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MILITARY AID, DIRECT INTERVENTION AND COUNTERTERRORISM

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Military Aid, Direct Intervention and Counterterrorism*

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Abstract

We present a model of transnational terrorism where two countries, home and foreign, face a terrorist threat based in the foreign country. The home country chooses how much to invest in defending itself or in reducing terrorist resources either indirectly by subsidising the foreign country or by directly by intervening itself. We use backward induction to solve a multiple stage game where the home country first commits to its policy decisions, then the foreign country chooses the effort it expends on reducing terrorist capability and finally, the terrorists decide their effort in attacking in the home or foreign country. In a numerical solution of the calibrated model, direct intervention only arises in equilibrium if foreign and home efforts are not close substitutes in the technology used to reduce the resources of the terrorist group. A higher relative military efficiency by the home country makes intervention more likely.

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Contents

1 Introduction

In military conflicts, as in some other activities, there are issues of strategic delegation: to what extent should one fight oneself or subsidize allies to fight for you. During the 18th century and the Napoleonic Wars, Britain not only defended itself against invasion by France, but repeatedly subsidized allies to fight either alongside her or instead of her. This strategy of supporting allies was partly a consequence of the fact that Britain could afford it, having an effective taxation system and good credit which allowed her to borrow. Ferguson (2001) argues that the combination of a Parliament, tax bureaucracy, national debt and central bank gave Britain a decisive military advantage over its main rival France: finance as much as firepower decided the fate of nations. This strategy also reflected relative military effectiveness. Britain's allies, continental powers with large standing armies, had a relative advantage in fighting France on land in Europe, compared to Britain an offshore naval power. To defeat France inland armies were needed, though the allies who provided these armies often did so at the cost of being overthrown, after defeat by France for instance.

Similar issues have arisen recently when the US has to choose the appropriate balance between direct intervention and indirect intervention through aid to a foreign allied government that faces a common enemy. In the early 1960s, the US had to decide whether to just subsidise the Government of South Vietnam to help it fight the Viet Cong and North Vietnam or also to commit US troops to Vietnam. After the 9/11 attacks in 2001, the US invaded Afghanistan and, with the help of the Northern Alliance, displaced the Taliban government. After the invasion, the US had the choice of fighting the Taliban directly or it could fight indirectly by providing military aid to foreign "allied" governments in Afghanistan or Pakistan, to encourage their efforts against the Taliban. The direct attacks on the Taliban could be done with boots on the ground or using unmanned aerial vehicles, UAVs, usually known as drones. The political economy of the US use of drones is discussed in Hall and Coyne (2014). Russia has faced the choice between just supporting proxies in other former Soviet republics or also committing its own troops as it did in Geogia in 2008 and Crimea in 2014. Thus historically, it has been very common for countries in conflict to have to make choices about the resources devoted to direct and indirect intervention.

The purpose of this paper is to examine the choice a home country makes between allocating resources to defence, direct attack (also called proactive or preemptive measures in the literature) and indirect attack by subsidising a foreign ally to fight a common enemy. We are particularly interested in examining the circumstances under which direct intervention becomes part of an equilibrium outcome. The degree of substitutability between the efforts of the home and foreign countries (e.g. between Britain as a naval power and Continental countries as land powers) will play a central role. To examine this issue we will adapt the game used by Bandyopadhyay *et al.* (2011), BSY, to model foreign aid as an element of counterterrorism policy. There is a large closely related literature. The choice between defence and attack is analysed in Sandler and Lapan (1988), Sandler and Siqueira (2006) and Bandyopadhyay and Sandler (2011) while Bandyopadhyay and Sandler (2014) contrast them in the context of immigration. Das and Chowdhury (2014) analyse the choice of attack or defense when a number of countries have a common terrorist enemy and also argue that some of the assumptions used in the reduced form approach of Sandler and Siqueira (2006) may not be robust to explicit modelling of terrorist behaviour. In particular, they question the assumption that an increase in defence (security-deterrence) by one country induces the terrorist organisation to focus more on other target countries. The issue of how terrorists change their targets in response to the choices made by the home and foreign countries will be an important issue in our model. Like BSY, we will label the enemy terrorists, but the enemy could be another country as in the case of France or North Vietnam above.

Home governments who are hegemonic powers, like twenty first century US or the eighteenth century UK, have a variety of instruments that can be used to influence the foreign government's efforts. These include general aid not tied to military effort; aid that is directly tied to military effort, including perhaps providing arms; various types of financial and trade sanctions; and regime change that replaces the foreign government by one more sympathetic to their interests. Our focus is on the use of military aid and to that end we abstract from the general aid that BSY allow for.

The offensive military efforts by both the home government (such as drone attacks) and by the foreign government may have unintended negative consequences, often described by the intelligence community as "blowback". Boyle (2013), discussing the blowback from the US use of drones argues “that drone strikes corrode the stability and legitimacy of local governments, deepen anti-American sentiment and create new recruits for Islamist networks aiming to overthrow these governments.” If the home country’s military aid or direct military action undermines the legitimacy of the foreign government, this may cause the population to be less supportive of the regime, thus less likely to provide the regime with information about the terrorists. Keeping control of the hearts and minds of the population is central to any counter-insurgency strategy. Alternatively the blowback may take the form of protests and regime change that replaces the foreign government with one that is less effective in defending both its own or the home countries interests. In either case, the probability of a successful terrorist attack, either on the foreign country or the home country, is increased. For convenience we label this blowback “regime change”, like BSY. Rosendorff and Sandler (1993) examine backlash or blowback effects of attacks on terrorists on their recruitment and on general grievance.

The enemy, the terrorists, have as their objective harming the home country or its foreign ally, by attacking them. The home country’s objective is to reduce the probability of a successful attack either on itself or its foreign ally. In some circumstances, for instance if there are diminishing returns to any specific form of military effort, which may be plausible for counter-terrorism (though not for, say, area bombing) then it is likely to be optimal to use all the forms available: defensive effort, direct intervention and indirect intervention through military aid to the foreign ally. In addition, it may be plausible to assume that the foreign ally will have some advantage in countering the terrorists if the terrorists are based there, as BSY argue. Clearly, given the often strained relationship between the US and its foreign allies, Afghanistan and Pakistan, the relative effectiveness of direct and indirect intervention can be a matter of dispute. There may also be complementarities, foreign military efforts increase the effectiveness of home military efforts and vice versa. We will be particularly interested in the factors that determine whether or not there is a corner solution with either no direct or no indirect intervention.

The budget constraints of the three agents are clearly a major factor. For the home country the budget may be endogenous in the longer run. In the case of Britain, Ferguson (2001) argued that war was a source of financial innovation and growth, but for cases like US counterterrorism it is probably adequate to assume an exogenous amount of national income available. In some cases, like the suitcases of US$100 bills the CIA passed to Afghan warlords in 2001, the cost is probably just that of printing the notes, since most large denomination US dollar notes stay abroad as a means of payment, rather than becoming a claim on US resources. The costs to the home government of direct military actions include both the budgetary costs (which depend on the force delivery technology available, e.g. drones are cheaper than troops on the ground) and the political costs, for instance if the home voters regard the death of home troops or the collateral damage to the foreign population as unacceptable. The resources available to the terrorists are a function of the military efforts of the home and foreign governments.

We will assume that the home government acts first, deciding its defensive effort, its direct military effort and its military aid to the foreign government. The foreign government acts next determining its military effort. Then there is a possible response by the foreign population, which we label regime change. Finally the terrorists choose the effort they devote to attacking the home and foreign countries and these attacks then succeed with some probability. To focus on the choice between direct and indirect attack we consider a complete information game. There is a literature on asymmetric information games with terrorists such as Lapan and Sandler (1993) and Arce and Sandler (2007), Arce and Sandler (2010).

In our model, we assume that the terrorists are the only enemies that the home and foreign government face. In practice, each may have multiple enemies. In particular, the foreign government may face a regional rival and divert the military aid, which the home government intended to be used against the terrorists, to use against the rival. In such circumstances, Boutton (2014) argues that the foreign government may have an incentive not to disarm terrorist groups, but rather to play up the threat from terrorism in order to continue receiving aid. For instance, he argues that Pakistan, seeing India as a greater threat than the Taliban, diverted a substantial portion of the US military aid, intended for use against the Taliban, to boost its military capability to fight India. He finds that while US foreign aid can help decrease terrorist activity in non-rivalrous states, the opposite is true in states with at least one rival. Boutton (2014) is primarily an empirical paper and does not provide a formal model of the process. Neither our model nor that of BSY allows for such an effect explicitly, but it could be implicitly allowed for through the equation determining the foreign countries military effort against the terrorists as a function of home country aid.

In a related paper, Dunne *et al.* (2006) prove that a defensive type of conflict can arise as an equilibrium result of the conflict between an incumbent and a contestant group to avoid military confrontation, which may encourage asymmetric conflict. In the present paper, terrorist capability is reduced by targeting their resources. There is substantial controversy over whether counter-insurgency warfare is best prosecuted by military means or trying to win "heart and minds" in order to lower population support for the terrorist group. In this paper, it is assumed that direct intervention by the home country makes the foreign government less popular, increasing the probability of a change of regime to one that is less capable of countering the terrorists. Dear (2014) examines the effectiveness of one form of direct military action: targeting the leaders of the terrorists. He gives a number of examples where even killing terrorist leaders can be counter-productive. For instance, the killing of a relatively moderate leader can lead to their replacement by a much more violent leadership as happened with Boko Haram after the Nigerian government killed Mohammad Yusuf in 2009.

One of the main contributions of this paper is to conduct a quantitative analysis of the equilibrium alongside analytical results. Depending on the specific circumstances, the strategic parameters can take a wide range of values, thus examining the sensitivity of the results to those parameter values is important for a number of reasons. Given the wide range of values that are possible, corner solutions are likely. The home country may not intervene directly, as BSY assume, or may not subsidize the foreign ally. We wish to characterize the parameter values that lead to such corner solutions. The calibration of the model also allows us to find the overall backward induction solution to the model. We are able to show that direct intervention is only likely to be part of the equilibrium result if the foreign and home effort are not close substitutes in their ability to reduce the resources of the terrorist group. The more effective the home country is at reducing terrorists resources, relative to the foreign country, the greater the likelihood of a direct intervention. There can also be ambiguity about the results unless one makes somewhat arbitrary assumptions about second derivatives. The numerical solutions allow us to characterize the effects of those assumptions.

The rest of the paper is structured as follows. Section 2 formally describes the game, sets out the equilibrium and provides some useful analytical results. Section 3 provides the quantitative analysis of the equilibrium calibrated to fit the outcomes to stylized facts. Section 4 concludes.

2 The Game

2.1 Elements of the Game

There are three players, the home country H who decides on defensive effort e^H to counter terrorist attacks at home, pro-active effort (direct intervention) e^{HF} and a military aid package to a foreign recipient F where the terrorists operate and train (indirect intervention). This package consists of a conditional aid αe^F where e^F is pro-active effort chosen by the foreign country, the second player, aimed at reducing the terrorists' capability. Direct intervention affects the probability of regime change according to

$$p^F = p^F(e^{HF}); \quad p_1^F > 0 \tag{1}$$

The third player is a terrorist organization who chooses attack effort aimed at the home country, a^H and the foreign country a^F subject to their resource constraint

$$a^H + a^F = M(e^F, e^{HF}); \quad M_1, M_2 < 0, \; M_{11}, M_{22} > 0 \tag{2}$$

Thus the choice variables are e^H, e^{HF} and α for the home country, e^F for the foreign recipient country and, a^H and a^F for the terrorists.[1]

Given these decisions, central to the model are the assumptions behind the way in which our two countries' efforts interact to determine the resources of the terrorists, $M(\cdot)$ in (2), the probabilities of a successful terrorist attack on country H and country F, how these are affected by a regime change and the probability of the latter happening in (1).

