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TRUTH-TRACKING JUDGMENT AGGREGATION OVER INTERCONNECTED ISSUES

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Truth-tracking judgment aggregation over interconnected issues*

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Abstract

This paper analyses the problem of aggregating judgments over multiple interconnected issues. Voters share a common preference for reaching true collective judgments, but hold private information about what the truth might be. Information conflicts may occur both between and within voters. Following Bozbay, Dietrich and Peters (2014), we assume strategic voting in a Bayesian voting game setting and we want to determine voting rules which induces an efficient Bayesian Nash equilibrium in truthful strategies, hence lead to collective judgments that efficiently incorporate all private information. Unlike in judgment aggregation problems with two independent issues where it is always possible to aggregate information efficiently, efficient information aggregation is not always possible with interconnected issues. We characterize the (rare) situations in which such rules exist, as well as the nature of these rules.

Keywords: judgment aggregation, private information, efficient information aggregation, strategic voting

JEL Classification Numbers: C70, D70, D71, D80, D82

1 Introduction

How should a group of individuals form a collective 'yes' or 'no' judgment on several issues given judgments of the group members? Judgment aggregation theory focuses on this question which has wide applications in many collective decision-making bodies, ranging from expert panels to juries, legislative committees to multi-member courts and so on. A typical example is the problem of the jury in a court trial, where the jury needs to form a collective 'yes' or 'no' judgment on whether the defendant

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has broken the contract and whether the contract is legally valid. The collective judgments on these issues typically determine a third issue, whether the defendant should be convicted. The issues for which a vote is taken can in principle be 'mutually interconnected' in such problems, i.e., a judgment made on one issue may restrict the judgment on the other. For instance, an expert panel might need to form 'yes' or 'no' judgments on the two issues of whether the CO_2 emissions are above a given threshold and whether there will be a critical temperature change, where a 'yes' judgment on the first issue requires a 'yes' judgment on the second issue; the United Nations security council might need to form judgments on whether a country will suffer a civil war, and whether it will suffer a severe socio-economic crisis, where the former implies the latter; the EU commission might need to form judgments on whether a candidate country implemented benchmark political criteria and whether the country's economy is unstable, where the negation of the former implies the latter and so on. As in the court trial example where the jury convicts the defendant or not, groups may take an action depending on the collective judgments, such as a large scale intervention in the UN example or providing additional funds to the candidate country in the EU commission example.

This paper takes a *truth-tracking* approach to judgment aggregation problems, where there are two interconnected issues in the agenda of a committee. We assume that there is an objective truth to be found, called the *state* (of the world). Individuals share a common preference for true (i.e., state-matching) collective judgments, but hold possibly conflicting private information about the state. We assume strategic voting, that is, rational behavior in the sense of a Bayesian Nash equilibrium of the corresponding game and we want to answer the following question: which voting rules (if any) lead to collective judgments which are efficient given all voters' private information? Such voting rules would give incentives for truthful voting, which cannot be taken for granted even when voters have no conflict of interest. That's because a voter can change the collective judgment on an issue only when she is pivotal, and being pivotal may eliminate the strategic incentive to vote truthfully. This observation is due to Austen-Smith and Banks (1996), where they study binary collective choice problems (which can be seen as a judgment aggregation problem with only one issue) with common interests and private information, and it gives rise to the analysis and design of voting rules which lead to truthful behavior of individuals as well as correct decisions.

We follow the approach of Bozbay, Dietrich and Peters (2014), who aim to determine which voting rules efficiently aggregate all private information in a similar setting when issues are independent. Given that interconnections are an important aspect of judgment aggregation theory and they are important in practice, this paper extends their work to agendas with interconnected issues.[1] It turns out that introdu-

[1]Interconnections are what make judgment aggregation non-trivial in the absence of strategic behavior, if a social-choice theoretic approach of aiming procedural fairness is taken rather than the truth-tracking approach. Indeed, if issues are independent, a separate yes/no vote can be taken for each issue and this never leads to inconsistent collective judgments. On the other hand, with private information and strategic behavior, the problem of designing a voting rule is non-trivial even if issues are independent (see Bozbay, Dietrich and Peters, 2014).

cing interconnections almost reverses the results. While with multiple independent issues and with a single issue, a voting rule which induces an efficient Bayesian Nash equilibrium in truthful strategies always exists, with interconnected issues such voting rule exists only under a strong condition relating the model parameters and preferences. This partial impossibility arises since information conflicts may occur within voters when we introduce interconnections. So, an individual might have private information conflicting the possible state of the world, although it is very unlikely in our model. When we focus on the specific kind of truth-tracking preferences, we see how strong the condition is. Under *simple preferences* where state-matching decisions are preferred to non-matching ones, it turns out that it is never the case that a voting rule supports truthful voting. Under *consequentialist preferences* where a voter prefers correct group actions to incorrect ones, the impossibility does not persist, although in almost all cases there is no rule which makes truthful voting an efficient equilibrium. Whenever it exists, this voting rule accepts both issues only when they are unanimously accepted. When issues are independent, quota rules, which decide each issue according to whether the proportion of 'yes' votes exceeds a particular threshold, can or should be used to guarantee efficient information aggregation. However, in case of two interconnected issues, there exists no such quota rule. Extending our analysis to many-issue case is not trivial. The number of possible types of interconnections and correspondingly, the number of ways to obtain a group action both grow rapidly with the number of issues and it is not clear how to obtain general results. Moreover, our impossibility results should persist with more than two issues as the problem gets much more complicated.

The paper proceeds as follows. In Section 2, we introduce our model. Section 3 contains a general existence result about efficient aggregation of information for any kind of common preferences, then focuses on the simple and consequentialist preferences, analyzing the implications of the general result. All proofs are given in the Appendix.

