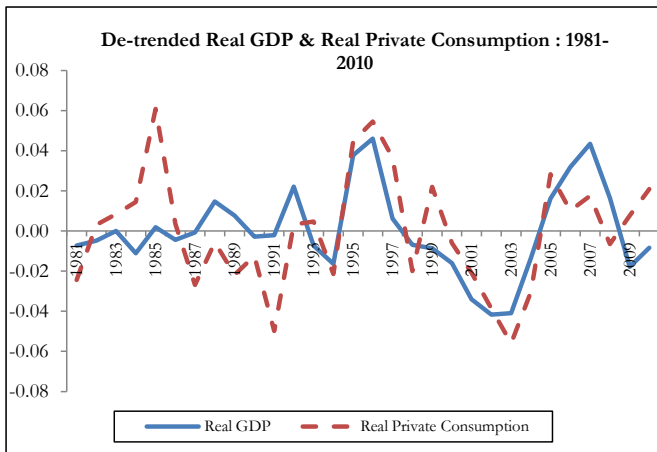


Figure 5 (iv)

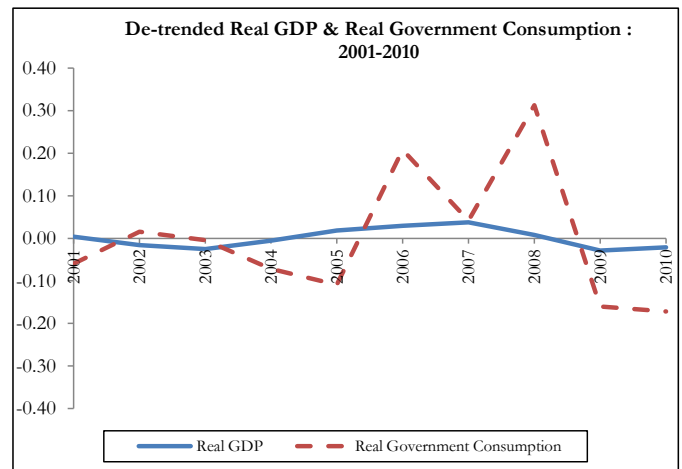
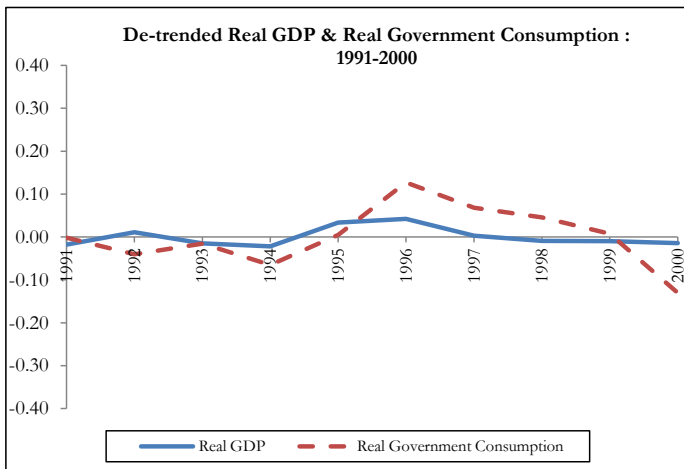
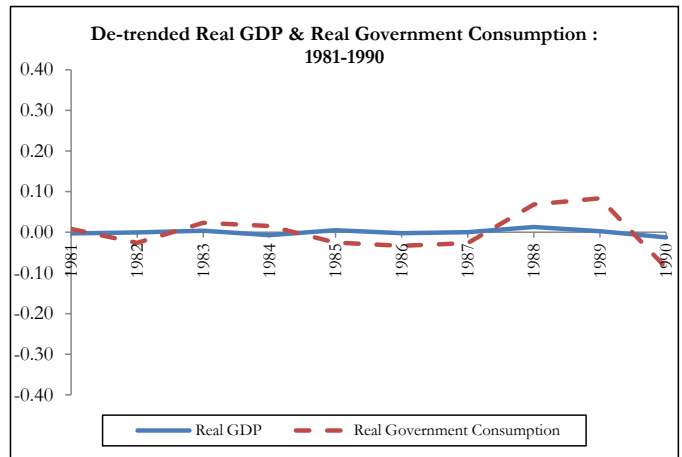
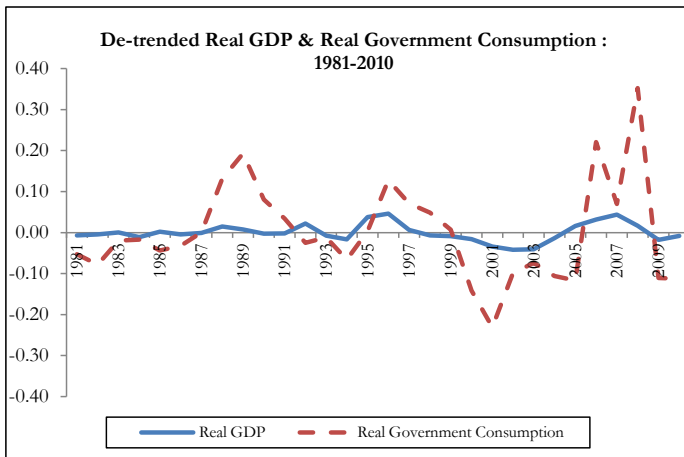


