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Shocking Stuff: Technology, Hours, and Factor Substitution

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Shocking Stuff: Technology, Hours, and Factor Substitution*

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Abstract

The response of hours to technology shocks is a key controversy in macroeconomics. We show that differences between RBC and NK models hinge on highly restrictive views of technology. We introduce CES production technologies and demonstrate that the response of hours depends on the factor-augmenting nature of shocks and the capital-labor substitution elasticity in both models. We develop analytical expressions to establish the thresholds determining its sign. This opens new margins for shock identification combining theory and VAR evidence. We discuss how our models provide new robust restrictions for empirical work, especially using the labor income share.

JEL Classification: E32, E23, E25.

Keywords: Technology Shocks, Hours Worked, RBC and NK models, Normalization, Factor Substitution, Factor Bias.

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1 Introduction

The reaction of hours worked to a technology shock has been a key and unresolved controversy in macroeconomics over the last decade. According to the canonical real business cycle (RBC) model, hours worked should rise after a (positive) productivity shock. However, in an influential paper, Galí (1999), using a structural VAR (SVAR), found the impact to be negative. This evidence has since been interpreted as favoring the New-Keynesian (NK) sticky-price model. Thereafter, testing and analyzing the technology hours response generated a substantial literature (for recent analysis from a variety of standpoints, see Basu et al. (2006), Alexopoulos (2011), Holly and Petrella (2012)). These differences in hours responses have been taken as a means of discriminating between different theories of business-cycle fluctuations. These theory differences, however, crucially hinge on a restrictive view of how production and technology relations work over the business cycle.

Our contribution is to analyze the consequences of richer supply and technology considerations in business-cycle models with particular focus on the reaction of hours after a technology shock. We do so by introducing Constant Elasticity of Substitution (CES) production techniques and factor-biased technology shocks in such models. With this less constrained specification, we show that the response of hours depends on the factor-augmenting nature of technology shocks and the capital-labor substitution elasticity in *both* RBC and NK models. Importantly, both can generate technology-hours responses of either sign. Nevertheless, these response signs can be pinned down to a set of core parameters for each model. They have threshold characteristics and we provide analytical expressions for the critical values of the elasticity of capital-labor substitution that would lead to a sign change. As a side product, we also show the methodological importance of supply-side *normalization* when technology is not Cobb-Douglas. The introduction of these hitherto unexplored factors opens up new margins for shock identification and model comparison combining theory and VAR evidence. Specifically, we discuss how the models provide us with new robust restrictions for empirical work, especially making use of the labor income share.

There are solid theoretical and empirical reasons for studying the effect of capital-labor substitution and biased technology shocks in business cycle models. Indeed, despite modern controversies, it is easy to forget that the effect of technical change on employment is a long-standing debate in economics – e.g., see Wicksell (1911)'s discussion of the historical "machinery question". The traditional Ricardian effect – defended by Hicks (1969) – supported the idea that technological advancement reduces employment in the short run, but increases it in the long run. The kind of mechanism envisaged, however, did not rest on the introduction of nominal rigidities that characterizes much of modern macroeconomics. It relied on aspects of the production process such as the degree to which different factors substitute or complement one another, and the extent to which technical change may be non neutral. Our approach is important precisely because modern business cycle models have largely abstracted from these aspects. They tend to impose aggregate (unitary elasticity) Cobb-Douglas production functions. This is unfortunate since this form is highly restrictive. Further, it cannot separately identify labor- and capital-augmenting technical progress.

Empirically, the Cobb-Douglas function is also routinely rejected by the data: Klump et al. (2007) and Chirinko (2008) suggest 0.4-0.6 as a benchmark elasticity range for the US. **Table 1** surveys studies which have reported both the estimated elasticity of substitution and factor-augmenting technical change (the latter either individually or jointly) for the US. Not only is the substitution elasticity typically below unity, but *both* forms of capital and labor augmentation are found to be present in the data (with the latter dominating), e.g., Sato (2006), Klump et al. (2007). Importantly, factor income shares also typically exhibit such protracted swings and trends over the business cycle as to be inconsistent with Cobb-Douglas (see Blanchard (1997), Jones (2003) and McAdam and Willman (2013)). Further, as Acemoglu (2009; chap. 15) points out, there is little reason to suppose that, over business cycles, technical change will be neutral.

The choice of highly restrictive production-technology relations may then be considered especially startling given the avowed interest of the literature in analyzing the cyclical importance of "technology" shocks. Contrast this with modern growth, public finance, labor and innovation literatures (e.g., Acemoglu (2009), Chirinko (2002), Acemoglu and Autor (2011)) where non-unitary substitution and factor-augmenting technical progress are key explanatory elements.

Our paper is related to a large and still evolving theoretical and empirical literature on the hours-technology correlation. Much of the first wave of research fell into one camp or another. There is, arguably, an increasing consensus fa-

Study	**Sample**	σ	$\gamma_N : \gamma_K$
	1890-1918	0.35	$\gamma_N - \gamma_N = 0.48$
Brown and Cani (1963)	1919-1937	0.10	$\gamma_N - \gamma_K = 0.62$
	1938-1958	0.11	$\gamma_N - \gamma_K = 0.36$
David and van de Klundert (1965)	1899-1960	0.32	2.2 : 1.5
Wilkinson (1968)	1899-1953	0.50	$\gamma_N - \gamma_K = 0.51$
Sato (1970)	1909-1960	$0.50 - 0.70$	2.0 : 1.0
Panik (1976)	1929-1967	0.76	$\gamma_N - \gamma_K = 0.27$
Antràs (2004)	1948-1998	0.80	$\gamma_N - \gamma_K = 3.15$
Klump et al. (2007)	1953- 1998	$0.50 - 0.70$	1.5 : 0.4
León-Ledesma et al. (2012)	1960- 2004	$0.40 - 0.70$	1.6 : 0.7

Table 1: Empirical Studies of the Aggregate Elasticity of Substitution and Technological Change in the US.

voring the existence of a negative response of hours to technology shocks (e.g. Canova et al. (2010)). A second (largely empirical) wave appears to favor time-varying correlations, e.g., Galí and Gambetti (2009). Some studies suggested that non-standard theoretical results could emerge. For example, it is known that an RBC model could also generate a negative technology-hours response if relative risk aversion was sufficiently high.[1] Rotemberg (2003) also showed that an RBC model with protracted technical diffusion could generate a negative technology-hours impact. Likewise, Francis and Ramey (2005) showed that hours fall in response to a labor-saving technology shock in an RBC model in the limit case of Leontief technology with a particular utility form. The limiting case of Leontief, however, tells us little about outcomes over more plausible technological configurations (i.e., positive substitution elasticities *as well as* factor-augmenting shocks). Moreover, although at business-cycle frequencies low substitution elasticities might be expected, *zero* is a uniquely strong assumption with the counter-

[1]This case is replicated in the online Appendix although we demonstrate that it holds only for gross complements.

factual (and essentially knife-edge) implication that output shares of capital and labor approach one-half.