1.1 Related Literature

A judgment aggregation problem is formulated in its present form by List and Pettit (2002, 2004), while the origins of the problem go back to works by Kornhauser and Sager (1986, 1993) in the area jurisprudence, to Guilbaud (1966), Wilson (1975) and Rubinstein and Fishburn (1986). The judgment aggregation literature contains several possibility and impossibility results generalizing the observation that majority judgments can be logically inconsistent (a phenomenon which is referred to as the *discursive dilemma*, generalizing Condorcet's classical voting paradox) when the classical social-choice theoretic approach is taken (see Dietrich 2006, 2007, 2010, 2015; Nehring and Puppe 2008, 2010; Nehring, Pivato and Puppe 2014; Dietrich and List 2007a, 2008, 2013; Dokow and Holzman 2010a, 2010b; Dietrich and Mongin 2010, Cuddy and Piggins 2013; for an introductory overview of judgment aggregation theory, see List and Polak 2010). Dietrich and List (2007b) analyze strategic voting in judgment aggregation but in a setting where voters have private values instead of private information. See also related work by Nehring and Puppe (2002, 2007).

Bovens and Rabinowicz (2006), List (2005) and Pivato (2011) are few contributions taking the truth-tracking approach, where they apply the Condorcet Jury Theorem to judgment aggregation without considering private information and strategic incentives. See List and Pettit (2011) for a philosophical analysis of the truth-tracking approach. This approach is well-established in the literature on binary collective choice problems, which is about voting on one issue and started with the seminal works by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). In this framework, voters are truth-trackers (with some degree of heterogeneity in preferences in the latter work) but they hold private information about what the truth might be. They show that it need not be rational for all voters to vote truthfully and taking this into account, they analyze asymptotic efficiency. Their observation gives rise to their analysis of voting rules which make truthful voting occur in equilibrium. Several works extend their analysis in different directions: Duggan and Martinelli (2001) and Meirowitz (2002) consider continuous rather than binary private information; Feddersen and Pesendorfer (1998), Coughlan (2000) and Gerardi (2000) focus on the unanimity rule in a similar setting and analyze its ability to protect the innocent in jury trials; Austen-Smith and Feddersen (2005, 2006) introduce a pre-voting deliberation stage when there are biases across voters.

We now discuss papers which combine multiple issue voting with the truth-tracking approach. The introduction of multiple issues to strategic voting and information aggregation problems is by Ahn and Oliveros (2012, 2013). While the first paper considers private values rather than common, the second takes a truth-tracking approach and is more relevant. They compare the asymptotic efficiency of two different mechanisms: a joint trial where both issues are resolved by a majority vote among the group, and a severed trial where each issue is decided by a majority vote among a subgroup. They show that neither of these procedures is generally more efficient than the other one if the group is large enough. Bozbay, Dietrich and Peters (2014) aim to design voting rules that efficiently use all private information and lead to truthful voting. Our paper can be seen as an extension of their work to interconnected issues. They find that in most cases, a quota rule should be used to guarantee efficient information aggregation. In our case, on the other hand, there exists no quota rule which makes truthful voting efficient. De Clippel and Eliaz (2015) consider a setting where a group's optimal action (such as convicting or acquitting the defendant) depends on whether some premises are judged to be true by the group. They compare premise-based voting with conclusion-based voting. Under the former, a vote is taken on each issue, and the outcomes determine the group action. Under the latter, the group votes directly on which action to take, without forming a group view on the issues. They show that premise-based voting is more efficient than conclusion-based voting, but that the difference vanishes asymptotically as the group size increases.

2 The Model

2.1 The judgment aggregation problem

We consider a group of voters, labeled $i = 1, ..., n$, where $n \geq 2$. This group needs a collective judgment on whether some proposition p or its negation $\bar{p}$ is true, and whether some other proposition q or its negation $\bar{q}$ is true. While doing so, voters know that the combination $\{\bar{p}, \bar{q}\}$ is not possible. Hence, the interconnection between the two propositions is encoded by '$\bar{p} \rightarrow q$'.[2] The three possible judgment sets are $\{p, q\}, \{p, \bar{q}\}, \{\bar{p}, q\}$, abbreviated by pq, $p\bar{q}$ and $\bar{p}q$, respectively. Similarly, $\{\bar{p}, \bar{q}\}$ is abbreviated by $\bar{p}\bar{q}$. Each voter votes for a judgment set in $\mathcal{J} = \{pq, p\bar{q}, \bar{p}q\}$. A collective decision in $\mathcal{J}$ is taken using a voting rule, defined as a function $f : \mathcal{J}^n \rightarrow \mathcal{J}$, which maps each voting profile $\mathbf{v} = (v_1, ..., v_n)$ to a decision $d \equiv f(\mathbf{v})$.

2.2 Truth-tracking preferences

There is one 'correct' judgment set in $\mathcal{J}$, which we call the *state (of the world)* and denote by s. The state is unobservable by voters. Voters have identical preferences, represented by a common utility function $u : \mathcal{J} \times \mathcal{J} \rightarrow \mathbb{R}$ which maps any decision-state pair (d, s) to its utility $u(d, s)$. The notion of truth-tracking requires the utility to be high if the decision is correct, however, multiplicity of issues and the structure of the problem allow for different specifications. We consider two kinds of preferences, namely, *simple* and *consequentialist* preferences. Under simple preferences, voters want to find out the state-matching decision. The utility function is given by

$$u(d, s) = \begin{cases} 1 & \text{if } d = s \text{ (correct decision)} \\ 0 & \text{if } d \neq s \text{ (incorrect decision).} \end{cases} \tag{1}$$

To define consequentialist preferences, we assume that there are two possible consequences of voting, which represent group action. A *consequence function* Co maps the set $\mathcal{J}$ to a two-element set of possible consequences. Consider the example of the EU Commission, having to decide whether to supply additional funds to a candidate country. This depends on the collective judgments on two issues: p: 'the country has implemented its benchmark political criteria' and q: 'the country's economy is unstable', where the negation of the former implies the latter. If both issues are judged to be true, the consequence is to supply the funds, so $\text{Co}(pq) =$'supply'. If only one of the issues is judged to be true, then the commission does not see the country as a good candidate for additional funds since they are either unnecessary or not deserved; so $\text{Co}(p\bar{q}) = \text{Co}(\bar{p}q) =$'no supply'. This consequence function with the property $\text{Co}(pq) \neq \text{Co}(p\bar{q}) = \text{Co}(\bar{p}q)$ is the only interesting consequence function up to isomorphism. Consequence functions which lead all decisions to the same consequence are degenerate and uninteresting. If the consequence function depends only on the decision between p and $\bar{p}$ (as in, $\text{Co}(pq) = \text{Co}(p\bar{q}) \neq \text{Co}(\bar{p}q)$), or only on