Figure 5 (v)

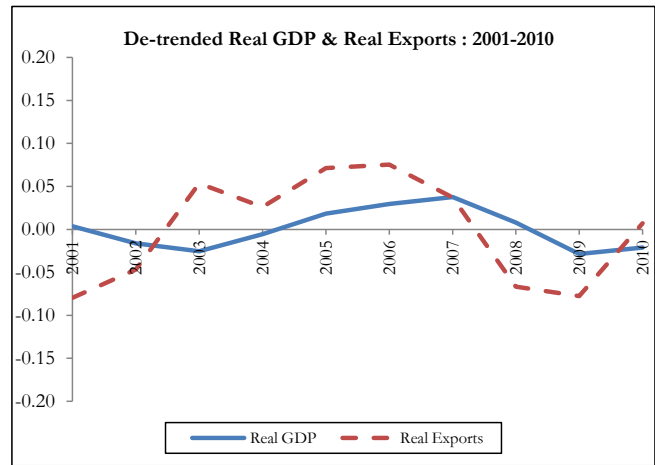
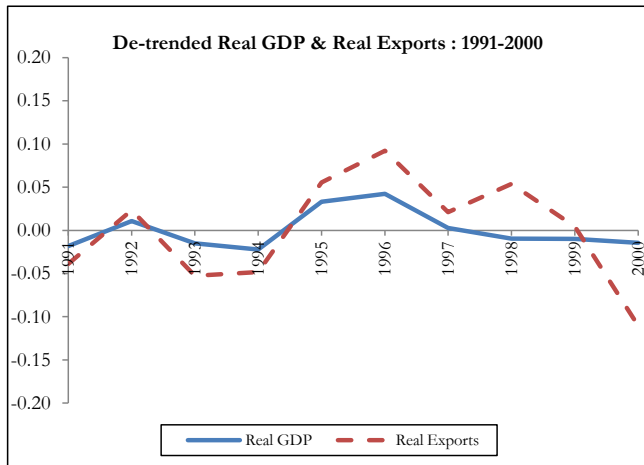
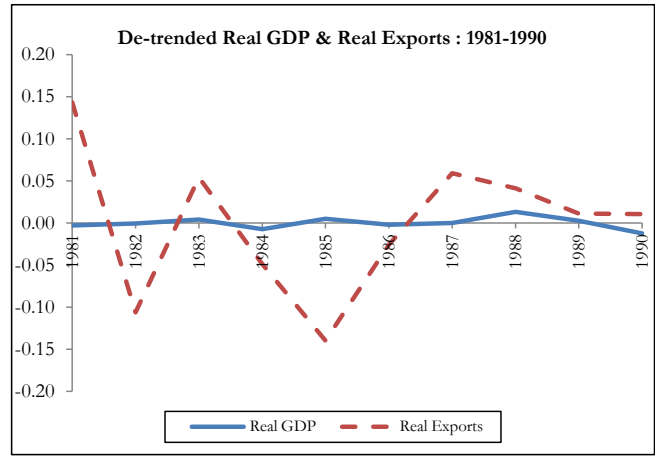
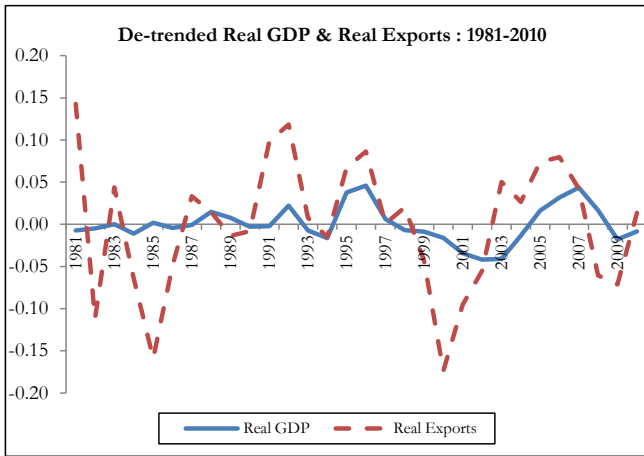