The paper proceeds as follows. The next section discusses the properties of the CES production function and the methodological issue of normalization. Section 3 presents the RBC and NK models. Section 4 discusses the calibration and presents some key simulation results. Section 5 derives threshold conditions (general and simplified) determining the sign of the response of hours to technology shocks. Section 6 discusses robustness and empirical identification issues arising from the models. Finally, we conclude.

2 The Normalized CES Production Function

The CES production function relating output (Y_t) to capital (K_t) and labor (H_t) can be represented as,

$$Y_t = Y_0 \left[\alpha_0 \left(\frac{K_t}{K_0} \Gamma_t^K \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{H_t}{H_0} \Gamma_t^H \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where α_0 is the distribution parameter, σ is the elasticity of capital-labor substitution, and Γ_t^K and Γ_t^H represent capital- and labor-augmenting technology respectively. Function (1) is represented here in "normalized" form, i.e., in index number form at $t = 0$, as will be discussed below.

The CES production function (1) nests Cobb-Douglas when $\sigma = 1$; the Leontief function (i.e., fixed factor proportions) when $\sigma = 0$; and a linear production function (i.e., perfect factor substitutes) when $\sigma \to \infty$. When $\sigma < 1$, we say that factors are gross complements in production and gross substitutes otherwise.

In business-cycle models factor substitutability and non-neutral technical change will matter in so far as they influence output, relative prices, income shares, and costs. Movements in these variables affect the inter-temporal decisions of consumers and firms. Some indications of the key role played by factor substitution can be gauged from the following. Assuming competitive markets and profit maximization, relative factor income shares and relative marginal products are (dropping time and normalization subscripts for convenience):

$$\Theta = \frac{rK}{wH} = \frac{\alpha}{1-\alpha}\left(\frac{\Gamma^K K}{\Gamma^N H}\right)^{\frac{\sigma-1}{\sigma}} \tag{2}$$

$$\iota = \frac{F_K}{F_H} = \frac{\alpha}{1-\alpha}\left[\left(\frac{K}{H}\right)^{-\frac{1}{\sigma}}\left(\frac{\Gamma^K}{\Gamma^H}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{3}$$

It is straightforward to show that the effect of technical bias and capital deepening on factor income shares and relative factor prices is related to whether factors are gross complements or gross substitutes:

$$sgn\left\{\frac{\partial\iota}{\partial\left(\Gamma^K/\Gamma^H\right)}\right\}, sgn\left\{\frac{\partial\Theta}{\partial\left(K/H\right)}\right\}, sgn\left\{\frac{\partial\Theta}{\partial\left(\Gamma^K/\Gamma^H\right)}\right\} = sgn\left\{\sigma-1\right\} \tag{4}$$

Hence, an increase in factor $J-$augmenting $(J = K, H)$ technical change "favors" factor J (i.e., implying $(\partial F_J/\partial\Gamma^J)/(\partial F_I/\partial\Gamma^J) > 1$, $J \neq I$, and raising J's income share for given factor proportions) if factors are gross substitutes $(\sigma > 1)$. The effects reverse if factors are gross complements. Thus, it is only in the gross-substitutes case that a factor J-augmenting change in technology is

J-biased. Naturally, the relations between the substitution elasticity, technical bias and factor shares evaporate under Cobb Douglas: factor income shares are constant and relative factor prices are purely determined by capital deepening. Equations (2)-(4) illustrate the importance of factor substitution and technical biases. The impact of technology shocks on factor payments depends on the substitution elasticity and the factor bias of the shock. This influences the dynamic response of interest and wages (and hence hours) to technology shocks. Note, though, statement (4) defines factor *demand* reactions to technology changes. They therefore abstract from labor-supply reactions.

It is important to stress that (1) is expressed in "normalized" form. At a simple level, one can think of normalization as removing the problem that arises from the fact that labor and capital are measured in different units. Under Cobb Douglas, normalization plays no role since, due to its multiplicative form, differences in units are absorbed by the scaling constant. The CES function, by contrast, is highly non-linear. Hence, unless correctly normalized, out of its three key parameters – the efficiency parameter, the distribution parameter and the substitution elasticity – only the latter is deep, that is, it is not a function of other parameters and can then be independently calibrated (see La Grandville (2009), León-Ledesma et al. (2010), Klump et al. (2012), Cantore and Levine (2012)). The other two parameters turn out to be affected by the value of the substitution elasticity and factor income shares. Accordingly: i) if one is interested in model sensitivity with respect to production parameters (as here), normalization is essential to have interpretable comparisons; and ii) without a proper normalization, nothing ensures that factor shares equal the distribution parameter, hence invalidating inference based on impulse-response functions (IRFs).

A logical way to proceed is then to choose a steady state and then calibrate the model using this as the normalization point. We can, for instance, set Y_0 and H_0 to unity. Since the real interest rate is determined by preferences and depreciation, we can then, given the income/factor income identity, $\underbrace{Y_0}_{=1} \equiv \underbrace{r_0 K_0}_{\alpha_0} + \underbrace{w_0 H_0}_{1-\alpha_0}$, define the steady-state capital stock as $K^* = \alpha_0 / r_0$. Here, α_0 and r_0 are the capital income share and real interest rate at the steady state. The real normalized/steady-state wage is solved as $w^* = 1 - \alpha_0$. This procedure ensures that the model is consistent, factor shares sum to unity and consumption plus investment equals output.

3 The Model(s)

The model used in subsequent analysis is a variant of the neoclassical growth model, where business cycles are primarily due to technology shocks. The economy is populated by a representative agent maximizing utility and providing capital and labor to the representative firm. We introduce adjustment costs to investment, monopolistic competition, and Calvo pricing in order to introduce real and nominal rigidities in the analysis and to allow a role for monetary policy. It can be shown that, by setting to zero investment adjustment costs, the markup, and price stickiness, the dynamics of this general model reduce to the standard RBC flex-price model.

To be consistent with the identification strategies used in the empirical literature and with the existence of a balanced growth path (BGP), we assume a unit root in (the log of) labor-augmenting technical progress (z_t^H).[2] That is, only labor-augmenting shocks can have a long run impact on productivity. Note that only labor-augmenting shocks can be permanent, as permanent capital-augmenting shocks would change steady state factor shares as is well known from the BGP theorem (see Uzawa (1961)). We then define the log capital-augmenting (z_t^K) and log Hicks-neutral ($z_t^{\mathbb{H}}$) technology shocks with a stable AR(1) process:

$$z_t^m = \rho z_{t-1}^m + \varepsilon_t^m, \ \rho < 1, m = K, \mathbb{H} \tag{5}$$

$$z_t^H = z_{t-1}^H + \varepsilon_t^H. \tag{6}$$

Given the presence of a unit root in the labor-augmenting shock, we stationarize the model by rescaling the real variables with the non-stationary process $e^{z_t^H}$. These rescaled real variables are denoted by a "ˆ". Moreover, we define the gross rate of growth of labor-augmenting technical progress as $g_t^H = e^{z_t^H}/e^{z_{t-1}^H}$.