[2] Any other type of interconnection between two issues (except bi-implication) is equivalent to this one up to isomorphism, so up to interchanging the roles of each proposition by its negation. Hence, studying this interconnection, we cover all possible interconnections between two propositions (except bi-implication, which is a trivial case).

the decision between q and $\bar{q}$ (as in, $\text{Co}(pq) = \text{Co}(\bar{p}q) \neq \text{Co}(p\bar{q})$), then the problem reduces to making a judgment on a single proposition-negation pair which has already been studied in the literature on binary collective choice with common interests. We define the consequentialist utility function as

$$u(d, s) = \begin{cases} 1 & \text{if } \text{Co}(d) = \text{Co}(s) \text{ (correct consequence)} \\ 0 & \text{if } \text{Co}(d) \neq \text{Co}(s) \text{ (incorrect consequence).} \end{cases} \quad (2)$$

Simple preferences may describe an environment where judgments on issues do not necessarily lead to a consequence or action, or where voters want to reach the right action through correct reasons. Note that incorrect decisions may lead to correct consequences. On the other hand, under consequentialist preferences, voters want to reach the correct consequence no matter whether the underlying premises are correct or not.

2.3 Private information and strategies

Each voter has a *type*, which is an element of $\mathcal{T} = \{pq, p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$ and is denoted by t generically. A voter's type represents evidence about whether p is true and whether q is true. For instance, the type $t = p\bar{q}$ represents evidence for p and for $\bar{q}$, and the type $t = \bar{p}\bar{q}$ represents evidence for $\bar{p}$ and for $\bar{q}$, which overall is conflicting information[3] since $\bar{p}\bar{q} \notin \mathcal{J}$. We write $\mathbf{t} = (t_1, ..., t_n) \in \mathcal{T}^n$ for a profile of voters' types.

Nature draws a state-types combination $(s, \mathbf{t})$ in $\mathcal{J} \times \mathcal{T}^n$ according to a probability measure denoted Pr. The prior probability of state $s \in \mathcal{J}$ is denoted

$$\pi_s = \Pr(s)$$

and is assumed to be in the interval $(0, 1)$.[4] If a proposition $r \in \{p, \bar{p}, q, \bar{q}\}$ represents (part of) voter i's type rather than (part of) the true state, we often write r_i for r. We write $\Pr(p_i|p)$ for the probability that voter i has evidence for p given that p is true. The probability of getting evidence for r given that r is true is denoted

$$a_r = \Pr(r_i|r)$$

and by assumption belongs to $(1/2, 1)$ and does not depend on the voter i.

We assume voters' types are independent given the state. Moreover, given the truth about p (i.e., either p or $\bar{p}$), a voter's evidence about p (i.e., either p_i or $\bar{p}_i$) is independent of the truth and the evidence about q; and similarly, given the truth

[3]The problem of conflicting private information may occur when voters draw their information from different sources (such as different experts or data sets) for each issue. Conflicting private information occurs only with low probability in the model.

[4]Note that there is some restriction on these prior probabilities due to the interconnection between two propositions. Let s_r denote the value of proposition $r \in \{p, q\}$ in the state s and π_r denote the probability of $r \in \{p, q\}$ being true. Then, $\Pr(s_p = \bar{p}|s_q = \bar{q}) = \Pr(s_q = \bar{q}|s_p = \bar{p}) = 0$, which imposes the following structure on the prior probabilities: $\pi_{pq} = \pi_p + \pi_q - 1$, $\pi_{p\bar{q}} = 1 - \pi_q$ and $\pi_{\bar{p}q} = 1 - \pi_p$. We can alternatively express our results in terms of prior probabilities of propositions rather than states.

about q, a voter's evidence about q is independent of the truth and the evidence about p. The joint distribution of the state and the types is then given by

$$\Pr(s, \mathbf{t}) = \Pr(s) \times \prod_{i=1}^{n} \Pr(t_i|s).$$

Each voter votes for a judgment in $\mathcal{J}$ based on her type. A *(voting) strategy* is a function $\sigma : \mathcal{T} \to \mathcal{J}$, mapping each type $t \in \mathcal{T}$ to the type's vote $v = \sigma(t)$. We write $\boldsymbol{\sigma} = (\sigma_1, ..., \sigma_n)$ for a profile of voters' strategies. We say that a strategy profile $\boldsymbol{\sigma}$ is *efficient* if for every type profile $\mathbf{t}$, the resulting decision $d = f(\sigma_1(t_1), ..., \sigma_n(t_n))$ is efficient, i.e., has maximal expected utility conditional on full information $\mathbf{t}$. We assume that a decision in $\mathcal{J}$ is efficient for some type profile. We exclude efficiency ties, i.e., those special parameter combinations such that some type profile leads to different efficient decisions (with different consequences when we assume consequentialist preferences). Hence, we exclude those instances where a voter is indifferent between two decisions except in the case that these decisions lead to the same consequence.

Our aim, besides efficiency, is to obtain truthful voting behavior in equilibrium. We mean informative voting by truthful behavior. We say that a strategy σ of a voter is *informative* if $\sigma(t) = t$ for all $t \in \mathcal{T} \setminus \{\bar{p}\bar{q}\}$ and $\sigma(\bar{p}\bar{q}) \in \{p\bar{q}, \bar{p}q\}$. Hence, a voter with informative strategy votes for her type if her type is non-conflicting, while she *partly* follows it when she has the conflicting evidence $t = \bar{p}\bar{q}$. Here, informativeness is open to behavior: one can choose between $p\bar{q}$ and $\bar{p}q$ under conflicting evidence. In a setting where information is never conflicting, following the evidence would be simply voting for the type. An informative voter in our setting follows the evidence as much as possible.[5]

3 A general (im)possibility

Given the setting described above, can we design voting rules which lead to efficient decisions as well as simple-minded, truthful voting behavior in equilibrium? For a rule to induce simple-minded, truthful behavior, informative voting should be rational in equilibrium, i.e., the profile of informative strategies must be a Nash equilibrium of the corresponding Bayesian game. Following a well-known result by McLennan (1998), for any voting rule, an efficient strategy profile is an equilibrium, and our objective is reduced to finding out when informative voting is efficient. Note that informative voting being efficient means that for any given type profile $\mathbf{t}$, *every* profile of corresponding informative strategies is efficient.

