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Mandates and Monetary Rules a New Keynesian Framework*

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Abstract

We develop a general mandate framework for delegating monetary policy to an instrument-independent, but goal-dependent central bank. The goal of the mandate consists of: (i) a simple quadratic loss function that penalizes deviations from target macroeconomic variables; (ii) a form of a Taylor-type nominal interest-rate rule that responds to the same target variables; (iii) a zero-lower-bound (ZLB) constraint on the the nominal interest rate in the form of an unconditional probability of ZLB episodes and (iv) a long-run (steady-state) inflation target. The central bank remains free to choose the strength of its response to the targets specified by the mandate. An estimated standard New Keynesian model is used to compute household-welfare-optimal mandates with these features. We find two main results that are robust across a number of different mandates: first, the optimized rule takes the form of a Taylor simple rule close to a price-level rule. Second, the optimal level of inflation target, conditional on a quarterly frequency of the nominal interest hitting the ZLB of 0.025, is close to the typical target annual inflation of 2% and to achieve a lower probability of 0.01 requires an inflation target of 3.5%.

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Keywords: New Keynesian Model, Mandates, Zero Lower Bound Constraint.

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Contents

1 Introduction

It is now generally accepted that monetary policy should be conducted within a framework of instrument independence based on the principles of commitment, accountability and transparency. By a credible commitment to future policies, central banks are then able to influence expectations that achieve the best trade-offs.[1]

In accordance with these principles, the inflation-targeting framework has in particular proven to be very popular. In what has been an influential paper with central banks, Rudebusch and Svensson (1999) model inflation targeting as allowing for concerns about the variability of real output. They examine two broad classes of rules: instrument and targeting rules. For the former the monetary instrument, the nominal interest rate, can be optimal in relation to a welfare criterion, or restricted to respond to particular target variables as in Levine and Currie (1987), Taylor (1993b) and Taylor (1999). It is now accepted in the literature that the optimal form of the latter, 'optimized simple rules', can closely mimic the former. A targeting rule is an assignment of a welfare loss function over deviations of goal variables from their targets (in effect bliss points). Then policy can again be conducted in the form of unrestricted optimal policy or restricted simple optimized rules.

In this paper we propose a general mandate framework that combines instrument and targeting rules in a consistent fashion. The mandate consists of four components: (i) a welfare objective delegated to the central bank in the form of a simple quadratic a loss function that penalizes deviations from target macroeconomic variables (as a benchmark we also consider the household utility as the objective); (ii) a form of a Taylor-type nominal interest-rate rule that responds to the *same* target variables specified in the loss function; (iii) a zero-lower-bound (ZLB) constraint on the nominal interest rate in the form of a specified unconditional probability of ZLB episodes and (iv) a long-run (steady-state) inflation target. The weights on the deviations from target variables in the mandate are then chosen to maximize the inter-temporal utility of the household. With these four features the mandate makes the central bank goal-dependent, but instrument-independent as it remains free to the choose the strength of its response to the targets in the rule. An estimated standard New Keynesian model of Smets and Wouters (2007) is used to compute the household-welfare-optimal mandates with these properties.

This paper relates to a strand of optimal mandate's literature which is reviewed below. The closest paper to ours is Debortoli *et al.* (2019) who develop a mandate framework in which a central bank is instrument-independent and goal-dependent. Our paper differs from this paper in a number of ways. First, we formalize the ZLB constraint on nominal interest rate as described above This small probability is interpreted as the tightness level of the ZLB. This approach

[1]See the central bank consensus in, for example, Yellen (2012).

enables us to employ a second-order perturbation solution combined with a penalty function to calculate the exact social welfare value used to design optimal mandates (see Kim *et al.* (2003)) thus avoiding a quadratic approximation. Second, we require the central bank to conduct its monetary policy in the form of an optimized interest rate rule with targets corresponding to those in the welfare goal mandate and with an imposed optimal shift in the steady state inflation rate. Finally, we sketch a more general mandate framework that can be applied to other aspects of macroeconomic policy. We now elaborate on these first two features leaving discussion of the general policy framework to a later section of the paper.

1.1 The ZLB Constraint through a Penalty Function

Under the ZLB occasionally binding constraint, standard perturbation methods are unavailable as the policy function is non-differentiable in the vicinity of the steady state. The option of a global solution by value function iteration methods is available, but suffers from the curse of dimensionality unless the state space is small. This problem becomes acute when the model solution is embedded in our mandate framework that involves the computation of optimized rules. A solution is to use an approximate perturbation-based method with a penalty function to impose the constraint in a continuously binding form.

There are two common approaches to this method: first, to add the penalty function to the policy-maker's welfare criterion (see Woodford (2003), Levine *et al.* (2008), Levine *et al.* (2012)); second, to adding the penalty function to the agents' welfare criteria in the model (see Den Haan and Wind (2012), Abo-Zaid (2015), Karmakar (2016). The general idea is that we allow anything to be feasible but let the objective function have some welfare penalty if the constraint is violated. More precisely, we allow the nominal interest rate hit the zero bound with a small probability which can be interpreted as the tightness level of the ZLB constraint.[2] In this paper, we follow this literature by adding a penalty term on the central bank's objective function. We also compare different mandate formations delegated to the central bank, namely: a so-called ZLB mandate which retains the household utility as a welfare measure, a number of simple quadratic loss function mandates, and an asymmetric functional form mandate.