The NK model (and its nested RBC form) is well known and can be introduced compactly.[3] Given the utility function

$$U_t = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \upsilon \frac{H_t^{1+\gamma}}{1+\gamma}, \tag{7}$$

where σ_c is the coefficient of relative risk aversion and γ is the inverse of the Frisch

[2] Assuming a unit root with drift process does not change any of our conclusions below.

[3] For the less familiar reader, our ECB working paper provides a fuller derivation of the two models: http://www.ecb.int/pub/pdf/scpwps/ecbwp1278.pdf

elasticity, the canonical model with CES technology is given by the following equations:

$$\widehat{C}_t^{-\sigma_c} = \beta \mathbb{E}_t \left\{ \frac{\widehat{C}_{t+1}^{-\sigma_c}}{g_{t+1}^H} \frac{1+r_t}{\pi_{t+1}} \right\} \tag{8}$$

$$\widehat{w}_t = \upsilon H_t^{\gamma} \widehat{C}_t^{\sigma_c} \tag{9}$$

$$\hat{F}(\cdot) = CES_t = Y_0 e^{z_t^{\mathbb{H}}} \left[\alpha_0 \left(\frac{\widehat{K}_{t-1}}{K_0 g_t^H} e^{z_t^K} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{H_t}{H_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{10}$$

$$\widehat{w}_t = mc_t (1-\alpha_0) \left(\frac{Y_0}{H_0 e^{z_t^{\mathbb{H}}}} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\widehat{Y}_t}{H_t} \right)^{\frac{1}{\sigma}} \tag{11}$$

$$r_t^K = mc_t \alpha_0 \left(\frac{Y_0}{K_0} e^{z_t^{\mathbb{H}}} e^{z_t^K} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\widehat{Y}_t g_t^H}{\widehat{K}_{t-1}} \right)^{\frac{1}{\sigma}} \tag{12}$$

$$\widehat{C}_t + \widehat{I}_t \leq \widehat{Y}_t \tag{13}$$

$$\hat{Y}_t = \hat{F}(\cdot) - \frac{\chi}{e^{z_t^H}} \tag{14}$$

$$\hat{K}_t - (1-\delta) \frac{\hat{K}_{t-1}}{g_t^H} = \hat{I}_t (1 - S(\hat{X}_t)) \tag{15}$$

$$\hat{X}_t = \frac{\hat{I}_t g_t^H}{\hat{I}_{t-1}} \tag{16}$$

$$1 = Q_t(1 - S(\hat{X}_t) - S'(\hat{X}_t)\hat{X}_t + \beta \mathbb{E}_t (Q_{t+1} \frac{\hat{C}_t^{\sigma_c}}{\hat{C}_{t+1}^{\sigma_c} g_{t+1}^H} S'(\hat{X}_{t+1}) \hat{X}_{t+1}^2) \tag{17}$$

$$Q_t = \beta \mathbb{E}_t (\frac{\hat{C}_t^{\sigma_c}}{\hat{C}_{t+1}^{\sigma_c} g_{t+1}^H} (r_{t+1} + Q_{t+1}(1-\delta)) \tag{18}$$

$$x1_t = \widetilde{p}_t^{-1-\eta} \widehat{Y}_t mc_t + \theta \beta \left[\frac{\widehat{C}_t^{\sigma_c}}{\widehat{C}_{t+1}^{\sigma_c} g_{t+1}^H} \pi_{t+1}^{\eta} \left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}} \right)^{-1-\eta} x1_{t+1} \right] \tag{19}$$

$$x2_t = \widetilde{p}_t^{-\eta} \widehat{Y}_t + \theta \beta \left[\frac{\widehat{C}_t^{\sigma_c}}{\widehat{C}_{t+1}^{\sigma_c} g_{t+1}^H} \pi_{t+1}^{\eta-1} \left(\frac{\widetilde{p}_t}{\widetilde{p}_{t+1}} \right)^{-\eta} x2_{t+1} \right] \tag{20}$$

$$x2_t = \frac{\eta}{\eta - 1} x1_t \tag{21}$$

$$\log\left(\frac{1+r_t}{1+r}\right) = \alpha_r \log\left(\frac{1+r_{t-1}}{1+r}\right) + \alpha_\pi \log\left(\frac{\pi_t}{\pi}\right) + \alpha_y \log\left(\frac{\widehat{Y}_t}{\widehat{Y}_t^f}\right) \tag{22}$$

where $C_t,\ w_t$ and r_t^K are, respectively, real consumption, real wages and the rental rate on capital. π_t is inflation, r_t the net nominal interest rate, I_t investment, Y_t output, Q_t Tobin's q, mc_t marginal costs, $\widetilde{p}_t$ is the optimized price level chosen by the firms able to re-optimize, $x1_t$ and $x2_t$ are two auxiliary variables used to write the NK Phillips Curve in non-linear form and Y_t^f is the output level that would prevail under flexible prices.

Parameters β, δ , and υ represent, respectively, the discount factor, the capital depreciation rate, and a leisure scaling constant. η is the elasticity of substitution between differentiated goods, χ represents fixed costs in production[4] and the probability of re-optimizing prices is $1-\theta$.

Equations (8) and (9) represent the household's optimal consumption and labor supply choices given utility function (7). Solving equation (9) for hours shows the familiar results that, after a shock, hours rise if real wages grow faster than consumption. Equations (10) to (12) are the CES production function and its factor derivatives in normalized form. Equations (13) and (14) describe the economy's resource constraint while (15)-(18) define the capital law of motion, investment and Tobin's q in the presence of quadratic investment adjustment costs, given by $S(X_t) = \psi/2\,(I_t/I_{t-1} - 1)^2$. The non-linear NK Phillips curve is defined by (19)-(21) and the model is closed by the Taylor rule, (22).

[4]These are chosen to ensure zero profits in steady state. This in turn guarantees that there is no incentive for other firms to enter the market in the long run, see Coenen et al. (2008).

4 Calibration and impulse-response analysis

4.1 Calibration

Table 2 reports the parameter values used for the RBC and the NK model. We set a discount rate of around 4% per year. The Frish elasticity of labor supply is set to 1. The normalized capital share is set to 0.4 in line with typical estimates. The investment adjustment cost parameter is set to 2.5 (a common benchmark, e.g., Christiano et al. (2005)) in the NK and to 0 in the RBC models. The price elasticity of demand, η, ensures a steady-state price mark-up, $\mu = \eta/\left(\eta - 1\right)$, of 20% over marginal costs in the NK model while in the RBC, by setting $\eta \to \infty$, we obtain no price mark-up. The depreciation rate of capital is 10% per year. The Calvo parameter implies a fixed-price duration of 4 quarters in the NK model. The substitution elasticity σ is set to range from 0.35 (lower bound) to 1 (Cobb-Douglas) and 1.35 (upper bound). These values traverse the key conditions of gross complements and gross substitutes. The persistence of Hicks and capital-augmenting technology shocks is set to 0.95. For simplicity, in our core calibration of the NK model, we assume monetary policy only responds to deviations of inflation from the steady state with a coefficient just respecting the Taylor principle.[5] Parameter v is set to equate the real wage expressions in (9) and (11), implying that $v = \left(\left(1-\alpha_0\right) r_0^{\sigma_c}\right)/\left(\left(r_0 - \delta\alpha_0\right)^{\sigma_c}\right)$. Both models are normalized around the same steady state point. Later, we perform extensive robustness tests around these benchmark values.