There always exist voting rules which make informative voting efficient when we consider multiple issue setting with no interconnections as Bozbay, Dietrich and Peters (2014) show. However, it turns out that this result does not persist when we introduce interconnections between the issues. Informative voting is efficient only under a strong condition over the model parameters.

[5] An alternative way to define informative strategy might be to impose voters with conflicting types to vote for the judgment set with the highest expected utility given the conflicting type, or to completely ignore the private information and vote for the judgment set with the highest prior probability. Our result for general preferences do not change under both definitions.

Condition 1: For any $\mathbf{t}, \mathbf{t}' \in \mathcal{T}^n$, if $\{i : t_i = pq\} = \{i : t'_i = pq\}$, then some decision $d \in \mathcal{J}$ is efficient for both $\mathbf{t}$ and $\mathbf{t}'$.

Theorem 1 *Consider an arbitrary common utility function $u : \mathcal{J}^2 \to \mathbb{R}$. There exists a voting rule for which informative voting is efficient if and only if Condition 1 holds.*

This condition is clearly strong: it requires that for all type profiles where the same group of voters have type pq, there must be a common efficient decision. We now narrow our focus to the specific kind of preferences to see further implications of this condition. We study simple and consequentialist preferences in turn in the following subsections, so that we can say more about the nature of voting rules making informative voting efficient whenever the condition is satisfied.

3.1 Simple preferences

We start by addressing simple preferences, defined by (1), where correct decisions are preferred to incorrect ones. By focusing on simple preferences, can we say more than the (partial) existential claim of Theorem 1? Here is the answer, which states the impossibility of efficient information aggregation under simple preferences.

Theorem 2 *Under simple preferences, there exists no voting rule for which informative voting is efficient.*

It turns out that Condition 1 never holds under simple preferences. This result is not surprising as we now illustrate. Consider two type profiles $\mathbf{t}$, $\mathbf{t}'$ where only voter 1 receives evidence for pq. Suppose further, all other voters in $\mathbf{t}$ have evidence for $p\bar{q}$ while all other voters in $\mathbf{t}'$ have evidence for $\bar{p}q$. Condition 1 requires that there is a common efficient decision for both type profiles, while this seems too demanding since the two type profiles are almost opposite to each other.

3.2 Consequentialist preferences

Does the impossibility we obtain for simple preferences persist under consequentialist preferences? We state our answer to this question after defining the following two coefficients.

$$A := \pi_{p\bar{q}} \left(\frac{1-a_{\bar{q}}}{a_q}\right)^n + \pi_{\bar{p}q} \left(\frac{1-a_{\bar{p}}}{a_p}\right)^{n-1} \frac{a_{\bar{p}}}{1-a_p},$$
$$B := \pi_{p\bar{q}} \left(\frac{1-a_{\bar{q}}}{a_q}\right)^{n-1} \frac{a_{\bar{q}}}{1-a_q} + \pi_{\bar{p}q} \left(\frac{1-a_{\bar{p}}}{a_p}\right)^n.$$

Theorem 3 *Under consequentialist preferences, there exists a voting rule for which informative voting is efficient if and only if the decision pq is efficient only for the unanimous type profile $\mathbf{t} = (pq, ..., pq)$ (which is the case if and only if $A, B > \pi_{pq}$).*

The theorem states that informative voting can be efficient under consequentialist preferences if pq is the efficient decision *only* when there is perfect evidence for pq. This is what Condition 1 reduces to under consequentialist preferences. To satisfy this condition, the prior probability of pq should be sufficiently low compared to prior probabilities of $p\bar{q}$ and $\bar{p}q$. For instance, if $\pi_{pq} = 0.2$, $\pi_{p\bar{q}} = \pi_{\bar{p}q} = 0.4$, $a_p = a_q = a_{\bar{p}} = a_{\bar{q}} = 0.6$ and $n = 3$, such a voting rule exists whereas no voting rule makes informative voting efficient if instead $\pi_{pq} = 0.3$ and $\pi_{p\bar{q}} = \pi_{\bar{p}q} = 0.35$. We present a simple characterization of voting rules which make informative voting efficient when the condition is satisfied.

Proposition 1 *Assume consequentialist preferences and Condition 1. A voting rule $f : \mathcal{J}^n \to \mathcal{J}$ makes informative voting efficient if and only if for every voting profile $\mathbf{v} \in \mathcal{J}^n$, the decision $f(\mathbf{v}) = pq$ if $\mathbf{v} = (pq, ..., pq)$ and $f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q\}$ otherwise.*

Proposition 1 describes a class of voting rules which accept pq only when there is *unanimous* agreement about both issues being true. Some of these rules characterized by Proposition 1 satisfy some natural properties which we define below:

- *Anonymity:* For all voting profiles $(v_1, ..., v_n) \in \mathcal{J}^n$ and all permutations $(i_1, ..., i_n)$ of the voters, $f(v_{i_1}, ..., v_{i_n}) = f(v_1, ..., v_n)$.

- *Monotonicity:* For all voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$, if for each r in $f(\mathbf{v})$ the voters who accept r in $\mathbf{v}$ also accept r in $\mathbf{v}'$, then $f(\mathbf{v}') = f(\mathbf{v})$. This property implies that increasing the support for the accepted proposition does not reverse the acceptance.

- *Neutrality:* For all voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$ for which there is no permutation $(i_1, ..., i_n)$ of the voters with $(v_{i_1}, ..., v_{i_n}) = (v'_1, ..., v'_n)$, if for every voter i the vote v_i contains p if and only if the vote v'_i contains q, then $f(\mathbf{v})$ contains p if and only if $f(\mathbf{v}')$ contains q. Informally, the two issues are treated symmetrically.