[1] Regarding partial and full derivatives, the following notation is adopted. Consider a function of two variable $f(x,y)$. Then $f_1 \equiv \frac{\partial f}{\partial x}$, $f_2 \equiv \frac{\partial f}{\partial y}$, $f_{11} \equiv \frac{\partial^2 f}{\partial x^2}$, $f_{22} \equiv \frac{\partial^2 f}{\partial y^2}$ and $f_{12} = f_{21} \equiv \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ in the usual way. For conciseness for a function of one variable, $f(x)$ we define $f_1 \equiv \frac{df}{dx}$ and $f_{11} \equiv \frac{d^2 f}{dx^2}$. To completely characterize the equilibrium we will need higher derivatives of the form $f_{111} \equiv \frac{\partial^3 f}{\partial x^3}$, $f_{112} \equiv \frac{\partial^3 f}{\partial x^2 \partial y}$ etc. Again for conciseness for a function of one variable, $f(x)$, we denote $f_{111} \equiv \frac{d^3 f}{dx^3}$.

The probability of a successful attack on country H, in the absence of regime change in country F, is given by

$$\sigma^H = \sigma^H(e^H, a^H); \quad \sigma_1^H < 0,\ \sigma_2^H > 0,\ \sigma_{21}^H < 0,\ \sigma_{11}^H > 0,\ \sigma_{22}^H < 0 \tag{3}$$

however, if regime change takes place, this is scaled up to

$$\tilde{\sigma}^H = (1+\eta)\sigma^H(e^H, a^H) \tag{4}$$

Whereas e^H is defensive, counter-terrorist effort in country F serves to reduce terrorist capacity according to (2) and has no impact on the probability of successful attack for a given level of attack effort. Thus the terrorists' success probability of an attack on F is

$$\sigma^F = \sigma^F(a^F); \quad \sigma_1^F > 0, \quad \sigma_{11}^F < 0 \tag{5}$$

and, by analogy with (4), if regime change takes place this is scaled up to

$$\tilde{\sigma}^F = (1+\eta)\sigma^F(a^F) \tag{6}$$

We can now write down the resource constraint of the terrorists and Country F, and the payoffs for all three players. Let T^H and T^{HF} be the costs inflicted on the H country by a successful terrorist attack at home and abroad respectively. Then, the expected national income of the H country is

$$\begin{aligned} Y^H &= \bar{Y}^H - \underbrace{(1-p^F)(\sigma^H T^H + \sigma^F T^{HF})}_{\text{E(costs) without regime change}} - \underbrace{p^F(\tilde{\sigma}^H T^H + \tilde{\sigma}^F T^{HF})}_{\text{E(costs) with regime change}} - \underbrace{(e^H + e^{HF} + \alpha e^F)}_{\text{military-aid expend.}} \\ &= \bar{Y}^H - \gamma(e^{HF}, \eta)[T^H \sigma^H(e^H, a^H) + T^{HF}\sigma^F(a^F)] - (e^H + e^{HF} + \alpha e^F) \\ &= U^H(e^H,\, e^{HF},\, \alpha,\, e^F,\, a^H,\, a^F) \end{aligned} \tag{7}$$

where $\bar{Y}^H$ is potential income having substituted for $\tilde{\sigma}^H$ and $\tilde{\sigma}^H$ from (4) and (6) and having defined

$$\gamma(e^{HF}, \eta) \equiv 1 + p^F(e^{HF})\eta \tag{8}$$

The payoff of the H country is $Y^H = U^H(e^H,\, e^{HF},\, \alpha,\, e^F,\, a^H,\, a^F)$ expressed in terms of the choice variables (e^H, e^{HF}, α) of the H country and the choice variables for the other players which have a direct impact on the H country's objective function: that is, e^F by the foreign country and (a^H, a^F) by the terrorists.

Similarly, we have the expected national income for country F

$$\begin{aligned} Y^F &= \bar{Y}^F - \gamma(e^{HF}, \eta)\sigma^F(a^F)T^F - e^F(1-\alpha) \\ &= U^F(e^F,\, e^{HF},\, \alpha,\, a^F) \end{aligned} \tag{9}$$

where T^F are the terrorist attack costs inflicted on the F country.

Finally, the aim of the terrorists is to inflict damage on the H and F countries with weights ϕ^H and $\phi^F = 1 - \phi^H$ respectively. Thus their payoff is

$$U^T = \gamma(e^{HF}, \eta)[\phi^H\{T^H\sigma^H(e^H, a^H) + T^{HF}\sigma^F(a^F)\} + \phi^F T^F \sigma^F(a^F)] = U^T(e^H, e^{HF},\, e^F,\, a^H,\, a^F) \tag{10}$$

2.2 Equilibrium

The equilibrium concept is a complete information backward induction outcome with country H as the leader, country F as the second mover and the terrorists as the third mover. Thus country H is able to commit with respect to the moves of country F and the terrorists, and country F can commit with respect to the terrorists. The backward solution can be described in the following way:

- **Stage 3**: Terrorists maximize $U^T(e^H, e^{HF},\, e^F,\, a^H,\, a^F)$ with respect to $a^H \geq 0$ and $a^F \geq 0$ given their resource constraint (2) and given actions undertaken at stages 2 and 1, e^F, e^{HF}, e^H and α.

- **Stage 2**: The F country maximizes $U^F(e^F,\, e^{HF},\, \alpha,\, a^F)$ with respect to $e^F \geq 0$ given the reaction function $a^F(e^H,\, e^{HF},\, e^F)$ from stage 1 and given e^H, e^{HF}, α.

- **Stage 1**: The H country maximizes $U^H(e^H,\, e^{HF},\, \alpha,\, e^F,\, a^H,\, a^F)$ with respect to $e^H \geq 0$, $e^{HF} \geq 0$, $\alpha \in [0,1]$ given reaction functions $a^F(e^H,\, e^{HF},\, e^F)$, $a^H(e^H,\, e^{HF},\, e^F)$ and $e^F(e^H,\, e^{HF}, \alpha)$.

The details of the first order conditions (FOCs) at each stage are as follows:

2.2.1 Stage 3

To maximize (10) with respect to $a^F \geq 0$ and $a^H \geq 0$ given the constraint $a^H + a^F \leq M(e^F, e^{HF})$ and previous actions e^H, e^{HF}, e^F, define the Lagrangian

$$\mathcal{L} = \sigma^H(e^H, a^H) + \tau\sigma^F(a^F) + \lambda^M(M(e^F, e^{HF}) - a^H - a^F) + \lambda^{a^H} a^H + \lambda^{a^F} a^F \tag{11}$$

where $\tau \equiv \frac{\phi^H T^{HF} + \phi^F T^F}{\phi^H T^H}$ and λ^M, λ^{a^H}, $\lambda^{a^F} \geq 0$ are multipliers. The FOCs are

$$\begin{aligned} \sigma_2^H(e^H, a^H) - \lambda^M + \lambda^{a^H} &= 0 & (12)\\ \tau\sigma_1^F(a^F) - \lambda^M + \lambda^{a^F} &= 0 & (13)\\ \lambda^M(M(e^F, e^{HF}) - a^H - a^F) &= 0 & (14)\\ \lambda^{a^H} a^H &= 0 & (15)\\ \lambda^{a^F} a^F &= 0 & (16)\\ a^H,\ a^F,\ M - a^H - a^F,\ \lambda^M,\ \lambda^{a^H},\ \lambda^{a^F} &\geq 0 & (17) \end{aligned}$$

which solving, gives the *reaction functions of the terrorists*

$$\begin{aligned} a^H &= a^H(e^H, e^{HF}, e^F) & (18)\\ a^F &= a^F(e^H, e^{HF}, e^F) & (19) \end{aligned}$$

Equations (18) and (19), or equivalently (12)–(17) constitute the stage 3 equilibrium given previous actions e^H, e^{HF}, e^F.

Clearly the capacity constraint must bind at the optimum so $\lambda^M = 0$. For an internal solution $a^H, a^F > 0$ we must also have that $\lambda^{a^H} = \lambda^{a^F} = 0$ so that

$$\sigma_2^H(e^H, a^H) = \tau\sigma_1^F(a^F) = \tau\sigma_1^F(M - a^H) \tag{20}$$

which equates the marginal utility from effort by the terrorist in countries H and F. The second order condition for the internal solution is

$$\sigma_{22}^H(e^H, a^H) + \tau\sigma_{11}^F(M - a^H) < 0 \tag{21}$$

which is guaranteed by the conditions σ^H_{22}, $\sigma^F_{11} < 0$.

Using the Implicit Function Theorem, it is straightforward to prove the following results for interior solutions. See Appendix B for details.

Proposition 1

$a^i_j(e^H, e^{HF}, e^F) < 0$ except $a^F_1(e^H, e^{HF}, e^F) > 0$.

An increase in either direct intervention effort or foreign government effort will certainly reduce the resources available to the terrorists and this will unambiguously reduce their attack effort both at home and abroad. An increase in defensive home effort however will discourage attack effort at home but it will encourage attack effort abroad.

Proposition 2

The signs of $\frac{\partial a^H}{\partial \phi^H}$ and $\frac{\partial a^F}{\partial \phi^H}$ are ambiguous but $\frac{\partial a^H}{\partial T^H} > 0$ and $\frac{\partial a^F}{\partial T^H} < 0$.

An increase in T^H, the cost inflicted on the home country by a successful attack, increases the attack effort the terrorists devote to the home country and reduce the effort devoted to the foreign country.

2.2.2 Stage 2

At the second stage, country F maximizes its objective function (9) with respect to it effort e^F, given the reaction functions (18) and (19) and previous actions by country H, e^H, e^{HF} and α. We define the Lagrangian

$$\mathcal{L} = \bar{Y}^F - \gamma(e^{HF}, \eta)\sigma^F(a^F)T^F - e^F(1-\alpha) + \lambda^{e^F} e^F \tag{22}$$

The FOCs are

$$\begin{aligned} \gamma(e^{HF}, \eta)\sigma^F_1(a^F)T^F a^F_3(e^H, e^{HF}, e^F) + 1 - \alpha - \lambda^{e^F} &= 0 \quad (23) \\ \lambda^{e^F} e^F &= 0 \quad (24) \end{aligned}$$

where to compute $a^F_3(e^H, e^{HF}, e^F)$, we differentiate the FOCs from stage 3 to obtain four additional equations to compute a^F_3, a^H_3, $\lambda^{a^F}_3$ and $\lambda^{a^H}_3$

$$\begin{aligned} \tau\sigma^F_{11}(a^F) - \sigma^H_{22}(e^H, a^H)a^H_3 - \lambda^{a^H}_3 + \lambda^{a^F}_3 &= 0 \quad (25) \\ \lambda^{a^H}_3 a^H + \lambda^{a^H} a^H_3 &= 0 \quad (26) \end{aligned}$$

$$\lambda_3^{a^F} a^F + \lambda^{a^F} a_3^F = 0 \quad (27)$$

$$M_1 - a_3^H - a_3^F = 0 \quad (28)$$

The FOCs and Second Order Conditions (SOCs) for an internal solution $e^F > 0$, $a^F > 0$, $a^H > 0$, $\lambda^{e^F} = \lambda^{a^H} = \lambda^{a^F} = \lambda_3^{a^H} = \lambda_3^{a^F} = 0$ are

$$\gamma(e^{HF}, \eta)\sigma_1(a^F)T^F a_3^F(e^H, e^{HF}, e^F) + 1 - \alpha = 0 \quad (29)$$

$$-\gamma(e^{HF}, \eta)T^F(\sigma_{11}(a^F)(a_3^F(e^H, e^{HF}, e^F))^2 + \sigma_1(a^F)a_{33}^F(e^H, e^{HF}, e^F)) < 0 \quad (30)$$

where

$$\begin{aligned}\sigma_{222}^H(e^H, a^H)\,(a_3^H(e^H, e^{HF}, e^F))^2 &+ \sigma_{22}^H(e^H, a^H)\,a_{33}^H(e^H, e^{HF} e^F)) \\ &= \tau[\sigma_{111}^F(a^F)\,(a_3^F(e^H, e^{HF}, e^F))^2 + \sigma_{11}^F a_{33}^F(e^H, e^{HF} e^F)] \\ a_{33}^H(e^H, e^{HF}, e^F) + a_{33}^F(e^H, e^{HF}, e^F) &= M_{11}(e^F, e^{HF}) \quad (31)\end{aligned}$$

provides two additional equations to evaluate a_{33}^H and a_{33}^F.