[2]Other papers compute the full non-linear solution - see for example Coibion *et al.* (2012), Dordal-i-Carreras *et al.* (2016), Holden (2016b) and Holden (2016a). But combined with optimized rules and other players in the mandate, this approach would pose a substantial computational challenge. Moreover our ZLB penalty-function mandate is a transparent implicit requirement for the central bank to stay within a bound on the standard deviation of the nominal interest rate as emphasized by Woodford (2003).

1.2 The Optimal Steady-State Inflation Target

A crucial feature of an welfare-optimized monetary policy rule is the optimal level of inflation target (the chosen inflation trend) in the optimized rule. In our sticky-price and sticky-wage model a positive trend inflation is costly to the economy through both steady-state effect and dynamic effects. The former is the more important so we examine this in some detail. For simplicity, consider the zero growth case $g = 0$ for which wage and price inflation are equal. Then from Appendix A.1 (which allows for the general $g > 0$ case) for price-setting the impact of trend inflation Π on the steady state is given by:

$$\frac{P^0}{P} = \left(\frac{1 - \xi_p \Pi^{(1-\gamma_p)(\zeta_p - 1)}}{1 - \xi_p}\right)^{\frac{1}{1-\zeta_p}}$$

$$\Delta_p = \frac{1 - \xi_p}{1 - \xi_p \Pi^{\zeta_p(1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p}$$

$$MC = \left(1 - \frac{1}{\zeta_p}\right) \frac{1 - \xi_p \beta \Pi^{\zeta_p(1-\gamma_p)}}{1 - \xi_p(1+g)\beta \Pi^{(\zeta_p - 1)(1-\gamma_p)}} \frac{P^0}{P}$$

where $\frac{P^0}{P}$ is the re-optimized Calvo-price for each retail variety, re-set with probability ξ_p, ζ_p is the price-elasticity of demand Δ_p is price dispersion across varieties, MC is the real marginal cost equal to the inverse of the price mark-up and β is the household discount factor.

For wage-setting we have analogous results:

$$\frac{W_n^O}{W_n} = \left(\frac{1 - \xi_w \Pi^{(1-\gamma_w)(\zeta_w - 1)}}{1 - \xi_w}\right)^{\frac{1}{1-\zeta_w}}$$

$$\Delta_w = \frac{1 - \xi_w}{1 - \xi_w \Pi^{(1-\gamma_w)\zeta_w}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w}$$

$$\frac{W_h}{W} = \frac{\left(1 - \xi_w \beta \Pi^{(1-\gamma_w)\zeta_w}\right)\left(1 - \frac{1}{\zeta_w}\right) \frac{W_n^O}{W_n}}{1 - \xi_w \beta \Pi^{(1-\gamma_w)(\zeta_w - 1)}}.$$

where $\frac{W_n^O}{W_n}$ is the re-optimized Calvo-nominal wage for each labour variety, re-set with probability ξ_w, ζ_w is the wage-elasticity of demand Δ_w is nominal wage dispersion across varieties and $\frac{W_h}{W}$ is the inverse of the wage mark-up over the wage rate $W_{h,t}$ at which households supply hours.

Thus for $\zeta_p > 1$, both the optimized price $\frac{P^0}{P}$ and price dispersion Δ_p *increase* with the trend inflation rate Π. However noting that the price mark-up is the inverse of the real marginal cost, i.e., equal to $= 1/MC$, we can see that the price response to the re-optimized price *decreases* with Π. Analogous results for $\zeta_w > 1$ hold for the optimized nominal wage, wage dispersion and the wage mark-up which is the inverse of $\frac{W_h}{W_n}$. Taking these results together we than have two effects of trend-inflation that increase distortions from sticky prices and wages and thus reduce welfare whereas the third effect results in the opposite. However numerical results based our estimated model confirms that the former easily outweigh the latter.[3]

The dynamic effect of trend-inflation was first studied by Ascari and Ropele (2007). They show it leads to richer inflation dynamics and a higher inflation volatility. High trend-inflation in other words is varying inflation. They also show that the Taylor principle is no longer sufficient to guarantee a unique rational expectations equilibrium in New Keynesian models for even moderate levels of inflation. Coibion *et al.* (2012) argue that a greater steady state inflation induces a higher level of forward-looking dynamic behavior when sticky-price firms are able to reset their prices. The more forward-looking is a firm, the greater is then anticipation of other firms raising the optimal reset price. These dynamic considers point to further welfare costs of high trend-inflation.

A positive trend inflation then increases distortions caused by sticky prices and wages and is welfare-reducing. On the other hand a positive inflation trend reduces the frequency of the nominal interest rate hitting the ZLB constraint. Christiano *et al.* (2011) argue that the zero-bound scenario involves a deflationary spiral which contributes to and accompanies a large fall in output, which leads to a high volatility and large output cost. Specifically, when output falls, marginal cost falls and price declines. With staggered pricing, a drop in price causes agents to expect a deflation in the future. With the nominal interest rate stuck at zero, the real interest rate rises, which leads to an increase in desired saving and a decrease in output. As a result, the cumulative fall in output required to reduce desired saving to zero is extremely significant.

Are there then welfare benefits from increasing trend-inflation given the desirability of avoiding such adverse zero-bound episodes? Should then the optimal level of inflation target set by the central bank be close to the typical target inflation of 2% or is the inflation target too blunt an instrument to efficiently reduce the severe costs of zero-bound episodes. These are the research questions we now pursue in our general mandate framework

1.3 More Related Literature

In our paper quadratic loss functions are seen as transparent simple mandates and not the best approximation of the household utility. But as is well-known, in the simple work-horse three-equation NK model with price stickiness Woodford (2003) shows that a quadratic function in inflation and the output gap is an accurate approximation up to second order. But this no longer applies to the Smets and Wouters (2007) we employ with capital, wage stickiness and less than full capacity utilization.