While some of the voting rules described in Proposition 1 are anonymous, monotonic and neutral, others fail to satisfy any of these properties. Once we impose anonymity, monotonicity and neutrality on the voting rules characterized in Proposition 1, we obtain the voting rule satisfying the following conditions as we state in our next proposition. For each $\mathbf{v} \in \mathcal{J}^n$,

$$f(\mathbf{v}) = pq \iff n_p^{\mathbf{v}} = n_q^{\mathbf{v}} = n \tag{3}$$

$$f(\mathbf{v}) = p\bar{q} \text{ if } n_p^{\mathbf{v}} > n_q^{\mathbf{v}} \tag{4}$$

$$f(\mathbf{v}) = \bar{p}q \text{ if } n_p^{\mathbf{v}} < n_q^{\mathbf{v}} \tag{5}$$

$$f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q\} \text{ if } n_p^{\mathbf{v}} = n_q^{\mathbf{v}} < n \tag{6}$$

Proposition 2 *Assume consequentialist preferences. A voting rule $f : \mathcal{J}^n \to \mathcal{J}$ which makes informative voting efficient is anonymous, monotonic and neutral if and only if it belongs to the class of rules defined by (3)-(6).*

Among the efficient aggregation possibilities, anonymity, monotonicity and neutrality can be attained. Note that the property of *independence*, which imposes that the decision on each proposition $r \in \{p, q\}$ only depends on the votes on r, is not satisfied by any rule defined in Proposition 2. This rules out *quota rules*[6] which are very common in the literature and which, given the optimal acceptance thresholds, almost always lead to efficient information aggregation when issues are independent or when there is a single issue. Following Proposition 1, we have the following corollary which points out the contrast between our results and those of independent issues.

Corollary 1 *There exists no quota rule $f : \mathcal{J}^n \to \mathcal{J}$ making informative voting efficient.*

This corollary applies to both kind of preferences as we already have a general impossibility in the case of simple preferences. The possibility of efficient information aggregation under consequentialist preferences rely mostly on the indifference of voters between the two judgment sets $p\bar{q}$ and $\bar{p}q$. The judgment sets $p\bar{q}$ and $\bar{p}q$ both lead to the same consequence, so the same utility for each consequentialist voter. Consider a type profile $\mathbf{t}$ with $t_i = p\bar{q}$ for all $i \in \{1, ..., n\}$ and a type profile $\mathbf{t}'$ with $t'_i = \bar{p}q$ for all $i \in \{1, ..., n\}$. Condition 1, the existence condition for efficient information aggregation, would require that the efficient decision for both $\mathbf{t}$ and $\mathbf{t}'$ is the same. This is of course not possible under simple preferences, while it follows under consequentialist preferences.

In the context of consequentialist preferences, our results raise the question of whether aggregating judgments on the consequence (or *conclusion*) rather than the underlying issues (or *premises*) would suffice for efficient information aggregation purposes. When issues are independent, it leads to valuable information loss to focus only on the consequence since an efficient mechanism uses all available information. See De Clippel and Eliaz 2015, for comparison of conclusion-based and premise-based procedures in their ability to efficiently aggregate information. However, our results show that the efficient mechanism – whenever it exists – does not distinguish between $p\bar{q}$ and $\bar{p}q$ which lead to the same consequence. A conclusion-based procedure where a yes/no vote is taken only on the consequence would equally be efficient if a 'yes' judgment is made if and only if all voters vote 'yes' under consequentialist preferences in our framework. This implies another stark difference between implications of interconnected and independent issues. We want to conclude with the following remark motivated by our results.

Remark 1 *Informative voting is not efficient almost in all cases with an agenda with mutually interconnected propositions.*

[6]Formally, a quota rule is given by two thresholds $m_p, m_q \in \{0, 1, ..., n+1\}$ with $m_p + m_q \leq n+1$, and for each voting profile it accepts p [q] if and only if at least m_p [m_q] voters accept it in the profile. The additional requirement of $m_p + m_q \leq n + 1$ is for leaving out $\bar{p}\bar{q}$ from possible outcomes. This requirement follows from Theorem 2(c) in Dietrich and List (2007c). Quota rules are monotonic, anonymous and *independent*, but not necessarily neutral. Whenever the acceptance thresholds for propositions are equal, they turn out to be neutral.

A Appendix: proofs

We introduce some preliminary derivations before we prove our results. The probability of the three states in $\mathcal{J}$ conditional on the *full* information $\mathbf{t} \in \mathcal{J}^n$ is given as follows, where k is the number of types that contain p in $\mathbf{t}$ and l is the number of types that contain q in $\mathbf{t}$.

$$\Pr(pq|\mathbf{t}) = \frac{\pi_{pq} a_p^k (1-a_p)^{n-k} a_q^l (1-a_q)^{n-l}}{\Pr(\mathbf{t})} \tag{7}$$

$$\Pr(p\bar{q}|\mathbf{t}) = \frac{\pi_{p\bar{q}} a_p^k (1-a_p)^{n-k} (1-a_{\bar{q}})^l a_{\bar{q}}^{n-l}}{\Pr(\mathbf{t})} \tag{8}$$

$$\Pr(\bar{p}q|\mathbf{t}) = \frac{\pi_{\bar{p}q} (1-a_{\bar{p}})^k a_{\bar{p}}^{n-k} a_q^l (1-a_q)^{n-l}}{\Pr(\mathbf{t})}. \tag{9}$$

Note that our assumption that $\bar{p}\bar{q}$ cannot be the state of the world imposes the following structure on the prior probabilities: $\pi_{pq} = \pi_p + \pi_q - 1$, $\pi_{p\bar{q}} = 1 - \pi_q$ and $\pi_{\bar{p}q} = 1 - \pi_p$. We use prior probabilities of states instead of prior probabilities of propositions in our derivations for simplicity of notation.

We are now ready to prove our results.

