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THE STATE OF DSGE MODELLING

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The State of DSGE Modelling

Revised Version*

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Abstract

This survey and assessment of the state of DSGE modelling is structured around six key criticisms levelled at the approach. The first is fundamental and common to macroeconomics and microeconomics alike – namely, problems with rationality and Expected Utility Maximization (EUM). The second is that DSGE models examine fluctuations about an exogenous balanced growth path and there is no role for endogenous growth. The third consists of a number of concerns associated with estimation. The fourth is another fundamental problem with any micro-founded macro-model – that of heterogeneity and aggregation. The fifth and sixth concerns focus on the rudimentary nature of earlier models that lacked unemployment and a banking sector.

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Contents

1 Introduction

There have been a number of recent assessments of the "state of macro" and the contribution of DSGE models - a list that is by no means exhaustive would include: Pesaran and Smith (2011), Blanchard (2009), Blanchard *et al.* (2010), Driffill (2011), Pesaran and Smith (2011), Blanchard *et al.* (2013), Blanchard (2016), Blanchard and Summers (2017), Vines and Wils (2018), Christiano *et al.* (2018) and Galí (2018).

This survey is structured around a number of criticisms regarding DSGE models and ways in which the community of DSGE macro-modellers are responding. We begin in Section 2 by providing a brief description of what has become known as DSGE modelling and how it emerged from earlier macro-economic frameworks. Six main areas of general criticism of the DSGE modelling approach are highlighted in this Section.

A technical Section 3 then sets out a widely used and referenced New Keynesian model developed by Smets and Wouters (2003) and Smets and Wouters (2007) that draws heavily on Christiano *et al.* (2005). Sections 4–9 proceed to the six criticisms, describing the response of the DSGE modelling community to them. Section 10 summarizes a sharp alternative agent based approach to the DSGE macroeconomic modelling that is motivated by these concerns, but provides very distinct solutions. Section 11 discusses how DSGE models can be used for macro-economic policy design and Section 12 concludes.

2 What Are DSGE Models?

DSGE or Dynamic Stochastic General Equilibrium macroeconomic models have exactly these three ingredient: they are micro-founded, modelling forward-looking economic agents (households, firms, banks, governments) making individually rational decisions over a time horizon, so they are dynamic; the economy features uncertainty in the form of exogenous random shocks, so they are stochastic; they are equilibrium models in the Nash sense that all agents are maximizing some measure of their inter-temporal welfare over time, given their environment of other maximizing agent. However they can feature non-market clearing prices and wages so they can be disequilibrium models in the Walrasian sense. We return to this issue later in Section 8.

The construction of a DSGE model requires the specification of agents' preferences, the economy's technological constraints and the set of exogenous shocks to which the economy is subjected. There is also an implicit or explicit assumption regarding the formation of expectations and the information available regarding future macroeconomic variable relevant to the agents. Until recently most DSGE models have assumed rational, model consistent expectations with information assumptions made clear as we proceed.

The agents decision rules are derived from the first order conditions of the dynamic optimization problem for each agent. Aggregating over agents and (usually) assuming that markets clear allow us to derive a system of non-linear stochastic difference equations, involving the endogenous variables, the parameters and a set of shocks. The purpose is then to find a stable and unique solution to the model, which requires an additional set of procedures.

Models are written in stationary form with variables written as deviations from a balanced-growth steady-state. Often DSGE models are log-linearized about this steady state and written in state-space form. Then standard computational methods (or analytical ones for small models) result in a linear rational expectations solution of the model. Non-linear models can be solved using a second or higher order approximation in the vicinity of the steady state. Global solution methods that avoid small-deviation approximation are also employed.

Parameter values are chosen using a combination of off-model estimation, calibration and systems estimation. Calibration can take the form of 'reverse-engineering' the steady state to pin down produce parameter values that will produce observed variables such as the "great ratios" (consumption and investment GDP ratios). Similarly calibration can pin down parameters defining shock processes by reverse-engineering second moments of data. Actual estimation of DSGE models use a systems approach, usually Bayesian, but also Generalized Method of Moments (GMM).

The road of DSGE modelling is full of twists and turns, but the different directions taken seem to have converged to what is still, to a large extent, the consensual synthesis. The models that are now the mainstay for policy analysis and forecasting depart significantly from previous approaches in that they strike a balance between internal consistency, empirical adherence and adequacy for policy analysis. By contrast, in the 1960s-70s macroeconomic models were mostly based on equation-by-equation estimation of what were essentially Keynesian reduced form behavioural equations, without explicit expectations. Large models were then constructed using these behavioural relationships as building blocks, alongside identities defining aggregate demand, trade balances and the government budget constraint.

The introduction of first adaptive and then rational expectations (RE) led to what proved to be a fatal blow for this generation of models – the Lucas Critique (Lucas (1976), seconded by Sims (1980) and Sargent (1981)). In the context of forward-looking agents with rational expectations, this critique showed that apparently stable empirical backward-looking relationship between, for example, consumption, post-tax income and real consumption, were not independent of the policy rule in place. The implication of this finding is that these (apparently structural) models were at best suitable for forecasting on the basis of a continuation of an existing policy and were, therefore, unfit for the purpose of examining the consequences of different policies.

There are a number of key criticisms levelled at DSGE models. The first is fundamental and common to macroeconomics and microeconomics alike – namely, problems with rationality and Expected Utility Maximization (EUM). The second is that DSGE models examine fluctuations about an exogenous balanced growth path and there is no role for endogenous growth. The third consists of a number of concerns associated with estimation. The fourth is another fundamental problem with any micro-founded macro-model – that of heterogeneity and aggregation. The fifth and sixth concerns focus on the rudimentary nature of earlier models that lacked unemployment and a banking sector.

We consider these six concerns in turn in subsequent sections 4–9, but first we examine what has become the gold standard for DSGE modelling developed by Smets and Wouters (2003) and Smets and Wouters (2007) which we refer to as the Smets-Wouters medium-sized New Keynesian model (SW).

3 The Smets-Wouters Model

In the SW model there are four sets of representative agents: households, final goods producers, trade unions and intermediate goods producers. The later two produce differentiated labour services and goods respectively and, in each period of time, consist of a group that is locked into an existing contract and another group that can re-optimize.[1]

3.1 Households

At time $t = 0$, household i maximizes its expected lifetime utility

$$\begin{aligned}\Omega(i) &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t(i), C_{t-1}, H_t^s(i)) \\ &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{[C_t(i) - \chi C_{t-1}]^{1-\sigma_c}}{1-\sigma_c} \exp\left[(\sigma_c - 1) \frac{(H_t^s(i))^{1+\psi}}{1+\psi} \right] \right] \end{aligned} \quad (1)$$

where $\mathbb{E}_t[\cdot]$ denotes rational expectations based on information available at time t, $C_t(i)$ is real consumption , $H_t^s(i)$ is hours supplied, β is the discount factor, χ controls external habit formation where C_{t-1} is aggregate consumption taken as given by household j, σ_c is the inverse of the elasticity of inter-temporal substitution (for constant labour), and ψ is the inverse of the Frisch labour supply elasticity. Preferences chosen by SW in (1) are compatible with balanced growth (see King *et al.* (1988)).[2]

The household's budget constraint in period t is given by

$$C_t(i) + I_t(i) + \frac{B_t(i)}{RPS_t R_{n,t} P_t} + T_t = \frac{B_{t-1}(i)}{P_t} + r_t^K K_{t-1}(i) + W_{h,t} H_t^s(i) + \Gamma_t \quad (2)$$

where I_t is investment into physical capital, B_t is government bonds held at the end of period t, $R_{n,t-1}$ is the nominal interest rate paid on government bonds held at the beginning of period t, RPS_t is an exogenous premium in the return on bonds that follows an AR1 process. T_t is lump-sum taxes, r_t^K is the real rental rate, $W_{h,t}$ is the real wage rate at which households supply labour that is homogeneous at this point to trade unions and Γ_t is the profit of intermediate firms distributed to households. Notice that we deviate from the original SW model and do not allow for variable capital utilization in the model.

[1]Our model is a slightly slimmed down version of Smets and Wouters (2007) in several respects which are pointed out as we proceed.