A number of papers simulate large-scale models in which a central bank commits to a version

[3] Coibion *et al.* (2012) come to the same conclusion.

of the Taylor rule to explore the optimal level of target inflation. Reifschneider and Williams (1999) and Günter *et al.* (2004) find a 2% inflation target to be an adequate buffer against the ZLB having noticeable adverse effects on the economy. However, these authors do not consider the costs associated with a higher average inflation rate.

A more recent strand of literature studies the optimal level of inflation target under the ZLB, e.g. Ascari and Ropele (2007), Coibion *et al.* (2012), Dordal-i-Carreras *et al.* (2016), Ngo (2017) and Andrade *et al.* (2019). Coibion *et al.* (2012), in particular, extend the standard linear-quadratic optimal analysis for a workhorse NK model with no capital and flexi-price with a zero net inflation steady state to the trend-inflation case. The model is calibrated to first and second moments of the data. They solve for the duration of the ZLB endogenously and show that the cost of a binding ZLB is significantly smaller than the costs of positive level of inflation target resulting in a low inflation target well below 2% per year for a given probability of 5% per quarter for the nominal interest rate hitting the ZLB.

But other papers argue that the 2% inflation target is too low and a target inflation of 4% would be adequate and would not harm an economy significantly (see Ball (2013)). However, Ascari and Sbordone (2014) and Kara and Yates (2017) argue that with a higher level of inflation target the determinacy region is significantly reduced. The latter paper in particular finds in a model of heterogeneity in price stickiness when trend inflation is 4% that the determinacy region in the model is almost non-existent. This result is particularly relevant for our results. In our mandates the central bank chooses an optimized form of the monetary rule which is constrained by the need for determinacy; by choosing an interest-rate rule with considerable persistence (in fact close to a price-level rule) the indeterminacy problem is avoided[4] and a 4% target with its associated low probability of ZLB episodes becomes possible.

1.4 Road-Map

The rest of the paper's structure is organized as follows: in the next section we briefly represent the full Smets-Wouter New Keynesian model (Smets and Wouters, 2007) which is estimated by Bayesian methods with different data series of the nominal interest rate, namely the standard Federal interest rate and the Shadow interest rate (Wu and Zhang, 2016); we then introduce the general monetary policy delegation game between different agents in the economy which leads to the main numerical results of the paper. A section then sketches a generalization of the framework for monetary-fiscal policy interactions before concluding comments complete the paper.

[4] This is now a well-known result: for example in a simple three-equation NK model Hommes *et al.* (2019) show analytically that persistence in the interest rate rule increases the determinate policy space of the feedback parameters of inflation and output.

2 A Non-Linear Smets-Wouters Model with Trend Inflation

Most papers using the Smets and Wouters (2007) model use the linearized form about a balanced-non-zero growth and effectively zero-net-inflation steady-state.[5] The non-linear form of the model with a trend net inflation is relatively unexplored, but is crucial for the welfare-analysis of this paper. The properties of the model in a non-zero-net inflation rate steady state, set out in Section (1.2), are crucial in this set-up. This section therefore sets the full non-linear form to be solved in the vicinity of a trend net inflation deterministic steady state.

There are four sets of representative agents: households, final goods producers, trade unions and intermediate goods producers. The later two produce differentiated labour services and goods respectively and, in each period of time, consist of a group that is locked into an existing contract and another group that can re-optimize.[6]

2.1 Households

At time $t = 0$, household i maximizes its expected lifetime utility

$$\begin{aligned}\Omega_0(i) &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t(i), C_{t-1}, H_t^s(i)) \\ &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{[C_t(i) - \chi C_{t-1}]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma - 1) \frac{(H_t^s(i))^{1+\psi}}{1+\psi} \right] \right] \end{aligned} \quad (1)$$

where $\mathbb{E}_t[\cdot]$ denotes rational expectations based on information available at time t, $C_t(i)$ is real consumption , $H_t^s(i)$ is hours supplied, β is the discount factor, χ controls external habit formation where C_{t-1} is aggregate consumption taken as given by household j, σ is the inverse of the elasticity of inter-temporal substitution (for constant labour), and ψ is the inverse of the Frisch labour supply elasticity. Preferences chosen by SW in (1) are compatible with balanced growth (see King *et al.* (1988)).

[5] This is achieved by assuming that Calvo price and wage contracts are fully indexed in the steady state, but only partially away from the steady state. A zero net inflation steady is convenient for linearization as it removes the steady state distortion from dispersion, but abstracts from the trend inflation rate effects that are central to this paper. Moreover the convenient indexing assumption is inconsistent with microevidence on price setting - see, for example, Linde and Trabandt (2018).

[6] Our model is a slightly slimmed down version of Smets and Wouters (2007) in one respects, we employ a Dixit-Stiglitz rather than Kimball aggregators over differentiated goods and labour types. We discuss this simplification later.