[5] A full set of robustness results to parameter changes is provided in the online Appendix. We also relax there the assumptions on monetary policy.

		RBC	NK
	Households		
β	Discount Factor	$1.04^{-1/4}$	$1.04^{-1/4}$
γ	Inverse Frisch Elasticity	1	1
υ	Leisure Scaling Constant	0.84	0.84
	Firms		
η	Price Elasticity of Demand	∞	6
μ	Mark-up	1	1.20
θ	Calvo Parameter	0	0.75
δ	Depreciation Rate	$1.1^{1/4}-1$	$1.1^{1/4}-1$
ψ	Investment Adjustment Costs	0	2.5
α_0	Normalized Capital Income Share	0.4	0.4
σ	Elasticity of Factor Substitution	$[0.35, 1, 1.35]$	$[0.35, 1, 1.35]$
ρ	Shock Auto-Regressive Parameter	0.95	0.95
	Monetary policy		
π	Steady State Inflation	n.a.	1
α_π	Response to Inflation	n.a.	1.1
α_y	Response to Output Gap	n.a.	0.0
α_r	Interest-Rate Smoothing	n.a.	0.0

Table 2: Parameter Calibration

4.2 Impulse Response Analysis

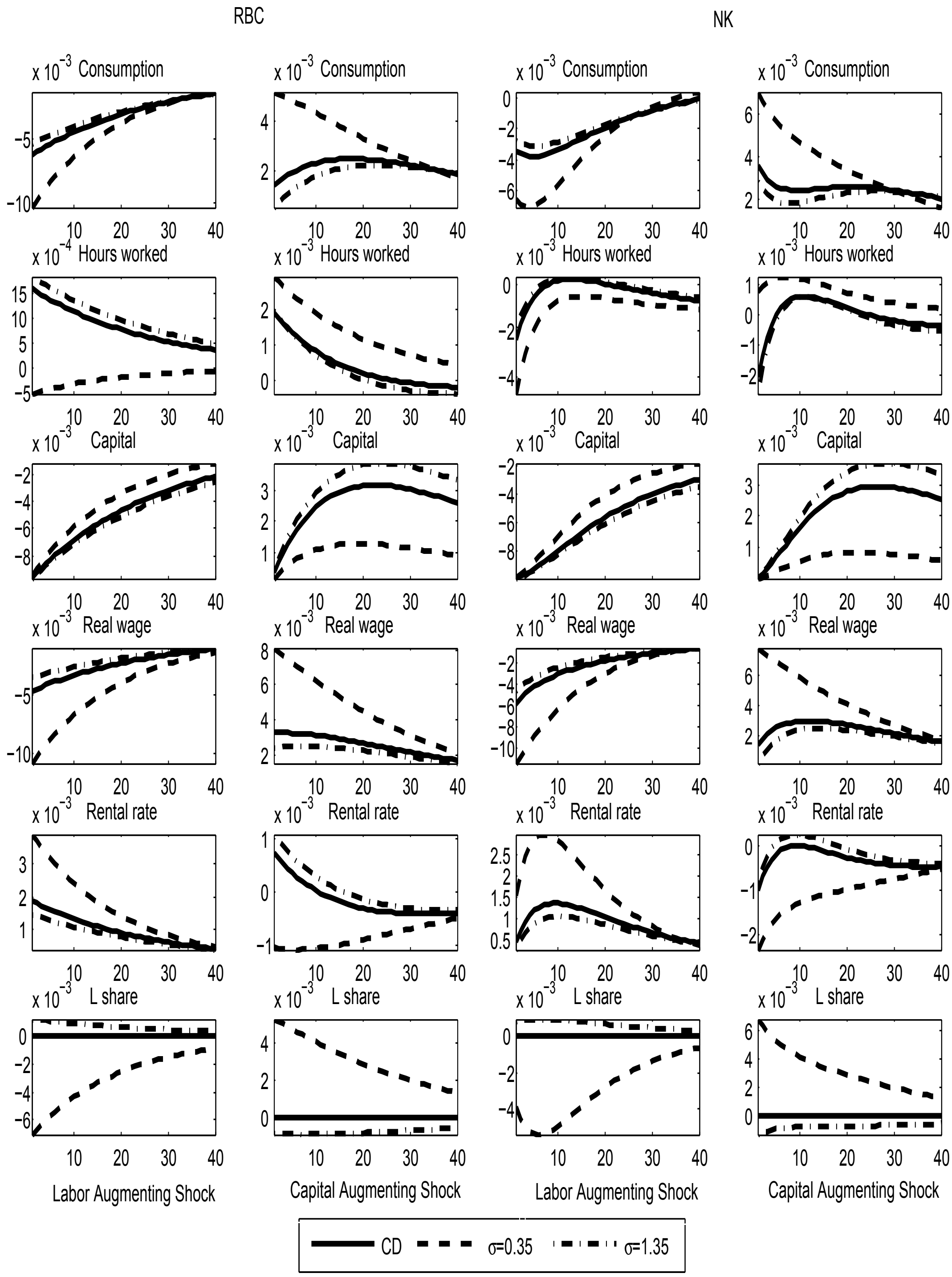
Figure 1 depicts the dynamic responses of selected variables to a one percentage point increase in ε_t^K and ε_t^H (i.e., capital and labor augmenting innovations) in the canonical RBC and NK models, respectively. All the IRFs for the real variables presented are for the re-scaled variables when these present stochastic trends, i.e. $\widehat{C}_t$, $\widehat{K}_t$, and $\widehat{w}_t$.[6]

The shocks are conducted against the parameters values shown in Table 2 with the substitution variations $\sigma \in \{0.35, 1, 1.35\}$. Variations in σ, recall, are admissible in our framework since we express the models in normalized form.

The effect of the positive technology shocks is to stimulate output, consump-

[6]We also computed dynamic responses for the Hicks-neutral shock $\varepsilon_t^{\mathbb{H}}$ (see online Appendix).

Figure 1: Impulse responses for the RBC and NK models



Note: Impulse responses in the model(s) of selected variables to a labor and capital-augmenting shock of one standard deviation. Responses are reported as percentage deviations from the non-stochastic steady state.

tion and investment.[7] Movements in factor income shares (excluding Cobb-Douglas where shares are constant), following Section 2, "favor" either factor depending on the source of the technology improvement and whether factors are gross complements or substitutes.