These FOCs lead to the *reaction function of country F* which can be written as

$$e^F = e^F(e^H, e^{HF}, \alpha) \quad (32)$$

The following result can now be established:

Proposition 3

The signs of $e_1^F(e^H, e^{HF}, \alpha)$ and $e_2^F(e^H, e^{HF}, \alpha)$ are ambiguous but, $e_3^F(e^H, e^{HF}, \alpha) > 0$.

Proof (The technical details of this proof are presented in Appendix A and Appendix B)

Using the Implicit Function Theorem

$$e_1^F = \frac{\gamma T^F\left(\sigma_{11}^F a_1^F a_3^F + \sigma_1^F a_{31}^F\right)}{Y_{e^F e^F}^F}.$$

For the SOC to hold we require $Y_{e^F e^F}^F < 0$ (the conditions for this to hold are discussed in the Appendix). If we assume third order derivatives of the probability function are zero then, $a_{31}^F = 0$ and the above would have a clear negative sign. However, in general the expression would have an ambiguous sign.

$$e_2^F = \frac{T^F a_3^F \left(\gamma_1 \sigma_1^F + \gamma \sigma_{11}^F a_2^F\right) + \gamma T^F \sigma_1^F a_{32}^F}{Y_{e^F e^F}^F}.$$

Even if we assume that third order derivatives are zero, we would still have an ambiguous sign for a_{32}^F. As further illustrated in Appendix B, the sign of a_{32}^F will depend on the sign of M_{12}.

Finally,

$$e_3^F = \frac{1}{-Y_{e^F e^F}^F} > 0.$$

Unlike BSY, the impact of an increase in home defensive effort on the foreign effort e_1^F is ambiguous. The reason is that although an increase in such effort would encourage terrorist attack effort at foreign ($a_1^F > 0$) and therefore, lower the incentive for foreign effort since $\sigma_{11}^F < 0$. In our analysis, we also include the impact that an increase in home protection effort has on the impact of foreign effort on foreign attack effort a_{31}^F, this will be in general ambiguous.

The new element in our analysis is the direct intervention. Its impact on the foreign effort, e_2^F, is nevertheless ambiguous, an interesting feature of this effect however is that it will be determined by how the direct intervention and foreign effort interact on the reduction of the terrorists' resources, M_{12}. Finally, an increase in the foreign effort subsidy, α, clearly encourages home effort ($e_3^F > 0$). A result which is in line with BSY.

2.2.3 Stage 1

Finally maximizing (7) with respect to e^H, e^{HF} and α respectively, given the reaction functions (18), (19) and (32) gives the three FOC for an *internal solution* $e^H, e^{HF}, \alpha > 0$:

$$\begin{aligned} &\gamma(e^{HF},\eta)[T^H(\sigma_1^H(e^H,a^H) + \sigma_2^H(e^H,a^H)\left(a_1^H(e^H,e^{HF},e^F) + a_3^H e_1^F\right) \\ + \quad & T^{HF}\sigma_1^F(a^F)\left(a_1^F(e^H,e^{HF},e^F) + a_3^F e_1^F\right)] + \left(1+\alpha e_1^F\right) = 0 \end{aligned} \tag{33}$$

The first element of the FOC for home effort in country H above represents the positive direct impact of increasing home defensive effort as it directly reduces the probability of successful attack at home $\sigma_1^H < 0$. However, the increase in home defensive effort e^H encourages terrorists foreign attack effort, $a_1^F > 0$, which in turn increases the probability of a successful attack on home country interests in the foreign country, $\sigma_1^F > 0$. In addition, an increase in home defensive effort affects the foreign country effort in an ambiguous way, this change will affect both the cost of the subsidy given to the foreign government (αe_1^F) and, indirectly, the attack efforts of the terrorists on countries H and F, $(a_3^H e_1^F$ and $a_3^F e_1^F)$. For the case when $e_1^F < 0$, $a_3^F e_1^F > 0$, hence causing an increase in the expected damage on national interests at foreign (i.e., the second term above is positive).

$$p_1^F(e^{HF})\eta[T^H\sigma^H(e^H,a^H)+T^{HF}\sigma^F(a^F)]+\gamma(e^{HF},\eta)[T^H\sigma_2^H(e^H,a^H)\left(a_2^H(e^H,e^{HF},e^F)+a_3^He_2^F\right)$$
$$+T^{HF}\sigma_1^F(a^F)\left(a_2^F(e^H,e^{HF},e^F)+a_3^Fe_2^F\right)]+(1+\alpha e_2^F)=0 \tag{34}$$

The first element on the LHS of the FOC for direct intervention effort above is positive, it represents the positive impact that direct intervention has on the probability of successful attack through its increase in the likelihood of regime change. The third element in the FOC represents the impact on the cost of a change in the direct intervention effort. The second element represents the impact of direct intervention on the terrorists attack effort, an increase in direct intervention effort decreases this attack effort directly ($a_2^H < 0$ and $a_2^F < 0$), but, it has an ambiguous indirect effect ($a_3^H e_2^F$, $a_3^F e_2^F$) whose sign depends on the sign of e_2^F. If home direct effort discourages the foreign government's effort, $e_2^F < 0$ the indirect effect will undermine the direct effect. As already discussed, the sign of e_2^F will be determined by how the home and foreign effort interact on the reduction of the terrorist resources.

$$\begin{aligned}\gamma(e^{HF},\eta)[T^H\sigma_2^H(e^H,a^H)a_3^H(e^H,e^{HF},e^F) \quad &+ \quad T^{HF}\sigma_1^F(a^F)a_3^F(e^H,e^{HF},e^F)]e_3^F(e^H,e^{HF},\alpha)] \\ +(e^F+\alpha e_3^F) \quad &= \quad 0 \end{aligned} \tag{35}$$

Given our results in stages 2 and 3, we know that the first element in the FOC for military subsidy above is negative. It represents the negative impact that the military subsidy to the foreign government, α, has on the expected damage on national interests by the terrorists. The subsidy induces an increase in military effort by the foreign government $\left(e_3^F > 0\right)$ and this reduces the terrorists attack effort both at foreign $\left(a_3^F < 0\right)$ and home $\left(a_3^H < 0\right)$ countries.

It seems intuitive to argue that anything that enhances the impact of direct effort on the probability of regime change should discourage such effort. Also the incentive to undertake direct effort, will be determined by the relative effectiveness of the military efforts of the foreign, e^F, and the direct intervention by the home government, e^{HF}, in reducing terrorist resources: M_1 and M_2 . Another important element will be the ease with which a military subsidy is able to induce foreign effort, T^F and ϕ^Fwill be important parameters to consider in this respect. If regime change caused a re-weighting of targets on the objective function of the terrorist towards home, lower ϕ^F, this would decrease the incentive for direct action as it would reduce the incentive of foreign to invest in effort.

3 Quantitative Analysis

So far, we have considered general functional forms. This has allowed us to highlight the different strategic effects present in our model. We have also been able to highlight that these effects often counteract each other producing ambiguous results for the impact of the home government direct intervention and defensive effort on the foreign effort. This is to be expected; the nature of such strategic interactions is likely to be sensitive to the particular circumstances of the conflict. In the present section, we introduce specific functional forms and calibrate the model using parameters that could describe particular conflicts in order to clarify the likely nature of the strategic effects at play in our framework. The calibration is particularly useful in illustrating the importance of the degree of substitution between home and foreign military efforts in determining whether direct intervention might turn out to be an equilibrium outcome and also, looking at the impact of the relative effectiveness of the two countries' efforts at reducing the terrorist resources. We choose our parameters to focus on that outcome. We present our results using figures which we will interpret in the light of our general model.

3.1 Choice of Functional Forms

There are three sets of functional forms to choose in order to conduct numerical solutions: the probability of regime change, the success probabilities and the terrorist capacity function. We consider these in turn:

Probability of regime change: $p^F = p^F(e^{HF})$; $(p^F)' > 0$ chosen so that $p^F \in (0,1)$. Choose a logit functional form

$$\log \frac{p^F}{1-p^F} = \alpha_p + \beta_p e^{HF} \tag{36}$$

This can be written

$$p^F = \frac{\exp(\alpha_p + \beta_p e^{HF})}{1+\exp(\alpha_p + \beta_p e^{HF})} \tag{37}$$

Terrorists' success probability of an attack on F: $\sigma^F = \sigma^F(a^F)$; $\sigma_1^F > 0$; $\sigma_{11}^F < 0$ chosen so that $\sigma^F \in [0,1)$ and $\sigma^F(0) = 0$.

By analogy with (37) choose

$$\sigma^F(a^F) = \frac{\exp(a^F) - 1}{1+\exp(a^F)} \tag{38}$$

Then

$$\begin{aligned}
\sigma_1^F(a^F) &= \frac{2\exp(a^F)}{(1+\exp(a^F))^2} > 0 \\
\sigma_{11}^F(a^F) &= \frac{2\exp(a^F)(1-\exp(a^F))}{(1+\exp(a^F))^3} < 0 \text{ if } a_F > 0
\end{aligned}$$

So one unit of terrorist capacity results in a success probability of $\frac{\exp(1)-1}{1+\exp(1)} = 0.4621$ in the F country and (from below) in the H country if no counterterrorist effort is expended. In other words a terrorist unit (or cell) results in $1/0.4621 = 2.164$ successful attacks.

Terrorists' success probability of an attack on H: $\sigma^H = \sigma^H(e^H, a^H)$; $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{21}^H < 0$, $\sigma_{11}^H > 0$, $\sigma_{22}^H < 0$.

We choose a contest success function of the general form

$$\sigma^H(e^H, a^H) = \frac{f(a^H)}{f(a^H) + f(\beta_\sigma e^H)}$$

where $f(\cdot)$ is an increasing function of normalized effort. A contest success function of this form fulfills the five axioms of Skaperdas (1996) for any $n \geq 2$ player contest. Hwang (2012) discusses the choice of $f(.)$.

In addition we impose the conditions

$$\begin{aligned} \sigma^H(e^H, 0) &= 0 \\ \sigma^H(e^H, a^H) &\to 0 \text{ as } e^H \to \infty \\ \sigma^H(e^H, a^H) &\to 1 \text{ as } a^H \to \infty \end{aligned}$$

for any $e^H, a^H \geq 0$. The choice $f(x) = \exp(x) - 1$ so that

$$\sigma^H(e^H, a^H) = \frac{\exp(a^H) - 1}{\exp(a^H) + \exp(\beta_\sigma e^H) - 2}$$

satisfies all these conditions.