The household's budget constraint in period t is given by

$$C_t(i) + I_t(i) + \frac{B_t(i)}{RPS_t R_{n,t} P_t} + T_t = \frac{B_{t-1}(i)}{P_t} + \left(r_t^K u_t(i) - a(u_t(i))\right) K_{t-1}(i) + W_{h,t} H_t^s(i) + \Gamma_t \quad (2)$$

where I_t is investment into physical capital, B_t is government bonds held at the end of period t, $R_{n,t-1}$ is the nominal interest rate paid on government bonds held at the beginning of period t, RPS_t is an exogenous premium in the return on bonds that follows an AR1 process. T_t is lump-sum taxes, r_t^K is the real rental rate, u_t is the utilisation rate of capital, IS_t is an investment specific technological shock (the inverse of the relative price of new capital in consumption terms), $a(u_t(i))$ is the physical cost of use of capital in consumption terms, $W_{h,t}$ is the real wage rate at which households supply labour that is homogeneous at this point to trade unions and Γ_t is the profit of intermediate firms distributed to households. Notice that we deviate from the original SW model and do not allow for variable capital utilization in the model. End of period capital stock, $K_t(i)$, accumulates according to

$$K_t(i) = (1-\delta)K_{t-1}(i) + (1 - S(X_t(i)))I_t(i)IS_t \quad (3)$$

where IS_t is an investment specific technological shock that follows an AR1 process, $X_t(i) = I_t(i)/I_{t-1}(i)$ is the growth rate of investment, and $S(\cdot)$ is an adjustment cost function such that $S(X) = 0$, $S'(X) = 0$, and $S''(\cdot) = 0$ where X is the steady state value of investment growth. For $S(X_t)$ in a symmetric equilibrium we choose the functional form: $S(X_t) = \phi_X(X_t - \bar{X}_t)^2$ where $\bar{X}_t$ is the balanced-growth steady-state trend. For $a(u_t)$ we choose the functional form: $a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$ with $u_t = u = 1$ in the steady state.

The solution to the household's problem imply the Euler Consumption equation, an arbitrage condition and a first order condition equating the marginal rate of substitution between leisure and consumption with the real wage:

$$\mathbb{E}_t[\Lambda_{t,t+1}(i)R_{t+1}] = 1 \quad (4)$$

$$\mathbb{E}_t[\Lambda_{t,t+1}(i)R_{t+1}^K] = 1 \quad (5)$$

$$-\frac{U_{C,t}(i)}{U_{H,t}(i)} = W_{h,t} \quad (6)$$

$$a'(u_t) = r_t^K \quad (7)$$

where $\Lambda_{t,t+1}(i) \equiv \beta \frac{U_{C,t+1}(i)}{U_{C,t}(i)}$ is the stochastic discount factor, $U_{C,t}(i) \equiv \frac{\partial U_t(i)}{\partial C_t(i)}$, $U_{H,t}(i) \equiv \frac{\partial U_t(i)}{\partial H_t(i)}$ are marginal utilities; $R_t = \frac{R_{n,t-1}}{\Pi_t}$ and $R_t^K = \frac{[r_t^K u_t - a(u_t) + (1-\delta)Q_t]}{Q_{t-1}}$ are the real gross returns on government bonds and physical capital respectively and Q_t is the price of capital (Tobin's Q).

2.2 The Labour Market

Households supply their homogeneous labour to trade unions that differentiate the labour services. A labour packer buys the differentiated labour from the trade unions and aggregate them into a composite labour using the Dixit-Stigliz aggregator[7] for aggregate labour supply

$$H_t = \left(\int_0^1 H_t(j)^{(\zeta_w - 1)/\zeta_w} dj \right)^{\zeta_w/(\zeta_w - 1)} dj \tag{8}$$

where ζ_w is the elasticity of substitution among different types of labour, and we index trade unions by j. The labour packer minimizes the cost $\int_0^1 W_{n,t}(j) H_t(j) dj$ of producing the composite labour service, where $W_{n,t}(j)$ denotes the nominal wage set by union j. This leads to the standard demand function

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}} \right)^{-\zeta_w} H_t^d \tag{9}$$

where $W_{n,t}$ is the aggregate nominal wage given by the Dixit-Stigliz aggregator

$$W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\zeta_w} dj \right]^{\frac{1}{1-\zeta_w}} \tag{10}$$

Sticky wages are introduced through Calvo contracts supplemented with indexation. At each period there is a probability $1 - \xi_w$ that trade union j can choose $W_{n,t}^O(j)$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - W_{h,t+k} \right] \tag{11}$$

subject to the demand function (9), where $\gamma_w \in [0, 1]$ is a wage indexation parameter.

The solution to the above problem is the first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - \frac{\text{MRSS}_{t+k}}{(1 - 1/\zeta_w)} W_{h,t+k} \right] = 0 \tag{12}$$

where we have introduced a mark-up shock MRSS_t to the marginal rate of substitution that follows an AR1 process. This leads to

$$\frac{W_{n,t}^O(j)}{W_{n,t}} = \frac{\frac{1}{1-1/\zeta_w} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) W_{h,t+k} \text{MRSS}_{t+k}}{W_{n,t} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \frac{1}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w}}$$

[7] See Dixit and Stiglitz (1977).Smets and Wouters (2007) generalize the aggregator to a Kimball form as in Kimball (1995) which introduces a variable mark-up even in the absence of wage stickiness. But as Klenow and Willis (2016) argues a significant difference between the two aggregators only emerges if one calibrates the model using an implausibly high price super-elasticity. See also Deak *et al.* (2020).

$$
= \frac{\frac{1}{1-1/\zeta_w}\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} \frac{\left(\frac{W_{n,t+k}}{W_{n,t}}\right)^{\zeta_w}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w \zeta_w}} W_{h,t+k} H^d_{t+k} \text{MRSS}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} \frac{\left(\frac{W_{n,t+k}}{W_{n,t}}\right)^{\zeta_w}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w (\zeta_w - 1)}} \frac{W_{n,t}}{P_{t+k}} \frac{P_t}{P_t} H^d_{t+k}} \tag{13}
$$

