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CES Technology and Business Cycle Fluctuations

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Abstract

This paper contributes to an emerging literature that brings the constant elasticity of substitution (CES) specification of the production function into the analysis of business cycle fluctuations. Using US data, we estimate by Bayesian methods a medium-sized DSGE model with a CES rather than Cobb-Douglas (CD) technology. The main empirical result is to confirm decisively the superiority of CES rather than CD production functions in terms of model fit. We estimate a elasticity of substitution of elasticity well below unity at 0.15-0.18 and in a marginal likelihood race assuming equal prior model probabilities, CES beats the CD production decisively. The marginal likelihood improvement is matched by the ability of the CES model to fit the data in terms of second moments and a comparison with a DSGE-VAR further confirms the ability of the CES model to reduce model misspecification. We find that the CES model performance is further improved when the estimation is carried out under the imperfect information assumption. The principle reason for our result is that the CES specification captures movements of factor shares at the business cycle frequency. Hence the main message for DSGE models is that we should dismiss once and for all the use of CD for business cycle analysis.

JEL Classification: C11, C52, D24, E32.
Keywords: CES production function, DSGE model, Bayesion estimation, imperfect information, DSGE-VAR

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Contents

1 Introduction

This paper extends the DSGE model developed by Christiano *et al.* (2005) and Smets and Wouters (2007) to allow for a richer and more data coherent specification of the production side of the economy. The idea is to enrich what has become the workhorse DSGE model by relaxing the usual Cobb-Douglas production assumption in favour of a more general CES function which allows for cyclical variations in factor shares, the estimation of the capital/labour elasticity of substitution and biased technical change.

The CES production function has been used extensively in many area of economics since the middle of the previous century (Solow (1956) and Arrow *et al.* (1961)). Thanks to La Grandville (1989), who introduced the concept of *normalization*, it has been extensively used in growth theory. Indeed La Grandville (1989) showed that it was possible to obtain a perpetual growth in income per-capita, even without any technical progress. Furthermore factor substitution and the bias in technical change feature an important role in many other areas of economics[1] but, until recently have been largely disregarded in business cycle analysis. On the empirical side León-Ledesma *et al.* (2010) show that normalization improves empirical identification.[2]

The concepts of biased technical change and imperfect factor substitutability between factors of production has been introduced in business cycle analysis by Cantore *et al.* (2014b). They show that the introduction of a normalized CES production function into an otherwise standard RBC and/or NK DSGE model significantly changes the response of hours worked to a technology shock under both price-setting mechanisms and that such response might change as well within each model depending on the parameters related to the production process (developing a threshold rule for the 'impact' of a technology shock on hours worked). They also show how the introduction of biased technical change and imperfect substitutability allow movement in factor shares which appear to fluctuate at business cycle frequencies in the data but are theoretically constant under the Cobb-

[1] The value of the substitution elasticity has been linked to differences in international factor returns and convergence (e.g., Klump and Preissler (2000), Mankiw (1995)); movements in income shares (Blanchard (1997), Caballero and Hammour (1998), Jones (2003)); the effectiveness of employment creation policies (Rowthorn (1999)), etc. The nature of technical change, on the other hand, matters for characterizing the welfare consequences of new technologies (Marquetti (2003)); labour-market inequality and skills premia (Acemoglu (2002)); the evolution of factor income shares (Kennedy (1964), Acemoglu (2003)) etc.

[2] They show that using a normalized approach permits to overcome the 'impossibility theorem' stated by Diamond *et al.* (1978) and simultaneously identify the elasticity of substitution and biased technical change.

Douglas specification. Indeed there is mounting evidence in the literature[3] that whilst constant factor shares might be a good approximation for growth models where the time span considered is very long, at business cycle frequencies those shares are not constant. This is clearly shown in Figure 1 for the US data used to estimate our model.

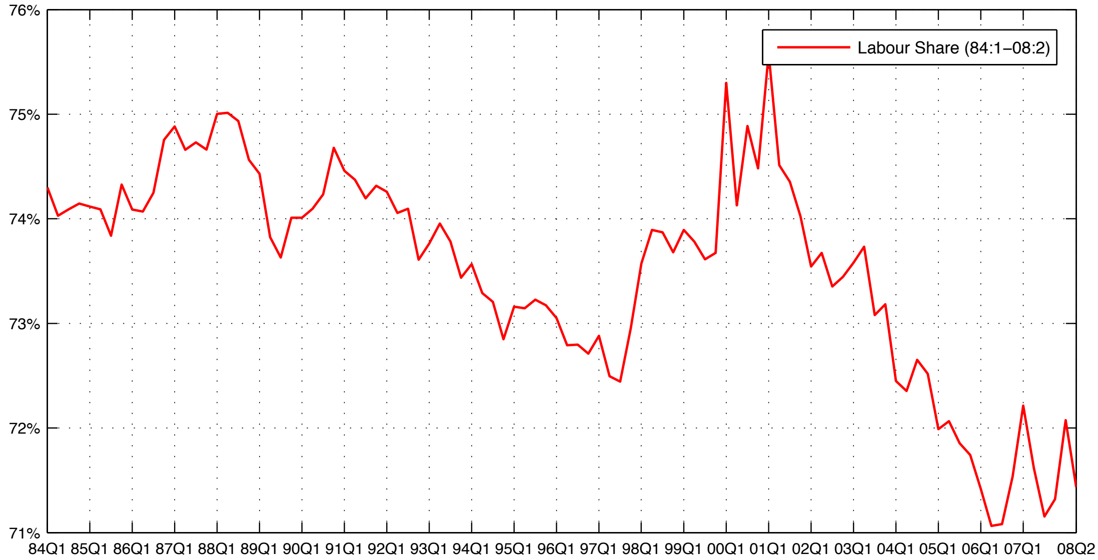


Figure 1: US Labour Share (Source: Department of Labor, U.S. Bureau of Labor Statistics)

Furthermore Cantore *et al.* (2012) test empirically the model(s) developed by Cantore *et al.* (2014b) using rolling-windows Bayesian techniques in order to check if the documented time-varying relation between hours worked, productivity and output (see Fernald (2007) and Galí and Gambetti (2009) among others) can be explained using the threshold rule.

Apart from Cantore *et al.* (2014b) and Cantore *et al.* (2012) most DSGE models continue to use the Cobb-Douglas assumption even if the empirical evidence provided through the years has ruled out the possibility of unitary elasticity of substitution (see among others Antràs (2004), Klump *et al.* (2007), Chirinko (2008) and León-Ledesma *et al.* (2010)). In this paper we show that the introduction of a CES production function in a medium-scale DSGE model in the spirit of Christiano *et al.* (2005) and Smets and Wouters (2007) makes it possible to exploit the movements of factor shares we observe in the data

[3]See for example Blanchard (1997), Jones (2003, 2005), McAdam and Willman (2013) and Ríos-Rull and Santaeulália-Llopis (2010).

to improve significantly the performance of the model. To the best of our knowledge, we are the first to compare the empirical implications of CD and CES production functions in a DSGE context.

The main results of our paper are first, in terms of model posterior probabilities, impulse responses, second moments and autocorrelations, the assumption of a CES technology significantly improves the model fit. Second, this finding is robust to the information assumption assumed for private agents in the model. Indeed allowing the latter to have the same (imperfect) information as the econometrician (namely the data) further improves the fit compared with the standard assumption that they have perfect information of all state variables including the shock processes. Third, using US data, we estimate by Bayesian-Maximum-Likelihood (BML) methods a elasticity of substitution of elasticity between the capital/labour ratio and the wage rate/capital cost ratio to be 0.15-0.18, a value broadly in line with the literature using other methods of estimation.[4]

The rest of the paper is organized as follows. Section 2 describes the model with particular attention paid to the normalization of the CES production function.[5] Section 3 sets out the BML estimation of the model and includes an investigation of the model performance under both the standard and the imperfect information assumptions. Section 4 examines the ability of the model to capture the main characteristics of the actual data as described by second moments and the impulse response functions of an estimated "DSGE-VAR" hybrid. Section 5 compares the variance decomposition of the structural shocks for the CES and Cobb-Douglas formulations. Section 6 concludes the paper.

2 The Augmented SW Model

Here we present, concisely the augmented SW model with a wholesale and a retail sector, Calvo prices and wages, CES production function, adjustment costs of investment and variable capital utilization. Figure 2 illustrates the model structure. The model equilibrium conditions are presented in non-linear form. The novel feature is the introduction of a CES production function in the wholesale sector, instead of the usual Cobb-Douglas form. This generalization then allows for the identification of both labour-augmenting

[4]See, for example, Table 2 in Rowthorn (1999), Chirinko (2008), León-Ledesma *et al.* (2010) and the survey by Klump *et al.* (2012).

[5]We utilize a normalization procedure of re-parametrization proposed by Cantore and Levine (2012).

and capital-augmenting technology shocks. As in Smets and Wouters (2007) we use a household utility function compatible with a balance growth path in the steady state, but we adopt a more standard functional form used in the RBC literature. However we do not adopt Kimball aggregators for final output and composite labour.[6] Again as in their paper we introduce a monopolistic trade-union that allows households to supply homogeneous labour. Then as long as preference shocks are symmetric, households are identical in equilibrium and the complete market assumption is no longer required for aggregation. The supply-side of the economy consists of competitive retail sector producing final output and a monopolistically competitive wholesale sector producing differentiated goods using the usual inputs of capital and work effort. Households consume a bundle of differentiated commodities, supply labour and capital to the production sector, save and own the monopolistically competitive firms in the goods sector. Capital producers provide the capital inputs into the wholesale sector.[7]

We set out the model first without specifying the form of the utility and production functions in order to obtain a flexible framework in which it will be easy to stick different functional forms.

The sequencing of decisions is as follows[8]

1. Each household supplies homogeneous labour at a price $W_{h,t}$ to a trade-union. Households choose their consumption, savings and labour supply given aggregate consumption (determining external habit). In equilibrium all household decisions are identical.

2. Capital producing firms convert final output into new capital which is sold on to

[6]The motivation for generalizing Dixit-Stiglitz aggregators is to bring estimates of price and wage contract lengths into line with micro-econometric evidence. In fact our estimates for US data are compatible with the simpler Dixit-Stiglitz formulation.

[7]There are other differences with Smets and Wouters (2007): (i) Our price and wage mark-up shocks follow an AR(1) process instead of the ARMA process chosen by SW; (ii) in SW the government spending shock is assumed to follow an autoregressive process which is also affected by the productivity shock; (iii) we have a preference shock instead of the risk-premium shock. Chari *et al.* (2009) criticized the risk premium shock arguing that has little interpretation and in unlikely to be invariant to monetary policy. We prefer our somewhat simpler set-up and we expect none of the differences to affect the main focus of the paper which is on the comparison between CD and CES production functions.

[8]Sequencing matters for the monopolistic trade-unions and intermediate firms who anticipate and exploit the downward-sloping demand for labour and goods respectively. Different set-ups with identical equilibria are common in the literature. Monopolistic prices can be transferred to the retail sector. When it comes to introducing financial frictions, for example, as in Gertler and Karadi (2011) the introduction of separate capital producers as in our set-up is convenient, but not essential in the SW model without such frictions.

intermediate firms.

3. A monopolistic trade-union differentiates the labour and sells type $N_t(j)$ at a price $W_t(j)$ to a labour packer in a sequence of Calvo staggered wage contracts. In equilibrium all households make identical choices of total consumption, savings, investment and labour supply.

4. The competitive labour packer forms a composite labour service according to a constant returns CES technology $N_t = \left(\int_0^1 N_t(j)^{(\zeta-1)/\zeta} dj\right)^{\zeta/(\zeta-1)}$ and sells onto the intermediate firm.

5. Each intermediate monopolistic firm f using composite labour and capital rented from capital producers to produce a differentiated intermediate good which is sold onto the final goods firm at price $P_t(f)$ in a sequence of Calvo staggered price contracts.

6. Competitive final goods firms use a continuum of intermediate goods according to another constant returns CES technology to produce aggregate output $Y_t = \left(\int_0^1 Y_t(f)^{(\mu-1)/\mu} df\right)^{\mu/(\mu-1)}$.

We now solve the model by backward induction starting with the production of final goods.

2.1 Final Goods

Each final goods firms minimizes the cost $\int_0^1 P_t(f)Y_t(f)df$ of producing the final output $Y_t = \left(\int_0^1 Y_t(f)^{(\zeta-1)/\zeta} df\right)^{\zeta/(\zeta-1)}$. This leads to the standard result for the Dixit-Stigliz aggregator

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} Y_t \tag{1}$$

$$P_t = \left[\int_0^1 P_t(f)^{1-\zeta} df\right]^{\frac{1}{1-\zeta}} \tag{2}$$

$$P_t Y_t = \int_0^1 P_t(f) Y_t(f) df \tag{3}$$

where P_t is an aggregate price index. Note that (1) and (3) imply (2).

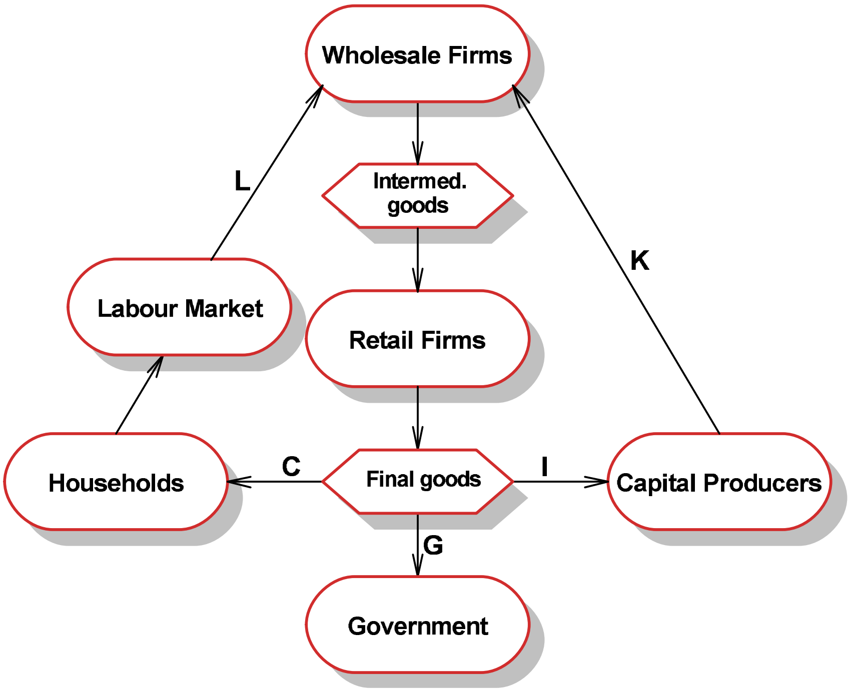


Figure 2: Model Structure

2.2 Intermediate Firms

In the intermediate goods sector each good f is produced by a single firm f using composite labour and capital with a technology:

$$Y_t(f) = (1-c)F(ZK_t, ZN_t, N_t, U_tK_t) \tag{4}$$

where c are fixed costs of production and U_t allows for variable capital utilization. The parameter c is pinned down by a free-entry condition that drives profits in the steady state to zero. Given that at this stage we do not specify the form of the production function we allow for all the possible specification of technology shocks. Calling ZK_t capital-augmenting and ZN_t labour-augmenting we are in the case of Hicks neutrality if $ZK_t = ZN_t > 0$, Solow neutrality if $ZK_t > 0$ and $ZN_t = 0$ and Harrod neutrality in the case of $ZK_t = 0$ and $ZN_t > 0$. Then minimizing costs $P_tRR_t^KU_t(f)K_t(f)+W_tN_t(f)$ leads to

$$\begin{aligned}\frac{W_t}{P_t} &\equiv MPL_t = MC_t(f)F_{N,t} &(5)\\ RR_t^K &\equiv MPK_t = MC_t(f)F_{K,t} &(6)\end{aligned}$$

where MPL_t and MPK_t are the marginal products of labour and capital respectively, RR_t^K is the real cost of capital. As usual the firm's cost minimizing real marginal costs $(MC_t(f))$ is given by the Lagrange multiplier related to the production function constraint.