With this functional form we have the following first and second partial derivatives

$$\begin{aligned} \sigma_1^H(e^H, a^H) &= -\frac{\beta_\sigma \exp(\beta_\sigma e^H)(\exp(a^H) - 1)}{(\exp(a^H) + \exp(\beta_\sigma e^H) - 2)^2} < 0 \text{ for all } a^H > 0 \\ \sigma_2^H(e^H, a^H) &= \frac{\exp(a^H)(\exp(\beta_\sigma e^H) - 1)}{(\exp(a^H) + \exp(\beta_\sigma e^H) - 2)^2} > 0 \text{ for all } e^H > 0 \\ \sigma_{11}^H(e^H, a^H) &= \frac{{\beta_\sigma}^2 \exp(\beta_\sigma e^H)(\exp(a^H) - 1)(2 + \exp(\beta_\sigma e^H) - \exp(a^H))}{(\exp(a^H) + \exp(\beta_\sigma e^H) - 2)^3} \\ &\geq 0 \text{ iff } 2 \geq \exp(a^H) - \exp(\beta_\sigma e^H) \\ \sigma_{22}^H(e^H, a^H) &= -\frac{\exp(a^H)(\exp(\beta_\sigma e^H) - 1)(2 - \exp(\beta_\sigma e^H) + \exp(a^H))}{(\exp(a^H) + \exp(\beta_\sigma e^H) - 2)^3} \\ &\leq 0 \text{ iff } 2 \geq \exp(\beta_\sigma e^H) - \exp(a^H) \\ \sigma_{12}^H(e^H, a^H) &= -\frac{\beta_\sigma \exp(\beta_\sigma e^H + a^H)(\exp(\beta_\sigma e^H) - \exp(a^H))}{(\exp(a^H) + \exp(\beta_\sigma e^H) - 2)^3} \\ \leq 0 & \qquad \text{for all } a^H, e^H \geq 0 \text{ if } \exp(\beta_\sigma e^H) - \exp(a^H) > 0 \end{aligned}$$

It follows that all the conditions $\sigma^H = \sigma^H(e^H, a^H)$; $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{21}^H < 0$, $\sigma_{11}^H > 0$, $\sigma_{22}^H < 0$ are satisfied iff $a^H, e^H \geq 0$, $\exp(\beta_\sigma e^H) > \exp(a^H)$ and

$$|\exp(\beta_\sigma e^H) - \exp(a^H)| < 2$$

That is iff normalized efforts by the H country and the terrorist in that country are not too far apart in equilibrium. These conditions impose the following bounds on σ^H

$$\frac{1}{2} - \frac{1}{2\exp(a^H)} < \sigma^H < \frac{1}{2}$$

Terrorists' Capacity: $M(e^F, e^{HF})$; $\quad M_1, M_2 < 0,\ M_{11}, M_{22} > 0.$
We choose a CES production function that allows for a different degrees of substitution between efforts e^F and e^{HF}:

$$M(e^F, e^{HF}) = \bar{M}\exp\left(-E\right)$$

where

$$E = \left((\beta_{MF}e^F)^\epsilon + (\beta_{MHF}e^{HF})^\epsilon\right)^{\frac{1}{\epsilon}} \qquad -\infty < \epsilon < 1$$

is a CES production function of anti-terrorist effort and $\frac{1}{1-\epsilon}$ is the elasticity of substitution between the two forms of effort, e^F and e^{HF}. For $\epsilon = 1$ we have the case of perfect substitutes whilst as $\epsilon \to -\infty$ we approach the Leontief case.

We noted in the discussion of stage 1 above that the relative effectiveness of home and foreign efforts is a major determinant of whether the home country undertakes direct military intervention. The crucial parameters representing this in the calibration are β_{MHF} and β_{MF}. These will reflect the technologies available to the two governments. For instance β_{MHF} would be large relative to β_{MF} if the home government has access to drone technology not available to the foreign government.

With this functional form we have that

$$\begin{aligned}
M_1 &= -\bar{M}\exp(-E)\frac{\partial E}{\partial e^F} < 0 \\
M_2 &= -\bar{M}\exp(-E)\frac{\partial E}{\partial e^{HF}} < 0 \\
M_{11} &= \bar{M}\exp(-E)\left(\left(\frac{\partial E}{\partial e^F}\right)^2 - \frac{\partial^2 E}{\partial (e^F)^2}\right) > 0
\end{aligned}$$

$$
\begin{aligned}
M_{22} &= \bar{M}\exp(-E)\left(\left(\frac{\partial E}{\partial e^{HF}}\right)^2 - \frac{\partial^2 E}{\partial (e^{HF})^2}\right) > 0 \\
M_{12} = M_{21} &= \bar{M}\exp(-E)\left(\frac{\partial E}{\partial e^{F}}\frac{\partial E}{\partial e^{HF}} - \frac{\partial^2 E}{\partial e^{F}\partial e^{HF}}\right)
\end{aligned}
$$

Thus M_1, $M_2 < 0$ and M_{11}, $M_{22} > 0$, but the sign of M_{12} is ambiguous since

$$
M_{12} = \bar{M}E\exp(-E)\frac{\beta_{MF}^{\epsilon}\beta_{MHF}^{\epsilon}(e^F)^{\epsilon-1}(e^{HF})^{\epsilon-1}}{((\beta_{MF}e^F)^{\epsilon} + (\beta_{MHF}e^{HF})^{\epsilon})^2}(E - 1 + \epsilon)
$$

Since $M_1 < 0$, $M_{12} > 0$ means that the higher the effort of one party (home or foreign) the lower the negative impact of the other party's effort on terrorist capacity M. For the case of perfect substitutes, $\epsilon = 1$ and $M_{12} > 0$. But for $\epsilon < 1$ there is a high level of capacity relative to its maximum at which $E = \log\frac{\bar{M}}{M} < 1 - \epsilon$ and $M_{12} < 0$. This condition can be written

$$
\frac{M}{\bar{M}} > \exp(-(1-\epsilon))
$$

At that point higher effort on one party actually increases the other party's negative impact on M.[2]

3.2 Calibration Strategy

The general idea of the calibration of parameter is to assume an observed baseline equilibrium. We then use such observations to solve for model parameters consistent with them. In general terms, our baseline equilibrium can be described in terms of a vector $\underline{X} = f(\underline{\theta})$ of outcomes where $\underline{\theta}$ is a vector of parameters. The calibration strategy is to choose a subset $\underline{X}_1$ of n observed outcomes to calibrate a subset $\underline{\theta}_1$ of n parameters. Partition $\underline{X} = [\underline{X}_1, \underline{X}_2]$ and $\underline{\theta} = [\underline{\theta}_1, \underline{\theta}_2]$. Then $\underline{\theta}_1$ is then found by solving

$$
[\underline{X}_1, \underline{X}_2] = f([\underline{\theta}_1, \underline{\theta}_2])
$$

for $\underline{X}_2$ and $\underline{\theta}_1$, given $\underline{X}_1$ and $\underline{\theta}_2$. If such a solution exists for economically meaningful parameter values (usually real positive numbers) $\underline{\theta}_1$, then a successful calibration has been achieved.

[2]But note that this result depends on the exponential form of the function M. If instead of (3.1) we choose a power function $M = \bar{M}E^{-1}$, then the sign of M_{12} is the same as $1 - \epsilon$ so for $\epsilon < 1$ we have that $M_{12} > 0$ unambiguously.

To implement this calibration strategy in the model we have seven parameters to be set associated with success probabilities and terrorist capacity: α_p, β_p, η, β_σ, $\bar{M}$, β_{MF} and β_{MHF}. Further parameters associated with costs of successful attacks are T^H, T^{HF}, T^F and ϕ^H. These are the parameters $\underline{\theta}$ that determine the actions of the players e^H, e^{HF}, α for the H country, e^F for the F country and a^H, a^F for the terrorists. Outcomes from these actions, also determined by $\underline{\theta}$, are the probabilities σ^H, $\tilde{\sigma}^H$, σ^F, $\tilde{\sigma}^F$, p^F and the capability M.

We can first pin down the maximum terrorist capacity $\bar{M}$ as follows. Consider a scenario in the F country where there is no counterterrorist effort ($e^F = e^{HF} = 0$). Then a *maximum success probability*, $(\sigma^F)^{max}$ is reached given by

$$(\sigma^F)^{max} = \frac{\exp(\bar{M}) - 1}{1 + \exp(\bar{M})} \Rightarrow \bar{M} = \log\left(\frac{1 + (\sigma^F)^{max}}{1 - (\sigma^F)^{max}}\right)$$

If we can observe $(\sigma^F)^{max}$, this then determines $\bar{M}$.

Second we impose $\phi^H = 1 - \phi^F$ and consider variations as different scenarios. For example $\phi^H = 0$ ($\phi^H = 1$) is the case where terrorists only target the F (H) country.

Third we construct fear factor outcomes from the equilibrium as follows. Consider a worst-case scenario where attacks in both countries are successful. Then the costs incurred are $T^H + T^{HF}$ for the home country and T^F for the foreign country which compares with expenditures $e^H + e^{HF} + \alpha e^F$ for the home country and $e^F(1-\alpha)$ in the foreign country. Then define '*fear factor*' parameters as the ratios of these costs

$$\begin{aligned} ff^H &= \frac{T^H + T^{HF}}{e^H + e^{HF} + \alpha e^F} \\ ff^F &= \frac{T^F}{e^F(1-\alpha)} \end{aligned}$$

for the home and foreign countries respectively. Thus if we impose the ratio $\frac{T^{HF}}{T^H}$ by observing (or just targeting) these fear factors we can pin down T^H and T^F from any equilibrium of e^H, e^{HF} and α.

The three parameters in the terrorist capacity function, ϵ, β_{MF} and β_{MHF} and β_σ in σ^H are crucial for determining the choice of effort by all parties in creating and reducing terrorist activity. We impose the elasticity ϵ and consider variations as scenarios. For our baseline $\epsilon = 1$, we then solve for parameters β_σ and β_{MF} to achieve target probabilities

σ^H and σ^F. This leaves β_{MHF} which we assume is equal to β_{MF}.

This leaves parameters determining the probability of regime change α_p and $\beta_p e^{HF}$ in (36) and η determining the effect of regime change on $\tilde{\sigma}^H$ in (4). We impose β_p and η and then calibrate α_p to achieve a target for p^F. This completes the calibration strategy. Table 1 summarizes the procedure.

Variable	Target Outcome
β_σ	Home Success Probability σ^H
β_{MF}	Foreign Success Probability σ^F
α_p	Probability of Regime Change p^F
Inflicted Costs T^H	Home Fear Factor ff^H
Inflicted Costs T^F	Foreign Fear Factor ff^F
Inflicted Costs Abroad T^{HF}	Assume $T^{HF} = T^H$
Direct Intervention Effect on Capacity β_{MHF}	Assume $\beta_{MHF} = \beta_{MF}$
Max Military Capacity $\bar{M}$	Max of probability σ^F

Table 1: Parameters to Calibrate and the Target Outcomes

3.3 Equilibrium Computation

We now present results for the following choice of imposed parameters values summarized in Table 1: $\phi^H = \phi^F = 0.5$, $\eta = 0.5$, $(\sigma^F)^{max} = 0.75$, $\beta_p = 0.1$ and $\epsilon = 1$. To calibrate the remaining parameters, we choose the following target outcomes: $\sigma^H = 0.1$, $\sigma^F = 0.2$, $p^F = 0.25$ and $ff^H = ff^F = 5$. With these targets we compute the parameters implied by the equilibrium as set out in Table 1. The results for the equilibrium and actual outcomes are set out in the first column of Table 2. The calibrated parameters turned out as: $\beta_\sigma = 1.25$, $\beta_{MF} = \beta_{MHF} = 2.2$, $T^H = T^F = 1.5$. As can be seen from the Table we were not able to hit the targets exactly but we came close.[3]

With these parameter values we find a Stage 1 equilibrium with $e^H = 0.38$, $\alpha = 0.21$ and $e^{HF} = 0$ and $e^F = 0.51$ at Stage 2 of the game. In this equilibrium success probabilities are $\sigma^H = 0.09$ and $\sigma^F = 0.22$. Thus the Home Country chooses not to intervene directly and about 9% of attacks are successful in the home country and 22% of attacks in the foreign country. In Figure 1 and 2 variations in α about this equilibrium are plotted. In

[3] One cannot assume that a solution to (3.2) exists for all equilibrium outcomes.