Proof of Theorem 1. To start with, we introduce some notation. Given a voting profile $\mathbf{v}$, let $\Theta(\mathbf{v})$ denote the set of all type profiles which possibly lead to $\mathbf{v}$ under informative voting. Given a type profile $\mathbf{t}$, let $\Omega(\mathbf{t})$ denote the set of all voting profiles which possibly result from $\mathbf{t}$ under informative voting. Consider a voting rule $f : \mathcal{J}^n \to \mathcal{J}$.

(1) First, let Condition 1 hold. Suppose there is an exogenously given ordering of judgment sets, and let f be the following voting rule: for all $\mathbf{v} \in \mathcal{J}^n$, $f(\mathbf{v}) = d \iff d$ is the highest ordered decision among all decisions which are efficient for some $\mathbf{t} \in \Theta(\mathbf{v})$. Consider any type profile $\hat{\mathbf{t}} \in \mathcal{T}^n$ and suppose informative voting. We want to show that $(*)$ for each $\mathbf{v} \in \Omega(\hat{\mathbf{t}})$, $f(\mathbf{v})$ is efficient for $\hat{\mathbf{t}}$. Let $\mathbf{v} \in \Omega(\hat{\mathbf{t}})$. One can show that all type profiles in $\Theta(\mathbf{v})$ share the same subvector restricted to pq. Since Condition 1 holds, there is some decision d which is efficient for all $\mathbf{t} \in \Theta(\mathbf{v})$, including $\hat{\mathbf{t}}$. It follows from Condition 1 that if any other decision $d' \neq d$ is efficient for *some* $\mathbf{t} \in \Theta(\mathbf{v})$, it is efficient for all $\mathbf{t} \in \Theta(\mathbf{v})$. Then, $(*)$ holds.

(2) Conversely, let f make informative voting efficient. Let $\mathbf{t}, \mathbf{t}'$ be two type profiles in $\mathcal{T}^n$ with $\{i : t_i = pq\} = \{i : t_i' = pq\}$. One has to show that $(**)$ there is $d \in \mathcal{J}$ which is efficient for both $\mathbf{t}, \mathbf{t}'$. By construction, for each $\mathbf{v} \in \Omega(\mathbf{t})$, $\mathbf{t}' \in \Theta(\mathbf{v})$; and similarly, for each $\mathbf{v}' \in \Omega(\mathbf{t}')$, $\mathbf{t} \in \Theta(\mathbf{v}')$. Then, $f(\mathbf{v})$ must be efficient for $\mathbf{t}'$ (as well as $\mathbf{t}$) and $f(\mathbf{v}')$ must be efficient for $\mathbf{t}$ (as well as $\mathbf{t}'$) since informative voting is efficient. So, $(**)$ holds. □

Proof of Theorem 2. By Theorem 1, it is sufficient to show that Condition 1 never holds under simple preferences. Suppose for a contradiction, it holds. Consider the two type profiles $\mathbf{t} = (p\bar{q}, ..., p\bar{q})$ and $\mathbf{t}' = (\bar{p}q, ..., \bar{p}q)$. Since $\{i : t_i = pq\} = \{i : t_i' = pq\}$ and Condition 1 holds, there is a decision which is efficient for both profiles.

Then, $p\bar{q}$ must be efficient for $\mathbf{t}$ since otherwise $p\bar{q}$ would not be efficient for any type profile which contradicts to non-degeneracy assumption. Similarly, $\bar{p}q$ must be efficient for $\mathbf{t}'$. Hence, $p\bar{q}$ and $\bar{p}q$ are both efficient given $\mathbf{t}$ or $\mathbf{t}'$, which contradicts to no-efficiency ties assumption. □

Proof of Theorem 3. Let the condition in Theorem 3 (that pq is only efficient for the unanimous type profile $\mathbf{t} = (pq, ..., pq)$) be called Condition 2.

(1) We first prove that Condition 2 implies that there is a voting rule for which informative voting is efficient and $A, B > \pi_{pq}$. Assume Condition 2 holds. This implies that Condition 1 holds. By Theorem 1, there is a voting rule which makes informative voting efficient. Let $\mathbf{t}, \mathbf{t}'$ be type profiles with one $p\bar{q}$ and one $\bar{p}q$ respectively while each of the rest of the types is pq. Without loss of generality, let $\mathbf{t} = (pq, ..., \bar{p}q)$ and $\mathbf{t}' = (pq, ..., p\bar{q})$. By Condition 1, $p\bar{q}, \bar{p}q$ are both efficient for each of the type profiles. Using (7) and (9), we can write the following:

$$E(u(p\bar{q}, S)|\mathbf{t}) > E(u(pq, S)|\mathbf{t}) \tag{10}$$

$$\Leftrightarrow \pi_{p\bar{q}} a_p^{n-1}(1 - a_p)(1 - a_{\bar{q}})^n + \pi_{\bar{p}q}(1 - a_{\bar{p}})^{n-1} a_{\bar{p}} a_q^n > \pi_{pq} a_p^{n-1}(1 - a_p) a_q^n \tag{11}$$

$$\Leftrightarrow \pi_{p\bar{q}} \left(\frac{1 - a_{\bar{q}}}{a_q}\right)^n + \pi_{\bar{p}q} \left(\frac{1 - a_{\bar{p}}}{a_p}\right)^{n-1} \left(\frac{a_{\bar{p}}}{1 - a_p}\right) > \pi_{pq}. \tag{12}$$

Similarly,

$$E(u(p\bar{q}, S)|\mathbf{t}') > E(u(pq, S)|\mathbf{t}') \tag{13}$$

$$\Leftrightarrow \pi_{p\bar{q}} a_p^n (1 - a_{\bar{q}})^{n-1} a_{\bar{q}} + \pi_{\bar{p}q}(1 - a_{\bar{p}})^n a_q^{n-1}(1 - a_q) > \pi_{pq} a_p^n a_q^{n-1}(1 - a_q) \tag{14}$$

$$\Leftrightarrow \pi_{p\bar{q}} \left(\frac{1 - a_{\bar{q}}}{a_q}\right)^{n-1} \left(\frac{a_{\bar{q}}}{1 - a_q}\right) + \pi_{\bar{p}q} \left(\frac{1 - a_{\bar{p}}}{a_p}\right)^n > \pi_{pq}. \tag{15}$$

So, $A, B > \pi_{pq}$.