Pricing by the firm follows the standard Calvo framework supplemented with indexation. At each period there is a probability of $1-\xi_p$ that the price is set optimally.[9] The optimal price derives from maximizing discounted profits. For those firms and workers unable to reset, prices are indexed to last period's aggregate inflation, with indexation parameter γ_p. With indexation parameter $\gamma_p \geq 0$, this implies that successive prices with no re-optimization are given by $P_t^0(f)$, $P_t^0(f)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}$, $P_t^0(f)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_p}$, ... For each intermediate producer f the objective is at time t to choose $\{P_t^0(f)\}$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(f)\left[P_t^0(f)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma} - P_{t+k}MC_{t+k}\right] \tag{7}$$

subject to $Y_{t+k}(f) = \left(\frac{P_t^0(f)}{P_{t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p}\right)^{-\zeta} Y_{t+k}$ (from (1)), where $\Lambda_{t,t+k} \equiv \beta\frac{U_{C,t+k}/P_{t+k}}{U_{C,t}/P_t}$, is the nominal stochastic discount factor over the interval $[t, t+k]$ and ζ is the elasticity of substitution across intermediate goods. Since firms are atomistic, the aggregate price index and the discount factor are given in their calculations.

This leads to the following first-order condition:

$$E_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(f)\left[P_t^0(f)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p} - MS_{p,t}P_{t+k}\mathrm{MC}_{t+k}\right] = 0 \tag{8}$$

where we introduced, as usual in the literature, a time varying mark-up of prices over marginal costs $MS_{p,t} = \frac{\zeta}{(\zeta-1)}eP_t$ with eP_t being the price mark-up shock process. Then by the law of large numbers the evolution of the price index is given by

$$P_{t+1}^{1-\zeta} = \xi_p\left(P_t\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}\right)^{1-\zeta} + (1-\xi_p)(P_{t+1}^0(f))^{1-\zeta} \tag{9}$$

[9]Thus we can interpret $\frac{1}{1-\xi_p}$ as the average duration for which prices are left unchanged.

2.3 Labour Packer

As with final goods firms, the labour packer minimizes the cost $\int_0^1 W_t(j)N_t(j)dj$ of producing the composite labour service $N_t = \left(\int_0^1 N_t(j)^{(\mu-1)/\mu}dj\right)^{\mu/(\mu-1)}$. This leads to the standard result for the Dixit-Stigliz aggregator

$$
\begin{aligned}
N_t(j) &= \left(\frac{W_t(j)}{W_t}\right)^{-\mu} N_t && (10)\\
W_t &= \left[\int_0^1 W_t(j)^{1-\mu}dj\right]^{\frac{1}{1-\mu}} && (11)\\
W_t N_t &= \int_0^1 W_t(j)N_t(j)dj && (12)
\end{aligned}
$$

where W_t is an aggregate wage index. Note that (10) and (12) imply (11).

2.4 Trade-Unions

Wage setting by the trade-union again follows the standard Calvo framework supplemented with indexation. At each period there is a probability $1-\xi_w$ that the wage is set optimally. The optimal wage derives from maximizing discounted profits. For those trade-unions unable to reset, wages are indexed to last period's aggregate inflation, with wage indexation parameter γ_w. Then as for price contracts the wage rate trajectory with no re-optimization is given by $W_t^0(j)$, $W_t^0(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}$, $W_t^0(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_w}$, $\cdots$. The trade union that buys homogeneous labour at a price $W_{h,t}$ and converts it into a differentiated labour service of type j. The trade union time t then chooses $W_t^0(j)$ to maximize

$$
E_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} N_{t+k}(j)\left[W_t^0(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - W_{h,t+k}\right] \tag{13}
$$

where $N_t(j)$ is given by (10) so that $N_{t+k}(j) = \left(\frac{W_t^0(j)}{W_{t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}\right)^{-\eta} N_{t+k}$ and η is the elasticity of substitution across labour varieties. By analogy with (8) this leads to the following first-order condition

$$
E_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} N_{t+k}(j)\Big[W_t^0(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - MS_{w,t}W_{h,t+k}\Big] = 0 \tag{14}
$$

where $MS_{w,t} = \frac{\eta}{(\eta-1)} eW_t$ is the time varying wage mark-up with eW_t being the wage mark-up shock process. Then by the law of large numbers the evolution of the wage index is given by

$$W_{t+1}^{1-\eta} = \xi_w \left(W_t \frac{\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}}{\frac{P_{t+1}}{P_t}} \right)^{1-\eta} + (1-\xi_w)(W_{t+1}^0(j))^{1-\eta} \tag{15}$$

2.5 Representation of Price-Wage Dynamics as Difference Equations

We now proceed to represent the price and wage dynamics as difference equations. This is necessary to set up the model in standard software such as DYNARE and is also convenient when it comes to linearizing the model about a steady state. Both sides of the foc for pricing (8) and wage (14), are of the form considered in Appendix A. Using the Lemma, first define

$$\Pi_{p,t} \equiv \frac{P_t}{P_{t-1}} = \pi_t + 1 \tag{16}$$

$$\frac{P_t^0}{P_t} \equiv \frac{J_{p,t}}{H_{p,t}} \tag{17}$$

$$\tilde{\Pi}_{p,t} \equiv \frac{\Pi_{p,t}}{\Pi_{p,t-1}^{\gamma_p}} \tag{18}$$

and then aggregate inflation dynamics are given by

$$H_{p,t} - \xi_p \beta E_t[\tilde{\Pi}_{p,t+1}^{\zeta-1} H_{p,t+1}] = Y_t U_{C,t} \tag{19}$$

$$J_{p,t} - \xi_p \beta E_t[\tilde{\Pi}_{p,t+1}^{\zeta} J_{p,t+1}] = MS_{p,t} Y_t MC_t U_{C,t} \tag{20}$$

$$1 = \xi_p \tilde{\Pi}_{p,t}^{\zeta-1} + (1-\xi_p)\left(\frac{J_{p,t}}{H_{p,t}}\right)^{1-\zeta} \tag{21}$$

For staggered wage setting, symmetrically, wage dynamics are given by defining:

$$\Pi_{w,t} \equiv \frac{W_t}{W_{t-1}} \Pi_{p,t} \tag{22}$$

$$\frac{W_t^0}{P_t} \equiv \frac{J_{w,t}}{H_{w,t}} \tag{23}$$

$$MS_{w,t} \equiv \frac{\mu}{\mu-1} eW_t \tag{24}$$

Aggregate wage dynamics are then given by

$$
\begin{aligned}
H_{w,t} - \xi_w \beta E_t \left[\Pi_{w,t+1}^{\mu} \left(\frac{\Pi_{p,t+1}}{\Pi_{p,t}^{\gamma_w}} \right)^{\mu-1} \right] H_{w,t+1} &= N_t U_{C,t} && (25) \\
J_{w,t} - \xi_w \beta E_t \left[\Pi_{w,t+1}^{\mu} \left(\frac{\Pi_{p,t+1}}{\Pi_{p,t}^{\gamma_w}} \right)^{\mu} \right] &= -MS_{w,t} N_t MU_t^N && (26) \\
\xi_w \left[\Pi_{w,t} \frac{\Pi_{p,t}}{\Pi_{p,t-1}^{\gamma_w}} \right]^{\mu-1} + (1-\xi_w) \left(\frac{\frac{J_{w,t}}{H_{w,t}}}{\frac{W_t}{P_t}} \right)^{1-\mu} &= 1 && (27)
\end{aligned}
$$

2.6 Capital Producers

Capital producing firms convert I_t of output into $(1 - S(X_t))I_t$ of new capital sold at a real price Q_t. They then maximize expected discounted profits

$$
E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}^{r} \left[Q_{t+k} Z I_{t+k} (1 - S\left(I_{t+k}/I_{t+k-1} \right)) I_{t+k} - I_{t+k} \right]
$$

where $\Lambda_{t,t+k}^{r} \equiv \beta \frac{U_{C,t+k}}{U_{C,t}}$ is the *real* stochastic discount factor over the interval $[t, t+k]$. This results in the first-order condition

$$
Q_t Z I_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[\Lambda_{t,t+1}^{r} Q_{t+1} Z I_{t+1} S'(X_{t+1}) \frac{I_{t+1}^2}{I_t^2} \right] = 1 \tag{28}
$$

Capital accumulation is given by

$$
K_{t+1} = (1-\delta) K_t + (1 - S(X_t)) I_t Z I_t; \tag{29}
$$

where δ is the depreciation rate, ZI_t is the investment specific shock, $X_t = \frac{I_t}{I_{t-1}}$ and $S()$ satisfies S', $S'' \geq 0\,;\ S(1+g) = S'(1+g) = 0$.

Demand for capital by firms must satisfy

$$
E_t[R_{t+1}] = \frac{E_t \left[F_{K,t} + (1-\delta) Q_{t+1} \right]}{Q_t} \tag{30}
$$

In (30) the right-hand-side is the gross return to holding a unit of capital in from t to $t+1$. The left-hand-side is the gross return from holding bonds, the opportunity cost of capital. We complete this set-up with the functional form

$$S(X) = \phi_X (X_t - (1+g))^2 \tag{31}$$

where g is the balanced growth rate.

Owners of physical capital can control the intensity at which capital is utilized in production. As in Christiano *et al.* (2005) and Smets and Wouters (2007) we assume that using the stock of capital with intensity U_t produces a cost of $a(U_t)K_t$ units of the composite final good. The functional form is chosen consistent with the literature:

$$a(U_t) = \gamma_1 (U_t - 1) + \frac{\gamma_2}{2}(U_t - 1)^2 \tag{32}$$

and satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. Note that $\frac{\gamma_1}{\gamma_2} = \frac{1-\phi}{\phi}$ in the Smets and Wouters (2007) set-up. In order to compare results we will estimate ϕ.

2.7 The Household Problem

The Household problem is standard and can be summarized by:

$$\begin{aligned}
\text{Utility} : U_t &= U(C_t, L_t) && (33)\\
\text{Euler} : U_{C,t} &= \beta E_t \left[R_{t+1} U_{C,t+1} \right] && (34)\\
\text{Labour Supply} : \quad \frac{U_{N,t}}{U_{C,t}} &= -MRS_t \equiv -\frac{W_{h,t}}{P_t} && (35)\\
\text{Leisure} : L_t &\equiv 1 - N_t && (36)
\end{aligned}$$

2.8 Monetary Authority, Aggregation and Equilibrium

Nominal and real interest rates are related by the Fischer equation

$$E_t[R_{t+1}] = E_t \left[\frac{R_{n,t}}{\Pi_{t+1}} \right] \tag{37}$$

where the nominal gross interest rate $R_{n,t}$ is a policy variable, typically given by a simple Taylor-type rule:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \alpha_R \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_Y \log\left(\frac{Y_t}{Y}\right) + \epsilon_{m,t} \tag{38}$$

where, we define the output gap as the deviation between the output and its steady-state value.

The resource constraint must take into account relative price dispersion across varieties and wage dispersion across firms. By writing $Y_t(f)^W = F(Z_t, N_t(j), U_t K_{t-1})$.[10] At firm level supply must equal demand:

$$(1-c)F(Z_t, \left(\frac{W_t(j)}{W_t}\right)^{-\mu} N_t, U_t K_{t-1}) = (C_t + I_t + G_t + a(U_t)K_{t-1}) \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} \quad (39)$$

Integrating across all firms, taking into account that the capital-labour ratio is common across firms and that the wholesale sector is separated from the retail sector we obtain

$$(1-c)F(Z_t, \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\mu} dj N_t, U_t K_{t-1}) = (C_t + I_t + G_t + a(U_t)K_{t-1}) \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} df \quad (40)$$

where the price dispersion is given by $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} df$ and wage dispersion is given by $\Delta_{w,t} = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\mu} dj$. As shown in Appendix B:

$$\Delta_{p,t} = \xi \tilde{\Pi}_t^{\zeta} \Delta_{p,t-1} + (1-\xi)\left(\frac{P_t^0}{P_t}\right)^{-\zeta} \quad (41)$$

$$\Delta_{w,t} = \xi_w \tilde{\Pi}_{w,t}^{\mu} \Delta_{w,t-1} + (1-\xi_w)\left(\frac{W_t^0}{W_t}\right)^{-\mu} \quad (42)$$

$$(43)$$

Then (39) takes the form:

$$Y_t = (1-c)\frac{Y_t^W}{\Delta_{p,t}\Delta_{w,t}} = C_t + I_t + G_t + a(U_t)K_{t-1} \quad (44)$$

2.9 The Normalized CES Production Function

The production function is assumed to be CES as in Cantore *et al.* (2014b) which nests Cobb-Douglas as a special case and admits the possibility of neutral and non-neutral technical change. Here we adopt the 're- parametrization' procedure described in Cantore

[10] Where by simplicity we call Z_t a vector containning each type of biased and un-biased technical change defined in (4).

and Levine (2012) in order to normalize the CES production function:

$$\begin{aligned} Y_t^W &= \left[\alpha_k (ZK_t U_t K_t)^{\psi} + \alpha_n (ZN_t N_t)^{\psi}\right]^{\frac{1}{\psi}} \; ; \psi \neq 0 \;\&\; \alpha_k + \alpha_N \neq 1 \\ &= (ZK_t U_t K_t)^{\alpha_k} (ZN_t N_t)^{\alpha_n} \; ; \psi \to 0 \;\&\; \alpha_k + \alpha_N = 1 \end{aligned} \tag{45}$$

where Y_t^W, K_t, N_t are wholesale output, capital and labour inputs respectively at time t and ψ is the substitution parameter and α_k and α_n are sometimes referred as distribution parameters. As explained earlier, the terms ZK_t and ZN_t capture capital-augmenting and labour-augmenting technical progress respectively. Calling σ the elasticity of substitution between capital and labour,[11] with $\sigma \in (0, +\infty)$ and $\psi = \frac{\sigma - 1}{\sigma}$ then $\psi \in (-\infty, 1)$. When $\sigma = 0 \Rightarrow \psi = -\infty$ we have the Leontief case, whilst when $\sigma = 1 \Rightarrow \psi = 0$ (45) collapses to the usual Cobb-Douglas case.

From the outset a comment on dimensions would be useful. Technology parameters are not measures of efficiency as they depend on the units of output and inputs (i.e., is not dimensionless[12]) and the problem of normalization arises because unless $\psi \to 0$, α_n and α_k in (45) are not shares and in fact are not dimensionless.

Marginal products of labour and capital are respectively

$$F_{N,t} = \frac{Y_t^W}{N_t} \left[\frac{\alpha_n (ZN_t N_t)^{\psi}}{\alpha_k ZK_t U_t K_t^{\psi} + \alpha_n (ZN_t N_t)^{\psi}} \right] = \alpha_n ZN_t^{\psi} \left(\frac{Y_t^W}{N_t} \right)^{1-\psi} \tag{46}$$

$$F_{K,t} = \frac{Y_t^W}{K_t} \left[\frac{\alpha_k ZK_t U_t K_t^{\psi}}{\alpha_k ZK_t U_t K_t^{\psi} + \alpha_n (ZN_t N_t)^{\psi}} \right] = \alpha_k (U_t ZK_t)^{\psi} \left(\frac{Y_t^W}{K_t} \right)^{1-\psi} \tag{47}$$

The equilibrium of real variables depends on parameters defining the RBC core of the model ϱ, σ_c, δ, ψ, α_k and α_n, and those defining the NK features. Of the former, ϱ, ψ and

[11] The elasticity of substitution for the case of perfect competition, where all the product is used to remunerate factor of productions, is defined as the elasticity of the capital/labour ratio with respect to the wage/capital rental ratio. Then calling W the wage and $R + \delta$ the rental rate of capital we can define the elasticity as follows:

$$\sigma = \frac{d\frac{K}{L}\frac{L}{K}}{d\frac{W}{R+\delta}\frac{R+\delta}{W}}.$$

See La Grandville (2009) for a more detailed discussion.