Figure 5 (vi)

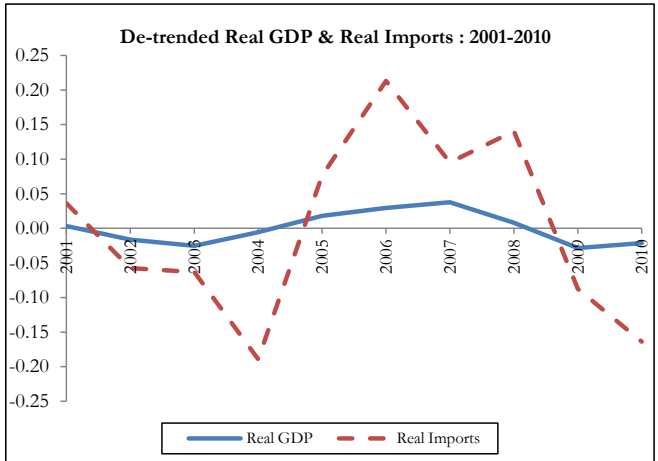
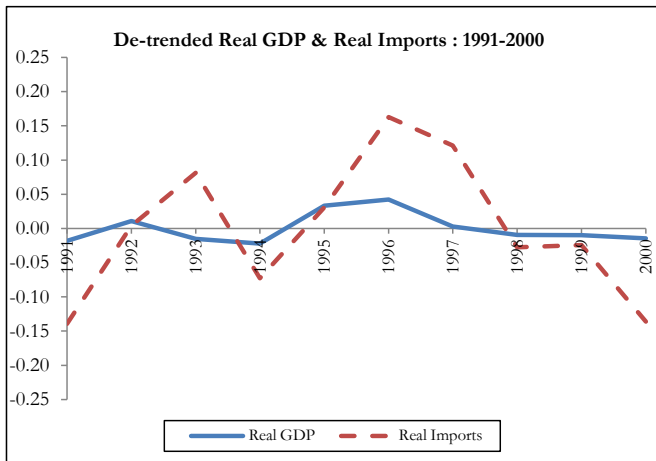
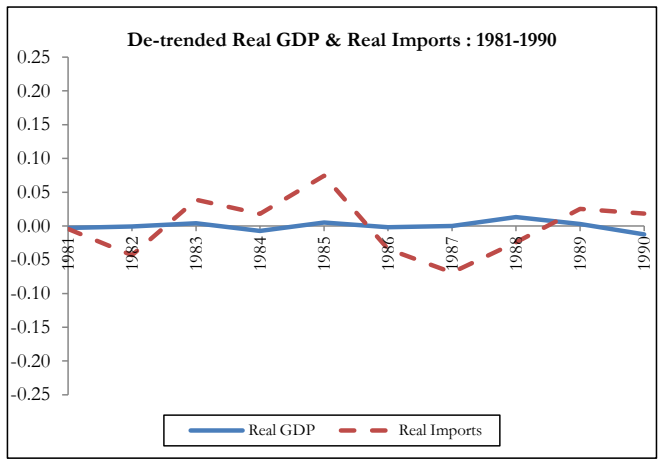
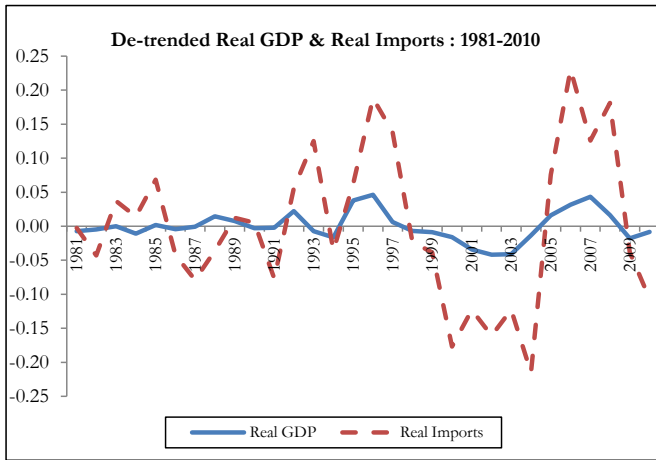