(2) We now prove that if there is a voting rule for which informative voting is efficient, Condition 2 holds. Consider a voting rule $f : \mathcal{J}^n \to \mathcal{J}$ and suppose f makes informative voting efficient. By Theorem 1, Condition 1 holds. Given a type profile $\mathbf{t} \in \mathcal{T}^n$, let $\Gamma(\mathbf{t})$ denote the set of type profiles which have the same subvector on pq as in $\mathbf{t}$. Let $n_r^{\mathbf{t}}$ denote the number of occurrences for a proposition r in a type profile $\mathbf{t}$. Now, take a type profile $\hat{\mathbf{t}} \in \mathcal{T}^n$ with k times pq where $1 \le k < n$. The proof proceeds in several steps.

Claim 1: There is a type profile $\mathbf{t} \in \Gamma(\hat{\mathbf{t}})$ with $n_p^{\mathbf{t}} = k$ and $n_q^{\mathbf{t}} = k$.

Any type profile with k times pq and $n - k$ times $\bar{p}\bar{q}$ satisfies this condition and one of these type profiles is obviously in $\Gamma(\hat{\mathbf{t}})$. Now, take $\tilde{\mathbf{t}} \in \mathcal{T}^n$ with $k - 1$ times pq.

Claim 2: There is a type profile $\mathbf{t} \in \Gamma(\tilde{\mathbf{t}})$ with $n_p^{\mathbf{t}} = k$ and $n_q^{\mathbf{t}} = k$.

One can easily see there is always a type profile with the exact same pq structure as $\tilde{\mathbf{t}}$ and with only one occurrence of $p\bar{q}$ and only one occurrence of $\bar{p}q$.

Claim 3: Under consequentialist preferences, for all $\mathbf{t}, \mathbf{t}' \in \mathcal{T}^n$ with $n_p^{\mathbf{t}} = n_p^{\mathbf{t}'}$ and $n_q^{\mathbf{t}} = n_q^{\mathbf{t}'}$, $E(u(d, S)|\mathbf{t}) = E(u(d, S)|\mathbf{t}')$ for each $d \in \mathcal{J}$.

The claim follows from the expressions (7)-(9). By Condition 1, there is a decision $d \in \mathcal{J}$ which is efficient for all $\mathbf{t} \in \Gamma(\hat{\mathbf{t}})$. Similarly, there is a decision $d \in \mathcal{J}$ which is efficient for all $\mathbf{t} \in \Gamma(\tilde{\mathbf{t}})$. Combining Claim 1, 2 and 3, one obtains that the same decision $d \in \mathcal{J}$ is efficient for all $\mathbf{t} \in \Gamma(\hat{\mathbf{t}})$ and all $\mathbf{t} \in \Gamma(\tilde{\mathbf{t}})$. Since this is true for all k with $1 \leq k < n$, there is a decision d which is efficient for all $\mathbf{t} \in \mathcal{T}^n \setminus \{(pq, ..., pq)\}$. By non-degeneracy assumption, pq is efficient for $\mathbf{t} = (pq, ..., pq)$. Hence, d must be in $\{p\bar{q}, \bar{p}q\}$ since otherwise pq would be efficient for all type profiles which contradicts to non-degeneracy assumption. Hence, Condition 2 holds.

(3) We finally prove that $A, B > \pi_{pq}$ implies Condition 2. Let $A, B > \pi_{pq}$. We first prove the following claim.

Claim 4: The expected utility of pq given a type profile $\mathbf{t}$ is an increasing function of $n_p^{\mathbf{t}}$ and $n_q^{\mathbf{t}}$.

The claim follows from the definition of the utility function and from $\Pr(S = pq|\mathbf{t})$ being an increasing function of $n_p^{\mathbf{t}}$ and $n_q^{\mathbf{t}}$. Let $\mathbf{t}, \mathbf{t}' \in \mathcal{T}^n$ be type profiles with one $p\bar{q}$ and one $\bar{p}q$ respectively while each of the rest of the types is pq. Without loss of generality, let $\mathbf{t} = (pq, ..., \bar{p}q)$ and $\mathbf{t}' = (pq, ..., p\bar{q})$. By (7) and (9), one has $E(u(p\bar{q}, S)|\mathbf{t}) > E(u(pq, S)|\mathbf{t})$ and $E(u(p\bar{q}, S)|\mathbf{t}') > E(u(pq, S)|\mathbf{t}')$. By Claim 4, it follows that $E(u(p\bar{q}, S)|\mathbf{t}) = E(u(\bar{p}q, S)|\mathbf{t}) > E(u(pq, S)|\mathbf{t})$ for all $\mathbf{t} \in \mathcal{T}^n \setminus \{(pq, ..., pq)\}$ which means $p\bar{q}, \bar{p}q$ are efficient for each $\mathbf{t} \in \mathcal{T}^n \setminus \{(pq, ..., pq)\}$. Thus, Condition 2 holds. □

Proof of Proposition 1. Consider a voting rule $f : \mathcal{J}^n \rightarrow \mathcal{J}$. Proof of the 'if' part is obvious and left to the reader. To show converse, let f make informative voting efficient. Note that Condition 1 reduces to decision pq being only efficient for the unanimous type profile $\mathbf{t} = (pq, ..., pq)$ under consequentialist preferences by Theorem 3. Then, for all voting profiles obtained by informative voting from any $\mathbf{t} \in \mathcal{T}^n \setminus \{(pq, ..., pq)\}$, $f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q\}$. By non-degeneracy assumption, pq is efficient for $\mathbf{t} = (pq, ..., pq)$. By f making informative voting efficient, $f(\mathbf{v}) = pq$ if $\mathbf{v} = (pq, ..., pq)$. □

Proof of Proposition 2. Consider a voting rule $f : \mathcal{J}^n \rightarrow \mathcal{J}$. Let $n_r^{\mathbf{v}}$ denote the number of occurrences of $r \in \{p, q\}$ in a voting profile $\mathbf{v}$.