[12] For example for the Cobb-Douglas production function in the steady state, $Y = K^{\alpha}(AN)^{1-\alpha}$, by dimensional homogeneity, the dimensions of A are (output per period)$^{\frac{1}{1-\alpha}}$ / ((person hours per period)$\times$ (machine hours per period)$^{\frac{\alpha}{1-\alpha}}$). For some this poses a fundamental problem with the notion of a production function - see Barnett (2004). Units can be chosen so that when $N = 1$ and $K = 1$, then $Y = 1$ implying $A = 1$. For the equilibrium to be independent of the choice of units, it follows that it must be independent of the steady state value A. This is readily demonstrated in what follows.

σ_c are dimensionless, δ depends on the unit of time, but unless $\psi = 0$ and the technology is Cobb-Douglas, α_k and α_n depend on the units chosen for factor inputs, namely machine units per period and labour units per period. To see this rewrite the wholesale firm's foc (5) and (6) in terms of factor shares

$$\frac{W_t N_t}{P_t^W Y_t^W} = \alpha_n Z N_t^{\psi} \left(\frac{Y_t^W}{N_t}\right)^{-\psi} \tag{48}$$

$$\frac{(R_t - 1 + \delta) K_t}{P_t^W Y_t^W} = \alpha_k (U_t Z K_t)^{\psi} \left(\frac{Y_t^W}{K_t}\right)^{-\psi} \tag{49}$$

where $P_t^W \equiv MC_t P_t$ is the price of wholesale output. Then we have

$$\frac{W_t N_t}{(R_t + \delta)} = \frac{\alpha_n}{\alpha_k} \left(\frac{ZK_t U_t K_t}{ZN_t N_t}\right)^{-\psi} \tag{50}$$

Thus α_n (α_k) can be interpreted as the share of labour (capital) iff $\psi = 0$ and the production function is Cobb-Douglas. Otherwise the dimensions of α_k and α_n depend on those for $\left(\frac{ZK_t U_t K_t}{ZN_t N_t}\right)^{\psi}$ which could be for example, (effective machine hours per effective person hours)$^{\psi}$. In our aggregate production functions we choose to avoid specifying unit of capital, labour and output.[13] It is impossible to interpret and therefore to calibrate or estimate these 'share' parameters.

There are two ways to resolve this problem; 're-parameterize' the dimensional parameters α_k and α_n so that they are expressed in terms of dimensionless ones all parameters to be estimated or calibrated (see Cantore and Levine (2012)), or 'normalize' the production function in terms of deviations from a steady state. We consider these in turn.

2.9.1 Re-parametrization of α_n and α_k

On the balanced-growth path (bgp) consumption, output, investment, capital stock, the real wage and government spending are growing at a common growth rate g driven by exogenous labour-augmenting technical change $\overline{ZN}_{t+1} = (1 + g)\overline{ZN}_t$, but labour input N is constant.[14] As is well-known a bgp requires either Cobb-Douglas technology or that

[13] Unlike the physical sciences where particular units are explicitly chosen so dimension-dependent parameters pose no problems. For example the fundamental constants such as the speed of light is expressed in terms of metres per second; Newtons constant of gravitation has units cubic metres per (kilogram $\times$ second^2) etc.

[14] If output, consumption etc are defined in per capita terms then N can be considers as the proportion of the available time at work and is therefore both stationary and dimensionless.

technical change must be driven solely by the labour-augmenting variety (see, for example, Jones (2005)). Then $ZK_t = ZK$ must also be constant along the bgp.

On the bgp let capital share and wage shares in the wholesale sector be α and $1-\alpha$ respectively. Then using (48) and (49) we obtain our *re-parameterization* of α_n and α_k:

$$\alpha_k = \alpha \left(\frac{\bar{Y}_t}{ZK\bar{U}_t\bar{K}_t} \right)^{\psi} \tag{51}$$

$$\alpha_n = (1-\alpha) \left(\frac{\bar{Y}_t}{\overline{ZN}_t N} \right)^{\psi} \tag{52}$$

Note that $\alpha_k = \alpha$ and $\alpha_n = 1-\alpha$ at $\psi = 0$, the Cobb-Douglas case.[15] This completes the stationarized equilibrium defined in terms of *dimensionless* RBC core parameters ϱ, σ_c, ψ,α and δ which depends on the unit of time, plus NK parameters. In (51) and (52) dimensional parameters are expressed in terms of other endogenous variables Y^W, N and K which themselves are functions of $\theta \equiv [\sigma, \psi, \alpha, \delta, \cdots]$. Therefore $\alpha_n = \alpha_n(\theta)$, and $\alpha_k = \alpha_k(\theta)$ which expresses why we refer to this procedure as reparameterization.

There is one more normalization issue: the *choice of units* at some point say $t = 0$ on the steady state bgp. We use for simplicity $\bar{Y}_0 = \overline{ZN}_0 = ZK = 1$[16] but, as it is straightforward to show that having expressed the model in terms of dimensionless parameters through re-parameterization makes the steady state ratios of the endogenous variables of the model independent of this choice.

2.9.2 The Production Function in Deviation Form

This simply bypasses the need to retain α_k and α_n and writes the dynamic production function in deviation form about its steady state as

$$\frac{Y_t^W}{\bar{Y}_t^W} = \left[\frac{\alpha_k (ZK_t U_t K_t)^{\psi} + \alpha_n (ZN_t N_t)^{\psi}}{\alpha_k (ZK\bar{U}_t\bar{K}_t)^{\psi} + \alpha_n (\overline{ZN}_t N)^{\psi}} \right]^{\frac{1}{\psi}} = \left[\frac{\alpha_k \left(\frac{ZK_t U_t K_t}{ZK\bar{U}_t\bar{K}_t} \right)^{\psi}}{\alpha_k + \alpha_n \left(\frac{\overline{ZN}_t N}{ZK\bar{U}_t\bar{K}_t} \right)^{\psi}} + \frac{\alpha_n \left(\frac{ZN_t N_t}{\overline{ZN}_t N_t} \right)^{\psi}}{\alpha_k \left(\frac{ZK\bar{U}_t\bar{K}_t}{\overline{ZN}_t N} \right)^{\psi} + \alpha_n} \right]^{\frac{1}{\psi}}$$

[15] And as argued before if $\alpha \in (0,1)$ $\alpha_k + \alpha_n = 1$ iff $\psi = 0$.

[16] By assuming $\bar{Y}_0 = 1$ we implicitely assume $\bar{Y}_0^W = \frac{\bar{Y}_0}{1-c}$.

Then from (51) and (52) we can write this simply as

$$\frac{Y_t^W}{\bar{Y}_t^W} = \left[(1-\alpha)\left(\frac{ZK_tU_tK_t}{ZK\bar{U}_t\bar{K}_t}\right)^{\psi} + \alpha\left(\frac{ZN_tN_t}{\overline{ZN}_tN}\right)^{\psi}\right]^{\frac{1}{\psi}} \tag{53}$$

as in Cantore *et al.* (2014b). The steady-state normalization now consists of $\overline{ZN}_0 = \bar{Y}_0 = ZK = 1$[17] and is characterized entirely by fixed shares of consumption, investment and government spending and by labour supply as a proportion of available time, all dimensionless quantities apart from the unit of time.

Using either of these two approaches, as showed by Cantore and Levine (2012), the steady state ratios of the endogenous variables and the dynamics of the model are not affected by the starting values of output and the two source of shocks ($\bar{Y}_0, \overline{ZN}_0$, ZK) which only represent choice of units. Crucially, this implies also that changing σ does not change our steady state ratios and factor shares, impulse response functions are directly comparable, and parameter values are consistent with their economic interpretation.

2.10 Utility Function

The household utility function is chosen to be compatible with a balanced-growth steady state and allows external habit-formation:

$$\begin{aligned}
U_t &= \frac{eB_t((C_t - \chi C_{t-1})^{(1-\varrho)}(1-N_t)^{\varrho})^{1-\sigma_c} - 1}{1-\sigma_c} && (54)\\
U_{C,t} &= eB_t(1-\varrho)(C_t - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)-1}((1-N_t)^{\varrho(1-\sigma_c)}) && (55)\\
U_{N,t} &= -eB_t\varrho(C_t - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)}(1-N_t)^{\varrho(1-\sigma_c)-1} && (56)
\end{aligned}$$

Where χ represents the habit formation parameter and eB_t a preference shock.

2.11 Shock Processes

To close the model we need to specify the law of motion of the shock processes

$$\begin{aligned}
\log ZK_t - \log ZK &= \rho_{ZK}(\log ZK_{t-1} - \log ZK) + \epsilon_{ZK,t} && (57)\\
\log ZN_t - \log \overline{ZN}_t &= \rho_{ZN}(\log ZN_{t-1} - \log \overline{ZN}_t) + \epsilon_{ZN,t} && (58)
\end{aligned}$$

[17] Which is almost identical to the one used in Cantore *et al.* (2014b) although they normalize as well hours worked to 1 using the accounting identity $\bar{Y} = (\bar{R}+\delta)\bar{K} + \bar{W}\bar{N}$.

$$\log ZI_t - \log ZI = \rho_{ZI}(\log ZI_{t-1} - \log ZI) + \epsilon_{ZI,t} \tag{59}$$

$$\log G_t - \log \overline{G}_t = \rho_G(G_{t-1} - \overline{G}_t) + \epsilon_{G,t} \tag{60}$$

$$\log eP_t - \log eP = \rho_P(eP_{t-1} - eP) + \epsilon_{P,t} \tag{61}$$

$$\log eW_t - \log eW = \rho_W(eW_{t-1} - eW) + \epsilon_{W,t} \tag{62}$$

$$\log eB_t - \log eB = \rho_W(eB_{t-1} - eB) + \epsilon_{B,t} \tag{63}$$

In total the model has these 7 AR(1) shocks plus the shock to the monetary policy rule.

3 Estimation

We estimate the linearized version of the model around zero steady state inflation by Bayesian methods using DYNARE. We use the same observable set as in Smets and Wouters (2007) in first difference at quarterly frequency but extend the sample length to the second quarter of 2008, the point before the outbreak of 2008 crisis. Namely, these observable variables are the log differences of real GDP, real consumption, real investment and real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. As in Smets and Wouters (2007), hours worked are derived from the index of average hours for the non-farm business sector and we divide hourly compensation from the same sector by the GDP price deflator to obtain the real wage. All series are seasonally adjusted. All data are taken from the FRED Database available through the Federal Reserve Bank of St.Louis and the US Bureau of Labour Statistics. The sample period is 1984:1-2008:2.

The corresponding measurement equations for the 7 observables are, using lower case letters to express variables in log-deviations from the steady state,:

$$dy = y_t - y_{t-1} + ctrend \tag{64}$$

$$dc = c_t - c_{t-1} + ctrend \tag{65}$$

$$di = i_t - i_{t-1} + ctrend \tag{66}$$

$$dw = w_t - w_{t-1} + ctrend \tag{67}$$

$$\Pi_{obs} = \pi_t + conspie \tag{68}$$

$$R_{obs} = rn_t + consrn \tag{69}$$

$$h_{obs} = n_t + conslab \tag{70}$$

The four observable are taken in first difference while inflation, nominal interest rate and hours worked are used in levels. We introduce a common trend to the real variables and a specific one to inflation, nominal interest rate and hours worked.

3.1 Bayesian Methodology

Bayesian estimation entails obtaining the posterior distribution of the model's parameters, say θ, conditional on the data. Using the Bayes' theorem, the posterior distribution is obtained as:

$$p(\theta|Y^T) = \frac{L(Y^T|\theta)p(\theta)}{\int L(Y^T|\theta)p(\theta)d\theta} \tag{71}$$

where $p(\theta)$ denotes the prior density of the parameter vector θ, $L(Y^T|\theta)$ is the likelihood of the sample Y^T with T observations (evaluated with the Kalman filter) and $\int L(Y^T|\theta)p(\theta)d\theta$ is the marginal likelihood. Since there is no closed form analytical expression for the posterior, this must be simulated.

One of the main advantages of adopting a Bayesian approach is that it facilitates a formal comparison of different models through their posterior marginal likelihoods, computed using the Geweke (1999) modified harmonic-mean estimator. For a given model $m_i \in M$ and common data set, the marginal likelihood is obtained by integrating out vector θ,

$$L\left(Y^T|m_i\right) = \int_\Theta L\left(Y^T|\theta, m_i\right) p\left(\theta|m_i\right) d\theta \tag{72}$$

where $p_i\left(\theta|m_i\right)$ is the prior density for model m_i, and $L\left(Y^T|m_i\right)$ is the data density for model m_i given parameter vector θ. To compare models (say, m_i and m_j) we calculate the posterior odds ratio which is the ratio of their posterior model probabilities (or Bayes Factor when the prior odds ratio, $\frac{p(m_i)}{p(m_j)}$, is set to unity):

$$PO_{i,j} = \frac{p(m_i|Y^T)}{p(m_j|Y^T)} = \frac{L(Y^T|m_i)p(m_i)}{L(Y^T|m_j)p(m_j)} \tag{73}$$

$$BF_{i,j} = \frac{L(Y^T|m_i)}{L(Y^T|m_j)} = \frac{\exp(LL(Y^T|m_i))}{\exp(LL(Y^T|m_j))} \tag{74}$$

in terms of the log-likelihood. Components (73) and (74) provide a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. Such comparisons are important in the assessment of rival models, as the model which attains the highest odds outperforms its rivals and is therefore favoured.

Given Bayes factors, we can easily compute the model probabilities $p_1, p_2, \cdots, p_n$ for n models. Since $\sum_{i=1}^{n} p_i = 1$ we have that $\frac{1}{p_1} = \sum_{i=2}^{n} BF_{i,1}$, from which p_1 is obtained. Then $p_i = p_1 BF(i,1)$ gives the remaining model probabilities.

3.2 Likelihood Comparison of Models

We compare four different model specifications in order to see if the introduction of factor substitutability and/or the biased technical change improves the fit of the estimation. In the first row of Table 1 we present the likelihood density of the model with the CD production function where only the labour-augmenting technology shock is present. In the second row we introduce the CES and calibrate the elasticity of substitution to 0.4, following the literature as in Cantore *et al.* (2014b) and Klump *et al.* (2012), and introduce the capital-augmenting shock whilst in rows 3 and 4 we estimate σ in a model with and without the latter shock. Strictly speaking a meaningful likelihood comparison that provides information about σ is only possible between row 1 and 3 (where we can compare like for like without adding a further exogenous shock).

Table 1 reveals that Models with the CES production function clearly outperforms its CD counterpart with a posterior probability of 100%. This suggests that incorporating a CES production function offers substantial improvements in terms of the model fitness to the data in the US economy. The differences in log marginal likelihood are substantial. For example, the log marginal likelihood difference between the first two specifications is 12.47 corresponding to a posterior Bayes Factor of 2.6041e+05. As suggested by Kass and Raftery (1995), the posterior Bayes Factor needs to be at least $e^3 \approx 20$ for there to be a positive evidence favouring one model over the other.