By the law of large numbers the evolution of the aggregate wage is given by

$$
W_{n,t}^{1-\zeta_w} = \xi_w \left(W_{n,t-1} \Pi_{t-1}^{\gamma_w}\right)^{1-\zeta_w} + (1-\xi_w)(W_{n,t}^O)^{1-\zeta_w} \tag{14}
$$

which can be written as

$$
1 = \xi_w \left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t^w}\right)^{1-\zeta_w} + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{1-\zeta_w} \tag{15}
$$

Wage dispersion is defined as $\Delta_{w,t} = \int (W_{n,t}(j)/W_{n,t})^{-\zeta_w} dj$. Assuming that the number of trade unions is large, we obtain the following dynamic relationship:

$$
\begin{aligned}
\Delta_{w,t} &= \xi_w \int_{not\ optimize} \left(\frac{W_{n,t-1}^O(j)\Pi_{t-1}^{\gamma_w}}{W_{n,t}}\right)^{-\zeta_w} dj + (1-\xi_w) \int_{optimize} \left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{-\zeta_w} dj \\
&= \xi_w \frac{(\Pi_t^w)^{\zeta_w}}{\Pi_{t-1}^{\zeta_w \gamma_w}} \Delta_{w,t-1} + (1-\xi_w)\left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{-\zeta_w}
\end{aligned} \tag{16}
$$

2.3 Firms in the Wholesale Sector

Wholesale firms employ a Cobb-Douglas production function to produce a homogeneous output

$$
Y_t^W = F(A_t, H_t^d, u_t K_{t-1}) = (A_t H_t^d)^\alpha (u_t K_{t-1})^{1-\alpha} - F_t \tag{17}
$$

where F_t are exogenous fixed costs growing in a balanced-growth steady in line with the other real varaibles. Profit-maximizing demand for factors results in the first order conditions

$$
W_t \equiv \frac{W_{n,t}}{P_t} = \alpha \frac{P_t^W}{P_t} \frac{Y_t^W + F_t}{H_t^d} \tag{18}
$$

$$
r_t^K = (1-\alpha) \frac{P_t^W}{P_t} \frac{Y_t^W + F_t}{u_t K_{t-1}} \tag{19}
$$

2.4 Firms in the Retail Sector

The retail sector uses a homogeneous wholesale good to produce a basket of differentiated goods for aggregate consumption

$$C_t = \left(\int_0^1 C_t(m)^{(\zeta_p - 1)/\zeta_p} dm \right)^{\zeta_p/(\zeta_p - 1)} \tag{20}$$

where ζ_p is the elasticity of substitution. For each m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize (20) given total expenditure $\int_0^1 P_t(m)C_t(m)dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_t(m)$ of the form

$$C_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta_p} C_t \Rightarrow Y_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta_p} Y_t \tag{21}$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta_p} dm \right]^{\frac{1}{1-\zeta_p}}$. P_t is the aggregate price index. C_t and P_t are Dixit-Stigliz aggregates – see Dixit and Stiglitz (1977).

Following Calvo (1983), we now assume that there is a probability of $1 - \xi_p$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then prices are indexed to last period's aggregate inflation, with indexation parameter γ_p. With indexation parameter $\gamma_p \geq 0$, this implies that successive prices with no re-optimization are given by $P_t^0(f)$, $P_t^0(f) \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_p}$, $P_t^0(f) \left(\frac{P_{t+1}}{P_{t-1}} \right)^{\gamma_p}$, ... For each retail producer m, given its real marginal cost (the inverse of the price mark-up)

$$MC_t = \frac{P_t^W}{P_t}, \tag{22}$$

the objective is at time t to choose $\{P_t^O(m)\}$ to maximize discounted profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - MC_{t+k} \right] \tag{23}$$

subject to

$$Y_{t+k}(m) = \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} \right]^{-\zeta_p} Y_t \tag{24}$$

The solution to this is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - \frac{1}{(1 - 1/\zeta_p)} MC_{t+k} \right] = 0 \tag{25}$$

which leads to

$$\frac{P_t^O(m)}{P_t} = \frac{\frac{1}{1-1/\zeta_p}\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \frac{\left(\frac{P_{t+k}}{P_t}\right)^{\zeta_p}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p \zeta_p}} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \frac{\left(\frac{P_{t+k}}{P_t}\right)^{\zeta_p - 1}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p(\zeta_p-1)}} Y_{t+k}} = \frac{P_t^O}{P_t} \tag{26}$$

in a symmetric equilibrium.

By the law of large numbers the evolution of the price index is given by

$$P_t^{1-\zeta_p} = \xi_p \left(P_{t-1}\Pi_{t-1}^{\gamma_p}\right)^{1-\zeta_p} + (1-\xi_p)(P_t^O(m))^{1-\zeta_p} \tag{27}$$

which can be written as

$$1 = \xi_p \left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P_t^O(m)}{P_t}\right)^{1-\zeta_p} \tag{28}$$

Price dispersion is defined as $\Delta_{p,t} = \int (P_t(m)/P_t)^{-\zeta_p} dm$. Assuming that the number of firms is large, we obtain the following dynamic relationship:

$$\begin{aligned} \Delta_{p,t} &= \xi_p \int_{not\ optimize} \left(\frac{P_{t-1}^O(m)\Pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\zeta_p} dm + (1-\xi_w)\int_{optimize} \left(\frac{P_t^O(m)}{P_t}\right)^{-\zeta_p} dm \\ &= \xi_p \frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p \gamma_p}} \Delta_{p,t-1} + (1-\xi_p)\left(\frac{P_t^O(m)}{P_t}\right)^{-\zeta_p} \end{aligned} \tag{29}$$