Figure 6

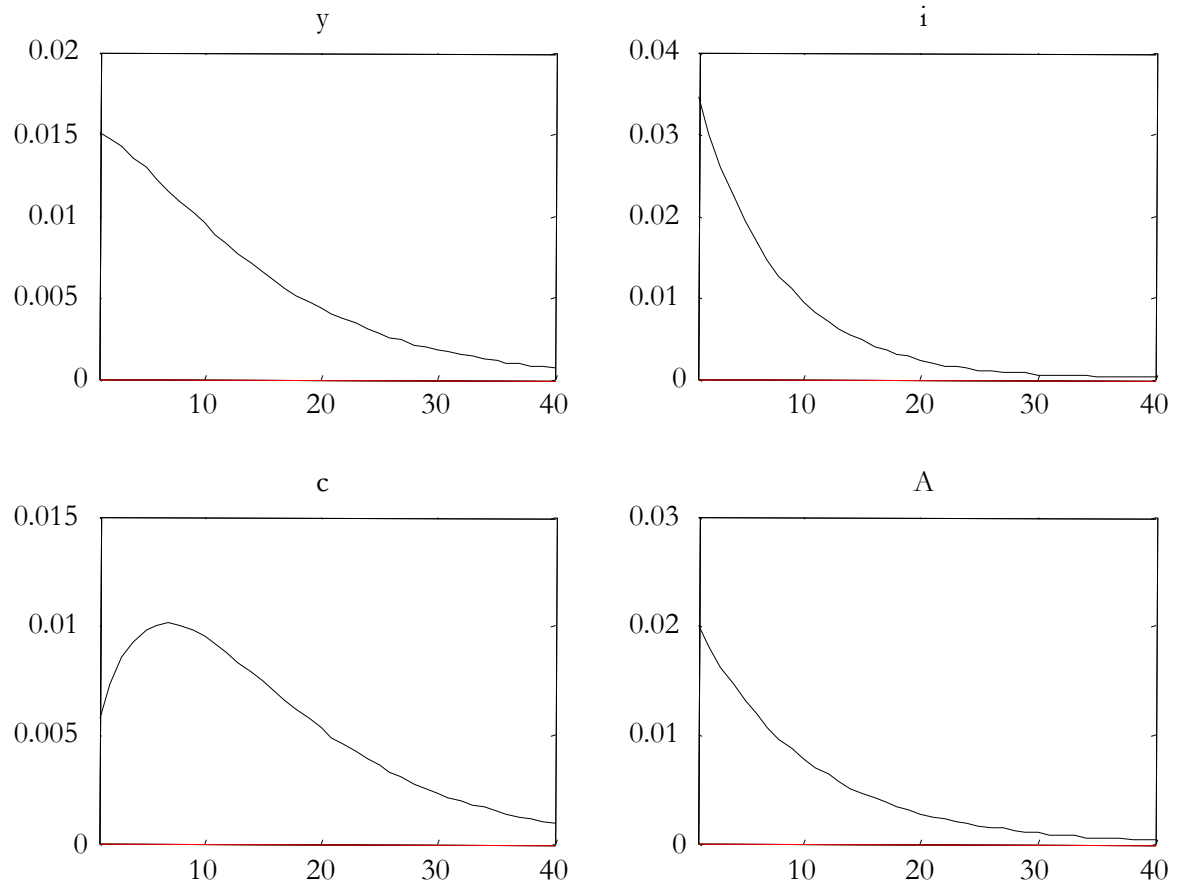


Figure 7 (i)

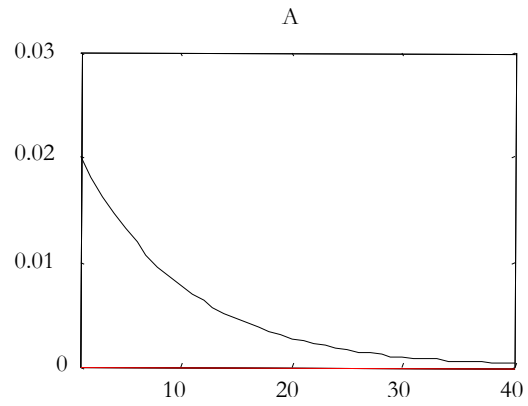