Model	σ	Technology shocks	Log data density	Difference with CD
CD	1	ZL	-544.60	0
CES0	calibrated=0.4	ZK & ZL	-532.13	12.47
CES1	estimated=0.15	ZL	-528.50	16.10
CES2	estimated=0.15	ZK & ZL	-528.31	16.29

Table 1: Marginal Likelihood Comparison Between CD and CES Specifications

3.3 Estimation under the Standard Information Assumptions

In this section we made the standard information assumption in solving rational expectations models that economic agents have perfect information about the realizations of current shocks and other relevant macroeconomic variables, alongside their knowledge of the model, parameter values and the policy rule, whereas the econometrician uses only observable data. In effect the private sector has more information than the econometrician, so we refer to this case as *asymmetric information* (AS).

The joint posterior distribution of the estimated parameters is obtained in two steps. First, the posterior mode and the Hessian matrix are obtained via standard numerical optimization routines. The Hessian matrix is then used in the Metropolis-Hastings (MH) algorithm to generate a sample from the posterior distribution. Two parallel chains are used in the Monte-Carlo Markov Chain Metropolis-Hastings (MCMC-MH) algorithm. Thus, 250,000 random draws (though the first 30% 'burn-in' observations are discarded) from the posterior density are obtained via the MCMC-MH algorithm, with the variance-covariance matrix of the perturbation term in the algorithm being adjusted in order to obtain reasonable acceptance rates (between 20%-30%).

Estimation results from posteriors maximization are presented in Appendix C. We used the same priors as Smets and Wouters (2007) for common parameters whereas we used a loose prior for the elasticity of substitution between capital and labour in order to see if the data are informative about the value of this parameter. A few structural parameters are kept fixed or calibrated based on some parameters being estimated in the estimation procedure, in accordance with the usual practice in the literature (see Table 2). This is done so that the calibrated parameters reflect steady state values of the observed variables.

First we focus on the posterior estimates obtained using the most general CES model, CES2. As shown in Tables 7 and 8, the point estimates under the CES assumption are tight and plausible. In particular, focusing on the parameters characterizing the degree of price stickiness and the existence of real rigidities, we find that the price indexation parameters are estimated to be smaller than assumed in the prior distribution (in line with those reported by Smets and Wouters (2007)). The estimates of the $\gamma' s$ imply that inflation is intrinsically not very persistent in the CES model specifications. The posterior mean

Calibrated parameter	Symbol	Value
Discount factor	β	0.99
Depreciation rate	δ	0.025
Growth rate	g	$\frac{\delta}{4}$
Substitution elasticity of goods	ζ	7
Substitution elasticity of labour	μ	7
Variable capital utilization	γ_1	$\frac{1}{\beta} + \delta - 1$
Fixed cost	c	$1 - MC = 0.1429$
Preference parameter	ϱ	$\frac{1-h}{1+h(c_y(1-\chi)/\alpha-1)}$
Implied steady state relationship		
Government expenditure-output ratio	g_y	0.2
Investment-output ratio	i_y	$\frac{(1-\alpha)\delta}{(1/\beta-1+\delta)}$
Consumption-output ratio	c_y	$1 - g_y - i_y$

Table 2: Calibrated Parameters

estimates for the Calvo parameters, ξ_p and ξ_w, imply an average price contract duration of around 2.31 quarters and an average wage contract duration of around 1.84 quarters, respectively. These results are in general consistent with the findings from empirical works on the DSGE modelling in the US economy. It is interesting to note that the risk-aversion parameter (σ_c) is estimated to be less than assumed in the prior distribution, indicating that the inter-temporal elasticity of substitution (proportional to $1/\sigma_c$) is estimated to be about 0.86 in the US, which is plausible as suggested in much of RBC literature. As expected, the policy rule estimates imply a fairly strong response (α_π) to expected inflation by the US Fed Reserve and the degree of interest rate smoothing (α_r) is fairly strong.

Figure 5 in Appendix C plots the prior and posterior distributions for the above CES model. The location and the shape of the posterior distributions are largely independent of the priors we have selected since priors are broadly less informative. Most of the posterior distributions are roughly symmetric implying that the mean and median coincide. According to Figure 5, there is little information in the data for some parameters where prior and posterior overlap.[18] Perhaps the most notable finding comes from the estimation of the parameter σ - our key parameter in the CES setting. As a result of assuming a very diffuse prior with large standard deviation, we find that the data is very informative about this parameter (as clearly shown in the figure, curves do not overlap each other and are very different) and the point estimate of σ in Table 7 is close to the plausible values.

[18] In particular parameters ρ_{ZK}, σ_c, γ_p, γ_w and α are weakly identified.**BO pi in Figure 5 should be α. Could we have Greek letters in the Figure as in the main paper.**

This further provides strong evidence to support the empirical importance of the CES assumption.

We now turn to the comparisons between parameter estimates under CD and CES specifications. Parameter posteriors that are quantitatively different[19] from the estimation using a Cobb-Douglas specification are underlined in Tables 7 and 8.

Starting with the parameters related to the exogenous shocks (Table 7) we notice that the estimated standard deviations of the newly introduced capital-augmenting technology shock is very small but, probably because of its introduction, the standard deviation of the investment specific shock reduces significantly (from 4.16 in the CD specification to 3.06 in the CES case).[20] We also notice that the estimated standard deviations of the mark-up shocks are lower under the CES specification and the standard deviation of the preference shock is slightly higher. The autoregressive parameters of the exogenous shocks are not affected significantly by the CES choice.

Posterior estimation of the investment adjustment costs parameter (ϕ^X) reduces by 0.75 points when we estimate the model under CES showing once again how introducing factor-biased technical change affects significantly the estimation of 'investment-related' parameters. The parameters of the utility function also appear to be affected by the choice of the production function (σ_c reduces by 0.94 and χ reduces by around 0.21). Regarding the parameters associated with sticky prices and wages both the probabilities of no price-adjustment (ξ_p) and no wage-adjustment (ξ_w) change significantly, decreasing from 0.77 to 0.57 and 0.60 to 0.46, respectively. Monetary policy weights (except the weight on inflation which increases slightly), real and nominal trends estimations are not affected by the introduction of factor substitutability and biased technical change.

3.4 Estimation under Symmetric Imperfect Information

In this section, which is based on Levine *et al.* (2012), we relax the extreme perfect information assumptions for the private sector (the standard asymmetric information (AI) case in Levine *et al.* (2012)) and assume that both private agents and the econometrician have the *same* imperfect information (II) set. We provide the estimation results from

[19] Difference in posteriors up to 0.05 were not considered quantitatively relevant here.

[20] The two shocks are clearly related and it is very likely that when ZK is absent ZI is capturing "capital-biased" technological progress.

posteriors maximization for Model CES2, the 'best' model in Table 1, under II. The following table provides a formal Bayesian comparison between CES2 under AI and II respectively.[21].

Model	σ	Technology shocks	Log data density	Difference with CD
CES2-AI	estimated=0.15	ZK & ZL	-528.31	16.29
CES2-II	estimated=0.18	ZK & ZL	-524.59	20.01

Table 3: Marginal Likelihood Comparison Between AI and II

It is clear from Table 3 that the assumption of imperfect information leads to a better fit as implied by the marginal likelihoods. Recalling Kass and Raftery (1995), a Bayes factor of 10-100 or a log data density range of [2.30, 4.61] is 'strong to very strong evidence'. For our Model CES2, there is 'strong' evidence in favour of the II assumption. The differences in log data density or the posterior odds ratio are substantive when comparing models assuming both CES and II with the model with CD. For example, the log marginal likelihood difference between Model CES2 under II and Model CD is 20.01. In order to choose the former over Model CD, we need a prior probability over Model CD 4.9004e+08($= e^{20.01}$) times larger than our prior probability over Model CES2 under II and this factor is decisive.

parameter	prior mean	post. mean CES (AI case)	5% CES	95% CES	Prior	pstdev CES
ρ_{ZL}	0.5	0.9470 (0.9443)	0.9059	0.9932	beta	0.2
ρ_{ZK}	0.5	0.4980 (0.4441)	0.1887	0.8325	beta	0.2
ρ_G	0.5	0.9613 (0.9631)	0.9418	0.9782	beta	0.2
ρ_{ZI}	0.5	0.7961 (0.7429)	0.6817	0.9117	beta	0.2
ρ_P	0.5	0.9672 (0.9744)	0.9399	0.9955	beta	0.2
ρ_W	0.5	0.9750 (0.9656)	0.9565	0.9938	beta	0.2
ρ_B	0.5	0.8728 (0.9311)	0.7977	0.9503	beta	0.2
ε_{ZL}	0.1	0.6905 (0.6833)	0.5944	0.7793	invg	2.0
ε_{ZK}	0.1	0.0804 (0.0744)	0.0240	0.1492	invg	2.0
ε_G	0.5	1.9947 (1.9904)	1.7650	2.2374	invg	2.0
ε_{ZI}	0.1	2.8668 (3.0647)	1.4762	4.2147	invg	2.0
ε_P	0.1	0.3771 (0.3756)	0.3093	0.4370	invg	2.0
ε_W	0.1	0.9781 (0.9482)	0.8328	1.1282	invg	2.0
ε_M	0.1	0.1604 (0.1579)	0.1371	0.1832	invg	2.0
ε_B	0.1	1.0863 (1.4997)	0.7681	1.3876	invg	2.0

Table 4: Posterior Results for the Exogenous Shocks (II)

[21]To complete the comparison of CD and CES under either AI or II we have also compared the two production function assumption under II. Similar results to those under AI were obtained and are not reported

parameter	prior mean	post. mean CES (AI case)	5% CES	95% CES	Prior	pstdev CES
σ	1	0.1788 (0.1542)	0.0735	0.2841	gamma	1
h	0.4	0.5343 (0.5136)	0.4184	0.6758	beta	0.1
ϕ	0.5	0.7959 (0.7860)	0.7052	0.8838	beta	0.15
ϕ^X	2	1.7097 (1.9210)	0.5106	2.6885	norm	1.5
σ_c	1.5	1.2609 (1.1571)	0.6997	1.8070	norm	0.3750
χ	0.7	0.3324 (0.3445)	0.2111	0.4341	beta	0.1
ξ_w	0.5	0.4384 (0.4577)	0.3373	0.5373	beta	0.1
ξ_p	0.5	0.5634 (0.5677)	0.4788	0.6538	beta	0.1
γ_w	0.5	0.4578 (0.4489)	0.2123	0.7005	beta	0.15
γ_p	0.5	0.3656 (0.3512)	0.1434	0.5742	beta	0.15
α	0.3	0.3489 (0.3553)	0.2629	0.4379	norm	0.05
α_π	1.5	2.3968 (2.3771)	2.1130	2.6680	norm	0.25
α_r	0.75	0.7959 (0.7911)	0.7598	0.8336	beta	0.1
α_y	0.125	0.0592 (0.0667)	0.0174	0.0974	norm	0.05
$conspie$	0.625	0.5462 (0.5732)	0.4783	0.6118	gamma	0.1
$ctrend$	0.4	0.4609 (0.4975)	0.4110	0.5113	norm	0.1

Table 5: Posteriors Results for Model Parameters (II)

We now turn to the comparisons between parameter estimates under AI and II (Tables 4 and 5). Overall, the parameter estimates are plausible and reasonably robust across information specifications, despite the fact that the II alternative leads to a better model fit based on the corresponding posterior marginal likelihood. It is interesting to note that assuming II for the private sector reinforces the evidence that the *ZK* and *ZI* shocks are related when CES is introduced. We notice that in the II case the estimated standard deviations of the capital-augmenting technology shock (*ZK*) is slightly larger and as a result the standard deviation of the investment specific shock (*ZI*) further reduces (from 3.06 under AI to 2.87 in the II case). This again confirms our finding early that when *ZK* is absent *ZI* is capturing "capital-biased" technological progress and the degree of which depends on whether the shocks are observed or not by the private sector. The other significant change in estimates from AI to II is from the investment adjustment costs parameter (ϕ^X) and this shows how assuming II helps provide evidence that introducing factor-biased technical change affects significantly the estimation of 'investment-related' parameters. Our model comparison analysis contains one important result suggesting that a combination of incorporating CES and with information set II offers a decisive improvement in terms of the model fit, dominates the standard CD model with a very large LL difference of around 20.

4 Model Validation

After having shown the model estimates and the assessment of relative model fit to its other rivals with different restrictions, we use them to investigate a number of key macroeconomic issues in the US. The model favoured in the space of competing models may still be poor (potentially misspecified) in capturing the important dynamics in the data. To further evaluate the absolute performance of one particular model against data, it is necessary to compare the model's implied characteristics with those of the actual data (and an identified VAR model).

In this section, we address the following questions: (i) can the models capture the underlying characteristics of the actual data? (ii) what are the impacts of the structural shocks on the main macroeconomic time series?

4.1 Standard Moment Criteria

Summary statistics such as first and second moments have been standard as means of validating models in the literature on DSGE models, especially in the RBC tradition. As the Bayes factors (or posterior model odds) are used to assess the relative fit amongst a number of competing models, the question of comparing the moments is whether the models correctly predict population moments, such as the variables' volatility or their correlation, i.e. to assess the absolute fit of a model to macroeconomic data.

To assess the contributions of assuming different specifications of production function in our estimated models, we compute some selected second moments and present the results in this section. Table 6 presents the (unconditional) second moments implied by the above estimations and compares with those in the actual data. In particular, we compute these model-implied statistics by solving the models at the posterior means obtained from estimation. The results of the model's second moments are compared with the second moments in the actual data to evaluate the models' empirical performance.

In terms of the standard deviations, Models CD and CES generate relatively high volatility (standard deviations) compared to the actual data (except for the interest rate and the CD production assumes constant labour share). Overall, the estimated models are able to reproduce broadly acceptable volatility for the main variables of the DSGE model and all model variants can successful replicate the stylized fact in the business cycle research that investment is more volatile than output whereas consumption is less volatile. In line with the Bayesian model comparison, the NK models with CES technology fit the data better in terms of implied volatility, getting closer to the data in this dimension (we highlight the 'best' model (performance) in bold). Note that all our CES models clearly outperform the CD model in capturing the volatilities of all variables except for hours) and CES2 with II does extremely well at matching the investment and real wage volatilities in the data. Furthermore by not imposing a constant labour share as in the CD model CES2 with II generates the standard deviation very close to the data but performs badly for hours. As suggested by the likelihood comparison, the differences in generating the moments between the CES specification with only the shock ZK and the CES with both ZK and ZL shocks are qualitatively very small.

			Standard Deviation					
Model	Output	Consumption	Investment	Real wage	Inflation	Interest rate	Hours	Labour share
Data	0.58	0.53	1.74	0.66	0.24	0.61	2.47	2.07
Model CD	0.78	0.69	1.87	0.99	0.43	0.44	**2.84**	0
Model CES1	0.69	0.66	1.82	0.73	0.43	0.50	5.76	3.83
Model CES2	0.69	0.66	1.81	0.73	0.43	0.50	5.79	3.85
CES2 with II	**0.69**	**0.66**	**1.80**	**0.72**	**0.36**	**0.50**	4.09	**2.59**
			Cross-correlation with Output					
Data	1.00	0.61	0.64	-0.11	-0.12	0.22	-0.25	-0.05
Model CD	1.00	0.76	0.57	0.59	**-0.11**	-0.37	0.11	-
Model CES1	1.00	0.43	0.63	0.13	-0.08	-0.11	0.03	-0.23
Model CES2	1.00	0.43	0.63	**0.13**	-0.07	**-0.11**	**0.03**	**-0.23**
CES2 with II	1.00	**0.47**	**0.63**	0.15	-0.06	-0.15	0.06	-0.28
			Autocorrelations (Order=1)					
Data	0.28	0.17	0.56	0.17	0.54	0.96	0.93	0.90
Model CD	0.37	0.42	**0.58**	0.50	0.69	0.89	0.93	-
Model CES1	0.29	0.32	0.61	0.38	0.79	0.94	0.99	0.99
Model CES2	0.29	**0.31**	0.61	0.37	0.79	**0.94**	0.99	0.99
CES2 with II	**0.28**	0.33	0.63	**0.36**	**0.66**	0.91	0.98	**0.98**

Table 6: Selected Second Moments of the Model Variants

Table 6 also reports the cross-correlations of the eight observable variables vis-a-vis output. All models perform successfully in generating the positive contemporaneous correlations of consumption and investment observed in the data. All CES models fit the output-investment correlation with the data very well. The highlighted numbers in this category together with the evidence above show that the feature of CES in the model is particularly important in characterizing the investment dynamics. However, as evidence from the implied volatilities confirms, the main shortcoming of all the models, including the preferred ones, is the difficulty at replicating the cross-correlations of output with hours and the real wage, and mimicking the volatility observed in the hours data. This is not a very surprising result because there are no labour market frictions assumed in all the models under investigation. All models fail to predict the positive correlation between output and interest rate and CES models have problem in replicating the negative contemporaneous cross-relation between inflation and output. This is consistent with the work of Smets and Wouters (2003) as they find that the implied cross-correlations with the interest rate and inflation are not fully satisfactory. Nevertheless, the results in general show that, in the models where the CES specification is present, cross-correlations of endogenous variables are generally closer to those in the actual data. It is the empirical relevance of the CES feature that helps to explain the better overall fit as found in the LL race.