2.5 Closing the Model

The model is closed with a resource constraint

$$Y_t = C_t + G_t + I_t + a(u_t)K_{t-1} \tag{30}$$

A monetary policy rule for the nominal interest rate is given by the following Taylor-type rule

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &\quad + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \end{aligned}$$

$$+ MPS_t, \tag{31}$$

where MPS_t is a monetary policy shock. Our rule is of the implementable form as proposed by Schmitt-Grohe and Uribe (2007) in that the nominal interest rate responds to deviations of output about its steady state rather than deviations about the flexi-price level of output (i.e., the output gap). The latter would encompass the original rules proposed by Taylor (1993b) and Taylor (1999) for which there is no interest-rate smoothing ($\rho_r = 0$) and $\theta_{dy} = 0$. In the more recent of these papers, parameter values $\theta_\pi = 1.5$ and $\theta_y = 1.0$ are proposed.[8]

Nominal and real interest rates are related by the Fischer equation

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t}\right] \tag{32}$$

Market clearing for the labour market implies

$$H_t = \int_0^1 H_t(j)dj = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\zeta_w} dj H_t^d = \Delta_{w,t} H_t^d \tag{33}$$

Market clearing for the final good market implies

$$Y_t^W = \Delta_{p,t} Y_t \tag{34}$$

For completeness we include a government budget constraint

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_{n,t}} \tag{35}$$

where G_t is government spending that follows an AR1 process. However in this Ricardian set-up without distortionary taxes the constraint plays no role in the equilibrium.

Finally the seven exogenous processes are AR1 and evolve according to:

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{36}$$

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{37}$$

$$\log MS_t - \log MS = \rho_{MS}(\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \tag{38}$$

$$\log MRSS_t - \log MRSS = \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \tag{39}$$

[8]Note forward-looking ‘inflation-forecasting rules could also be considered but these are prone to a severe indeterminacy constraint that results in welfare-inferior outcomes (see Batini *et al.* (2006)).

$$\log IS_t - \log IS = \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{40}$$

$$\log RPS_t - \log RPS = \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \tag{41}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{42}$$

3 Bayesian Estimation

This section sets out the Bayesian estimation of the model using standard techniques.[9] The model is linearized computationally about the non-net inflation positive growth deterministic state set out above. Before presenting the results, we first describe the measurement equations, the data, the methodology (briefly) and identification. We also highlight the information assumptions made in solving for a RE equilibrium that are usually only implicit in Bayesian estimation exercises.

3.1 Data and Measurement Equations

Our observables used in the estimation are: GDP per capita growth (`dyobs`), consumption expenditure per capita growth (`dcobs`), investment per capita growth (`dinvobs`), real wage growth (`dwobs`), percentage deviation of hours worked per capita from mean (`labobs`), monetary policy rate (`robs`), inflation rate (`pinfobs`). The corresponding measurement equations are:

$$\texttt{dyobs} = \log\left((1+g)\frac{Y_t^c}{Y_{t-1}^c}\right) \tag{43}$$

$$\texttt{dcobs} = \log\left((1+g)\frac{C_t^c}{C_{t-1}^c}\right) \tag{44}$$

$$\texttt{dinvobs} = \log\left((1+g)\frac{I_t^c}{I_{t-1}^c}\right) \tag{45}$$

$$\texttt{dwobs} = \log\left((1+g)\frac{W_t^c}{W_{t-1}^c}\right) \tag{46}$$

$$\texttt{labobs} = \frac{H_t^d - H^d}{H^d} \tag{47}$$

$$\texttt{robs} = R_{n,t} - 1 \tag{48}$$

$$\texttt{pinfobs} = \log(\Pi_t) \tag{49}$$

[9] We used Dynare 5.7 for these results.

The steady state values of the observables are `dyobs` $=$ `dinvobs` $=$ `dcobs` $=$ `dwobs` $=$ $\log(1+g)$, `labobs` $=0$, `robs` $=R_n-1$, and `pinfobs` $=log(\Pi)$.

The original data are taken from the FRED Database available through the Federal Reserve Bank of St.Louis. The data consists of 7 quarterly time series, namely log output growth (*dyobs*), log consumption growth (*dcobs*), log investment growth (*dinvobs*), log wage growth (*dwobs*), labour hours supply (*laobs*), the net inflation (*pinfobs*), and finally the policy rate measurement (*robs*). Since our focus on the ZLB we also provide a new estimation with the Wu-Xia Shadow interest rate replacing the FED rate, *robs* - see Wu and Zhang (2016) and Wu and Zhang (2019). The sample period is 1958:1-2017:4. There is a pre-sample period of 4 quarters so the observations actually used for the estimation go from 1959:1-2017:4, 240 observations.

3.2 Bayesian Methodology, Identification and Information Assumptions

It is necessary to confront the question of parameter identifiability in DSGE models before taking them to the data, as model or parameter identification is a prerequisite for the informativeness of different estimators, and their effectiveness when one uses the models to address policy questions. Hence, we follows Iskrev (2010) Iskrev and Ratto (2010) to perform formal identification checks on the reduced form parameters and structure or deep parameters. Overall, our identification analysis using DYNARE (Juillard, 2003) shows that the Jacobian and Hessian matrices of the mapping from the reduced form of the estimated parameters into the first and second order moments of the observable variables are full rank. Thus, our estimated model is locally identifiable given the priors and observable data sample. Details of this analysis are shown in the Appendix.