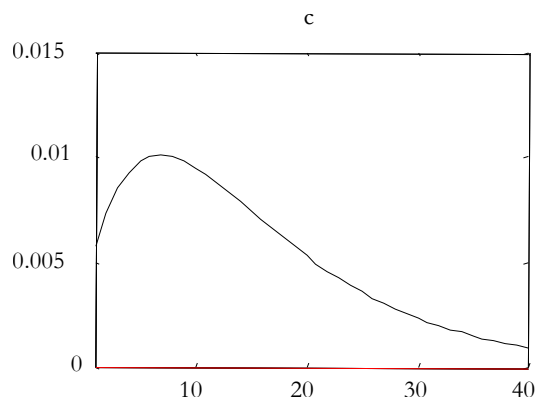
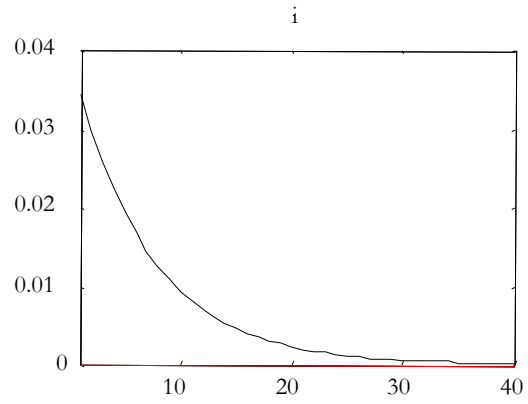
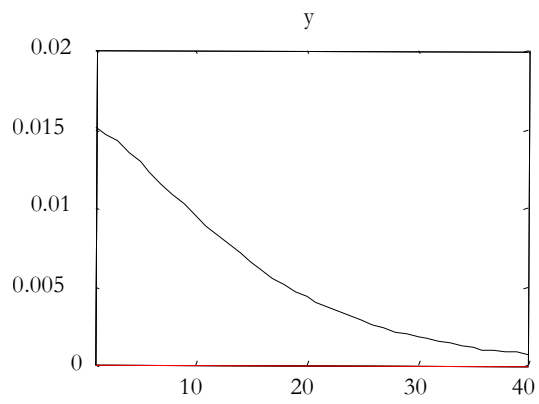


Figure 7 (ii)