To summarize this section, overall Bayesian Maximum-likelihood based methods suggest that the ability of the model's second moments to fit those of the data generally match the outcome of the likelihood race. The CES model assuming the II set delivers a better fit to the actual data for most of the second moment features in Table 6. However, as noted above, the differences in the second moments of the two competing CES variants are very small.

4.2 Autocorrelation Functions

We have so far considered autocorrelation only up to order 1. To further illustrate how the estimated models capture the data statistics and persistence in particular, we now plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model variants in Figure 3.

The CES models all stay within the 95% uncertainty bands and II CES2 model performs a little better than its AI counterpart. The inflation autocorrelations generated by CD model lie outside the 95% uncertainty bands. Of particular interest is that, when assuming CES production, the implied autocorrelograms fit very well the observed autocorrelation of inflation, interest rate, investment and real wage, whilst the CD model generates much less sluggishness and is less able to match the autocorrelation of inflation, interest rate and real wage observed in the data from the second lag onwards. Overall

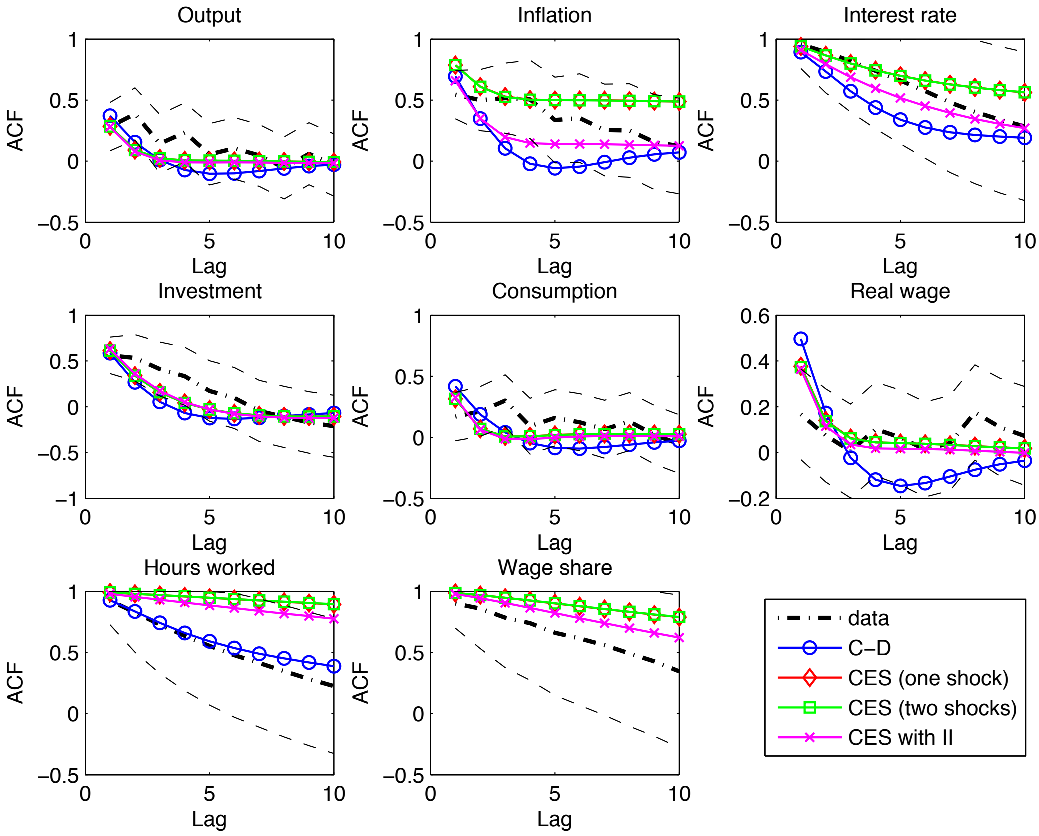


Figure 3: Autocorrelations of Observables in the Actual Data and in the Estimated Models*

* The approximate 95 percent confidence bands are constructed using the large-lag standard errors (see Anderson (1976)).

we find that, with nominal price stickiness in the models and highly correlated estimated price markup shocks, inflation persistence can be captured closely in DSGE models when CES production is assumed.

When it comes to output, all models perform well in matching the observed output persistence. However, the hours is more autocorrelated in all models than in the data, but now the model with the CD feature gets much closer to the data for higher order autocorrelations. All models match reasonably well the autocorrelations of investment and consumption. To summarize, the results for higher order autocorrelations for the most part show that the DSGE models under the more general CES production function are better at capturing the main features of the US data, strengthening the argument that the assumption of CES helps to improve the model fit to data.

4.3 DSGE-VARs and Impulse Responses

An alternative way of validating the model performance is to follow Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007) and to compare it with a hybrid model that is a combination of an unrestricted VAR and the VAR implied by the estimated DSGE model. We then go on to compare the estimated DSGE model and this 'DSGE-VAR' in terms of their impulse response functions (IRFs). We also investigate the impact on IRFs of changing the production function and information assumption. Since we have demonstrated that there is little difference between the CES variants in terms of matching the data, this exercise is only performed for the 'best' CES model (i.e. CES2 with AI and II).

The DSGE-VAR approach uses DSGE model itself to construct a prior distribution for the VAR coefficients so that DSGE-VAR estimates are tilted toward DSGE model restriction, thus identifying the shocks for the IRFs. This method constructs the DSGE prior by generating dummy observations from the DSGE model, and adding them to the actual data and leads to an estimation of the VAR based on a mixed sample of artificial and actual observations. The ratio of dummy over actual observations (called the hyper-parameter λ) controls the variance and therefore the weight of the DSGE prior relative to the sample. For extreme values of this parameter (0 or ∞) either an unrestricted VAR or the DSGE is estimated. If λ is small the prior is diffuse. When $\lambda = \infty$, we obtain a VAR approximation[22] of the log-linearized DSGE model. As λ becomes small the cross-equation restrictions impled by the DSGE model are gradually relaxed. The empirical performance of a DSGE-VAR will depend on the tightness of the DSGE prior. Details on the algorithm used to implement this DSGE-VAR are to be found in Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007).

We fit our VAR to the same data set used to estimate the DSGE model. We consider a VAR with 4 lags.[23] We use a data-driven procedure to determine the tightness of prior endogenously based on the marginal data density. Our choice of the optimal λ is 1.0 for all models, and this is found by comparing different VAR models using the estimates of

[22] The accuracy of the approximation depends on the invertibility of the DSGE model's moving average components and on the number of autoregressive lags included (Del Negro *et al.* (2007)).

[23] This choice of the lag length maximizes the marginal data density associated with the DSGE-VAR($\hat{\lambda}$).

the marginal data density (Figure 4). In particular, we iterate over a grid that contains the values of $\lambda = [0.43; 0.8; 1; 1.2; 1.4; 1.5; 2; 5; 10; \infty]$, we find that $\lambda = 1.0$ has the highest posterior probability for all models. Note that 0.43 is the smallest λ value for which we have a proper prior. Overall, the DSGE-VAR(4)'s with $\lambda = 1.0$ have the highest posterior probability.[24] This implies that the mixed sample that is used to estimate the VAR has a higher weight on the DSGE model (artificial observations) than on the VAR (actual observations). $\hat{\lambda}$ represents how much the economic model (DSGE) is able to explain the real data. More importantly, the results from comparing across different models show that Models CES consistently outperform Model CD and CD is strongly rejected in favour of CES when the weight on the DSGE model becomes higher(when λ tends to ∞).

The improved performance of the CES models over the CD applies at the optimum λ, and the log of the marginal likelihood difference (LL) is around 8. This implies a Bayes Factor of $\exp(8) \simeq 2981$ favouring the CES models. Beyond the optimum the LL falls far more rapidly for the CD model reaching a difference close to that for the actual linearized DSGE model reported earlier. Overall the LL plots then confirm the fact and the degree to which the CES models are less misspecified. The LL differences, $LL(\hat{\lambda}) - LL(DSGE)$, provide measures of the degree of the overall mis-specification for all our models – here $[LL(\hat{\lambda}) - LL(DSGE)]_{CD} > [LL(\hat{\lambda}) - LL(DSGE)]_{CES(\sigma=0.4)} > [LL(\hat{\lambda}) - LL(DSGE)]_{CES2} > [LL(\hat{\lambda}) - LL(DSGE)]_{CES2-II}$

Turning to the impulse responses, Figures 6–13 in Appendix D depict the mean responses corresponding to a positive one standard deviation shock. The endogenous variables of interest are the observables in the estimation and each response is for a 20 period (5 years) horizon. All DSGE impulse responses are computed simulating the vector of DSGE model parameters at the posterior mean values reported in Tables 4, 5, 7 and 8. The impulse responses for VAR(4) are obtained using the DSGE-VAR identification procedure using the best-fitting DSGE-VAR(4) with $\lambda = 1$. The surface between the black dashed lines in each panel covers the space between the first and ninth posterior deciles of the VAR's IRFs.

Overall, we find that the sign and magnitude of the DSGE and VAR impulse responses are quite similar implying that the DSGE model mimics the DSGE-VAR model in, at least,

[24] Alternatively, one can simply find the 'optimal' $\hat{\lambda}$ by estimating the parameter λ as one of the deep parameters (see, for more details, Adjemian *et al.* (2008)).

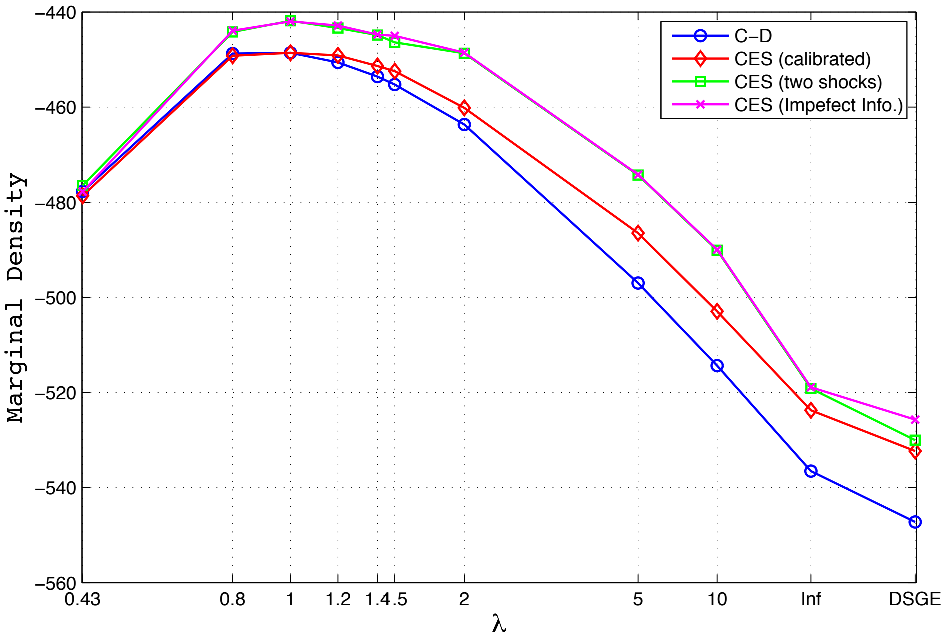


Figure 4: Marginal Likelihood as a Function of λ

some dimensions. Clearly the most important difference comes from fluctuations in factor shares under the CES specification. Fluctuations of shares translate as well in different IRFs of interest rate and wage in the two models. Indeed in Figure 6 (labour-augmenting shock) we can see how the response of the nominal interest rate is significantly different. Figure 8 (the investment shock) turns out be very interesting, given the highlighted difference in the estimation of parameters related to investment in section 3.3. We note how both, wage and interest rate, present a more sluggish response to an investment specific shock under CES and, as a result, a quite different response of consumption and inflation. Although they have similar shapes, the IRFs under CES are quantitatively different. In particular, the shock amplifies the initial responses of some variables. The disagreement in the IRFs to this particular shock can be explained by the large estimate of the shock standard deviation reported in Tables 4 and 7.

If we look closely at the responses to monetary policy shock (Figure 9), we find that both in the models and in the data, consumption, investment and output display a hump-shaped response to the policy shock. For this shock the IRFs from both CES and CD models are in agreement with the data and when comparing the performance from CES and CD the difference is quantitatively small. Turning to the IRFs to productivity shock (labour-augmenting), using both the CES and CD settings is able to provide reasonable responses. More importantly, if we compare each response from each DSGE model with that from the VAR model, we find that overall the discrepancy between VAR and DSGE is relatively smaller under the CES production assumption. This suggests that the DSGE model misspecification is larger with the CD production than with the CES. If we study carefully the responses to the other shocks, we can generally find the similar conclusion that CES helps reduce the discrepancy although the IRFs to the investment-specific shock are the exception. In Figure 10 we can see how the reaction of wage and interest rate are once again very different after a government spending shock. Indeed it the introduction of CES specification increases the magnitude of the responses a lot. As a result of those differences in the dynamics of factor prices we notice how investment and consumption also present an increase in the amplification of the government spending shock. For price and wage mark-up shock (Figures 12 and 13) we notice non-negligible differences in the responses of interest rate, inflation and hours worked.

Some interesting differences between information sets AI and II emerge from the analysis. We see pronounced hump-shaped responses to a technology shock for the CES model for the II case (for example in Figure 7 with the capital-augmenting technology shock). This reflects the *learning-about-the-shock* effect: that is arising from the rational learning of the private sector about the unobserved shock using Kalman updating (Levine *et al.* (2012)). As a result in this learning process we observe additional persistence from the responses to the shocks. It is interesting to find that the learning process presents more sluggish responses of interest rate to an investment specific shock and inflation to the mark-up shocks under CES2 and II.

To sum up, there also exists some evidence from IRFs in favour of the CES assumption in DSGE models, but the evidence from the IRFs is not as strong as that obtained by comparing the moments and the marginal likelihood comparison amongst models which more clearly reject the CD specification. Overall in terms of matching IRFs both CES models (with AI and II) provide a good fit of the transmission mechanism from all model shocks.

5 Variance Decomposition of Business Cycle Fluctuations

This section investigates the contribution of each of the structural shocks to the forecast error variance of the observable variables in the models, i.e. the underlying sources of fluctuations, at various horizons. The results are based on the models' posterior distribution reported in Tables 7 and 8. The results are summarized in Figures 14 and Tables 9 and 10 in Appendix D.