In a linear set-up about the deterministic steady state, the model's unique RE solution can be characterized by a standard transition equation:

$$s_t = A(\theta)s_{t-1} + B(\theta)u_t \tag{50}$$

and the set of measurement equations in sub-section 3.1 is written in a compact form as follows:

$$y_t = C(\theta)s_t + D(\theta)\omega_t \tag{51}$$

where $u_t = (\epsilon_{A,t}, \quad \epsilon_{G,t}, \quad \epsilon_{MCS,t}, \quad \epsilon_{MRSS,t}, \quad \epsilon_{IS,t}, \quad \epsilon_{MPS,t}, \quad \epsilon_{RPS,t})$ is a vector of structural and noisy shocks, ω_t is a vector of the observation noise, s_t is a vector of stationary variables, and y_t is a vector of observables as presented in the subsection (3.1). $A(\theta), B(\theta), C(\theta), D(\theta)$ are matrices of reduced form parameters.

The standard Kalman Filter is used to calculate the likelihood for a given sample of data of the observable vector. The details of the Kalman Filter from the state space of the form (50) and (51) given the priors, $p(\theta)$ (consisting of initial mean, $s_{0|0}$, and initial co-variance $P_{0|0}$) of the estimated parameters set θ, and the history of observations $y_t, y_{t-1}, ..., y_0$ are presented in Appendix C.

The posterior distribution is then updated using Bayes's rule: $p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}$. Our focus is on the posterior distribution of $p(\theta|y)$ that summarise what we know about θ, such as (posterior) means, medians, modes, etc (and respective standard deviations). Knowing this allows Bayesian inference expressed as $E[g(\theta)|y]$, where $g(\theta)$ is a function of interest:

$$E[g(\theta)|y] = \int g(\theta)p(\theta|y)d\theta$$

There is one more issue to address regarding the *assumed information set* of the agents in the model. Most DSGE models are still solved and/or estimated on the assumption that agents are simply provided with perfect information (henceforth PI) regarding the states including the exogenous processes, effectively as an endowment. If we drop this implausible assumption we must consider a signal extraction problem under imperfect information (II) for the agents in the model analogous to that we have considered for the econometrician. Fortunately we can retain the PI solution if we restrict ourselves to a class of models which are 'A-invertible' meaning that agents can infer the structural shocks from the information set assumed to be that of the econometrician. Levine *et al.* (2019) provide an A-invertibility condition that generalizes the "Poor Man's Invertibility Condition" of Fernandez-Villaverde *et al.* (2007) and show that in the Smets and Wouters (2007) of this paper with seven shock processes and seven observables is indeed A-invertible. It follows that II and PI solutions coincide and the standard information assumption is valid in our model.

3.3 Estimation results

The table 1 indicates the priors, the estimated posterior mode of the parameters obtained directly from the maximization of the posterior distribution, and the mean of the posterior distribution of the parameters obtained through the Metropolis-Hastings sampling

Table 1: Estimated results (posterior mean with a number of draws equal to 20000)

Parameters	Notations	**Prior**			**Post. Mode**		**Post. Mean**
		pdf	Mean	Std	Post. Mode	Sdt.	Post. Mean
Technology shock	ϵ_A	**IG**	0.001	0.02	0.0076	0.0004	0.0078
Government spending shock	ϵ_G	**IG**	0.001	0.02	0.0420	0.0019	0.0419
Markup shock	ϵ_{MCS}	**IG**	0.001	0.02	0.0113	0.0008	0.0116
Wage Markup shock	ϵ_{MRSS}	**IG**	0.001	0.02	0.0218	0.0027	0.0250
Investment shock	ϵ_{IS}	**IG**	0.001	0.02	0.0033	0.0002	0.0033
Monetary shock	ϵ_{MPS}	**IG**	0.001	0.02	0.0022	0.0002	0.0023
Preference shock	ϵ_{RPS}	**IG**	0.001	0.02	0.0186	0.0021	0.0186
AR1 technology shock	ρ_A	**B**	0.50	0.20	0.9739	0.0050	0.9744
AR1 gov. spending shock	ρ_G	**B**	0.50	0.20	0.9547	0.0094	0.9551
AR1 mark-up shock	ρ_{MCS}	**B**	0.50	0.20	0.9897	0.0067	0.9747
AR1 Wage Markup shock	ρ_{MRSS}	**B**	0.50	0.20	0.9510	0.0097	0.9495
AR1 Investment shock	ρ_{IS}	**B**	0.50	0.20	0.3179	0.0470	0.3208
AR1 Monetary shock	ρ_{MPS}	**B**	0.50	0.20	0.9760	0.0098	0.9693
AR1 Preference shock	ρ_{RPS}	**B**	0.50	0.20	0.9766	0.0088	0.9745
Investment adj cost	ϕ_X	**N**	2	0.75	0.3267	0.0606	0.3526
Inverse intertemporal EOS	σ_c	**N**	1.5	0.375	1.2561	0.0991	1.2212
Internal Habit	χ	**B**	0.5	0.1	0.2395	0.0384	0.2605
Weight on Leisure in utility	ψ	**N**	2	0.75	1.1927	0.2855	1.5056
Calvo's price	ξ_p	**B**	0.50	0.10	0.4748	0.0327	0.4754
Calvo's wage	ξ_w	**B**	0.50	0.10	0.3924	0.0466	0.4279
Price indexation	γ_p	**B**	0.50	0.10	0.2799	0.0746	0.3126
Wage indexation	γ_w	**B**	0.50	0.10	0.6482	0.0881	0.6369
Capital utilisation	γ_2	**B**	0.50	0.15	0.7933	0.0472	0.7907
Profit	F	**N**	0.25	0.250	0.5134	0.0749	0.4680
Feedback inflation	θ_π	**N**	2	0.25	2.5878	0.0284	2.6360
Lagged interest rate	ρ_r	**B**	0.70	0.10	0.7441	0.1516	0.7509
Feedback output gap	θ_y	**N**	0.125	0.05	-0.0382	0.0074	-0.0418
Feedback output growth	θ_{dy}	**N**	0.125	0.05	0.1980	0.0435	0.1967

algorithm of our estimation[10].