Figures 3 and 4 we plot variations in e^H and in Figures 5 and 6 variations in e^{HF} for the case of $\epsilon = 1$.

The plots in Figures 1 and 2 confirm the results from the general model. An increase in the foreign military subsidy α encourages foreign effort and this in turn will decrease the incentive that the terrorists have to invest in attack effort both home and foreign. As a result, the terrorist attack success probabilities decrease. Note that the Home welfare loss function is minimized at the baseline equilibrium value of $\alpha = 0.23$.

Figures 3 and 4 clarify the ambiguous effect that changes in home defensive effort has on foreign effort. Our plot indicates an initially positive and then declining impact of defensive effort on foreign effort. Note that for low values of home defensive effort, the plot for terrorist attack effort in country H, a^H, has a positive slope with the slope becoming negative later (this is also the case for the impact of e^H on a^F which is illustrated by the slope turning from negative to positive for higher levels of defensive effort). Our Appendix demonstrates that the sign of the impact of home defensive effort on the home attack effort and foreign attack effort is reversed when $\sigma_{21}^H > 0$. This will happen if $e^H < \frac{a^H}{\beta_\sigma}$ or $e^H < \frac{a^H}{2.2}$. Note that the Home welfare loss function is minimized at the baseline equilibrium value of $e^H = 0.52$.

In Figures 3 and 4, we also see the indirect impact that defensive effort has on the attack efforts of the terrorist through its impact of foreign effort. As the Figures show, this impact is ambiguous as well, for low values of defensive effort increases in this effort will encourage foreign effort, a clear crowding out effect develops for higher levels of defensive effort. The indirect effect reinforces the impact that defensive effort has on foreign attack effort a^F, however, it generates a counteracting force for the direct impact of e^H on a^H. The sign of direct effect however prevails as described in our previous paragraph. Although our setting is different from BSY, they also get a crowding out effect of defensive effort on foreign effort under a $\sigma_{21}^H < 0$ assumption.

Figures 5 and 6 clarify the ambiguous effect of direct intervention of foreign effort. For the case where direct intervention and foreign effort interact as perfect substitutes in the lowering of terrorist resources, we have that direct intervention crowds out foreign effort (see Figure 6). As seen in the theoretical framework, the impact of direct intervention of attack efforts was negative for both foreign and home attack effort as it reduced the resources available to the terrorists, however, the crowding out of foreign effort counteracts the first effect as this in turn increases the terrorist resources. As the plots show, the impact that these effects together have on attack efforts and therefore attack success rates is negligible. Note that in this case we have a corner solution as the welfare loss function is minimized at $e^{HF} = 0$.

An important result so far is that the Stage 1 equilibrium involves no direct military intervention ($e^{HF} = 0$). We have seen from the analysis that the sign of $\frac{\partial e^F}{\partial e^{HF}}$ is ambiguous. In fact with our parameter setting we see that $\frac{\partial e^F}{\partial e^{HF}} < 0$ so military intervention by the home country crowds out anti-terrorist effort by the foreign country and is counterproductive. But what happens if we reduce the degree of substitution between e^F and e^{HF} by lowering ϵ? With $\epsilon = 0.5$, Figures 7 and 8 show this now produces a Stage 1 equilibrium with some military intervention with $e^{HF} = 0.06$ (where welfare loss function is now minimized). Therefore, the nature of the technology by which H and F influence terrorist capacity is crucial for the choice of direct intervention. Figures 9 and 10 present 3-dimensional plots of the equilibria in these two cases.

Next we explore the corner solution at which direct military intervention is welfare-reducing for the home country by constructing a measure of the home versus foreign relative military efficiency defined by $\beta \equiv \frac{\beta_{MHF}}{\beta_{MF}}$. Up to now we have set $\beta = 1$. Figure 11 then plots β against the threshold value of ϵ at which the corner solution to the equilibrium, $e^{HF} = 0$, occurs. We see that as β increases, we can have quite modest complementarity between home and foreign effort to see direct intervention emerge as a possible equilibrium.

Columns 2–4 of Table 2 set out the full equilibrium for the case of imperfect substitution between e^F and e^{HF} with $\epsilon = 0.7, 0.5, 0.25$. We see that in these equilibria there is steady reduction of military aid to 0 and with some substitution by the H country towards expenditure on both defensive effort and direct intervention. The former disincentivises and the latter crowds out anti-terrorist effort e^F by the F country. Terrorism ceases owing to the reduction of their capacity and the success probability falls to zero in the H country, Eventually for $\epsilon = 0.25$ the success probability falls in the F country as well.

Variable	Value	Value	Value	Value
Elasticity ϵ	1.0	0.7	0.5	0.25
Home expenditure e^H	0.52	0.59	0.57	0.56
Military Aid α	0.23	0.09	0.02	0.00
Expenditure on Direct Intervention e^{HF}	0	0.09	0.04	
Foreign Expenditure e^F	0.61	0.54	0.33	0.15
Home Success Probability σ^H	0.11	0.00	0.00	0.00
Foreign Success Probability σ^F	0.20	0.25	0.18	0.05
Home Fear Factor ff^H	3.9	4.1	4.3	4.4
Home Expected Cost to Expenditure $\sigma^H ff^H$	0.43	0	0	0
Foreign Fear Factor ff^F	3.2	3.1	4.6	9.9
Foreign Expected Cost to Expenditure $\sigma^F ff^F$	0.64	0.78	0.83	0.50
Probability of Regime Change p^F	0.250	0.251	0.252	0.251

Table 2: Stage 1 Computed Equilibrium: e^F and e^{HF} perfect substitutes and imperfect ($\epsilon \in [0, 1]$). $\phi^H = 0.5$, $\eta = 0.5$, $(\sigma^F)^{max} = 0.75$

4 Conclusions

This paper presents a model of a conflict in which two countries, home and foreign, under threat of terrorist attack, interact non-cooperatively with the objective of limiting the expected damage done by the terrorists. Whereas the terrorists follow an offensive strategy, with the objective of causing damage to both countries, the two countries follow a defensive strategy, with the objective of limiting the expected damage. The two countries face different types of threat. The *foreign* country can only be damaged by terrorist attacks in their own territory. The *home* country, has national interests in both countries which can be damaged by the terrorists.

The two countries have different policy instruments. The foreign country just decides the level of effort it expends on limiting the resources available to the terrorists to carry out their attacks. The home country decides its effort on defence to protect its national territory, its military subsidy to encourage the foreign government's efforts to reduce the terrorists resources and its own direct intervention in the foreign country to reduce terrorist resources.

We model the interaction between the countries and the terrorist group as a multiple stage game where the home country first commits to their policy decisions, then the foreign government does, finally, the terrorist group decides how much effort to put into terrorist actions against the home or foreign country. We solve the game using backward induction.

The objective of our analysis is to identify the elements in the interaction among the different players which will explain the circumstances under which direct intervention will be part of an equilibrium. Our theoretical model shows different effects at play which often counteract each other, a feature which is characteristic of many conflicts. Our modeling strategy expands the BSY framework in a number of ways, the main difference is that we allow for direct intervention and investigate its interaction with foreign effort. As we find the solution to the model, unlike BSY we do not restrict ourselves by presuming that the third order derivatives of probability functions are zero.

Our backward induction method allows us to show that an increase in either direct intervention effort or foreign government effort will unambiguously reduce terrorist attack effort both at home and abroad, on the foreign country. An increase in defensive home effort however will discourage terrorist attack effort at home but will encourage attack effort abroad. As we proceed to the second stage we find that the whereas the military subsidy to the foreign government has a clear positive impact on foreign effort. The impact of both defensive and direct intervention efforts are ambiguous. However, our use of calibration allows us to resolve the sign of these effects and find the overall equilibrium.

Our calibration results confirm the positive impact of the military subsidy on foreign effort and identify a negative impact of direct intervention of foreign effort for both imperfect and perfect substitution in the two efforts in the reduction of terrorist resources. The crowding out the foreign effort is stronger if the two efforts are closer substitutes.

More importantly, the calibration of the model also allows us to find the overall backward induction solution to the model. For these parameters, we are able to show that direct intervention is only likely to be part of the equilibrium result if the foreign and home effort are not good substitutes in the technology used to reduce the resources of the terrorist group. Direct intervention will become more likely as the effectiveness of the home country in reducing terrorists resources, relative to the foreign country, increases.

Within the framework of this game, there is scope to examine the effect of a number of exogenous factors that change the incentive for the home country to intervene directly, including the probability of blowback which strengthens the terrorists, for instance through regime change.

Like BSY we consider a complete information game but, given the tensions in the relations between the US and both Afghanistan and Pakistan, an interesting topic of future research would be a game where the home or foreign country cannot monitor the effort or type of the other country from their observed actions. Brauer and van Tuyl (2008), chapter 3, examine the principal-agent problem faced by Renaissance Italian Cities who hired Condottieri to fight for them. In this case there were significant problems of moral hazard and adverse selection and the Condottieri often found it more profitable to attack the city that hired them, rather than that city's enemies.

Appendix A. Reaction Function Derivatives

To complete the equilibrium we require expression for $a_i^H(e^H, e^{HF}, e^F)$, $a_i^F(e^H, e^{HF}, e^F)$ and $e_i^F(e^H, e^{HF}, \alpha; i = 1, 3)$. a_3^F and a_3^H have already been obtained at Stage 2 as detailed in the main text. First, let $\tau = \frac{(\phi^H T^{HF} + \phi^F T^F)}{\phi^H T^H}$ be the ratio of F to H weighted costs inflicted by the terrorists. Then differentiating stage 3 condition (20) with respect to e^H, e^{HF} and e^F gives respectively

$$\begin{aligned}
\sigma_{21}^H(e^H, a^H) + \sigma_{22}^H(e^H, a^H)\, a_1^H(e^H, e^{HF} e^F) &= \tau\, \sigma_{11}^F(a^F)\, a_1^F(e^H, e^{HF}, e^F) & (39)\\
a_1^H(e^H, e^{HF}, e^F) + a_1^F(e^H, e^{HF}, e^F) &= 0 & (40)\\
\sigma_{22}^H(e^H, a^H)\, a_2^H(e^H, e^{HF} e^F) &= \tau\, \sigma_{11}^F(a^F)\, a_2^F(e^H, e^{HF}, e^F) & (41)\\
a_2^H(e^H, e^{HF}, e^F) + a_2^F(e^H, e^{HF}, e^F) &= M_2(e^F, e^{HF}) & (42)\\
\sigma_{22}^H(e^H, a^H)\, a_3^H(e^H, e^{HF} e^F) &= \tau\, \sigma_{11}^F(a^F)\, a_3^F(e^H, e^{HF}, e^F) & (43)\\
a_3^H(e^H, e^{HF}, e^F) + a_3^F(e^H, e^{HF}, e^F) &= M_1(e^F, e^{HF}) & (44)
\end{aligned}$$

Given functional forms for the probabilities p^F, σ^H, σ^F and M considered in the quantitative analysis section, we have so far 12 equations in 6 choice variables e^H, e^{HF}, α for country H, e^F for country F, a^H and a^F for terrorists; and 7 reaction function derivatives a_1^H, a_1^F, a_2^H, a_2^F, a_3^H, a_3^F and e_3^F.