[2]An alternative functional form for utility found in the literature from Jaimovich and Rebello (2008) controls the wealth effect and encompasses at two extremes KPR preferences with strong wealth effects and Greenwood *et al.* (1988) preferences that removes the effect. A further alternative are recursive preferences, as in Epstein and Zin (1989), which they allow for a distinction between the parameter governing risk aversion, and the inter-temporal elasticity of substitution. In KPR preferences in (1) these are both equal to $\frac{1}{\sigma_c}$.

End of period capital stock, $K_t(i)$, accumulates according to

$$K_t(i) = (1-\delta)K_{t-1}(i) + (1 - S(X_t(i)))I_t(i)IS_t \tag{3}$$

where IS_t is an investment specific technological shock that follows an AR1 process, $X_t(i) = I_t(i)/I_{t-1}(i)$ is the growth rate of investment, and $S(\cdot)$ is an adjustment cost function such that $S(X) = 0$, $S'(X) = 0$, and $S''(\cdot) = 0$ where X is the steady state value of investment growth. For $S(X_t)$ in a symmetric equilibrium we choose the functional form: $S(X_t) = \phi_X(X_t - \bar{X}_t)^2$ where $\bar{X}_t$ is the balanced-growth steady-state trend.

The solution to the household's problem imply the Euler Consumption equation, an arbitrage condition and a first order condition equating the marginal rate of substitution between leisure and consumption with the real wage:

$$\begin{aligned} \mathbb{E}_t[\Lambda_{t,t+1}(i)R_{t+1}] &= 1 && (4)\\ \mathbb{E}_t[\Lambda_{t,t+1}(i)R^K_{t+1}] &= 1 && (5)\\ -\frac{U_{C,t}(i)}{U_{H,t}(i)} &= W_{h,t} && (6) \end{aligned}$$

where $\Lambda_{t,t+1}(i) \equiv \beta\frac{U_{C,t+1}(i)}{U_{C,t}(i)}$ is the stochastic discount factor, $U_{C,t}(i) \equiv \frac{\partial U_t(i)}{\partial C_t(i)}$, $U_{H,t}(i) \equiv \frac{\partial U_t(i)}{\partial H_t(i)}$ are marginal utilities; $R_t = \frac{R_{n,t-1}}{\Pi_t}$ and $R^K_t = \frac{[r^K_t + (1-\delta)Q_t]}{Q_{t-1}}$ are the real gross returns on government bonds and physical capital respectively and Q_t is the price of capital (Tobin's Q).

3.2 The Labour Market

Households supply their homogeneous labour to trade unions that differentiate the labour services. A labour packer buys the differentiated labour from the trade unions and aggregate them into a composite labour using the Dixit-Stigliz aggregator[3] for aggregate labour supply

$$H_t = \left(\int_0^1 H_t(j)^{(\zeta_w-1)/\zeta_w}dj\right)^{\zeta_w/(\zeta_w-1)} \tag{7}$$

where ζ_w is the elasticity of substitution among different types of labour, and we index trade unions by j. The labour packer minimizes the cost $\int_0^1 W_{n,t}(j)H_t(j)dj$ of producing

[3] See Dixit and Stiglitz (1977).Smets and Wouters (2007) generalize the aggregator to a Kimball form as in Kimball (1995) which introduces a variable mark-up even in the absence of wage stickiness.

the composite labour service, where $W_{n,t}(j)$ denotes the nominal wage set by union j. This leads to the standard demand function

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}} \right)^{-\zeta_w} H_t^d \tag{8}$$

where $W_{n,t}$ is the aggregate nominal wage given by the Dixit-Stigliz aggregator

$$W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\zeta_w} dj \right]^{\frac{1}{1-\zeta_w}} \tag{9}$$

Sticky wages are introduced through Calvo contracts supplemented with indexation. At each period there is a probability $1 - \xi_w$ that trade union j can choose $W_{n,t}^O(j)$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - W_{h,t+k} \right] \tag{10}$$

subject to the demand function (8), where $\gamma_w \in [0, 1]$ is a wage indexation parameter.[4]

The solution to the above problem is the first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - \frac{\text{MRSS}_t}{(1 - 1/\zeta_w)} W_{h,t+k} \right] = 0 \tag{11}$$

where we have introduced a mark-up shock MRSS_t to the marginal rate of substitution that follows an AR1 process.

3.3 Final Good Producers

The representative final good producer uses a continuum of intermediate goods $Y_t(m)$ to produce a homogeneous final good given by the Dixit-Stiglitz output aggregator

$$Y_t = \left(\int_0^1 Y_t(m)^{(\zeta_p-1)/\zeta_p} dm \right)^{\zeta_p/(\zeta_p-1)} \tag{12}$$

Final goods producers are perfectly competitive and choose output to maximize profits $P_t Y_t - \int_0^1 P_t(m) Y_t(m) dm$, where $P_t(m)$ is the price of intermediate good m and P_t is the

[4] A variant of the SW model is provided by Galí (2015a) that introduces insider-outsider labor markets and hysteresis into the trade-union's objective (10), resulting in a model that better accounts for the high persistence of European unemployment.

aggregate price index given by the Dixit-Stiglitz price aggregator analogous to (9)

$$P_t = \left[\int_0^1 P_t(m)^{1-\zeta_p} dm\right]^{\frac{1}{1-\zeta_p}} \tag{13}$$

This implies the standard demand function

$$Y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} Y_t. \tag{14}$$

3.4 Intermediate Good Producers

Intermediate good m is produced using the technology

$$Y_t(m) = (A_t H_t^d(m))^{\alpha} K_{t-1}(m)^{1-\alpha} \tag{15}$$

where $H_t^d(m)$ and $K_t(m)$ are labour and capital demand, respectively. Labour productivity is decomposed into a cyclical component and a deterministic trend $A_t = \bar{A}_t A_t^c$, where the cyclical component follows an AR1 process.[5]

Intermediate producer m minimize the cost $W_t H_t^d(m) + r_t^K K_{t-1}(m)$, of producing a given level of output given by (15) where W_t and r_t^K are the real wage and return on capital, respectively. The Lagrangian for this optimization is $W_t H_t^d(m) + r_t^K K_{t-1}(m) + MC_t(Y_t - (A_t H_t^d(m))^{\alpha} K_{t-1}(m)^{1-\alpha}$ given Y_t, noting that the Lagrange multiplier is the real marginal cost MC_t. This implies that at the optimum

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{\alpha} (r_t^K)^{1-\alpha} / A_t^{\alpha} \tag{16}$$

which is the same for all intermediate producers.

Analogous to price-setting and again following Calvo (1983) we assume that each period the price of retail good m is set optimally to $P_t^0(m)$ with probability $1 - \xi_p$. If the price is not re-optimized, then prices are indexed to last period's aggregate inflation, with indexation parameter γ_p. Each retail producer m chooses $P_t^0(m)$ to maximize discounted

[5] Smets and Wouters (2003) introduce fixed costs and variable capacity utilization into the production function (15).

profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^0(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - MC_{t+k} \right] \tag{17}$$

where $\gamma_p \in [0,1]$ is a price indexation parameter, subject to demand (14) given the stochastic discount factor $\Lambda_{t,t+k}$ of the households.

The solution to the above problem is the first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^0(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - \frac{1}{(1-1/\zeta_p)} MC_{t+k} \mathrm{MCS}_{t+k} \right] = 0 \tag{18}$$

where we have introduced a mark-up shock MCS_t to the real marginal cost that follows an AR1 process.

Price inflation is defined by $\Pi_t^p \equiv \frac{P_t}{P_{t-1}}$ and wage inflation by $\Pi_t^w \equiv \frac{W_{n,t}}{W_{n,t-1}}$. To accommodate indexing in price and wage setting, we define $\tilde{\Pi}_t^p(\gamma) \equiv \frac{\Pi_t^p}{(\Pi_{t-1}^p)^\gamma}$. Then price and wage dynamics are succinctly given by

$$\begin{aligned} 1 &= \xi \tilde{\Pi}_t^p(\gamma_p)^{\zeta_p - 1} + (1-\xi) \left(\frac{P_t^O}{P_t} \right)^{1-\zeta_p} && (19) \\ 1 &= \xi_w \left(\frac{\Pi_t^w \tilde{\Pi}_t^p(\gamma_w)}{\Pi_t^p} \right)^{\zeta_w - 1} + (1-\xi_w) \left(\frac{W_t^O(j)}{W_{n,t}} \right)^{1-\zeta_w} && (20) \end{aligned}$$

Given (19) and (20), the dynamic behaviour of the real wage $W_t \equiv \frac{W_{n,t}}{P_t}$ is now also determined.