In the short run, within a year (t=1,4), movements in real GDP are primarily driven by the exogenous government spending shock and supply-side shocks (with the dominant influence of around 70%). For instance, most of the unexpected output fluctuations are mainly explained by the government spending shock (around 30-40% depending on the model specifications) and the two mark-up shocks (around 10-20% from each shock). Within the one-year horizon, the government spending shock dominate, accounting for the biggest part of the output forecast error variance under Model CD.

Not surprisingly, in the medium to long run the supply shocks and the exogenous spending shock together continue to dominate, but the contribution of government spending shock to output variability become smaller from medium to long run and the wage mark-up shock explain a bigger part of the long-run variations in output. This is especially the case when the model adopts the CES function form. In contrast, the monetary policy shock and preference shock have little impact on for output variability, regardless of forecast horizon. Based on the estimation sample, the investment shock and labour-augmenting productivity shock are found to be moderate factors behind both short-run and longer-run movements in output (account for around 10%-11% and around 7%-8%, respectively). In terms of determining the main driving forces of output, we compare all three specifications under investigation and find similar and consistent results. Results from the CES model with two technology shocks show that, in line with its estimated standard deviation, the capital-augmenting technology shock offers a qualitatively very small impact to the output fluctuations.

Under the estimated interest rate rule we find that the monetary policy shock is by far the most determinant influence to the nominal interest rate in the short run (1 quarter), which explains around 40% of its variance under the assumption of CES technology. The second largest component is the investment shock. In fact, this shock starts to dominate from the medium to long run and the contributions of policy shock declines quite sharply toward the longer horizon. However, models with CES specification tells a slightly different story. They show that the main driving factor in the long-run development of nominal interest rate is mark-up shocks. As expected, the preference shock explains a big part of consumption variation in the short-medium run, whilst the investment shock contributes the largest fraction of investment movements in the short run (within a year). In terms of explaining the consumption and investment fluctuations, we do not find notable differences whether CD or CES is assumed.

Interestingly, the CD model suggests that the shocks that explain most of inflation variance in the short run are the two mark-up shocks but the investment shock becomes more influential from medium to long run (nearly 20%). In contrast, our estimated CES models show that inflation fluctuations are mostly affected by the policy shock in the short and medium runs but the the main driving factor becomes the wage mark-up shock, dominates the investment shock, in the long run. There are only limited effects on inflation from the productivity shocks and various demand shocks. One possible reason for this is, according to Smets and Wouters (2007), that the estimated slope of the NK Phillips curve is small so that only large and persistent changes in the marginal cost will have an impact on inflation. Finally, the short to medium run contributions of the selected shocks to the forecast error variance of hours worked are broadly similar across the three models. In the long run, there are different results between the CES and CD assumptions. In particular, the model adopting the CD production suggests that the two mark-up shocks both contribute significantly to the variation in hours worked whereas, when we use the more general CES setting in our DSGE model, the wage mark-up shock clearly becomes the completely dominant force behind the long-run movements in hour worked from the mid-80's onwards. This finding from the CES model seems to be more plausible.

Overall, the results in this exercise mainly show that, over the sample period, the supply-side shocks account for much of the medium to long-run variance which is in line with the business cycle literature and identified VAR studies in industrialized economies. The disturbances from government expenditures are also important at explaining the dynamics of macro-variables in the US economy.

6 Conclusions

This paper contributes to a rapidly rising literature that brings the CES specification of the production function into the analysis of business cycle fluctuations. Whilst other papers including Cantore *et al.* (2012) and Cantore *et al.* (2014b) have explored this issue, this is the first one to confirm decisively the importance of CES rather than CD production functions by a comparison, in a Bayesian framework, of marginal likelihoods. Indeed in a marginal likelihood race our estimated best CES model (estimated under II) with an elasticity well below unity at 0.18 beats the CD production function by a substantial log-likelihood of 20.01. Assuming equal prior model probabilities, this implies that posterior model probabilities are 4.8517e+08:1 in favour of the CES.[25] The principle reason for this result is that movements of factor shares with the CES specification help substantially to fit the data. The marginal likelihood improvement is matched by the ability of the CES model to get closer to the data in terms of second moments, especially the volatilities of output, consumption and the real wage, and the autocorrelation functions for inflation and the nominal interest rate. A comparison with a DSGE-VAR further confirms the ability of the CES model to reduce model misspecification. The main message then for DSGE models is that we should dismiss once and for all the use of CD for business cycle analysis.

Despite these positive results, one area where the CES model remains a concern in terms of model misspecification is in the second moments involving wages and hours. For example both CD and CES models fail to reproduce the negative correlation between output and hours; furthermore the CES model produces far too much persistence in hours. As pointed out by Rowthorn (1999), a low capital-labour substitutability is crucial for understanding unemployment persistence. This suggests that future research should also look more closely at the labour market and introduce search-match frictions and unemployment alongside CES production.[26]

We pose the question of whether the superior fit of CES over CD production functions is robust to the information assumptions and whether imperfect information (II) can further improve the model fit using the CES specification. Indeed we find this is the case. Our

[25] But see Geweke and Amisano (2011) for an alternative to a marginal likelihood race proposing instead a 'prediction pool' consisting of an optimal linear combination of the marginal likelihoods for the two models.

[26] See Cantore *et al.* (2014a) using a simpler calibrated model.

model comparison analysis suggests that a combination of incorporating CES and with information set II offers a decisive improvement in terms of the model fit, dominates the standard CD model with a very large LL difference. When using II we also find that the poor performance from the CES models in terms of capturing hours volatility can be improved. Imposing the II assumption provides strong support to the empirical relevance of introducing factor-biased technical change and CES production into DSGE models, and can help improve the potential model misspecification, based on the moment analysis and estimated DSGE-VARs.

Our CES specification allows us to introduce a capital-augmenting shock alongside the labour-augmenting variety. However we find this does not bring about an improvement in the model fit and the contribution of the a capital-augmenting shock in the variance decomposition is small. We have noted the well-known result that a balanced-growth path (bgp) requires either CD technology or that technical change must be driven solely by the labour-augmenting variety. This raises an obstacle to the prospect of unifying business cycle analysis with long-term endogenous growth based on CES technology. One possible way forward is to follow León-Ledesma and Satchi (2011); they provide a model of optimal choice of CES production technology that results in a bgp with both labour and capital-augmenting technical change. Then CES prevails in the short-run but CD in the long run, thus allowing a capital-augmenting technical change contribution to long-run growth. These authors provide an alternative production function with these properties. A possible line for further research would be to incorporate this into the SW-type model of this paper and to assess its empirical performance.

References

Acemoglu, D. (2002). Technical Change, Inequality and the Labor Market. *Journal of Economic Literature*, **40**(1), 7–72.

Acemoglu, D. (2003). Labor- and capital-augmenting technical change. *Journal of the European Economic Association*, **1**, 1–37.

Adjemian, S., Darracq-Paries, M., and Moyen, S. (2008). Towards a monetary policy evaluation framework. ECB Working Paper No. 942.

Anderson, O. (1976). *Time Series Analysis and Forecasting: the Box-Jenkins Approach*. Butterworths, London.

Antràs, P. (2004). Is the US Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *Contributions to Macroeconomics*, **4**, Article 4.

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. (1961). Capital-labor substitution and economic efficiency. *Review of Economics and Statistics*, **43**(3), 225–250.

Barnett, W. (2004). Dimensions and Economics: Some Problems. *Quarterly Journal of Austrian Economics*, **7**(1), 95–104.

Blanchard, O. J. (1997). The Medium Run. *Brookings Papers on Economic Activity*, **2**, 89–158.

Caballero, R. J. and Hammour, M. L. (1998). Jobless growth: Appropriablity, factor substitution and unemployment. *Carnegie-Rochester Conference Proceedings*, **48**, 51–94.

Cantore, C. and Levine, P. (2012). Getting normalization right: Dealing with ?dimensional constants? in macroeconomics. *Journal of Economic Dynamics and Control*, **36**(12), 1931–1949.

Cantore, C., Ferroni, F., and León-Ledesma, M. A. (2012). The dynamics of hours worked and technology. Banco de España Working Papers 1238, Banco de España.

Cantore, C., Levine, P., and Melina, G. (2014a). A fiscal stimulus and jobless recovery. *The Scandinavian Journal of Economics*. Forthcoming.

Cantore, C., León-Ledesma, M., McAdam, P., and Willman, A. (2014b). Shocking stuff: Technology, Hours, and Factor Substitution. *Journal of the European Economic Association*, **12**(1), 108 – 128.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2009). New keynesian models: Not yet useful for policy analysis. *American Economic Journal: Macroeconomics*, **1**(1), 242–66.

Chirinko, R. S. (2008). Sigma: The Long and Short of It. *Journal of Macroeconomics*, **30**(2), 671–686.

Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, **113**, 1–45.

Del Negro, M. and Schorfheide, F. (2004). Priors from general equilibrium models for vars. *International Economic Review*, **45**, 643–673.

Del Negro, M., Schorfheide, F., Smets, F., and Wouters, R. (2007). On the fit of new keynesian models. *Journal of Business and Economic Statistics*, **25**(2), 123–162.

Diamond, P. A., McFadden, D., and Rodriguez, M. (1978). Measurement of the elasticity of substitution and bias of technical change. In M. Fuss and D. McFadden, editors, *Production Economics, Vol. 2*, pages 125–147. Amsterdam and North Holland.

Fernald, J. G. (2007). Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics*, **54**(8), 2467–2485.

Galí, J. and Gambetti, L. (2009). On the sources of the great moderation. *American Economic Journal: Macroeconomics*, **1**(1), 26–57.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, **58**(1), 17–34.

Geweke, J. (1999). Using Simulation Methods for Bayesian Econometric Models:Inference, Development, and Communication. *Econometric Reviews*, **18**, 1–127.

Geweke, J. and Amisano, G. (2011). Optimal prediction pools. *Journal of Econometrics*, **164**, 130141.

Jones, C. I. (2003). Growth, capital shares, and a new perspective on production functions. mimeo, Stanford University.

Jones, C. I. (2005). The shape of production functions and the direction of technical change. *Quarterly Journal of Economics*, **120**(2), 517–549.

Kass, R. E. and Raftery, A. E. (1995). Bayes Factors. *Journal of the American Statistical Association*, **90**, 773–795.

Kennedy, C. (1964). Induced bias in innovation and the theory of distribution. *Economic Journal*, **74**, 541–547.

Klump, R. and Preissler, H. (2000). CES production functions and economic growth. *Scandinavian Journal of Economics*, **102**, 41–56.

Klump, R., McAdam, P., and Willman, A. (2007). Factor Substitution and Factor Augmenting Technical Progress in the US. *Review of Economics and Statistics*, **89**(1), 183–92.

Klump, R., McAdam, P., and Willman, A. (2012). The normalized ces production function: Theory and empirics. *Journal of Economic Surveys*, **26**(5), 769–799.

La Grandville, O. de. (1989). In Quest of the Slutzky Diamond. *American Economic Review*, **79**, 468–481.

La Grandville, O. de. (2009). *Economic Growth: A Unified Approach*. Cambridge University Press.

León-Ledesma, M. A. and Satchi, M. (2011). The choice of ces production techniques and balanced growth. Studies in Economics 1113, Department of Economics, University of Kent.

León-Ledesma, M. A., McAdam, P., and Willman, A. (2010). Identifying the elasticity of substitution with biased technical change. *American Economic Review*, **100**(4), 1330–57.

Levine, P., Pearlman, J., Perendia, G., and Yang, B. (2012). Endogenous Persistence in an Estimated DSGE Model under Imperfect Information. *Economic Journal*, **122**(565), 1287 – 1312.

Mankiw, N. G. (1995). The growth of nations. *Brookings Papers on Economic Activity*, **1**, 275–310.

Marquetti, A. A. (2003). Analyzing historical and regional patterns of technical change from a classical-marxian perspective. *Journal of Economic Behavior and Organization*, **52**, 191–200.

McAdam, P. and Willman, A. (2013). Medium run redux. *Macroeconomic Dynamics*, **17**(04), 695–727.

Ríos-Rull, J.-V. and Santaeulália-Llopis, R. (2010). Redistributive shocks and productivity shocks. *Journal of Monetary Economics*, **57**(8), 931–948.

Rowthorn, R. (1999). Unemployment, wage bargaining and capital-labour substitution. *Cambridge Journal of Economics*, **23**, 413–425.

Smets, F. and Wouters, R. (2003). An estimated Stochastic Dynamic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, **1(5)**, 1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, **97**(3), 586–606.

Solow, R. M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, **70**, 65–94.

Appendix

A Expressing Summations as Difference Equations

In the first order conditions for Calvo contracts and expressions for value functions we are confronted with expected discounted sums of the general form

$$\Omega_t = E_t\left[\sum_{k=0}^{\infty}\beta^k X_{t,t+k}Y_{t+k}\right] \tag{A.1}$$

where $X_{t,t+k}$ has the property $X_{t,t+k} = X_{t,t+1}X_{t+1,t+k}$ (for example an inflation, interest or discount rate over the interval $[t, t+k]$).

Lemma

Ω_t can be expressed as

$$\Omega_t = X_{t,t}Y_t + \beta E_t\left[X_{t,t+1}\Omega_{t+1}\right] \tag{A.2}$$

Proof

$$
\begin{aligned}
\Omega_t &= X_{t,t}Y_t + E_t\left[\sum_{k=1}^{\infty}\beta^k X_{t,t+k}Y_{t+k}\right] \\
&= X_{t,t}Y_t + E_t\left[\sum_{k'=0}^{\infty}\beta^{k'+1} X_{t,t+k'+1}Y_{t+k'+1}\right] \\
&= X_{t,t}Y_t + \beta E_t\left[\sum_{k'=0}^{\infty}\beta^{k'} X_{t,t+1}X_{t+1,t+k'+1}Y_{t+k'+1}\right] \\
&= X_{t,t}Y_t + \beta E_t\left[X_{t,t+1}\Omega_{t+1}\right] \qquad \Box
\end{aligned}
$$

B Proof of Price and Wage Dispersion Results

For prices and without indexation, in the next period, ξ_p of these firms will keep their old prices, and $(1-\xi_p)$ will change their prices to P^O_{t+1}. By the law of large numbers, we assume that the distribution of prices among those firms that do not change their prices is the same as the overall distribution in period t. It follows that we may write

$$
\begin{aligned}
\Delta_{p,t+1} &= \xi_p \int_{m\ no\ change} \left(\frac{P_t(m)}{P_{t+1}}\right)^{-\zeta} + (1-\xi_p)\left(\frac{P^0_{t+1}}{P_{t+1}}\right)^{-\zeta} \\
&= \xi_p \left(\frac{P_t}{P_{t+1}}\right)^{-\zeta} \int_{m\ no\ change} \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} dm + (1-\xi_p)\left(\frac{P^0_{t+1}}{P_{t+1}}\right)^{-\zeta} \\
&= \xi_p \left(\frac{P_t}{P_{t+1}}\right)^{-\zeta} \int_{m} \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} dm + (1-\xi_p)\left(\frac{P^0_{t+1}}{P_{t+1}}\right)^{-\zeta} \\
&= \xi_p \Pi^{\zeta}_{t+1}\Delta_{p,t} + (1-\xi_p)\left(\frac{P^0_{t+1}}{P_{t+1}}\right)^{-\zeta} \qquad \text{(B.1)}
\end{aligned}
$$

The generalization to indexation is straightforward.