Factor shares and relative prices (marginal productivities) behave as anticipated. Differences in consumption and investment responses corresponding to alternative substitution values are in turn explained by, inter alia, investment reactions to changes in relative marginal productivities of capital and labor.

Where qualitative differences may arise lies in the hours response. These arise not only across the models but also *within* them. Whilst capital-augmenting technology-hours impacts are positive in the RBC model, labor-augmenting technology-hours impacts are negative in the NK one. This general pattern is what macroeconomists might expect. However, results also confirm that the models are capable of generating technology-hours impacts *of either sign*. For example, for the RBC model, when $\sigma = 0.35$, a negative impact results in the labor augmenting case. For the NK model, the same elasticity induces a positive response in the capital augmenting case.[8]

This figure, note, only gives a snapshot of our results; it confirms that both models can generate a variety of different response signs even for entirely stan-

[7]Note again that, since we report the non-stationary variables in re-scaled form, they appear relative to the (higher) new steady state. The figures for non re-scaled variables show an increase in all cases.

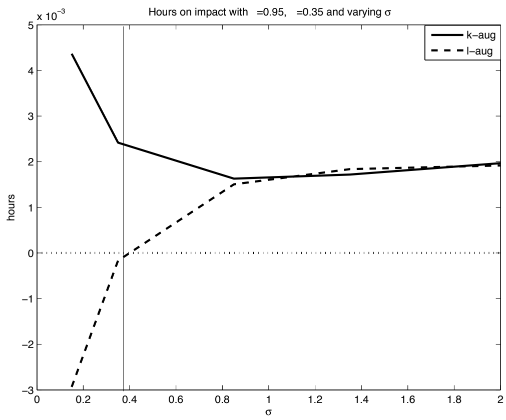
[8]Note that the negative hours response for the RBC model with labor-augmenting shocks and the positive one for capital-augmenting shocks in the NK model are relatively small. However, this is simply a construct of our core calibration. Our threshold conditions below will show more generalized cases: e.g., these responses are stronger the lower the value of the substitution elasticity.

dard parameterizations. In the following section we present a generalized interpretation of the technology-hours response in the models. It transpires that the response has threshold characteristics such that technology-hours responses may change sign (a key determinant of which is the value of the substitution elasticity and the source of the technology shock).

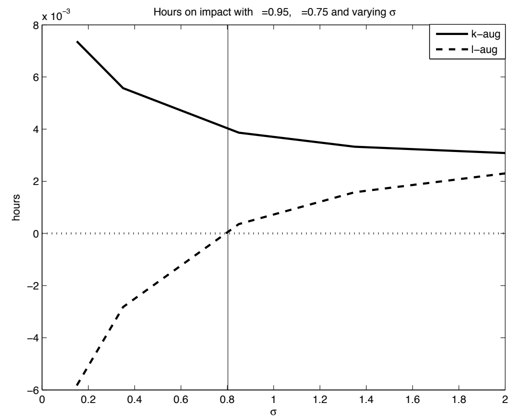
To anticipate those more general interactions, we perform some additional analysis. **Figures 2a** to **2d** analyze the impact response of hours along a $\sigma \in (0, 2]$ support, with a steady-state capital income share of $\alpha_0 = 0.35$ (just below our baseline of 0.4) and $\alpha_0 = 0.75$. The latter value, though counter-factual, does at least illustrate bluntly the threshold characteristics involved. For the RBC model, **2a** and **2b** shows that for all σ values sufficiently above α_0, the labor-augmenting/hours sign flips from negative to positive. For the NK model, it is the capital-augmenting shock that switches sign but in this case as the capital share increases (compare plots (c) to (d)), the threshold point (i.e., the vertical line) moves inwards.

5 Technology and Hours: Threshold conditions

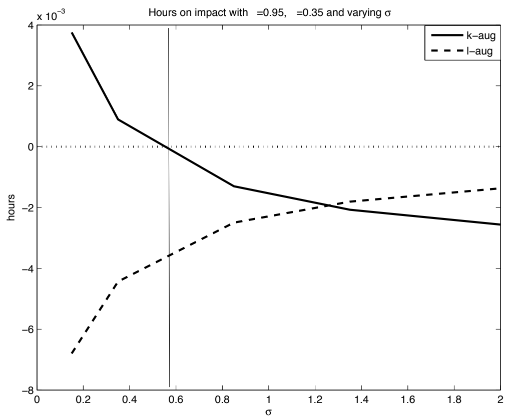
Thus for both models the hours-technology sign response may change over empirically relevant parameter values. We now derive analytical conditions determining the sign of that impact. We do so first for the more general NK case. The condition reveals some important features usually bypassed by the literature. To enhance the intuition, we then present the condition for the simpler RBC case.



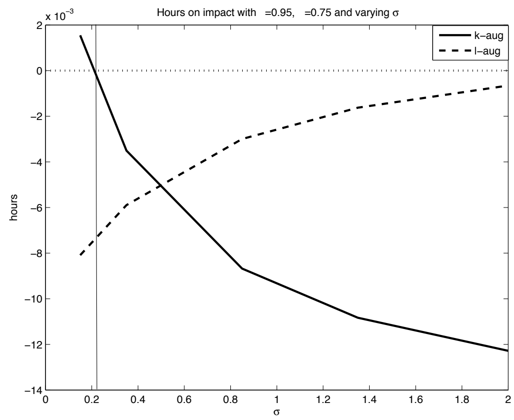
(a) Sensitivity for σ with $\alpha = 0.35$, RBC Model



(b) Sensitivity for σ with $\alpha = 0.75$, RBC Model



(c) Sensitivity for σ with $\alpha = 0.35$, NK Model



(d) Sensitivity for σ with $\alpha = 0.75$, NK Model

Figure 2: Changes in σ and α

These conditions can additionally be interpreted very intuitively in the labor demand / labor supply space.

5.1 The *General* Hours-Technology condition

Without loss of generality, we normalize around the following steady-state: $Y_0 = H_0 = 1 \Rightarrow K_0 = \frac{\alpha_0}{r_0}$, $w_0 = 1 - \alpha_0$ and $C_0 = Y_0 - \delta K_0 = (r_0 - \delta\alpha_0)/r_0$.[9] This implies that, around our baseline steady state, $dY_t = dY_t/Y_0 = d\log Y_t$.