3.5 Output and Labour Market Clearing

The output and labour market clearing conditions must take into account relative price dispersion across varieties and wage dispersion across firms. Integrating across all firms, taking into account that the capital-labour ratio is common across firms and that the wholesale sector is separated from the retail sector we obtain aggregate demand for intermediate (wholesale) goods necessary to produce final retail goods as

$$Y_t = (C_t + I_t + G_t) \int_0^1 \left(\frac{P_t(m)}{P_t} \right)^{-\zeta_p} dm \tag{21}$$

where labour market clearing gives total demand for labour, H_t^d, as

$$H_t = \int_0^1 H_t(j)dj = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\zeta_w} dj\, H_t^d = \Delta_t^w H_t^d \tag{22}$$

where the price dispersion is given by $\Delta_t^p = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\zeta_p} df$ and wage dispersion is given by $\Delta_t^w = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\zeta_w} dj$. Then we have

$$\Delta_t^p = \xi_p + \tilde{\Pi}_t^{\zeta_p} \Delta_{t-1}^p + (1-\xi_p)\left(\frac{P_t^O}{P_t}\right)^{-\zeta_p} \tag{23}$$

$$\Delta_t^w = \xi_w \tilde{\Pi}_{w,t}^{\zeta_w} \Delta_{t-1}^w + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\zeta_w} \tag{24}$$

3.6 The Government

A monetary policy rule for the nominal interest rate is given by the Taylor-type rule

$$\begin{aligned}\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &\quad + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \mathrm{MPS}_t,\end{aligned} \tag{25}$$

where Π_t is inflation, and MPS_t is a monetary policy shock that follows an AR1 process.

The government budget constraint is

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_{n,t}} \tag{26}$$

where G_t is government spending that follows an AR1 process.

3.7 Under-employment

All households are employed so there us no unemployment in the model. However the trade-union setting of the wage paid by producers results in under-employment given by $H_t^s - H_t^d$. The rate of under-employment is then given by

$$u_t = \frac{H_t^s - H_t^d}{H_t^s} \tag{27}$$

3.8 Aggregation

In a representative agent equilibrium with identical households, final producers, trade-unions able to re-optimize wage contracts, those who are unable to do so, price-setting intermediate firms able to re-optimize price contracts and those who are unable to do so, this then completes the model and determines the dynamics of aggregate consumption, investment, Tobin's Q, the real wage, capital stock and under-employment; gross real returns on bonds and capital; price and wage inflation given the nominal interest rate rule and seven exogenous AR1 shock processes RPS_t, G_t, IS_t, A^c_t, MRSS_t, MCS_t and MPS_t.

3.9 Impulse Responses

Table 1 shows the choice of parameter values in the simulations that follow. These are broadly in line with the priors that would be chosen in the Bayesian estimation of the model discussed in Section 6.

parameter	Symbol	Value
Discount factor	β	0.99
Depreciation rate	δ	0.025
Risk aversion	σ_c	1.5
Inflation adjustment costs	ϕ_X	2.0
Government Spending	$g_y = \frac{G}{Y}$	0.2
Labour share	α	0.67
Frisch Parameter	ψ	2.0
Price substitution elasticity	ζ_p	7.00
Wage substitution elasticity	ζ_w	3.00
Price Calvo contract	ξ_p	0.75
Wage Calvo contract	ξ_w	0.75
Price indexing	γ_p	0.5
Wage indexing	γ_w	0.5
Taylor Rule persistence	ρ_r	0.7
Taylor Rule inflation	θ_π	1.5
Taylor Rule output	θ_y	0.3
Taylor Rule output growth	θ_{dy}	0.3
Persistence of shocks	ρ_i	0.7
Standard deviation of shocks (%)	$\sigma_i = \mathrm{sd}(\epsilon_i)$	1.00

Table 1: Parameter Values

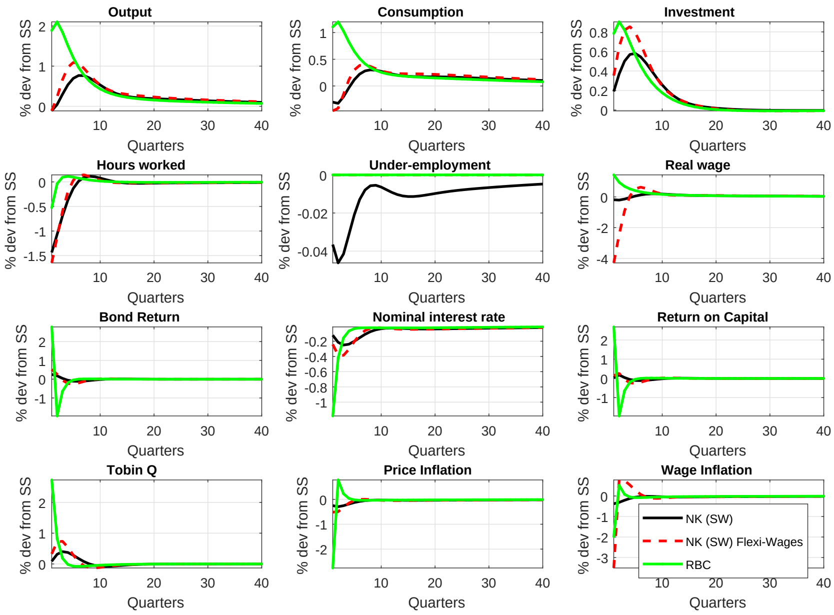

Figure 1: **RBC-NK SW Model Comparison: IRFs to a Labour Productivity Shock to A_t^c**

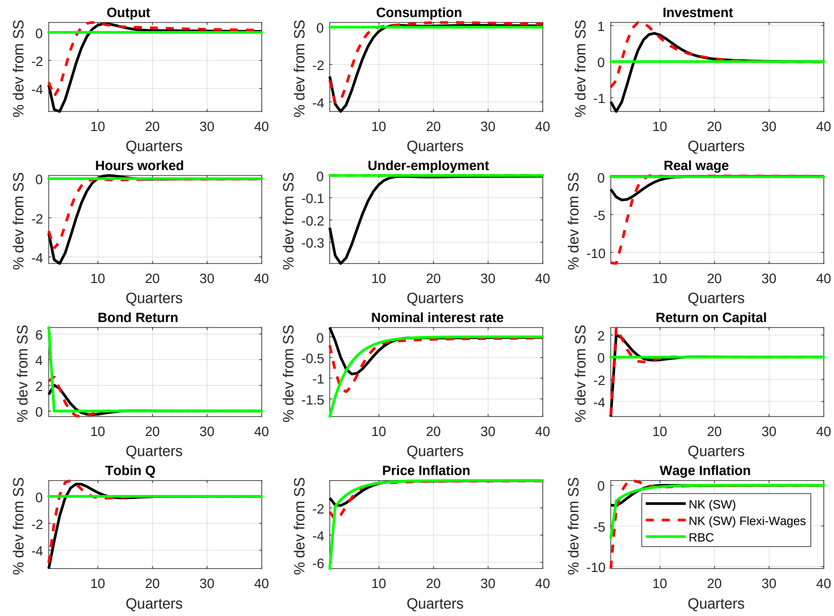

Figure 2: **RBC-NK SW Model Comparison: IRFs to a Monetary Policy Shock to $\epsilon_{MPS,t}$**

Figures 1 and 2 show the impulse response functions for a technology (A_t^c) and monetary policy shock (MPS_t) respectively.[6] In order to highlight the Keynesian frictions we compare three models: first the full SW model set of above with both price and wage stickiness (NK-SW). Second, we make wages flexible by setting $\xi_w = 0$ and removing the wage mark-up by letting $\mu \to \infty$, (model NK-Flexi-Wages). Finally, we also make prices flexible by setting $\xi_p = 0$ and removing the price mark-up by letting $\zeta \to \infty$. This third case is then the standard RBC model and therefore we refer to it as model RBC.