(1) First, let f be defined by (3)-(6). Clearly, f is anonymous. It follows from Proposition 1 that informative voting is efficient with f since for all $\mathbf{v} \in \mathcal{J}^n$, $f(\mathbf{v}) = pq$ if and only if $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} = n$; so, if and only if $\mathbf{v} = (pq, ..., pq)$. To show monotonicity of f, take two voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$ such that for all $r \in f(\mathbf{v})$, the voters who vote for r in $\mathbf{v}$ also vote for r in $\mathbf{v}'$.

Case 1: $f(\mathbf{v}) = pq$. Then $\mathbf{v} = (pq, ..., pq)$. By definition, $\mathbf{v}' = \mathbf{v}$ and $f(\mathbf{v}') = pq$.

Case 2: $f(\mathbf{v}) = p\bar{q}$. The definition of f implies either $n_p^{\mathbf{v}} > n_q^{\mathbf{v}}$ or $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} < n$; and the definition of $\mathbf{v}'$ implies $n_p^{\mathbf{v}'} \geq n_p^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} \leq n_q^{\mathbf{v}}$. Suppose $n_p^{\mathbf{v}} > n_q^{\mathbf{v}}$. Then, $n_p^{\mathbf{v}'} > n_q^{\mathbf{v}'}$ and $f(\mathbf{v}') = p\bar{q}$. Next, suppose $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} < n$. If $\mathbf{v}' \neq \mathbf{v}$, one has $n_p^{\mathbf{v}'} > n_p^{\mathbf{v}}$ or $n_q^{\mathbf{v}'} < n_q^{\mathbf{v}}$ which means $n_p^{\mathbf{v}'} > n_q^{\mathbf{v}'}$ and $f(\mathbf{v}') = p\bar{q}$.

Case 3: $f(\mathbf{v}) = \bar{p}q$. One can show that $f(\mathbf{v}') = \bar{p}q$ analogously to Case 2.

It remains to show neutrality of f. Take two voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$ such that $\mathbf{v}_r = \mathbf{v}'_{r'}$ for every distinct $r, r' \in \{p, q\}$ and there is no permutation of voters $(i_1, ..., i_n)$ with $(v_{i_1}, ..., v_{i_n}) = (v'_1, ..., v'_n)$. We have to show that (*) f accepts r in $\mathbf{v}$ if and only if f accepts r' in $\mathbf{v}'$. We distinguish 3 cases:

Case 1: $f(\mathbf{v}) = pq$. It is clear that $\mathbf{v}' = \mathbf{v}$, and $f(\mathbf{v}') = pq$.

Case 2: $f(\mathbf{v}) = p\bar{q}$. By definition of f, either $n_p^{\mathbf{v}} > n_q^{\mathbf{v}}$ or $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} < n$. One can see that the latter is not possible since then one could find a permutation of voters $(i_1, ..., i_n)$ with $(v_{i_1}, ..., v_{i_n}) = (v'_1, ..., v'_n)$. Suppose $n_p^{\mathbf{v}} > n_q^{\mathbf{v}}$. By definition of $\mathbf{v}'$, whenever p (q) is accepted in $\mathbf{v}$, q (p) is accepted in $\mathbf{v}'$. This means $n_p^{\mathbf{v}'} < n_q^{\mathbf{v}'}$ and $f(\mathbf{v}') = \bar{p}q$. So, f accepts p in $\mathbf{v}$ and q in $\mathbf{v}'$, and it accepts $\bar{q}$ in $\mathbf{v}$ and $\bar{p}$ in $\mathbf{v}'$. Hence, (*) holds.

Case 3: $f(\mathbf{v}) = \bar{p}q$. One can show that $f(\mathbf{v}') = \bar{p}q$ analogously to Case 2.

(2) Conversely, let f be anonymous, monotonic and neutral, and make informative voting efficient. We have to show that $(*)$ f is defined by (3)-(6). By Proposition 1 and informative voting being efficient, $f(\mathbf{v}) = pq$ if and only if $\mathbf{v} = (pq, ..., pq)$, equivalently $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} = n$. Now, take a voting profile $\mathbf{v} \in \mathcal{J}^n \setminus \{(pq, ..., pq)\}$.

Case 1: $n_p^{\mathbf{v}} > n_q^{\mathbf{v}}$. Suppose for a contradiction, $f(\mathbf{v}) = \bar{p}q$. Let $\mathbf{v}'$ be a voting profile with $n_p^{\mathbf{v}'} = n_q^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} = n_p^{\mathbf{v}}$. We start by proving the following claim.

Claim: For each combination of $k, l \in \{0, ..., n\}$, there is only one voting profile $\mathbf{v} \in \mathcal{J}^n$ with $n_p^{\mathbf{v}} = k$ and $n_p^{\mathbf{v}} = l$ up to the permutations of votes.

The claim follows from the fact that all votes containing $\bar{p}$ are $\bar{p}q$, and similarly, all votes containing $\bar{q}$ are $p\bar{q}$. Hence, subtracting number of p (q) occurrences in a profile from n gives the exact number of $\bar{p}q$ $(p\bar{q})$ votes. Then, there is only one voting profile with $n_p^{\mathbf{v}}$ times q and $n_q^{\mathbf{v}}$ times p up to permutations of votes. Hence, by neutrality and anonymity, $f(\mathbf{v}') = p\bar{q}$. However, by monotonicity of f, $f(\mathbf{v}') = \bar{p}q$ since $n_p^{\mathbf{v}'} \leq n_p^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} \geq n_q^{\mathbf{v}}$, a contradiction. Then, $f(\mathbf{v}) = p\bar{q}$.

Case 2: $n_p^{\mathbf{v}} < n_q^{\mathbf{v}}$. One can show that $f(\mathbf{v}) = \bar{p}q$ analogously to Case 1.

Case 3: $n_p^{\mathbf{v}} = n_q^{\mathbf{v}} < n$. By Proposition 1 and informative voting being efficient, $f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q\}$.

So, $(*)$ is true. □

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