It remains to find expressions for e_i^F, $i = 1, 3$. To do this first differentiate (29) with respect to e^H, e^{HF} and α to obtain respectively:

$$\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a_3^F(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a_{33}^F(e^H,e^{HF},e^F)e_1^F(e^H,e^{HF},\alpha)\right] = 0 \tag{45}$$

$$\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a_3^F(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a_{33}^F(e^H,e^{HF},e^F)e_2^F(e^H,e^{HF},\alpha)\right] = 0 \tag{46}$$

$$\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a_3^F(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a_{33}^F(e^H,e^{HF},e^F)e_3^F(e^H,e^{HF},\alpha)\right]-1 = 0 \tag{47}$$

Finally, differentiating (28) and (29) with respect to e^F, we have

$$\begin{aligned}\sigma_{222}^H(e^H,a^H)\,(a_3^H(e^H,e^{HF},e^F))^2 \;&+\; \sigma_{22}^H(e^H,a^H)\,a_{33}^H(e^H,e^{HF}e^F))\\ &=\; \tau[\sigma_{111}^F(a^F)\,(a_3^F(e^H,e^{HF},e^F))^2+\sigma_{11}^F a_{33}^F(e^H,e^{HF}e^F))]\end{aligned} \tag{48}$$

$$a_{33}^H(e^H,e^{HF},e^F)+a_{33}^F(e^H,e^{HF},e^F) = M_{11}(e^F,e^{HF}) \tag{49}$$

(45)–(49) provide five additional equations for a_{33}^H, a_{33}^F and e_i^F, $i=1,3$ completing the equilibrium.

Appendix B. Proofs of Analytical Results

Stage 3

As in BSY, terrorists choose their attack effort distribution across their base country, foreign and home, a^H and a^F that maximize their objective function subject to their resource constraint. For the interior solution we find the tangency condition:

$$\phi^H T^H \sigma_2^H(e^H,a^H) = \left(\phi^H T^{HF}+\phi^F T^F\right)\sigma_1^F(a^F)$$

Substituting budget constraint

$$a^F = M(e^F,e^{HF}) - a^H$$

we get:

$$\phi^H T^H \sigma_2^H(e^H, a^H) - [\phi^H T^{HF} + \phi^F T^F] \frac{\partial \sigma^F(M(e^F, e^{HF}) - a^H)}{\partial a^F} = 0$$

The following results can be established using the Implicit Function Theorem (IFT) and the assumptions over the properties of the probabilities of successful attack $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{11}^H > 0$, $\sigma_{22}^H < 0$, $\sigma_1^F > 0, \sigma_{11}^F < 0$.

Following BSY, we define

$$D = -\left[\phi^H T^H \sigma_{22}^H(e^H, a^H) + [\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F(a^F)\right] > 0$$

Now, using the IFT, we obtain the following expressions for the impact of the different country efforts on the attack efforts of the terrorists:

$$a_1^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{21}^H(e^H, a^H)}{D} = \frac{\phi^H T^H \sigma_{21}^H}{D} < 0 \Leftrightarrow \sigma_{21}^H < 0$$

$$a_1^F(e^H, e^{HF}, e^F) = -a_1^H(e^H, e^{HF}, e^F) = -a_1^H > 0 \Leftrightarrow \sigma_{21}^H < 0$$

$$a_2^H(e^H, e^{HF}, e^F) = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F(a^F) M_2(e^F, e^{HF})}{-D} = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F M_2}{-D} < 0$$

$$a_2^F(e^H, e^{HF}, e^F) = M_2(e^F, e^{HF}) - a_2^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{22}^H(e^H, a^H) M_2(e^F, e^{HF})}{-D} = \frac{\phi^H T^H \sigma_{22}^H M_2}{-D} < 0$$

$$a_3^H(e^H, e^{HF}, e^F) = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F(a^F) M_1(e^F, e^{HF})}{-D} = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F M_1}{-D} < 0$$

$$a_3^F(e^H, e^{HF}, e^F) = M_1(e^F, e^{HF}) - a_3^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{22}^H(e^H, a^H) M_1(e^F, e^{HF})}{-D} = \frac{\phi^H T^H \sigma_{22}^H M_1}{-D} < 0.$$

In addition,

$$\phi^H T^H \sigma_2^H(e^H, a^H) - [\phi^H T^{HF} + \phi^F T^F] \frac{\partial \sigma^F(M(e^F, e^{HF}) - a^H)}{\partial a^F} = 0$$

$$\frac{\partial a_3^H}{\partial \phi^H} = \frac{T^H \sigma_2^H - T^{HF} \sigma_1^F}{D}$$

$$\frac{\partial a_3^F}{\partial \phi^H} = -\frac{T^H \sigma_2^H - T^{HF} \sigma_1^F}{D}$$

Note that, $\sigma_2^H > 0$ and $\sigma_1^F > 0$, hence, the above will depend on parameters.

$$\frac{\partial a_3^H}{\partial T^H} = \frac{\phi^H \sigma_2^H}{D} > 0.$$

$$\frac{\partial a_3^F}{\partial T^H} = -\frac{\phi^H \sigma_2^H}{D} < 0.$$

Stage 2

The FOC for an internal solution $e^F > 0$ can be written as

$$-\gamma(e^{HF}, \eta)\sigma_1^F(a^F)a_3^F(e^H, e^{HF}, e^F)T^F - 1 + \alpha = 0$$

which leads to the *reaction function of country F* as

$$e^F = e^F(e^H, e^{HF}, \alpha)$$

The second order condition implies

$$Y^F_{e^F e^F} = -\gamma(e^{HF}, \eta)T^F \left[\sigma_{11}^F(a^F)\left(a_3^F\right)^2 + \sigma_1^F(a^F)a_{33}^F\right] = -\gamma T^F \left[\sigma_{11}^F \left(a_3^F\right)^2 + \sigma_1^F a_{33}^F\right] < 0.$$

Note that

$$a_{33}^H = \frac{\tau\sigma_{111}^F(-a_3^H)^2 - \sigma_{222}^H\,(a_3^H)^2 + \tau\sigma_{11}^F M_{11}}{\sigma_{22}^H + \tau\sigma_{11}^F}$$

$$a_{33}^F(e^H, e^{HF}, e^F) = M_{11} - \frac{\tau\sigma_{111}^F(-a_3^H)^2 - \sigma_{222}^H\,(a_3^H)^2 + \tau\sigma_{11}^F M_{11}}{\sigma_{22}^H + \tau\sigma_{11}^F}$$

or

$$a_{33}^F(e^H, e^{HF}, e^F) = \frac{\sigma_{22}^H\, M_{11} + (-a_3^H)^2\left(\sigma_{222}^H - \tau\sigma_{111}^F\right)}{\sigma_{22}^H + \tau\sigma_{11}^F}.$$

Now, we know that for the second order condition to hold we need

$$Y^F_{e^F e^F} = -\gamma T^F \left[\sigma^F_{11}\left(a^F_3\right)^2 + \sigma^F_1 a^F_{33}\right] = -\gamma T^F \left[\sigma^F_{11}\left(a^F_3\right)^2 + \sigma^F_1 a^F_{33}\right] < 0.$$

For the above to hold, we need $a^F_{33} >> 0$. For that, we need $M_{11} >> 0$, even if we assumed third order derivatives to be equal to zero.

For the comparative statics results we first need to find explicit expressions for a^F_{31} and a^F_{32}. For that, we differentiate (43) and (44) with respect to e^H and e^{HF} and substitute into each other:

First, a^F_{31}

$$\begin{aligned}
\sigma^H_{22}(e^H, a^H)\left(M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right) &= \tau\, \sigma^F_{11}(a^F)\, a^F_3(e^H, e^{HF}, e^F) \\
a^H_3(e^H, e^{HF}, e^F) &= M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)
\end{aligned}$$

$$\left(\sigma^H_{221}(e^H, a^H) + \sigma^H_{222}(e^H, a^H) a^H_1\right)\left(M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right) + \sigma^H_{22}(e^H, a^H)\left(-a^F_{31}(e^H, e^{HF}, e^F)\right)$$

$$= \tau\, \sigma^F_{111}(a^F) a^F_1\, a^F_3(e^H, e^{HF}, e^F) + \tau\, \sigma^F_{11}(a^F)\, a^F_{31}(e^H, e^{HF}, e^F)$$

$$a^F_{31} = \frac{\left(\sigma^H_{221} + \sigma^H_{222} a^H_1\right) a^H_3 - \tau\, \sigma^F_{111} a^F_1\, a^F_3}{\tau\, \sigma^F_{11} + \sigma^H_{22}}$$

In general, the sign of the above is ambiguous. However, if we assumed third order derivatives are zero, as BSY do, it would be zero.

Second, a^F_{32}

$$\sigma^H_{22}(e^H, a^H)\left[M_{12}(e^F, e^{HF}) - a^F_{32}(e^H, e^{HF}, e^F)\right] + \sigma^H_{222}(e^H, a^H) a^H_2\left[M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right]$$

$$= \tau\, \sigma^F_{111}(a^F) a^F_2\, a^F_3(e^H, e^{HF}, e^F) + \tau\, \sigma^F_{11}(a^F)\, a^F_{32}(e^H, e^{HF}, e^F).$$

We rewrite to get

$$a^F_{32}(e^H, e^{HF}, e^F) = \frac{\sigma^H_{22} M_{12} + \sigma^H_{222} a^H_2\left[M_1 - a^F_3\right] - \tau\, \sigma^F_{111} a^F_2\, a^F_3}{\sigma^H_{22} + \tau\, \sigma^F_{11}}.$$

Once more, the sign of the above is ambiguous, but in this case even if we assume that third order derivatives are zero, we are still left with

$$a_{32}^F(e^H, e^{HF}, e^F) = \frac{\sigma_{22}^H M_{12}}{\sigma_{22}^H + \tau\, \sigma_{11}^F}$$

The sign of the above will be positive as long as $M_{12} > 0$, since all the other terms are negative.

Now we can proceed to comparative statics result. Using the IFT and results from third stage, $a^H(e^H, e^{HF}, e^F)$ and $a^F(e^H, e^{HF}, e^F)$,we get:

First,

$\frac{\partial e^F}{\partial e^H} = e_1^F = \frac{Y^F_{e^F e^H}}{-Y^F_{e^F e^F}}$

$= \frac{-\gamma(e^{HF},\eta)T^F\sigma_{11}^F(a^F)a_1^F a_3^F(e^H,e^{HF},e^F)-\gamma(e^{HF},\eta)T^F\sigma_1^F a_{31}^F}{-Y^F_{e^F e^F}}$

$= \frac{\gamma T^F\left(\sigma_{11}^F a_1^F a_3^F+\sigma_1^F a_{31}^F\right)}{Y^F_{e^F e^F}}.$

If we assumed third order derivatives are zero as BSY do, then $a_{31}^F = 0$ and the above would have a clear negative sign. Otherwise the sign will be ambiguous.

Second,

$$\frac{\partial e^F}{\partial \alpha} = e_3^F = \frac{Y^F_{e^F\alpha}}{-Y^F_{e^F e^F}} = \frac{1}{-Y^F_{e^F e^F}} > 0.$$

Third,

$\frac{\partial e^F}{\partial e^{HF}} = e_2^F = \frac{Y^F_{e^F e^{HF}}}{-Y^F_{e^F e^F}}$

$= \frac{-\gamma_1(e^{HF},\eta)T^F\sigma_1^F(a^F)a_3^F(e^H,e^{HF},e^F)-\gamma(e^{HF},\eta)T^F\sigma_{11}^F(a^F)a_2^F a_3^F(e^H,e^{HF},e^F)-\gamma T^F\sigma_1^F a_{32}^F}{-Y^F_{e^F e^F}}$

$= \frac{T^F a_3^F\left(\gamma_1\sigma_1^F+\gamma\sigma_{11}^F a_2^F\right)+\gamma T^F\sigma_1^F a_{32}^F}{Y^F_{e^F e^F}} > 0.$

As already discussed, even if we assume that third order derivatives are zero, we would still have an ambiguous sign for a_{32}^F which would depend on the sign of M_{12}. As $\sigma_1^F > 0$, we would need $a_{32}^F < 0$ to not get an ambiguous sign above, for zero third order derivatives this would happen if $M_{12} < 0$.