First consider the impulse responses to a technology shock. All three models then see a rise in output, investment, Tobin's Q and an immediate fall in both price and wage inflation. The two Keynesian price and wage frictions in models NK-SW and NK-Flexi-Wages have a dampening effect on this rise in output, investment, Tobin's Q with model NK-Flexi-Wages lying between RBC and NK. The fall in the nominal interest rate is in accordance with the interest rate rule (25).

Interesting differences emerge in the labour market and the response of consumption: hours worked (H_t^d in the models) falls in all three cases which follows from the strong wealth in the choice of utility function noted in footnote 2. The negative wealth effect on the supply of hours of households H_t^s is offset by the rise in demand H_t^d on the supply side, but this is dampened in the models NK-SW and NK-Flexi-Wages. In fact with investment adjustment costs the tightening of monetary policy sees the real marginal cost fall and the wage-hours demand curve actually shifts to the left resulting in a drop in the real wage. Hence hours actually worked, H_t^d, fall by less in the RBC model.[7] In the Keynesian models, H_t^s falls, H_t^d rises and so under-employment $u_t = (H_t^s - H_t^d)/H_t^s$ falls. But in the RBC model the labour market clears, $H_t^s = H_t^d$ and so $u_t = 0$.

Now consider a monetary policy shock to the shock process MPS_t in the interest rate rule (25). The impulse responses demonstrate the most basic difference between the RBC and the two Keynesian models: Keynesian price and wage frictions create a real effect of monetary policy. For the RBC model only nominal variables, the nominal interest rate, price and wage inflation, and the bond return in the first period, a surprise effect, are affected and fall with a tightening of monetary policy. By contrast in models NW-SW and NK-Flexi-Wage we see a drop in output, consumption, investment, hours worked, the

[6] AR1 processes in log-linear form are $x_{it} = \rho_i x_{it-1} + \epsilon_i$.

[7] This relates to the hours-technology debate - see Galí and Rabanal (2005) and Whelan (2009).

real wage and Tobin's Q. The more Keynesian is the model (wage and price-stickiness rather than just price-stickiness) the greater is this effect on output, consumption and investment.

This completes our review of the SW model. The six feature to emphasize for the rest the Chapter are: rational expectations and, in an environment of uncertainty, expected utility maximization by households and firms; preferences consistent with deterministic exogenous balanced growth path about which the model is solved; a model waiting to be estimated by a systems method; a four-agent model (households, final goods producers, two sets of intermediate goods produces one able to re-optimize prices but the other locked into existing contract) each modelled as representative; underemployment, but no unemployment; and finally a arbitrage condition for the return on capital and bonds that rules out financial frictions. We now turn to an examination of how the current generation of DSGE models have developed in these six dimensions.

4 Rationality

The assumption of rationality in general and that of rational expectations in macro-models in particular has naturally generated a lively debate in economics and the social sciences. We consider these two aspects in turn.

4.1 Rationality in General

The assumption of perfect rationality has come under scrutiny since the 1950s when Herbert A. Simon claimed that agents are not realistically so rational so as to aspire to pay-off maximization. Instead he proposed 'bounded rationality' as a more realistic alternative to the assumption of rationality, incorporating players' inductive reasoning processes. This is the route that the Agent-Based models take (see, Section 10). Certainly, experimental studies of decision-making show human behaviour to be regularly inconsistent and contradictory to the assumption of perfect rationality. However the question that arises is whether economic agents can *learn* to be rational, so rationality may well be a reasonable empirical postulate to describe behaviour near a long-run steady state. We return to this question in Section 4.2 below.

Models can only be beaten by alternative models. A model of irrationality has to pin down why one decision is preferred to another and here we observe that analytically tractable theories of the inconsistency and irrationality in human behaviour simply have not yet been fully developed. Hence our best analytical models are based on the rationality assumption as we unfortunately have nothing superior on offer. However we can be more positive than that at least when it comes to competitive behaviour. Darwinian selection helps rational (that is, profit-maximizing) firms (profit-maximizing) to succeed in competition.

Perhaps the most convincing argument for adopting the rationality assumption is provided by Myerson (1999). If we view the aim of social sciences is not only to predict human behaviour in the abstract, but also, crucially, to analyze social institutions and assess proposals for their reform, it is useful to evaluate these institutions under the assumption of perfect rationality. In this way, we can solve for flaws as either defects in the institutional structure (and thereby institutional reform is the required solution) or as flaws in the rationality of the agents (which begs for improved education and/or provision of information for individuals). Accordingly this has become a logical and useful assumption for economists in order to see with more clarity when social problems must be solved by institutional reform. This argument can be refined to illustrate why this individual perfection assumption should be one of intelligent rational maximization, as in the models of non-cooperative game theory. Thus an argument for reform of social institutions (rather than for re-education of individuals) is most persuasive when it is based on a model which assumes that individuals intelligently understand their environment and rationally act to maximize their own welfare.

Even if we accept utility maximization, in an uncertain environment there still is an issue of whether it should be *expected* utility maximization (EUM). An alternative supported by experiments is *Prospect Theory* pioneered by Kahneman and Tversky (1979) and Kahneman and Tversky (1992). Prospect theory takes into account the empirical finding of experiments that people behave as if extremely improbable events are impossible and extremely probable events are certain (see Shiller (1999) and Barberis (2013)). Prospect theory can explain phenomena such as the equity premium puzzle. Woodford (2012) shows that reference-dependent choice of the kind captured by prospect theory may be understood as an efficient approach to rational choice with limited information processing capacity as in the 'rational inattention' literature discussed below in Section 4.2. However it is extremely difficult in incorporate into general equilibrium modelling; in the words of Shiller "EUM can be a workhorse for some sensible research".

4.2 Rational Expectations in Macroeconomics

Staying broadly within the rational expectations paradigm a number of refinements are on offer that assume that agents are not able to perfectly observe states that define the economy. Levine *et al.* (2012a) propose a general framework for introducing information limitations at the point agents form expectations. The 'rational inattention' literature (Sims (2005), Luo and Young (2009), Luo (2008)) and the 'sticky information' approach of Reis (2009) also fits into this agenda. The basic idea is that agents can process information subject to a constraint that places an upper bound on the information flow. Borrowing from information theory (which in turn borrows from statistical physics) the idea is formalized by an upper bound on the decrease in entropy that ensues as agents proceed from a prior to a posterior of a signal.

A more radical deviation from rational expectations is provided by the statistical rational learning literature pioneered by Evans and Honkapohja (2001a). This introduces a specific form of bounded rationality in which utility-maximizing agents make forecasts in each period based on standard econometric techniques such as least squares. In many cases this learning behaviour converges to a rational expectations equilibrium and much of this literature studies the conditions for this to happen.

In this genre of models there then exists a choice of learning model: *Euler* versus the *anticipated utility* approach (following Kreps (1998)) – henceforth EL and AU. In both approaches agents cannot form model-consistent expectations. Under EL agents forecast their own one-period ahead decisions whereas under AU agents form beliefs over the future infinite time horizon of aggregate states and prices which are exogenous to their decisions. The two approaches then differ with respect to what agents learn about – their own future one-period ahead decision for EL and variables exogenous to the agents for AU.

AU, also known the "infinite time-horizon" framework, is closely related to the "internal rationality" (IR) approach of Adam and Marcet (2011). Under both IR and AU agents maximize utility, given their constraints and a consistent set of probability beliefs about payoff-relevant variables that are *external*. Then with IR, beliefs take the form of a well-defined probability measure over a stochastic process (the 'fully Bayesian' plan). See Eusepi and Preston (2011) for an RBC BR model with AU, Preston (2005), Woodford (2013) and Hommes *et al.* (2019) who adopt a behavioural NK framework and Branch and McGough (2018) for a recent discussion of EL versus AU. Cogley and Sargent (2008) compares the IR vs AU and find that AU can closely approximate the fully Bayesian optimization. There are other agent-level learning alternatives such as shadow value and finite-horizon learning. See Branch *et al.* (2013), Woodford (2018) and Evans and McGough (2018) for recent reviews.