Consider now the full NK model (of which the RBC model is a limiting case). Although the labor supply function is common to both models, a crucial difference is that Calvo price stickiness, $\theta \in (0,1)$, activates a marginal cost channel into labor demand. Zero profits in the steady state further implies $\chi = Y_0/(\eta - 1)$. Taking this into account, together with normalization, we can derive the following labor demand (23) and labor supply (24) schedules:

$$d\log w_t = d\log mc_t + \frac{\alpha_0}{\sigma}dz_t^K + \frac{\sigma - \alpha_0}{\sigma}dz_t^H - \frac{\alpha_0}{\sigma}d\log H_t^D \tag{23}$$

$$d\log w_t = \gamma d\log H_t^S + \sigma_c d\log C_t \tag{24}$$

The impact effect of technology shocks on *equilibrium* hours is then given by:

$$\frac{d\log H_t}{dz_t^H} = \frac{1}{\Upsilon}\left[\frac{d\log mc_t}{dz_t^H} + \frac{\sigma - \alpha_0}{\sigma} - \sigma_c\frac{d\log C_t}{dz_t^H}\right] \tag{25}$$

$$\frac{d\log H_t}{dz_t^K} = \frac{1}{\Upsilon}\left[\frac{d\log mc_t}{dz_t^K} + \frac{\alpha_0}{\sigma} - \sigma_c\frac{d\log C_t}{dz_t^K}\right] \tag{26}$$

[9] Full derivations of some of the conditions in this section can be found in the supplementary Appendix.

Given that $\Upsilon = (\gamma\sigma + \alpha_0)/\sigma > 0$, the sign of the impact of technology on hours depends on the sign of the bracketed terms in (25) and (26). The first two elements in both expressions reflect the impact of shocks through labor demand shifts: changes in the marginal cost and changes in the marginal product of labor (through both increased output and factor substitution). The third element reflects the impact of shocks through changes in labor supply decisions. Recall that labor supply increases only if wages change more than consumption (weighted by relative risk aversion).

Regarding this third element, for a given shock dz_t^j $(j = H, K)$ around the steady state, we can exploit the decomposition,

$$\frac{d\log C_t}{dz_t^j} = \frac{d\log C_t}{d\log Y_t} \cdot \frac{d\log Y_t}{dz_t^j} = \frac{dC_t}{dY_t}\frac{Y_0}{C_0} \cdot \frac{d\log Y_t}{dz_t^j} = \frac{mpc_t}{apc} \cdot \frac{d\log Y_t}{dz_t^j} \tag{27}$$

where $mpc_t \in \mathbb{R}$ is the *marginal* propensity to consume, and $apc > 0$ is the *average* propensity to consume in the baseline steady state. The marginal propensity to consume – though endogenous to the model – nonetheless proves an intuitive lens through which to interpret the results.

Making use of (27), we are now in a position to establish general threshold values for σ that would imply a change in the sign of the response in hours,

$$\frac{d\log H_t}{dz_t^K} > 0 \quad \text{if} \quad \sigma - \frac{\alpha_0}{\sigma_c \frac{mpc_t}{apc}\mu\alpha_0 - \frac{d\log mc_t}{dz_t^K}} < 0 \tag{28}$$

$$\frac{d\log H_t}{dz_t^H} > 0 \quad \text{if} \quad \sigma - \frac{\alpha_0}{1 + \frac{d\log mc_t}{dz_t^H} - \sigma_c \frac{mpc_t}{apc}\mu(1-\alpha_0)} > 0 \tag{29}$$

where, as before, $\mu = \eta/(\eta - 1) > 1$.

Although these conditions appear unwieldy, they identify the channels of interest as well as admitting some special, more lucid, cases. The key margins for the conditions are the $\sigma - \alpha_0$ wedge (the intuition of which we discuss below), risk aversion and marginal consumption reactions. The latter two, though, are linked since the higher is σ_c, the less able are agents to smooth consumption and, ceteris paribus, the higher is mpc_t.[10]

5.2 The *Simple* Hours-Technology condition

Consider the RBC model with no investment adjustment costs, perfect competition and no price stickiness: $\phi \to 1$ and $d \log mc_t = 0$. The *simplified* threshold conditions for the substitution elasticity then become:

$$\frac{d \log H_t}{dz_t^K} > 0 \quad \text{if} \quad \sigma - \frac{1}{\sigma_c \frac{mpc_t}{apc}} < 0 \tag{30}$$

$$\frac{d \log H_t}{dz_t^H} > 0 \quad \text{if} \quad \sigma - \underbrace{\alpha_0 \left[1 - \sigma_c \frac{mpc_t}{apc} (1 - \alpha_0)\right]^{-1}}_{\varpi} > 0 \tag{31}$$

Conditions (30) and (31) have an analogous interpretation: both link the hours response to the location of the elasticity of substitution. Note the following:

i) In general, we would expect mpc_t to be positive but below unity;

ii) The substitution-capital share distance, $\sigma - \alpha_0$, can be interpreted as the degree of complementary between labor demand and labor augmenting tech-

[10] We also derive these threshold conditions using a non-separable utility function in consumption and leisure (see online Appendix). A full set of quantitative results, not reported, was also carried out for this case.

nical progress.[11]

With this, we can now rationalize the responses of hours found in **Figure 1** for the RBC model. If the consumption smoothing motive is strong, then mpc_t will be small. This implies that $d\log H_t/dz_t^K > 0$ for almost all empirically relevant values of σ; hence the outcomes in Figure 1.

For $d\log H_t/dz_t^H$ we have a positive response if a wedge is driven between the substitution elasticity and the capital share: $\sigma > \alpha_0\varpi$. As we shall discuss below, ϖ yields a natural interpretation of the extent to which labor-technology complementarity on the production side is diluted or amplified by corresponding developments on labor supply. To illustrate, if consumption smoothing was such that $mpc_t \to 0$, then $\varpi \to 1$, and the threshold value would simply be $\sigma > \alpha_0$ (i.e., any value of the substitution elasticity even marginally above the capital income share would generate a positive hours-technology impact for labor-augmenting shocks). In effect, this case would imply that the shock has a negligible effect on labor supply and that all changes in hours are essentially demand driven. For the labor-augmenting case, the shock has a factor substitution effect on labor demand equal to $(\sigma-1)/\sigma$, and an output effect equal to $(1-\alpha_0)/\sigma$. The sign of

[11] The proof is as follows. Given a linear homogenous production function:

$$Y_t = F\left(\Gamma_t^K K_t, \Gamma_t^H H_t\right) = \Gamma_t^H H_t f\left(\kappa_t\right)$$

if labor-augmenting technical progress raises labor demand we would have,

$$\frac{\partial^2 Y}{\partial \Gamma^H \partial H} = f\left(\kappa\right) - \kappa f'\left(\kappa\right) + \kappa^2 f''\left(\kappa\right) > 0 \tag{f1}$$

Exploiting the definition of the substitution elasticity, $\sigma = -\frac{f'(\kappa)\left[f(\kappa)-\kappa f'(\kappa)\right]}{\kappa f(\kappa) f''(\kappa)}$ and noting that $\alpha = \kappa f'\left(\kappa\right)/f\left(\kappa\right)$, it follows that $\sigma - \alpha = f'\left(\kappa\right)/f\left(\kappa\right)\left[f\left(\kappa\right) - \kappa f'\left(\kappa\right) + \kappa^2 f''\left(\kappa\right)\right]$, which, when abstracting from $f'\left(\kappa\right)/f\left(\kappa\right) > 0$, retrieves (f1). $\square$

the sum of these effects on labor demand will then depend on $sgn\,[\sigma - \alpha_0]$. This then rationalizes the results found in **Figures 2a** and **2b**.