C Posterior Distribution

parameter	prior mean	post. mean CD (SW07)	post. mean CES	5% CES	95% CES	Prior	pstdev CES
ρ_{ZL}	0.5	0.9600 (0.95)	0.9443	0.9009	0.9924	beta	0.2
ρ_{ZK}	0.5	N/A (N/A)	**0.4441**	0.1134	0.7595	beta	0.2
ρ_G	0.5	0.9509 (0.97*)	0.9631	0.9449	0.9829	beta	0.2
ρ_{ZI}	0.5	0.6785 (0.71)	0.7429	0.6232	0.8629	beta	0.2
ρ_P	0.5	0.6877 (0.89*)	**0.9744**	0.9527	0.9965	beta	0.2
ρ_W	0.5	0.9124 (0.96*)	0.9656	0.9343	0.9965	beta	0.2
ρ_B	0.5	0.3765 (N/A)	**0.9311**	0.0929	0.9623	beta	0.2
ε_{ZL}	0.1	0.7258 (0.45)	0.6833	0.5894	0.7718	invg	2.0
ε_{ZK}	0.1	N/A (N/A)	**0.0744**	0.0247	0.1356	invg	2.0
ε_G	0.5	2.1599 (0.53*)	1.9904	1.7479	2.2286	invg	2.0
ε_{ZI}	0.1	4.1634 (0.45)	**3.0647**	1.7292	4.3950	invg	2.0
ε_P	0.1	0.6097 (0.14*)	**0.3756**	0.3092	0.4386	invg	2.0
ε_W	0.1	1.1304 (0.24*)	**0.9482**	0.8004	1.0957	invg	2.0
ε_M	0.1	0.1505 (0.24)	0.1579	0.1360	0.1809	invg	2.0
ε_B	0.1	1.3639 (N/A)	1.4997	1.0848	1.9270	invg	2.0

Table 7: Posterior Results for the Exogenous Shocks

parameter	prior mean	post. mean CD (SW07)	post. mean CES	5% CES	95% CES	Prior	pstdev CES
σ	1	1 (1)	**0.1542**	0.0603	0.2434	gamma	1
h	0.4	0.5970 (N/A)	0.5136	0.3741	0.6509	beta	0.1
ϕ	0.5	0.8832 (0.54)	0.7860	0.7145	0.8625	beta	0.15
ϕ^X	2	2.6754 (2.87)	**1.9210**	0.8640	2.9677	norm	1.5
σ_c	1.5	2.0932 (1.38*)	**1.1571**	0.5539	1.7211	norm	0.3750
χ	0.7	0.5553 (0.71*)	**0.3445**	0.2191	0.4620	beta	0.1
ξ_w	0.5	0.6016 (0.7)	**0.4577**	0.3606	0.5537	beta	0.1
ξ_p	0.5	0.7770 (0.66)	**0.5677**	0.4808	0.6484	beta	0.1
γ_w	0.5	0.4340 (0.58)	0.4489	0.2047	0.6855	beta	0.15
γ_p	0.5	0.3062 (0.24)	0.3512	0.1325	0.5577	beta	0.15
α	0.3	0.2052 (0.19)	**0.3553**	0.2721	0.4367	norm	0.05
α_π	1.5	2.2379 (2.04)	2.3771	2.0900	2.6506	norm	0.25
α_r	0.75	0.8227 (0.81)	0.7911	0.7523	0.8301	beta	0.1
α_y	0.125	0.050 (0.08)	0.0667	0.0331	0.1007	norm	0.05
conspie	0.625	0.5502 (0.78)	0.5732	0.5069	0.6371	gamma	0.1
ctrend	0.4	0.4640 (0.43)	0.4975	0.4584	0.5379	norm	0.1

Table 8: Posteriors Results for Model Parameters

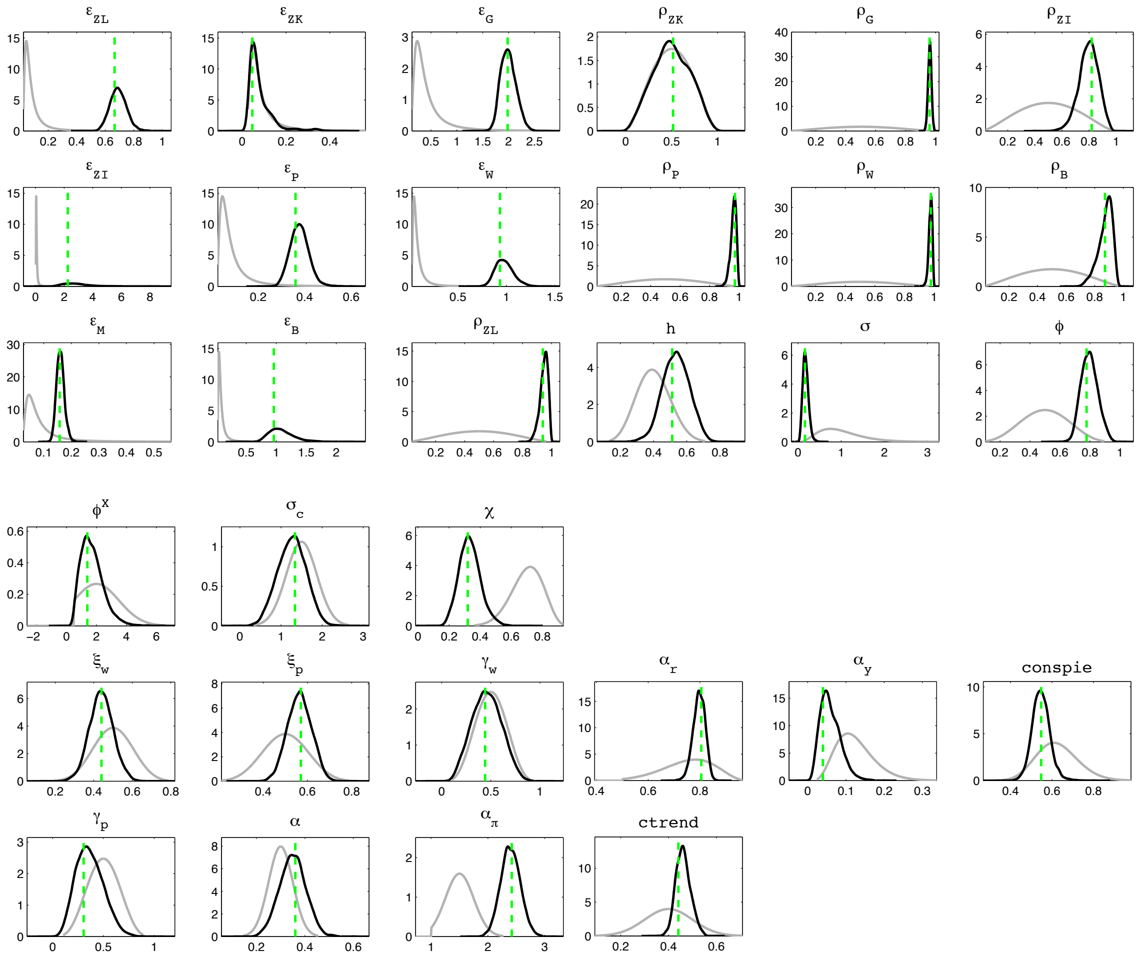


Figure 5: Priors and Posteriors distributions (Model CES2-II)

D Figures

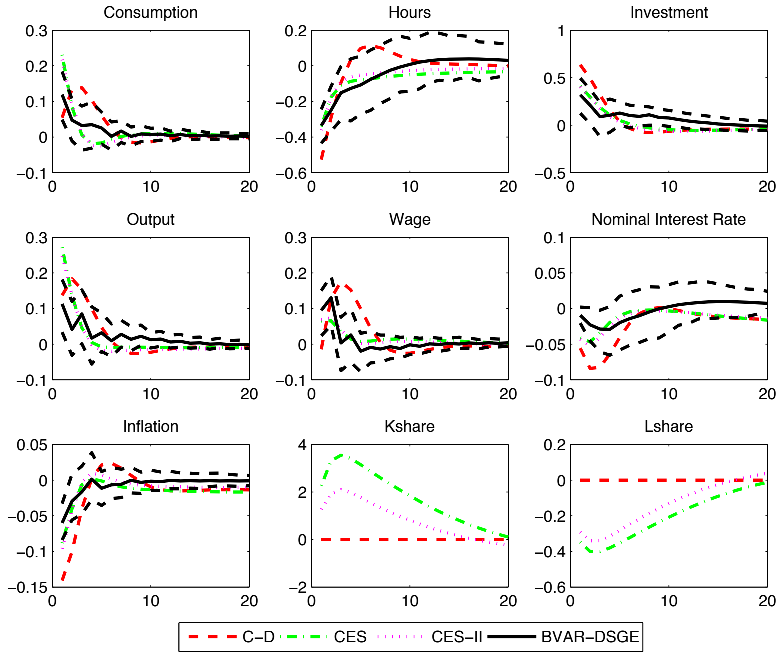


Figure 6: Bayesian IRFs - Labour-augmenting shock$^{\diamondsuit}$

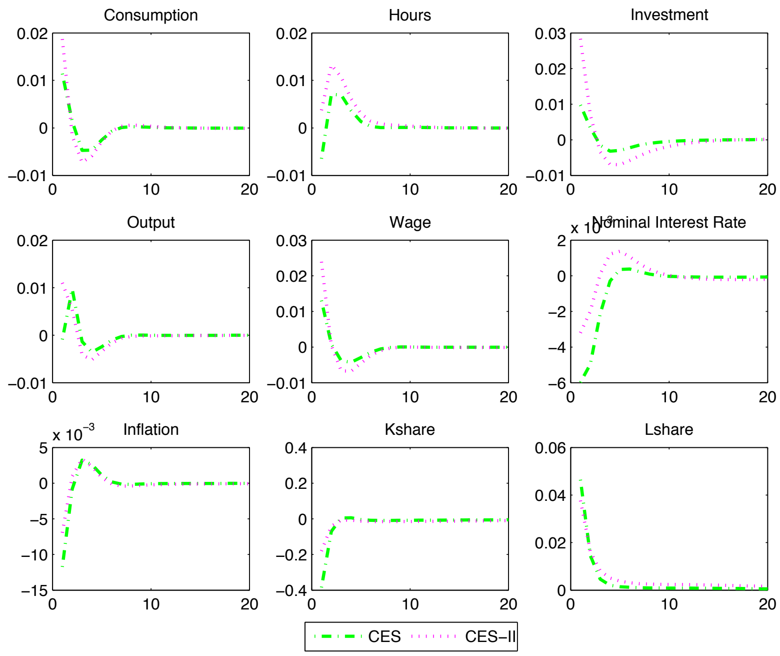


Figure 7: Bayesian IRFs - Capital-augmenting shock

$^{\diamondsuit}$ BVAR-DSGE($\hat{\lambda} = 1.1$): the dashed lines are the first and ninth posterior deciles of the VAR's IRFs. The bold black curve is the posterior mean of the VAR's IRFs.