A considerable advance in the literature on bounded rationality came in Brock and Hommes (1997), which embeds a simple heterogeneous expectations mechanism into a cobweb model of partial equilibrium. As in the standard cobweb model, firms have to forecast the equilibrium price before they set their output level. To do so, they have the choice of using a simple adaptive expectations predictor at zero cost, or perfect foresight at positive cost. The authors argue that firms will choose predictors that result in higher net profits, where the probability of choosing a given predictor is determined by a logit model. This choice is justified by an appeal to the discrete choice model described in Manski and McFadden (1981), which is widely used in microeconomics and econometrics. A similar approach is used in the reinforcement learning literature described in Young (2004).

The insights of Brock and Hommes (1997) were slowly incorporated into the New Keynesian literature in the early 2000s. Branch and McGough (2004) studied the impact of heterogeneous expectations on the existence of sunspot equilibria in rational expectations models, and Branch and Evans (2007) examined discrete choice dynamics of the form considered in Brock and Hommes (1997) in the context of a simple macroeconomic model. Heterogeneous expectations in a New Keynesian framework, albeit without discrete choice, were then examined in considerable detail in Branch and McGough (2009).

Heterogeneous expectations and discrete choice were fully incorporated into the New Keynesian three equation model around the time that the USA and Europe were recovering from the effects of the 2008 financial crisis, in the papers of Branch and McGough (2010) and De Grauwe (2011). The basic framework has become known as the behavioural New Keynesian model, largely the result of De Grauwe's book length treatment of the subject, (De Grauwe, 2012). Via the expectational mechanism of Brock and Hommes (1997), the behavioural New Keynesian (BNK) approach incorporates bounded rationality and heterogeneity into the standard New Keynesian (NK) three equation model. This embeds an intuitive form of complexity into the standard approach, where strategy switching generates recurring bouts of instability. Specifically, households and firms have the choice between two (or more) predictors of output and inflation. Both predictors can be simple non-rational rules as in De Grauwe (2011), or one can be rational as in Hommes *et al.* (2019). Around the steady state of a BNK model, both predictors are equally accurate, but one predictor becomes increasingly accurate relative to the other as the economy moves away from the steady state. Any exogenous shock that moves the economy away from the steady state can then lead to agents rapidly switching from one predictor to the other. This creates an endogenous amplification effect which can explain the existence of excess kurtosis and stochastic volatility observed in macroeconomic data. A full review of the behavioural New Keynesian model literature is provided by Calvert Jump and Levine (2019).

5 Integrating Endogenous Growth and Business Cycles

Turning to our second limitation – the lack of a role for endogenous growth. As Lucas (1987) pointed out the welfare gains from eliminating business cycle fluctuations in the standard RBC model are very small and are dwarfed by the gains from increased growth. It is true that adding New Keynesian frictions significantly increases the gains from stabilization policy, but they still remain small compared with those from increased growth.

5.1 DSGE Models with Endogenous Long-term Growth

Recently a number of papers have introduced long-run growth into DSGE models. Wang and Wen (2011) and Annicchiarico *et al.* (2011) do so within the simple AK approach. This literature establishes the existence of a relationship between growth and volatility with the important policy implications that monetary rules designed to stabilize short-run fluctuations affects the long-run balanced-growth path of the economy.

5.2 DSGE Models with Endogenous Technological Change

Closer to existing DSGE models is a literature that models of R&D led endogenous productivity about a balanced-growth path that remains exogenous, but new intermediate goods arrive exogenously. Leading examples of this are Comin and Gertler (2006), Comin (2009), Holden (2011), Comin *et al.* (2014), and Comin *et al.* (2016). We focus on the most recent of these cited papers as it suggests a general method of incorporating endogenous technical change and medium-term cycles into a range of DSGE models.

Comin *et al.* (2016) then develops and estimates a macroeconomic model modelled to allow for endogenous technology via R&D and adoption. The endogenous productivity mechanism is based on Comin and Gertler (2006), which uses the approach to connect business cycles to growth. The Comin/Gertler work, in turn, is a variant of the Romer (1986) expanding variety model of technological change, modelled to include an endogenous pace of technology adoption.

At the heart of the model is an aggregate production function for the final good which is the product of a productivity term that reflects endogenous variation and one reflecting exogenous variation. In modelling the former the paper focuses on two types of productivity enhancing investments: (i) the creation of new technologies through research and development (R&D) and (ii) the diffusion of new technologies via adoption expenditures. Thus the model allows for both endogenous and endogenous movements in total factor productivity and the empirical strategy is to let the data determine the importance of each. R&D led changes in productivity can encompass the basic NK model with only the exogenous form and a likelihood race can assess the empirical relevance of each form.

6 Empirical Concerns

Our third limitation centres on the empirical dimension. Although Bayesian Maximum-Likelihood estimation is a giant step forward from the calibration methods of earlier RBC models there are a number of concerns. Many of the empirical issues are discussed in the reviews Fernandez-Villaverde (2009), Fernandez-Villaverde *et al.* (2016) and the monograph Herbst and Schorfheide (2015). Concerns are associated with identification, choice of priors, pre-filtering of data, the relationship between VARS and the solution of DSGE models and non-linearities including those associated with occasionally binding constraints. We consider these in turn.

6.1 Identification

Any likelihood-based system estimation faces a potential identification problem originating from the complexity of mapping from many parameters to data and a resulting flat likelihood function. Techniques for testing for weak and strong non-identification issues is active area of research; see Canova and Sala (2009), Iskrev (2008), Ratto (2008) and Koop *et al.* (2013) for example, research that is feeding into toolboxes available in Dynare.

Bayesian rather than classical maximum likelihood estimation does address this problem, if we had confidence in the priors. This brings us to the choice of priors.

6.2 Priors

In many cases, the justification for the choice of priors reflects more the prior that some previous eminent researcher has got her priors right, often for a different model specification anyway. Del Negro and Schorfheide (2008) proposes an easily implementable method to obtain prior distributions for DSGE model parameters from data for moments of observable variables.

They divide the parameters into three groups, which reflect the information used to construct the prior. The first group contains the parameters that determine the steady states. The second group includes the utility, technology, and policy parameters governing the DSGE models endogenous propagation mechanism. For many of these parameters prior information comes from unrelated data sets, e.g. the prior for the labor supply elasticity parameter comes from micro-level studies on labor supply, the one for the price stickiness parameters from studies on price changes, etc. The third group consists of parameters describing the propagation mechanism of exogenous shocks. They propose a method of “endogenous priors” that translates priors about reasonable magnitudes for second moments of observables into a joint prior distribution for these parameters. Such priors may come from pre-sample evidence, for instance, and are assumed to be invariant across different DSGE model specifications.

6.3 Pre-filtering of Data

Another critique of Bayesian estimation is the method of pre-filtering the data.DSGE models are usually estimated with a two-step approach: data are first filtered and then structural parameters are estimated. This means that the choice of the statistical filter might be arbitrary and can affect the structural estimation. An alternative is to implement a hybrid one-step framework which links the observables to the model counterparts via a flexible specification which does not require that the cyclical component is solely located at business cycle frequencies and allows the non-cyclical component to take various time series patterns, see Filippo (2011), Canova (2013) and Cantore *et al.* (2015)

6.4 The Relationship between VARS and DSGE Models

Fernandez-Villaverde *et al.* (2007) show that for case where the number of shocks equals the number of observations then under quite weak 'invertibility' conditions the solution to the DSGE model can be approximated by a finite VAR. Levine *et al.* (2019) shows that these conditions become stronger in DSGE models where under rational expectations agents have imperfect information of the model's state variables. Given that many DSGE models have more shocks than observations (including measurement errors ensures this) this brings into question a common practice of validating a DSGE model by comparing its impulse response functions with those of a data VAR, still more the estimation of such models carried out by matching these two sets of outcomes. Meenagh *et al.* (2018) provides a survey of this 'indirect inference' approach to estimating DSGE models.

Del Negro and Schorfheide (2004) and Del Negro *et al.* (2007) propose a method of either validating the model performance or improving the fit using a combination of an unrestricted VAR and the VAR implied by the estimated DSGE model. The DSGE-VAR approach then uses the DSGE model itself to construct a prior distribution for the VAR coefficients so that DSGE-VAR estimates are tilted toward DSGE model restriction, thus identifying the shocks for the IRFs.