For the NK case, in turn, we need to consider the effects of shocks on real marginal cost. For the parameter values used (and for most parameterizations), real marginal cost falls after a technology shock. Looking at expressions (29) and (28), it is easy to see that we can now find a switching sign for the capital-augmenting shock. For the labor-augmenting shock, sign changes may still happen, but it is most likely that σ will be below the threshold (confirming **Figure 1**). The online Appendix provides an in-depth analysis of these effects in both models interpreted from the intuitive viewpoint of labor demand and supply.

6 Robustness and empirical implications

6.1 Robustness

We carried out a wide range of robustness exercises. We make them available in the Appendix with supplementary material. For brevity, we merely comment them here.

Variations in the Frisch elasticity (as expected under separable utility) and in the policy rule parameters have no impact on the earlier interpretation. Interestingly, given the strength of debates on nominal rigidities, Calvo frictions do not contribute to sign reversals either. Variations in the persistence of the capital-augmenting technology shock are, with a few exceptions, again sign neutral. Regarding risk aversion, we find that there are high enough values (for gross complements) or low enough values (for gross substitutes) of σ_c that can make the response of hours change.[12] Finally, on investment adjustment costs, it appears these need only be modest in value before the thresholds are hit. In general, thus, the results simply reflect quantitatively the mechanisms discussed earlier and confirm that our results are robust to alternative policy and preference specifications.

6.2 Empirical implications

Our theoretical results highlight the fact that the sign response of hours depends on the factor bias of technology shocks and, amongst other parameters, the relationship between the elasticity of substitution and the capital share. However, the hours-technology literature has crucially hinged upon empirical results from SVAR models. Although our focus is mainly theoretical, we analyze two important empirical aspects related to this literature. Firstly, if we take as given a sign response of hours from SVAR empirical estimates, we can study which of the baseline models (RBC or NK) is more likely to generate these responses. Secondly, and perhaps illustrating better the advantages of our approach, we can study whether our model can help identify different technology shocks by providing us with robust theory restrictions.

To answer these two questions, we consider the following experiment. We generate a range of IRFs drawing jointly from uniform distributions for the relevant parameters as in Canova and Paustian (2011). Labeling $\mathbb{M}$ the structure

[12] Variations in σ_c are only possible with stationary shocks, and this robustness exercise was carried out with and AR(1) process for z_t^H.

presented in (8)-(22), we consider 4 different sub-models (see **Table 3**): an RBC model with an elasticity of substitution $\sigma \in (0,1]$, $\mathbb{M}_1$, an RBC model with $\sigma \in (1,2]$, $\mathbb{M}_2$, a NK model with $\sigma \in (0,1]$, $\mathbb{M}_3$, and a NK model with $\sigma \in (1,2]$, $\mathbb{M}_4$. That is, we consider both RBC and NK models together with combinations of gross complementarity and gross substitutability. The range of values for the uniform distributions of the parameters are presented in **Table 4**, and draw from Canova and Paustian (2011). This allows us to assess, given the existing uncertainty about the true values of these parameters, the likelihood of different sub-models generating either positive or negative impact responses.

Submodel	**Description**	**Parameter restrictions**
$\mathbb{M}_1$	RBC with $\sigma \in (0,1)$	$\xi = 0$, $\eta = \infty$, $\psi = 0$
$\mathbb{M}_2$	RBC with $\sigma \in (1,2)$	$\xi = 0$, $\eta = \infty$, $\psi = 0$
$\mathbb{M}_3$	NK with $\sigma \in (0,1)$	n.a.
$\mathbb{M}_4$	NK with $\sigma \in (1,2)$	n.a.

Table 3: The sub-models.

Parameter	**Description**	**Support**
β	Discount factor	0.99
δ	Depreciation rate	0.025
η	Goods elasticity of substitution	(5,7)
σ_c	Risk aversion	1
γ	Inverse Frish elasticity	(0,5)
ξ	Calvo probability	(0,0.9)
α	Labor Share	(0.1,0.4)
σ	Elasticity of Factor substitution	see Table 3
ψ	Adjustment costs to Investment	(0,5)
ρ^K	Persistence in z_t^K	(0.5,0.99)
α_r	Inertia in Taylor rule	(0.25,0.95)
α_π	Response to inflation in Taylor rule	(1.05,2.50)
α_y	Response to the output gap in Taylor rule	(0,0.5)

Table 4: Support for parameters' calibration.

Figure 3 displays the median and 90% confidence intervals for the IRF for hours from 1,000 draws of the model and for both the (permanent) labor-augmenting and the (temporary) capital-augmenting shocks. Assuming both shocks are correctly identified, we can see that the likelihood of obtaining a negative impact response to the permanent shock is higher in the NK model. The RCB model can generate negative responses under the gross complements sub-model, as discussed above. However, both within that sub-model and looking across $\mathbb{M}_1$ and $\mathbb{M}_2$, the impact response in the RBC model is not robustly negative. Similar conclusions apply to the NK model when looking at the temporary capital-augmenting shock.

The bulk of empirical evidence on the hours-technology correlation has analyzed the impact effects of permanent shocks. In that sense, if we take as given the dominant view of a negative impact response as in Canova et al. (2010), then the NK model, probabilistically, dominates the RBC model. This conclusion, it has to be noted, assumes no strong priors about the true values of certain key parameters since they have been generated from uniform distributions on a wide support. If, for instance, we have a stronger prior that σ is normally distributed around the "Chirinko interval" according to existing empirical evidence, then the likelihood of a negative response in the RBC model would increase. This negative response, however, would still be un-robust. Another conclusion arising from this comparison is that, jointly, $\mathbb{M}_1$ and $\mathbb{M}_3$, where $\sigma \in (0, 1)$, dominate models where $\sigma \in (1, 2)$ both in terms of signs and the magnitude of the responses typically found in empirical work. Our model, however, now contains two possible technology shocks with substantially different effects on hours. Separate identification of both shocks would then help model comparison. We tackle this next.