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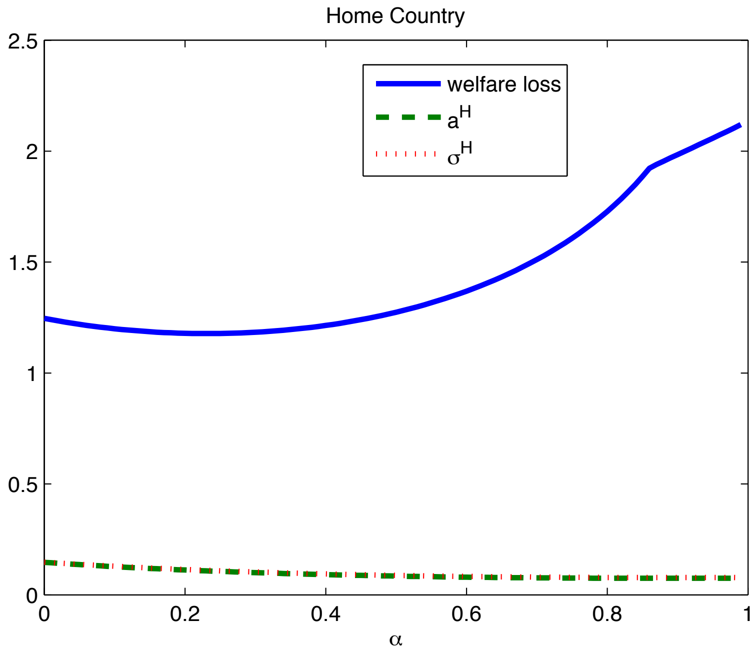

Figure 1: **Stage 2: Response to α for Home Country.** $e^H = 0.38$, $e^{HF} = 0$.

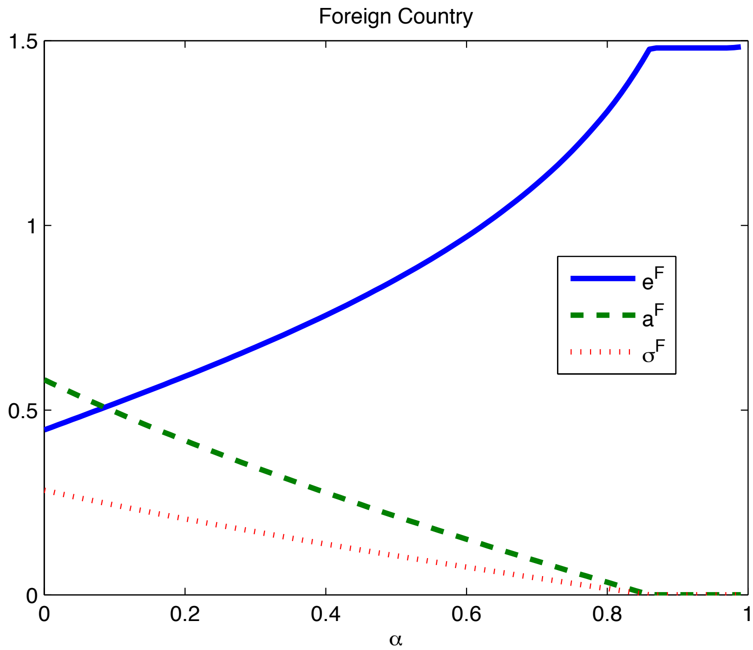

Figure 2: **Stage 2: Response to α for Foreign Country.** $e^H = 0.38$, $e^{HF} = 0$

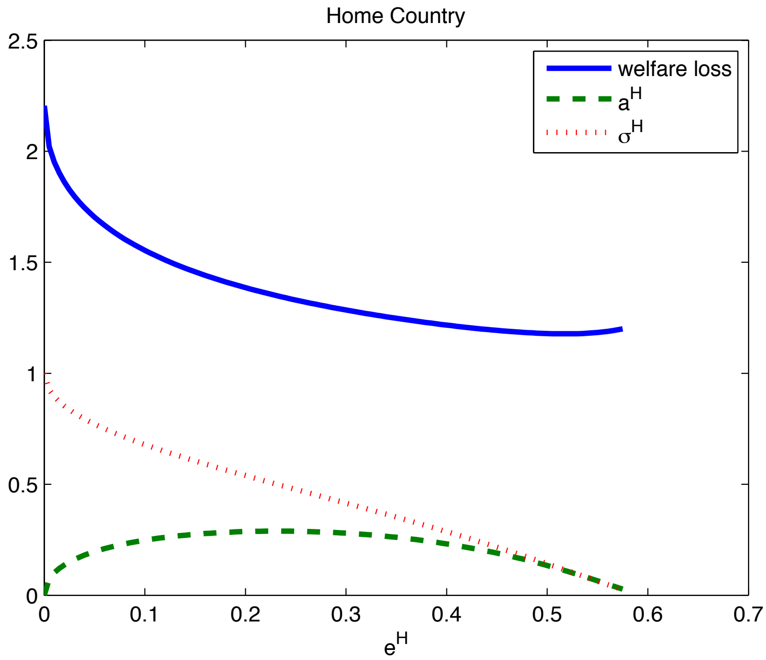

Figure 3: **Stage 2: Response to e^H for Home Country.** $\alpha = 0.21$, $e^{HF} = 0$

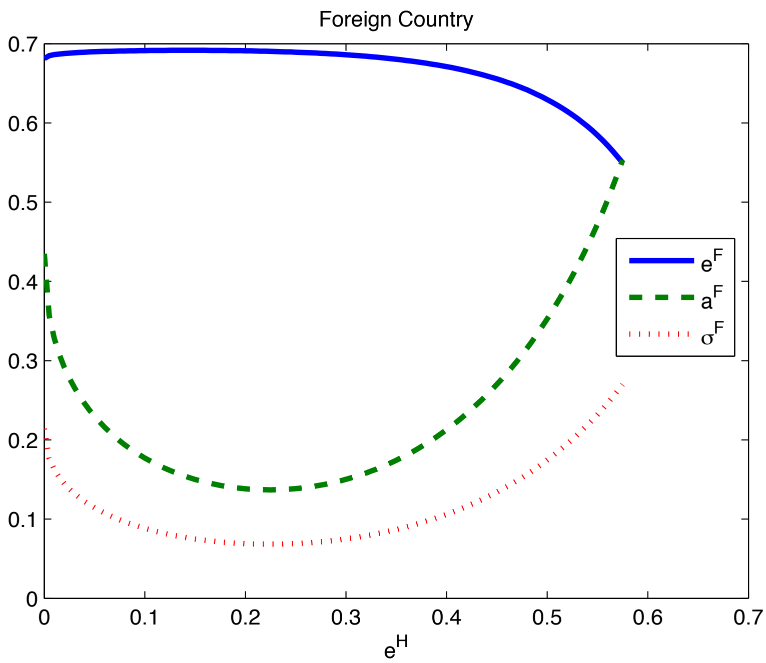

Figure 4: **Stage 2: Response to e^H for Foreign Country.** $\alpha = 0.21$, $e^{HF} = 0$

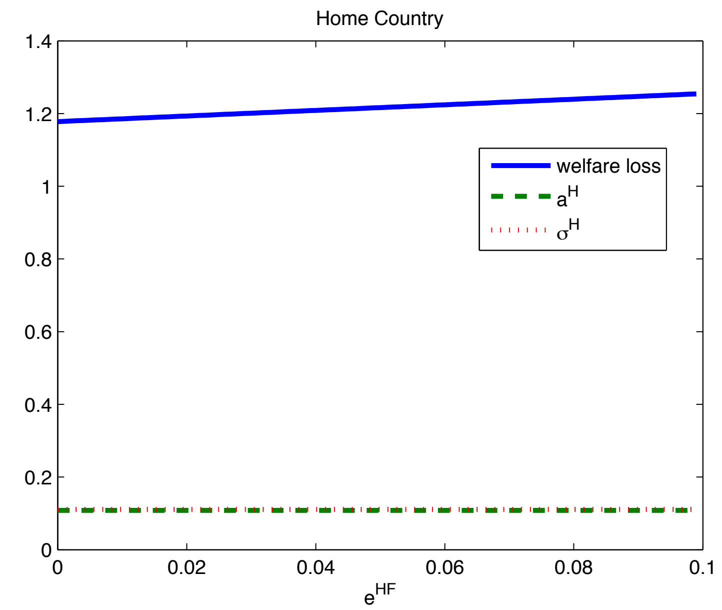

Figure 5: **Stage 2: Response to e^{HF} for Home Country. e^{H}, e^{HF} Perfect Substitutes.** $\alpha = 0.21$, $e^{H} = 0.38$

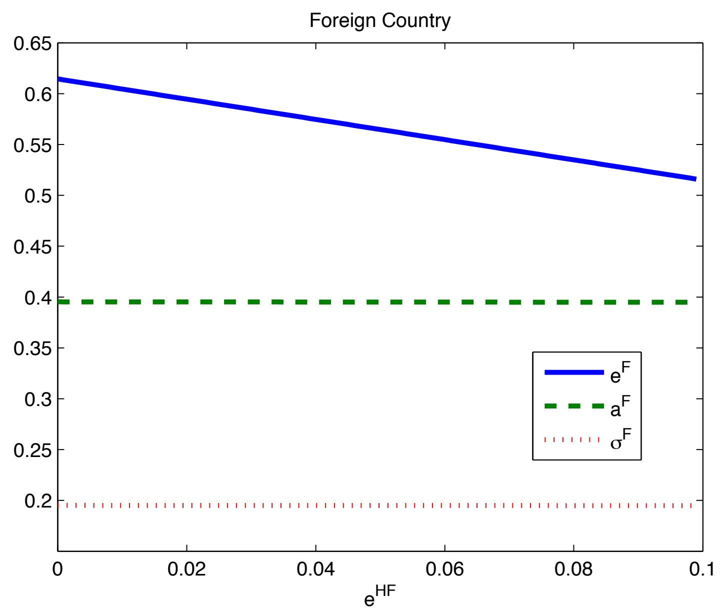

Figure 6: **Stage 2: Response to e^{HF} for Foreign Country. e^{F}, e^{HF} Perfect Substitutes.** $\alpha = 0.21$, $e^{H} = 0.38$