This method constructs the DSGE prior by generating dummy observations from the DSGE model, and adding them to the actual data and leads to an estimation of the VAR based on a mixed sample of artificial and actual observations. The ratio of dummy over actual observations (called the hyper-parameter λ) controls the variance and therefore the weight of the DSGE prior relative to the sample. For extreme values of this parameter (0 or ∞) either an unrestricted VAR or the DSGE is estimated. If λ is small the prior is diffuse. When $\lambda = \infty$, we obtain a VAR approximation of the log-linearized DSGE model. As λ becomes small the cross-equation restrictions implied by the DSGE model are gradually relaxed. Alternatively, one can simply find the 'optimal' value of λ by estimating this parameter as one of the deep parameters (see, for more details,). The optimal λ then represents how much the DSGE model is able to explain the real data.Details on the algorithm used to implement this DSGE-VAR are to be found in Del Negro and Schorfheide (2004), Del Negro *et al.* (2007) and Adjemian *et al.* (2008).

6.5 Non-Linearities

The aftermath of the Great Recession has seen the proliferation of DSGE models endowed with various forms of non-linearities. DSGE models are now used to explore numerous topics such as the effects of the nominal interest rate zero lower bound, occasionally binding constraints, uncertainty shocks, higher volatility, big or asymmetric shocks, risk-premia and behavioral models with highly non-linear learning mechanisms. In a linear world, DSGE model solutions can be represented by linear state-space systems of equations with Gaussian random variables. Since, in general, linear combinations of Gaussian variables are still Gaussian, it is possible to use the Kalman filter compute the likelihood function, and, thus, to make inference by means of Bayesian estimation methods. By contrast, higher-approximations result in non-linear state-space representations. Therefore, it is no longer possible to map the path of state variables over time, and consequently standard estimation techniques cannot be used for estimating the deep parameters of the model.

The literature offers two different approaches to make inference on the parameters of a non-linear DSGE model depending on whether or not filtering techniques are used to approximate the possible evolution of the state variables. Among the non-filtering-based estimators, the GMM method was adapted in various ways to match the moments of non-linear models. However with this method the econometrician needs to make an arbitrary choice about which moments should be assigned higher priority. By contrast filtering techniques are needed to approximate the likelihood of being in a certain state given the observables, and ultimately use Bayesian inference in a non-linear context. There are two approaches to approximate the posterior density of the model, global and local approaches.

Global approaches, such as the particle filter (PF), can produce unbiased estimates of the likelihood. Fernandez-Villaverde and Rubio-Ramirez (2004), Fernandez-Villaverde and Rubio-Ramirez (2015) have adapted the Metropolis-Hasting algorithm to estimate non-linear models using the PF. However the computational burden is very high and grows exponentially grows with high dimensional state-space models - the curse of dimensionality problem.

Although relying on restrictive assumptions on the distribution of latent states, local methods enable linear filtering techniques to be applied on non-linear models; this reduces the computational efforts and guarantees higher precision relative to the PF with few particles. The cubature Kalman filter - Arasaratnam and Haykin (2009)- assumes the posterior distribution of the states to take a priori a Gaussian form, which results in a gain in efficiency because the non-linear functions needs to be evaluated in a limited number of efficiently selected nodes.

In Kollmann (2013) an estimation method is adopted that avoids the need for a filter altogether by restricting the model to have the number of observed variables (used for estimation) to be equal to the number of exogenous shocks in the DSGE model. Exogenous innovations are extracted recursively by inverting the observation equation, which allows easy computation of the sample likelihood.

Kollmann (2017) then considers the general case where there are more shocks than observables (inevitable if observations are made with measurement errors). He uses a second-order perturbation approximation and the pruning scheme of Kim *et al.* (2008) under which second-order terms are replaced by products of the linearized solution. Unless the pruning algorithm is used, second-order approximated models often generate exploding simulated time paths. Pruning is therefore crucial for applied work based on second-order (or higher) approximated models. Kollmann assumes that the pruned second-order approximated model is the true data generating process (DGP). The method exploits the fact that the pruned system is linear in a state vector that consists of variables solved to second- and first-order accuracy, and of products of first-order accurate variables. Having set up the model in a linear state-space form standard, Bayesian estimation can then be used.

Meyer-Gohde (2014) in what he calls 'risky linear approximations' reconciles the linear framework with risk by constructing approximations of the policy functions of DSGE models that are linear in states but that account for risk in the points and slopes used to construct the linear approximation. Then two different approximations are constructed, one around the stochastic steady state and one around the ergodic mean. The method is sufficiently efficient to be used in estimation. Due to the linearity in states and under the assumption of normally distributed shocks, the Kalman filter is operational for the risky linear approximation. The paper finds that the risky linear approximation using the Kalman filter is as equally successful as standard perturbation PF estimation, both with the state space and nonlinear moving average policy function representations, in identifying parameters outside the reach of standard linear approximations.

Arioli (2018) assesses the performance these local approximation filtering techniques in estimating non-linear DSGE models. The main focus consists in evaluating the accuracy and efficiency of these filtering techniques through a Monte-Carlo study on a small New-Keynesian model with a trend in inflation.

7 Heterogeneous Agents and Aggregation

Finally we turn to what is perhaps the most important challenge for DSGE macroeconomics –that of heterogeneous agents and aggregation. The first generation of DSGE models, the RBC models stayed within the representative agent paradigm. The next wave of New Keynesian models made only a small deviation from this framework by assuming consumers have access to complete markets. Although they may differ in their initial tastes and are subject to staggered wage contracts and idiosyncratic shocks, they still face a common budget constraint. Then the economy in aggregate does not depend on the distribution of individual qualities.

7.1 HA and HANK Models

Contemporary dynamic general equilibrium modelling has gone a long way in answering critiques of representative agent assumptions, particularly with the work on incomplete market models that started in the real business cycle literature (e.g. Aiyagari (1994), Krusell and Smith (1998a); see Heathcote *et al.* (2009) for a survey). Overlapping generations models also remain important - although more in growth theory than short run macroeconomics - and there also exist DSGE models that incorporate asset market segmentation and multiple countries (see e.g. Alvarez *et al.* (2002) for a model which incorporates both of these elements).

A growing literature has emerged in recent years that aims at re-examining some important macro questions through models that assume the presence of idiosyncratic shocks to individuals income, together with the existence of incomplete markets and borrowing constraints. Those features are often combined with the kind of nominal rigidities and monetary non-neutralities that are the hallmark of New Keynesian models. Following Kaplan *et al.* (2016) we refer to those models as HANK models (for "Heterogenous Agent New Keynesian" models).

The seminal contribution to the heterogeneous agents (HA) literature is by Krusell and Smith (1998b). This paper investigates how movements in the distribution of income and wealth affect the macroeconomy. They construct a calibrated version of the stochastic growth model with partially uninsurable idiosyncratic risk and movements in aggregate productivity. They make two contributions.

First, they develop a solution method referred to as *aggregate approximation* that has become very popular in the literature. The main computational difficulty of dynamic heterogeneous-agent models is the dependence of aggregate variables on the income and wealth distributions of agents. In order to predict prices, for example, consumers need to keep track of these distributions. Krusell and Smith show that in equilibrium, despite the fact that the state of the economy at any point in time is an infinite-dimensional object, all aggregate variables can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock.

Second, they find a significant departure from permanent income behavior which stands in contrast to standard representative-agent models. Adding only uninsurable idiosyncratic risk to the representative-agent model implies an unrealistic wealth distribution in their model: both the mass of poor agents and the concentration of wealth among the very richest is below what we can be observed in the data. When they introduce preference (discount factor) heterogeneity to the model, then it succeeds quite well in replicating the observed wealth distribution. Although, aggregate wealth is mainly in the hands of the rich in the model, poor agents have a large influence on aggregate consumption. Since these agents are also impatient on average, they can be characterized as hand-to-mouth consumers which leads to the observed departure from permanent income behavior.