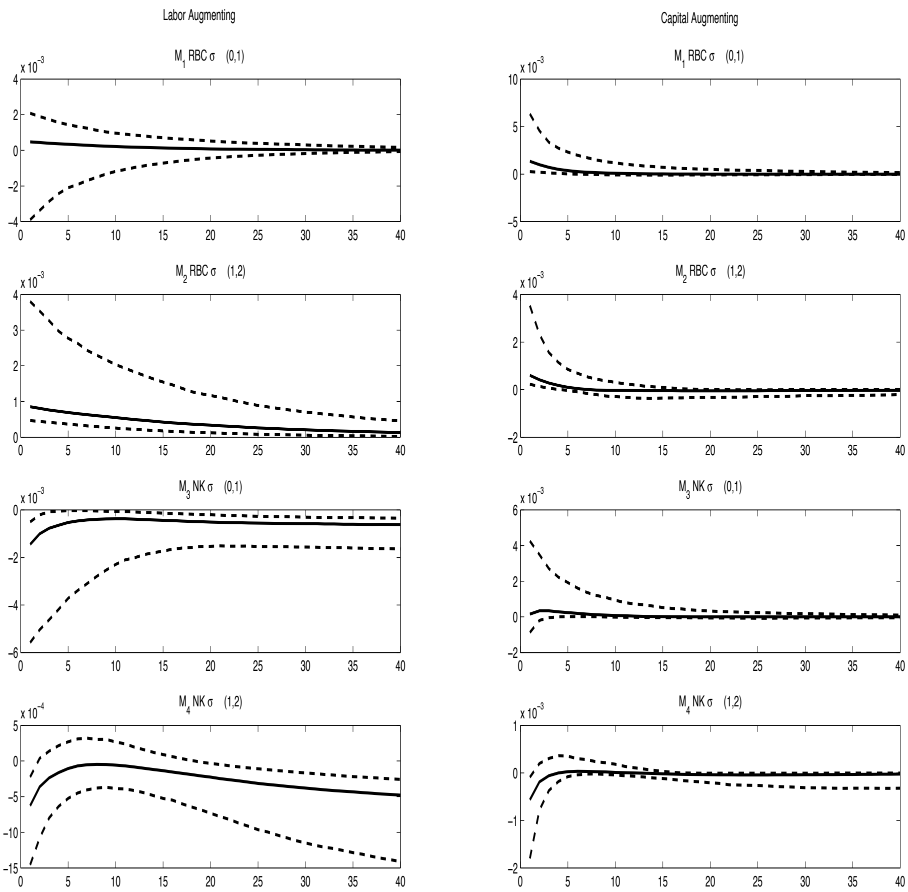


Figure 3: Median and 90% confidence intervals for the permanent labor-augmenting and the temporary capital-augmenting shocks in four sub-models

We look at whether the range of 4 sub-models considered can generate robust sign restrictions for application in VAR models that would sharpen inference and help identifying the two technology shocks in the models. Because in small scale VAR models data may not speak clearly, imposing more sign restrictions on variables that are not of direct interest can help identify shocks. In this case, the inclusion of variables other than productivity and hours can be guided by identification considerations. **Table 5** reports the signs of the impact response intervals for the two shocks across sub-models, for selected variables, using the draws from the uniform parameter distributions from Table 4.

Variable	z_t^H				z_t^K			
	$\mathbb{M}_1$	$\mathbb{M}_2$	$\mathbb{M}_3$	$\mathbb{M}_4$	$\mathbb{M}_1$	$\mathbb{M}_2$	$\mathbb{M}_3$	$\mathbb{M}_4$
$\hat{Y}$	-	?	-	-	+	+	+	+
$\hat{C}$	-	-	-	?	+	?	+	+
h	?	+	-	-	+	+	?	-
$\hat{w}$	-	-	-	-	+	+	+	?
$\hat{I}$	+	+	?	-	?	+	?	+
r^K	+	+	+	+	?	+	-	?
r	n.a.	n.a.	-	-	n.a.	n.a.	-	-
mc	n.a.	n.a.	-	-	n.a.	n.a.	-	-
π	n.a.	n.a.	-	-	n.a.	n.a.	-	-
LS	-	+	-	+	+	-	+	-

Table 5: A "+" ("-") indicates that at least 90% of the impact response interval is positive (negative); a "?" indicates that the impact response interval lies on both sides of the zero line.

The results yield several interesting robust responses. For instance, output always increases on impact (over and above its steady state) for the temporary shock, which is not the case for all sub-models for the permanent shock (relative to its new steady state). The opposite can be observed for the real user cost of capital (r^K), which always increases after a permanent shock. Potentially, however, using these two robust responses for VAR sign restrictions may be problematic. First, in a VAR with labor productivity and hours, the inclusion of output is redundant as it is already determined by the joint dynamics of H_t and Y_t/H_t. Second, the user cost may be difficult to measure if appropriate data on taxes, depreciation, and changes in the relative price of investment are not readily available.

A more promising avenue, however, might be the introduction of the share of labor in factor income. With Cobb-Douglas, the share would be independent of technology shocks. This is not the case in our model with CES technology. In the last row of Table 5, we can see that the response of the labor share *robustly* takes opposite signs in response to the two shocks and across all models. That is, irrespective of the value of σ, as long as it differs from unity, we can impose the restriction that the response of the labor share takes the opposite sign for temporary and permanent technology shocks. Introducing the labor share in a VAR model can thus aid identification as it allows us to impose the robust restriction that the *product* of the impact responses to temporary and permanent shocks is always negative. Thus, a SVAR including hours, productivity, and the labor share that combines long-run restrictions and a sign restriction on the labor share as explained above would then be able to separately identify the two technology shocks and non-technology shocks.

7 Conclusions

We re-examined the impact of technology shocks on hours worked in business cycle models. The usual interpretation being that, in a Real Business Cycle model, hours increase after a positive technology shock but initially fall in a New Keynesian one. This difference has been taken as a means of discriminating between different theories of business-cycle fluctuations and remains a key controversy in macroeconomics. Our contribution to this controversy has been to analyze the consequences of richer supply and technology considerations in business-cycle models.

Given the evidence, we believe it is no longer defensible for business-cycle models to ignore non-unitary capital-labor substitution elasticities and, by implication, factor-biased technology shocks. Cobb-Douglas is typically rejected by the data and factor income shares display important business-cycle fluctuations.

With the introduction of Constant Elasticity of Substitution (CES) supply side, we show that the response of hours depends on the factor-augmenting nature of technology shocks and the capital-labor substitution elasticity in *both* RBC and NK models. We demonstrate that both models can generate technology-hours responses of either sign. These response signs have threshold characteristics. In each model and shock case, we showed that there exists some value of the elasticity of substitution whereby a given technology-hours impact changed sign. The key margins – other than the substitution elasticity itself and the factor bias of technology shocks – are the capital share, risk aversion, and the reaction of the marginal propensity to consume. Variations in these margins prove to be relevant when determining the sign of the response.

The introduction of these hitherto unexplored factors reveals new margins for shock identification and model comparison combining theory and VAR evidence. We show that, when distinguishing between temporary and permanent technology shocks – a constraint induced by balanced growth path considerations – the probability of observing negative responses to the permanent shock differs substantially across models. We also show that the use of the labor income share in empirical work offers novel robust sign restrictions for shock identification. Specifically, the reaction of the labor share to labor-augmenting (permanent) and capital-augmenting (temporary) technology shocks robustly take opposite signs across a variety of models.

Our approach opens important new avenues for research. For instance, the approach offers a benchmark toolkit to analyze the cyclical properties of the labor share, the time-variation in the response of hours to technology shocks, and the sensitivity of, for instance, fiscal and monetary policy shocks to changes in the supply side specification.

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