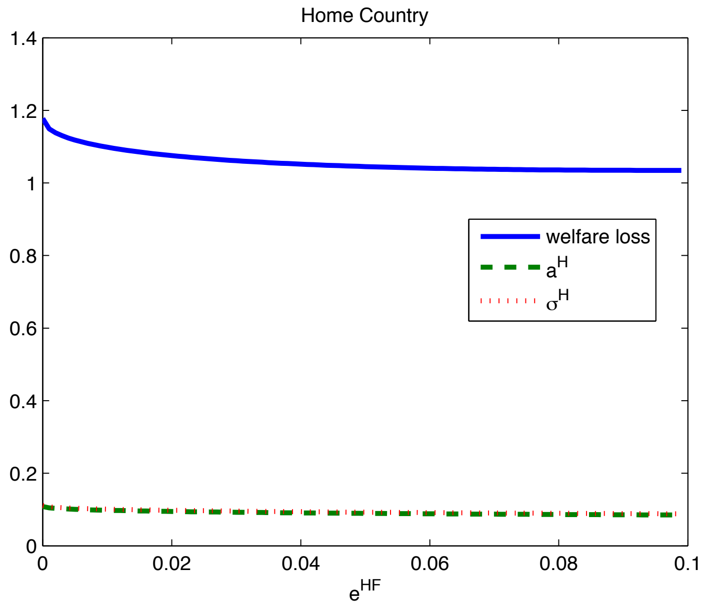

Figure 7: **Stage 2: Response to e^{HF} for Home Country. e^F, e^{HF} Imperfect Substitutes ($\epsilon = 0.5$).** $\alpha = 0.21$, $e^H = 0.38$

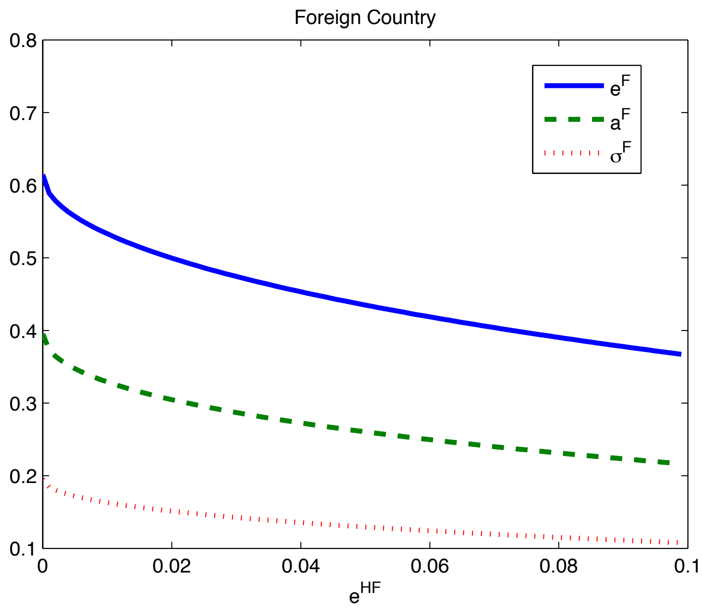

Figure 8: **Stage 2: Response to e^{HF} for Foreign Country. e^F, e^{HF} Imperfect Substitutes ($\epsilon = 0.5$).** $\alpha = 0.21$, $e^H = 0.38$

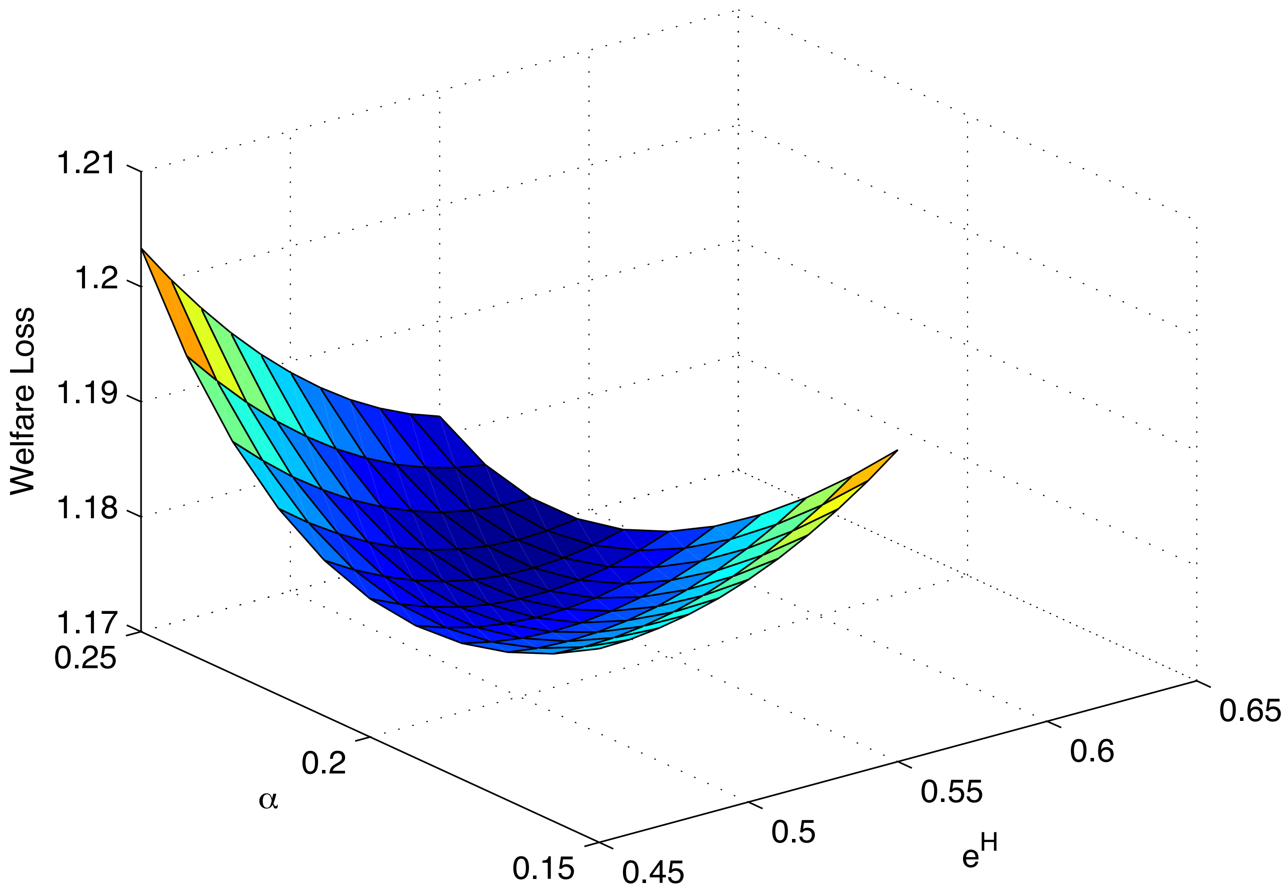

Figure 9: **Stage 1: Optimal Choice of e^H and α with $e^{HF} = 0$ (e^H and e^{HF} Perfect Substitutes).**

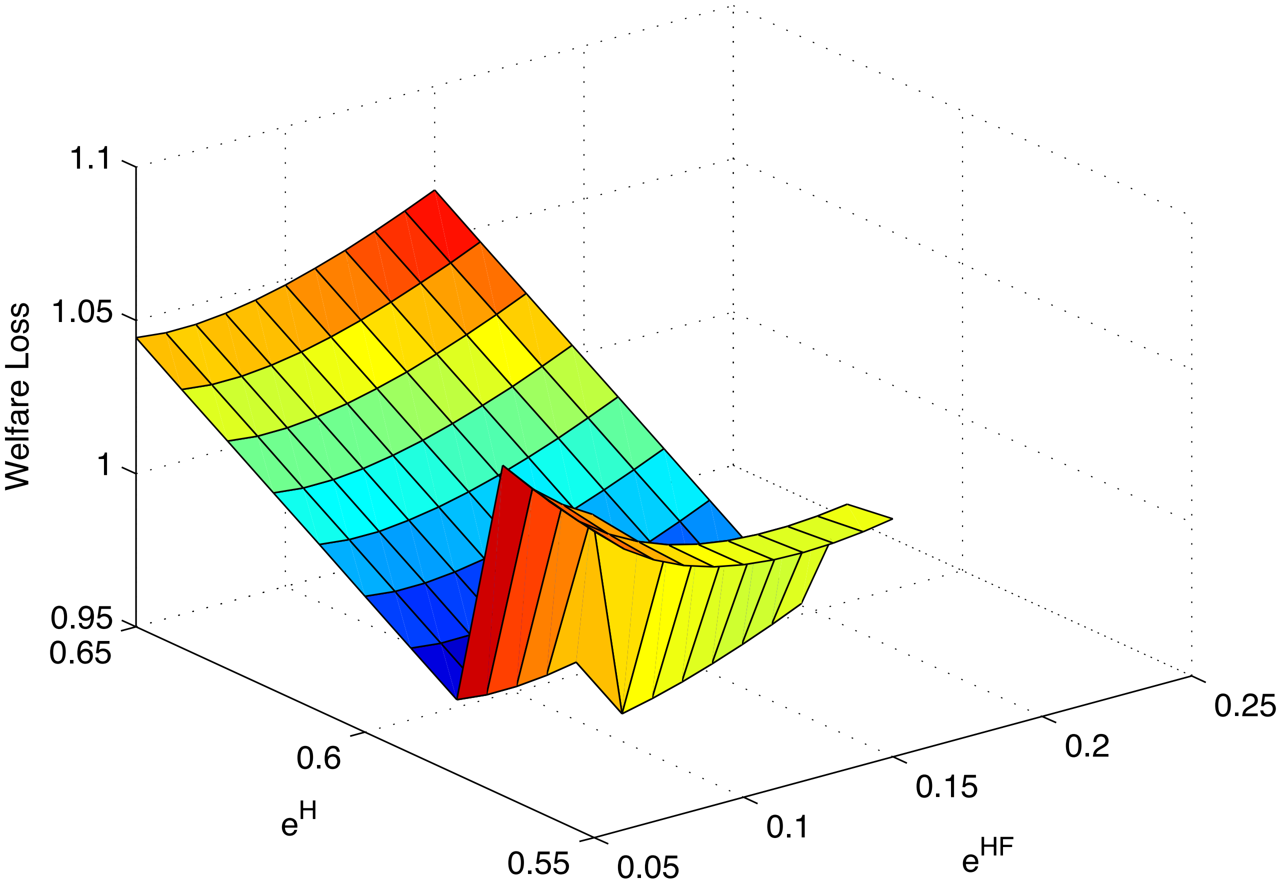

Figure 10: **Stage 1: Optimal Choice of e^H and e^{HF} Imperfect Substitutes ($\epsilon = 0.5$) with $\alpha = 0.2$.**

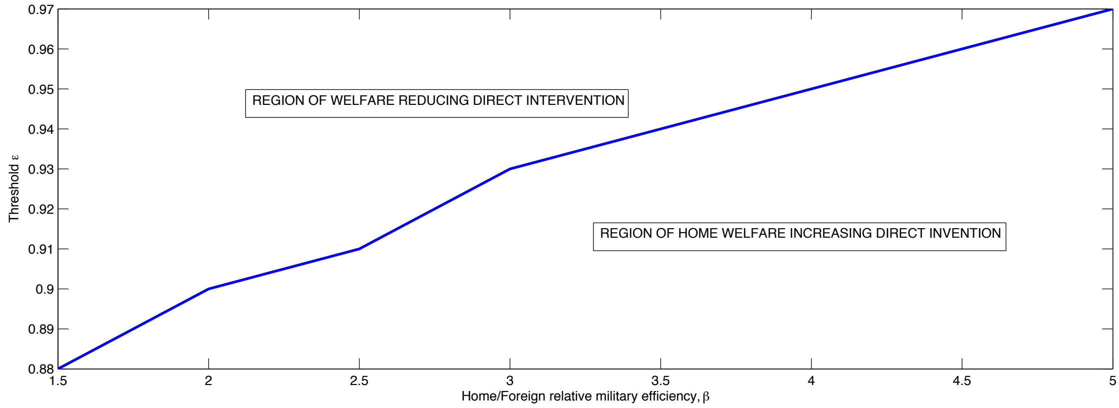

Figure 11: **Stage 2: Threshold of ϵ as Relative Home/Foreign Military Efficiency Increases.** $\alpha = 0.21,\ e^H = 0.38$