Several lessons have been drawn from the HA and HANK literature. Taking into account agents heterogeneity has been shown to be important in order to understand the transmission of monetary policy. Several authors have emphasized how the transmission of monetary policy and its aggregate effects may vary significantly depending on the prevailing fiscal policy, as the latter determines how the implementation of monetary policy affects the distribution of individual income and wealth among agents with different marginal propensities to consume.

But solving for the equilibrium of HANK economies requires the use of computational techniques that keep track of the wealth distribution and tackle the difficulties arising from the presence of occasionally binding borrowing constraints. This limits their usefulness for large and even medium-size NK models.

7.2 TANK Models

Debortoli and Galí (2018) provides a simpler two-agent NK (TANK) framework that is computationally easy to implement and even allows for some analytical results. Both these features help to understand and quantify the implications of heterogeneity for aggregate fluctuations. It distinguishes between two types of households at each point in time, which are labelled as "unconstrained" or "constrained", depending on whether their consumption satisfies or not a consumption Euler equation. Having made that distinction, the paper identifies three dimensions of heterogeneity that explain differences in aggregate fluctuations between a HANK economy and its representative agent counterpart (RANK, for short): (i) changes in the average consumption gap between constrained and unconstrained households, (ii) variations in consumption dispersion within unconstrained households, and (iii) changes in the share of constrained households . They show that the previous three factors are captured through additive "wedges"showing up in a log-linearized Euler equation for aggregate consumption, and which determine the differential behavior of a HANK economy relative to its RANK counterpart. Furthermore, by tracing their responses to aggregate shocks, they are able to assess the quantitative significance of each of those heterogeneity factors in shaping aggregate output fluctuations.

A further contribution of this paper is to assess the ability of Two Agent New Keynesian (TANK) models to approximate the role of heterogeneity in richer HANK models. The two types of consumers constrained and unconstrained - have constant shares in the population, while allowing only for aggregate shocks (i.e. disregarding idiosyncratic shocks). A unconstrained subset of households are assumed to have full access to financial markets (including markets for stocks and bonds), while constrained households are assumed to behave in a "hand-to-mouth" fashion, consuming their current income at all times. This will be the case if they do not have access to financial markets, find themselves continuously against a binding borrowing constraint, or display a pure myopic behavior.

Debortoli and Gali show that TANK model, which only captures one dimension of heterogeneity (the one which we refer to as the gap component), approximates reasonably well the predictions of a baseline HANK model regarding the effects of aggregate shocks on aggregate variables, as well as its predictions regarding the consequences of changes in the environment, once the fraction of constrained (and the transfer rule) are calibrated accordingly. Nonetheless, it should be clear that a simple TANK model will never be able to address many other questions involving heterogeneity (such as the effects of monetary policy on income and wealth distribution and, possibly, welfare) for which a richer HANK model is needed.

8 Unemployment and Disequilibrium in DSGE Models

Until recently as with their RBC antecedents the New Keynesian forms still omitted involuntary unemployment. We are now seeing labour markets models with unemployment in both RBC and DSGE NK models (for the latter, see for example Blanchard and Galí (2010), Thomas (2008) and Cantore *et al.* (2014)).

We now turn to the remaining area highlighted earlier, concerning the criticism that DSGE models fail to deal with disequilibrium. As Howitt (2012) puts it,

> "...the macroeconomic learning literature of Sargent (1999), Evans and Honkapohja (2001b) and others goes a long way towards understanding disequilibrium dynamics. But understanding how the system works goes well beyond this. For

in order to achieve the kind of coordinated state that general equilibrium analysis presumes, someone has to find the right prices for the myriad of goods and services in the economy, and somehow buyers and sellers have to be matched in all these markets. "

8.1 Search and Match in DSGE Models

Given the above, the behavioural NK models surveyed here can incorporate this type of disequilibrium, but do not usually do so. In addition, this basic critique of temporary equilibrium is often conflated with criticisms of the more restrictive Walrasian equilibria used in early DSGE models and real business cycle models, which tends to be replaced with search and matching, buffer stocks, and imperfect competition in contemporary models. The incorporation of search and matching mechanisms started with the contributions of Mortensen and Pissarides (1994) and Pissarides (2000), and there has been significant progress in embedding Mortensen-Pissarides search-matching (MPSM) frictions into otherwise standard DSGE models. Examples include Campolmi *et al.* (2010), Faia *et al.* (2010) and Monacelli *et al.* (2010). Many models featuring MPSM frictions focus only on the extensive margin, but there also are examples of models (for instance Thomas (2008), Krause *et al.* (2008) and Cantore *et al.* (2014)) that model the intensive margin in addition to the unemployment rate.

8.2 Disequilibrium

Regarding output market frictions and buffer stocks of goods and services, only recently have these been incorporated into RBC or DSGE models. Examples are Khan and Thomas (2007), which provides a micro-founded theory of inventories that succeeds in reproducing stylized facts regarding inventory investment in the USA, and Den Haan (2014), which combines an inventory model with a MPSM model of labour markets. These are equilibrium models in both the Nash sense and the sense described above, but disequilibrium models in the Walrasian sense. They also assume rational expectations; combining the goods and labour market frictions in these models with bounded rationality as above is a possible route for behavioural NK models to take.

9 A Financial Sector and Financial Friction

Another major lacuna in the earlier generation of DSGE models was been the absence of a banking sector. The monetary transmission mechanism existed simply through one nominal interest on a riskless bond, set by the central bank. The seminal work on financial frictions by Bernanke *et al.* (1999) introduced a risk premium paid by firms with an implicit intermediary financial institution. But it is only very recently that a comprehensive banking sector has appeared – see Gertler and Kiyotaki (2010a) as a representative example of this development.

Since the recent financial crisis, the number of papers studying the importance of financial frictions on macroeconomic outcomes and policy implications has grown considerably, commonly building on the mechanisms proposed in the Kiyotaki and Moore (2007) (KM) collateral constraints model, or the Bernanke *et al.* (1999) (BGG) costly state verification model. In KM the propagation and amplification comes from the fluctuations in asset prices, while in BGG from fluctuation of agents net worth. The KM approach was extended to study the effects of financial constraints on the banking sector in Gertler and Kiyotaki (2010b) (GK) where the limited commitment problem of KM introduces an agency problem between depositors and banks; when the value of bank capital declines, the borrowing constraint tightens and limits the amount of deposits the bank can raise and subsequently, the level of investment. Another extension proposed in Gertler and Karadi (2011) uses this approach to analyse the role of unconventional monetary policy. It is assumed the central bank can perform financial intermediation at a cost, but when the borrowing constraint tightens sufficiently, this cost is less than the inefficiency introduced by the agency problem. Macro-prudential regulation is studied in Gertler *et al.* (2012) where the government offers banks a subsidy per unit of outside equity issued and finances the subsidy with a tax on total assets. In equilibrium the tax is set to make the subsidy revenue neutral and is chosen so as to internalize the external benefits of outside equity issuance.

The KM and BGG approaches have both been applied to the housing market. Impatient households post housing as collateral as in KM to secure mortgage loans in Iacoviello and Neri (2010) where the mechanism of Iacoviello (2005) is focused on the demand-side of the economy, and shown to have important effects on the business cycle. The constraints arise in Forlati and Lambertini (2011) due to a BGG costly state verification mechanism which is applied to household credit by assuming households observe a private housing-value shock that can lead to default when households are insolvent. The authors emphasise increased housing investment risk in highly leveraged economies.

The large influence of the BGG and KM approaches to the financial frictions literature might be partly due to the simplicity of applying the frictions to a representative agent, rational expectations model, solved using linear approximation techniques. It has been argued in Holden *et al.* (2016) that this rules out *ex ante* the possibility of explaining a number of key stylized facts, such as the large positive skew in the interest spreads, and negative skews in investment. There have been a number of papers that do study the non-linear and asymmetric effects of financial frictions which are much better suited to explain such phenomena, usually using global solution methods, or the perturbation based methods of Holden (2016) and Guerrieri and Iacoviello (2015).