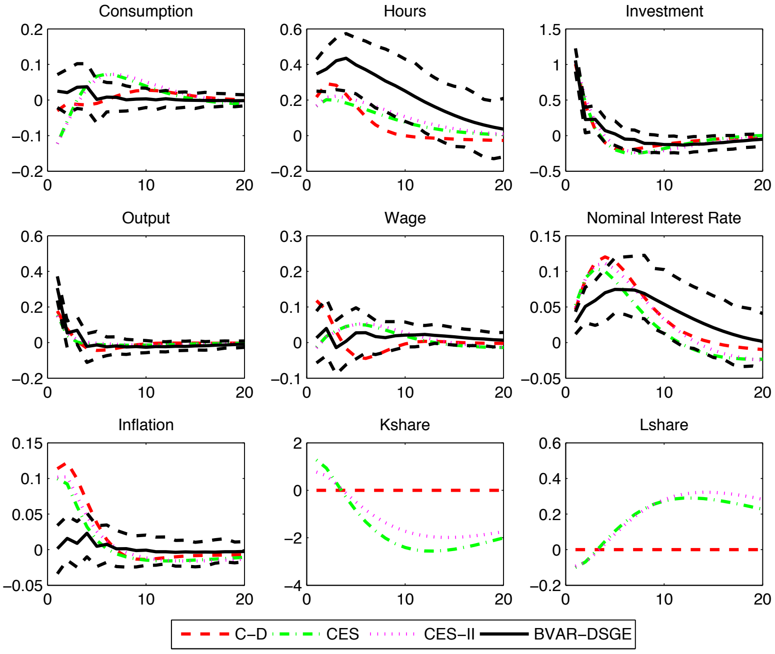


Figure 8: Bayesian IRFs - Investment-specific shock

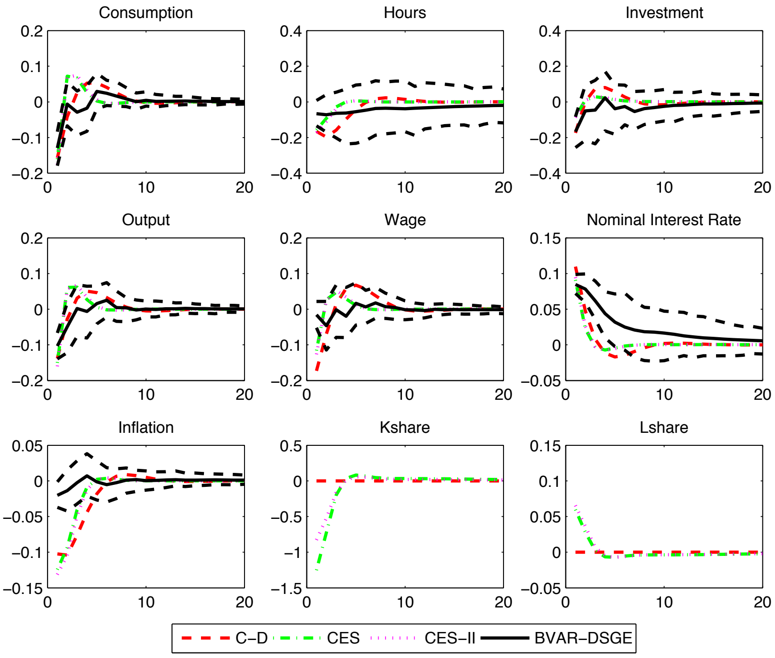


Figure 9: Bayesian IRFs - Monetary policy shock

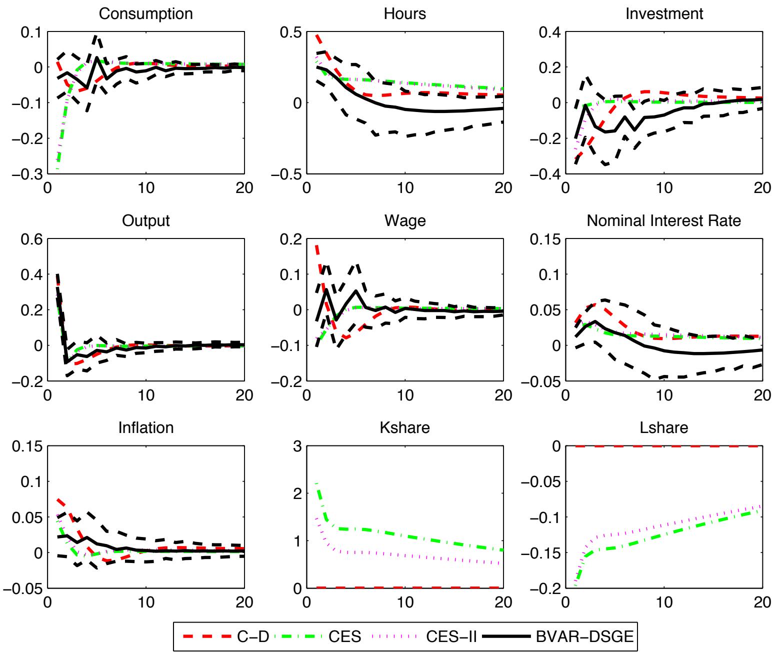


Figure 10: Bayesian IRFs - Government spending shock

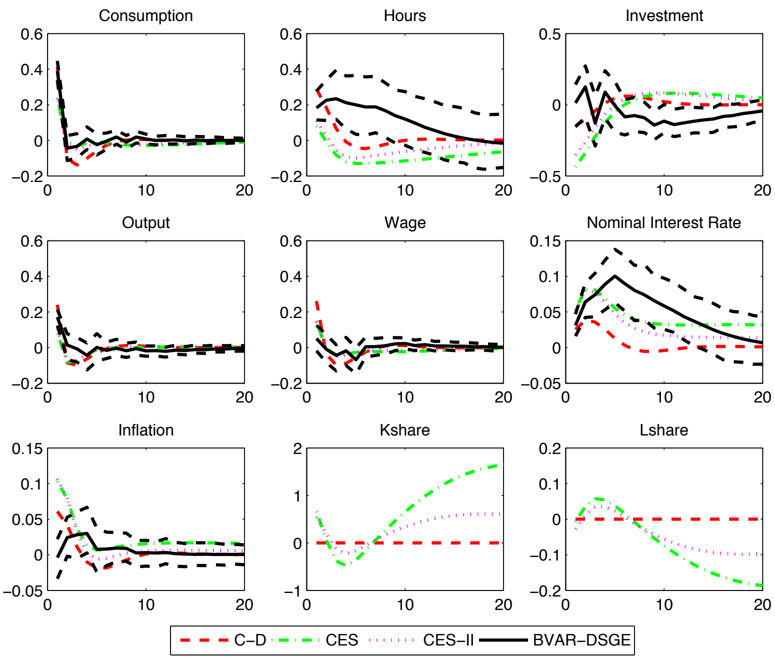


Figure 11: Bayesian IRFs - Preference shock

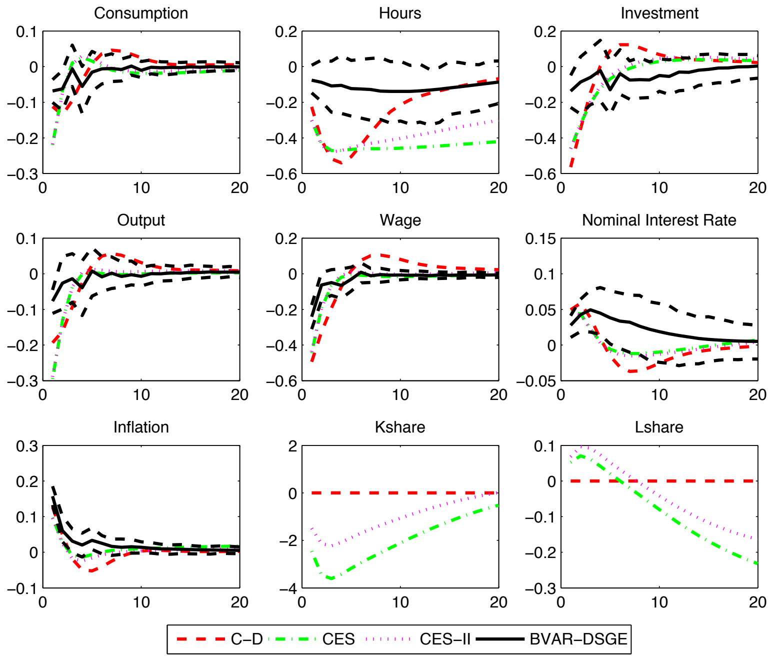


Figure 12: Bayesian IRFs - Price mark-up shock

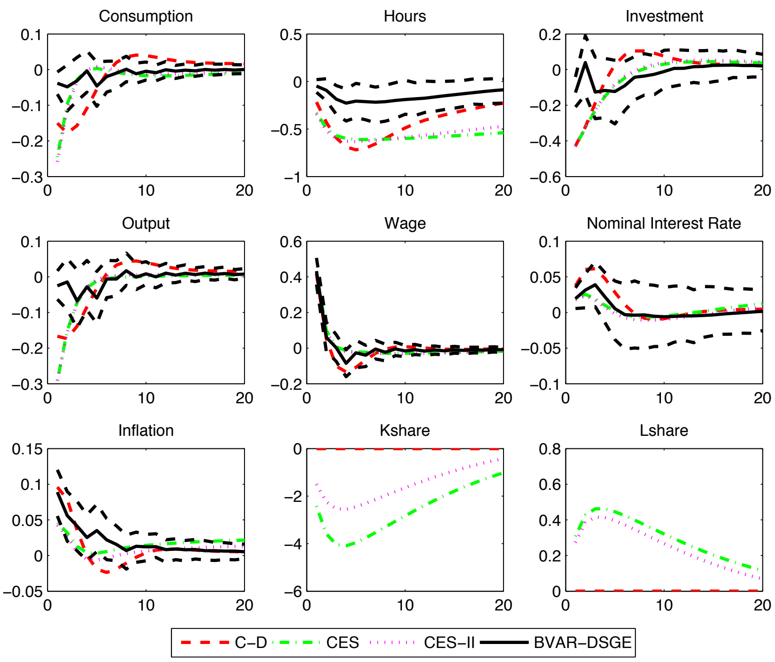
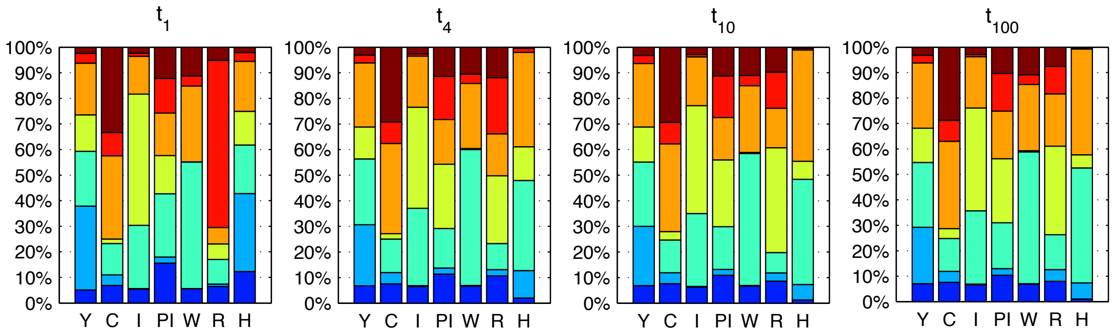
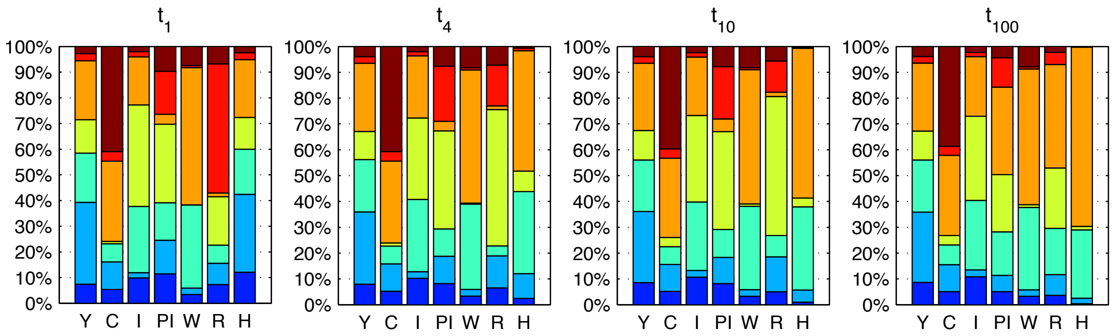


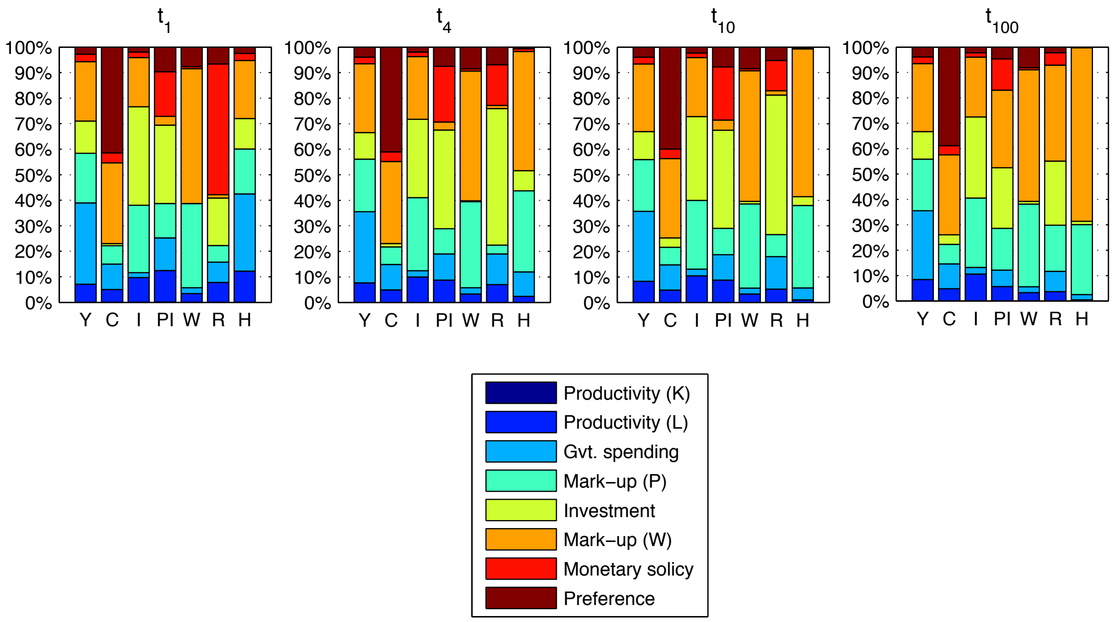
Figure 13: Bayesian IRFs - Wage mark-up shock



(a) C-D



(b) CES (one shock)



(c) CES (two shocks)

Figure 14: Variance Decomposition

		Shocks of the Estimated DSGE Models							
Forecast horizon	Observable variables	Productivity (K)	Productivity (L)	Government spending	Mark-up (price)	Investment	Mark-up (wage)	Monetary policy	Preference
t=1	Output	0.00	10.12	22.68	25.02	7.97	26.64	5.95	1.61
	Consumption	0.04	8.91	23.44	17.08	3.14	21.44	8.46	17.49
	Investment	0.00	4.77	0.00	12.74	57.90	13.50	0.38	10.71
	Inflation	0.22	10.53	2.41	22.72	12.92	8.06	22.66	20.48
	Real wage	0.05	2.75	5.46	36.17	0.34	49.75	1.57	3.92
	Interest rate	0.17	6.56	3.16	13.94	11.53	4.13	43.91	16.60
	Hours worked	0.01	15.85	21.60	21.75	7.48	26.56	5.38	1.37
t=4	Output	0.02	10.14	19.29	24.25	6.61	28.95	6.22	4.51
	Consumption	0.05	8.85	22.17	16.63	3.98	22.40	10.11	15.81
	Investment	0.00	5.74	0.01	15.59	46.06	18.29	0.30	14.00
	Inflation	0.15	8.56	1.65	16.45	15.47	11.29	23.26	23.18
	Real wage	0.05	2.87	5.43	37.01	1.15	48.04	1.70	3.74
	Interest rate	0.08	6.35	2.62	10.02	28.31	7.25	12.60	32.77
	Hours worked	0.01	3.99	8.86	31.43	6.17	46.90	1.34	1.29
t=10	Output	0.02	10.17	19.16	24.05	7.02	28.85	6.18	4.54
	Consumption	0.05	8.39	21.16	15.92	8.02	21.14	9.51	15.81
	Investment	0.00	5.26	0.02	14.07	50.39	16.76	0.28	13.22
	Inflation	0.14	8.77	1.53	16.64	14.83	14.47	21.00	22.62
	Real wage	0.05	2.76	5.24	35.22	2.88	47.77	1.62	4.45
	Interest rate	0.06	5.56	2.75	7.67	28.95	7.39	9.55	38.07
	Hours worked	0.00	1.68	6.31	32.87	3.42	52.94	0.51	2.28
t=100	Output	0.02	10.29	18.84	23.91	7.00	29.34	6.04	4.57
	Consumption	0.05	8.28	20.65	16.14	8.31	21.28	9.11	16.17
	Investment	0.00	5.55	0.02	14.27	47.79	17.81	0.26	14.29
	Inflation	0.08	7.92	0.91	29.09	9.51	24.34	11.95	16.21
	Real wage	0.05	2.83	5.11	34.42	3.26	47.96	1.54	4.82
	Interest rate	0.03	6.31	1.73	21.47	16.35	21.26	4.44	28.41
	Hours worked	0.00	0.77	4.24	45.15	1.40	46.83	0.20	1.40

Table 9: Variance Decomposition - Model CES (in Percent)$^{\diamondsuit}$

$^{\diamondsuit}$ All the variance decomposition is computed from the model solutions (order of approximation = 1). The results are based on the models' posterior distribution.

Forecast horizon	Observable variables	Shocks of the Estimated DSGE Models: Productivity (K)	Productivity (L)	Government spending	Mark-up (price)	Investment	Mark-up (wage)	Monetary policy	Preference
t=1	Output	0.00	3.46	43.84	10.14	11.01	10.43	4.63	16.48
	Consumption	0.00	1.62	0.12	5.00	0.42	12.27	8.55	72.02
	Investment	0.00	10.76	3.69	13.32	57.16	12.65	1.33	1.08
	Inflation	0.00	19.96	3.63	27.93	11.31	24.25	8.88	4.05
	Real wage	0.00	1.37	1.98	40.26	1.35	41.45	3.38	10.20
	Interest rate	0.00	10.28	3.05	14.01	7.05	12.07	50.67	2.87
	Hours worked	0.00	24.55	34.46	6.96	8.57	9.59	3.47	12.41
t=4	Output	0.00	9.50	31.80	12.61	8.36	20.13	3.53	14.08
	Consumption	0.00	7.45	1.98	8.88	0.28	26.66	5.68	49.08
	Investment	0.00	14.69	5.18	14.09	47.21	16.44	1.22	1.17
	Inflation	0.00	17.87	3.38	18.26	17.98	27.14	12.29	3.07
	Real wage	0.00	6.11	1.97	43.57	1.93	34.20	3.08	9.14
	Interest rate	0.00	16.14	5.78	12.10	24.89	24.08	13.22	3.79
	Hours worked	0.00	6.47	13.50	21.83	9.51	42.42	2.59	3.67
t=10	Output	0.00	9.29	30.15	13.51	8.99	20.92	3.79	13.35
	Consumption	0.00	7.47	2.10	9.51	0.65	27.38	6.16	46.72
	Investment	0.00	13.33	4.82	14.08	48.50	16.66	1.20	1.41
	Inflation	0.00	17.82	3.34	18.65	18.30	26.45	12.13	3.30
	Real wage	0.00	6.29	2.14	43.05	2.06	34.30	3.48	8.68
	Interest rate	0.00	14.77	5.71	9.85	34.94	21.72	10.02	2.98
	Hours worked	0.00	3.37	6.93	21.10	5.06	60.59	1.22	1.73
t=100	Output	0.00	9.42	29.38	13.58	8.78	22.12	3.70	13.01
	Consumption	0.00	7.46	2.14	9.42	0.99	28.89	5.95	45.15
	Investment	0.00	13.56	5.03	14.09	47.74	17.04	1.17	1.37
	Inflation	0.00	25.67	3.30	16.50	16.55	24.67	10.45	2.86
	Real wage	0.00	6.43	2.09	44.02	2.03	33.54	3.40	8.48
	Interest rate	0.00	25.00	6.78	8.48	29.90	19.26	8.15	2.44
	Hours worked	0.00	3.21	7.48	19.43	4.54	62.88	1.01	1.45

Table 10: Variance Decomposition - Model CD (in Percent)$^\diamond$

$^\diamond$ All the variance decomposition is computed from the model solutions (order of approximation = 1). The results are based on the models' posterior distribution.