10 Agent-Based Macro-Models

These fundamental concerns discussed up to now are now driving research into an Agent-Based (AB) alternative. This approach represents economic agents as well as various social and environmental phenomena as autonomous virtual entities that interact during simulation experiments following pre-defined rules. In standard macroeconomic models agents' decisions consist of behavioural or, in the case of DSGE models, micro-founded first-order conditions satisfying a dynamic optimization problem, that are continuous functions of the current and past state of the economy. The AB approach provides a potentially more flexible way of modelling the cognitive capabilities of decision makers and their responses to both the macro- and individual micro-environment. In AB economies can represent out-of-equilibrium behaviour and be regarded as "evolving systems of autonomous interacting agents" (Tesfatsion (2003)). Hence, while DSGE assumes that agents have very sophisticated computational capabilities and they live in very simple environments, AB models assume that people use simple behavioural rules to cope with complex and dynamic environments (Howitt (0121)).

ACE models (again see LeBaron and Tesfatsion (2008)) certainly tackle aggregation head-on and dispense with the rationality problem by ditching rational expectations altogether. But should central banks go down this path for their models? To quote LeBaron and Tesfatsion they (ACE models) "raise some practical complications for the applied econometrician... computational methods such a method of moments might be too computationally costly to undertake ... Researchers at central banks might never decide to fit giant ACE macro models to data."

This comment may prove to be too pessimistic. Although macroeconomic AB models still remain few in number, interest in this area has been growing recently. Perhaps the most developed AB macroeconomic model is Dawid *et al.* (2011) (see also Ashraf *et al.* (2011b)) which has the standard economic agents found in current DSGE models (consumption and capital goods producers, households, banks and government). There is also an increasing number of AB models of banks and interbank payment systems both in the academic literature (Arciero *et al.* (2009), Markose *et al.* (2011), Ashraf *et al.* (2011a)) and the working paper series of central banks (Galbiati and Soramaki (2008)). A good reference to a rapidly growing literature is Leigh Tesfatsion at Iowa State who maintains a large software database, bibliography and link repository at http://www2.econ.iastate.edu/tesfatsi/ace.htm. Dilaver *et al.* (2018) provides a review and synthesis of this literature and recent attempts to incorporate insights from ACE into DSGE models thus bridging the gap between the approaches.

11 Macroeconomic Policy

This section first considers the general optimal policy problem where the policymaker has a number of instruments and sets out to maximize a general discounted welfare criterion subject to the constraints of a DSGE model. It then moves onto the optimal design of robust policy.

11.1 Macroeconomic Policy Design using DSGE Models

If the policymaker is able to commit, the setting of instruments can be conducted in terms of the ex ante optimal policy. If the expected discounted household utility is chosen as the welfare criterion this becomes the well-known Ramsey problem. Note that the Ramsey solution is not the same thing as the *social planner's problem* in any model with some market failure. A problem with such a solution is that it involves a complex rule even for quite simple NK models. Much of the optimal policy literature therefore focuses on simple Taylor-type commitment rules that are optimized so as to come close to mimicking the Ramsey solution and this is the approach of this course.

In the absence of an ability to commit the policymaker must set policy to be time-consistent. Then the expected discounted household utility is maximized at time t, subject to the model constraints, in the knowledge that the same maximization procedure will be used to minimize at time $t+1$. A dynamic programming solution then seeks a stationary *Markov Perfect* solution of the equilibrium. See Currie and Levine (1993) and Söderlind (1999) for a LQ treatment of this problem under perfect information and Levine *et al.* (2012b) under imperfect information. But see Dennis and Kirsanova (2013) for the possibility of multiple equilibria. A comparison of the welfare outcomes of optimal policy under commitment and discretion provides a measure of the gains from commitment. See Levine *et al.* (2008) for such an assessment for monetary policy using a Bayesian-estimated SW model.

11.2 Robust Policy Across Contrasting Models

All models are wrong, but some are useful. **Box (1979).**

DSGE models estimated by Bayesian-Maximum-Likelihood methods can be considered as probability models in the sense described by Sims (2007) and be used for risk-assessment and policy design. We now consider the general policy question: how when faced with the existence of multiple competing and contrasting models, all of which are believed to be misspecified, should policymakers set macroeconomic policy? Building on Levine and Pearlman (2010), Cogley *et al.* (2011) and Levine *et al.* (2012c), Deak *et al.* (2019) proposes general framework that uses a pool of contrasting models for the policy design problem. The methodology uses Bayesian estimation to weight alternative models to design optimized Taylor-type rules that are robust in a sense described below. A crucial requirement is to provide a k-period ahead predictive density, given macro-economic data. The predictive density characterizes out-of-sample observations that have not been used up to that point in time to estimate the posterior density of the parameter vector. As such this provides predictions about future observations that fully incorporate the information regarding within-model uncertainty (defined below) in the data.

Two further requirements to apply the methodology are that the models in the pool to share the same policy instrument under investigation and to have a welfare criterion to rank alternative policies. The models in the pool do not need to share the estimated parameter vector, nor even the observables; they can be nested as well as non-nested. Thus the methodology can be applied to a wide range of macroeconomic models from mainstream DSGE, behavioural to agent-based, and indeed to other non-macroeconomic settings as long as these three requirements are met. The challenge for new models is to meet these basic conditions to be useful for guiding policy.

This methodology of then studies robust policy design where the policymaker faces three forms of uncertainty. The first derives from uncertain future shocks. This is a standard problem in the optimal policy literature. The second is parameter uncertainty within each competing model. The third source of uncertainty, "across-model uncertainty", we have alluded to: the existence of multiple competing models at her disposal. The paper then investigates the welfare consequences of a standard Taylor-type monetary policy rules using three medium-scale New Keynesian DSGE models: the Smets-Wouters model in Smets and Wouters (2007), the workhorse model widely used in policy-making institutions for forecasting and policy analysis, and the other two models that add variants of financial frictions. Hence, the model pool can be motivated by considering a policy maker who is uncertain how to incorporate financial frictions into a DSGE model or if they should be incorporated at all.

12 Concluding Comments

The opening comment in Blanchard (2016) expresses the view: "I see the current DSGE models as seriously flawed, but they are eminently improvable and central to the future of macroeconomics". This survey has highlighted a number of perceived flaws and described research that is addressing these concerns. I conclude by highlighting three challenges for the construction, estimation and policy analysis of DSGE models.

First, much of the DSGE literature has focused on the closed economy, but a more recent growing body of research now extends the models to open economies; see Galí (2015b) and Schmitt-Grohe and Uribe (2017) for excellent textbook treatments. However the long tradition of comparing model properties to stylized facts has been thus far less common for emerging economies, which usually face a data-related triple-whammy: short spans of data, quarterly data of low quality for some variables and several crises, structural breaks and changes in policy regimes.

Nevertheless, an RBC-DSGE literature focusing on emerging economies is gradually developing and a few stylized facts, alongside some empirical debates, have emerged. These include: higher output volatility than developed economies; deeper and steeper recessions, but faster recoveries; Consumption and real wage volatility and real wage exceeding output volatility; real interest rates: counter-cyclical and leading the cycle; net exports strongly counter-cyclical and more volatile; dramatic "sudden stops" in capital inflows; a weaker role of trade openness as a transmission mechanism; the importance of informality. Modelling features discussed up to now that are particularly relevant include the credit-constrained and Ricardian household distinction and therefore TANK models; Cross country financial frictions; and the need to integrate growth and business cycles. Some first steps in addressing these issues are Batini *et al.* (2007), Gabriel *et al.* (2012), Veigh (2013) and Anand *et al.* (2015).

Secondly, there appears to be a consensus that Bayesian systems estimation is the way to proceed, and the case is strengthened for emerging economies with less reliable macro-data. However the development of Bayesian estimation techniques that can incorporate the full range of non-linearities in DSGE models remains an major challenge. For example occasionally binding constraints arise in a number of ways in DSGE models: financial constraints discussed in Section 9 need not be continuously binding; policy constraints include the zero-lower bound on the nominal interest rate, a lower bound on the capital requirement of macro-prudential regulation and an upper bound on government debt. Although the solution of non-linear DSGE models is computationally possible, techniques that are sufficiently robust and efficient for the medium-sized models now emerging in the literature are as yet unavailable.

Thirdly, the use of the robustness methodology set out above has potential for arriving at a consensus on robust policy prescriptions for the full range of monetary, fiscal and macro-prudential policy issues outlined in Blanchard *et al.* (2010), Blanchard *et al.* (2013) and Blanchard and Summers (2017).

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