Overall, all estimated parameters are significantly different from zero. Most of the persistent shocks are estimated to have an autoregressive parameter that lies above 0.95, with the exception of the estimated persistent parameter on investment shock standing at 0.31. In addition, we find that the estimated price indexation parameter is smaller than the mean assumed in their prior distribution, $\gamma_p = 0.28$, but wage indexation is higher at $\gamma_w = 0.65$. Moreover, the estimated Calvo's price and wage parameters are relatively small at $\xi_p = 0.47$ and $\xi_w = 0.39$, but taken together these estimates represent a significant

[10] Results on the 5th, 50th, and 95th percentile of the posterior distribution of the parameters obtained through the Metropolis-Hastings sampling algorithm can be provided on request.

degree of price and wage stickiness and departure from full indexation. Thus a positive trend inflation leads to a significant welfare cost owing to both steady-state and dynamic costs discussed in Section 1.2. Besides, other estimated parameters' values are consistent with the results from Smets and Wouters (2007).

4 The Delegation Game

Consider a model with stationarized variables as above, but for notational convenience we drop the superscript c. Recall the nominal interest rate rule in 'implementable form':

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \tag{52}$$

which for optimal policy purposes we re-parameterize as

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) \tag{53}$$

which allows for the possibility of an integral rule with $\rho_r = 1$

Let $\rho \equiv [\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}]$ be the policy choice of feedback parameters that defines the form of the rule. The equilibrium is solved by backward induction in the following three-stage delegation game.

1. **Stage 1**: The policymaker chooses a per period probability of hitting the ZLB and designs the optimal loss function in the mandate.

2. **Stage 2**: The optimal steady state inflation rate consistent with stage 1 is chosen.

3. **Stage 3**: The CB receives the mandate in the form of a welfare criterion and rule of the form (53). Welfare is then optimized with respect to ρ resulting in an optimized rule.

This delegation game is solved by backwards induction as follows:

4.1 Stage 3: The CB Choice of Rule

Given a steady state inflation rate target, Π, the Central Bank (CB) receives a mandate to implement the rule (53) and to maximize with respect to ρ a modified welfare criterion

$$\begin{aligned}\Omega_t^{mod} &\equiv \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\left(U_{t+\tau} - w_r\left(R_{n,t+\tau} - R_n\right)^2\right)\right] \\ &= \left(U_t - w_r\left(R_{n,t} - R_n\right)^2\right) + \beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{mod}\right] \qquad (54)\end{aligned}$$

One can think of this as a mandate with a penalty function $P = w_r\left(R_{n,t} - R_n\right)^2$, penalizing the variance of the nominal interest rate with weight w_r.[11]

Following Den Haan and Wind (2012), an alternative mandate that only penalizes the zero interest rate in an asymmetric fashion is $P = P(a_t)$ where the OBC is $a_t \equiv R_{n,t} - 1 \geq 0$ with

$$P = P(a_t) = \frac{\exp(-w_r a_t)}{w_r} \qquad (55)$$

and chooses a large w_r. $P(a_t)$ then has the property

$$\begin{aligned}\lim_{w_r \to \infty} P(a_t) &= \infty \text{ for } a_t < 0 \\ &= 0 \text{ for } a_t > 0\end{aligned}$$

Thus $P(a_t)$ enforces the ZLB approximately but with more accuracy as w_r becomes large. Stages 3–1 then proceed as before, but we now confine ourselves to a large w_r which will enable Π to be close to unity.

Both the symmetric and asymmetric forms of a ZLB mandate result in a probability of hitting the ZLB

$$p = p(\Pi, \rho^*(\Pi, w_r)) \qquad (56)$$

where $\rho^*(\Pi, w_r)$ is the optimized form of the rule given the steady state target Π and the weight on the interest rate volatility, w_r.

[11] This closely follows the approximate form of the ZLB constraint of Woodford (2003) and Levine *et al.* (2008).

4.2 Stage 2: Choice of the Steady State Inflation Rate Π

Given a target low probability $\bar{p}$ and given w_r, $\Pi = \Pi^*$ is chosen so satisfy

$$p(R_{n,t} \leq 1) \equiv p(\Pi^*, \rho^*(\Pi^*, w_r)) \leq \bar{p} \tag{57}$$

This then achieves the ZLB constraint

$$R_{n,t} \geq 1 \text{ with high probability } 1 - \bar{p} \tag{58}$$

where $R_{n,t}$ is the nominal interest rate.

4.3 Stage 1: Design of the Mandate

The policymaker first chooses a per period probability $\bar{p}$ of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the actual household intertemporal welfare

$$\Omega_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} U_{t+\tau} \right] = U_t + \beta(1+g)^{1-\sigma} \mathbb{E}_t \left[\Omega_{t+1} \right] \tag{59}$$

with respect to w_r.

This three-stage delegation game defines an equilibrium in choice variables w_r^*, ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (58).

5 The ZLB Delegation Mandate

Before considering transparent simple qudratic mandates. we first examine the numerical solution of the three-stage delegation game in the estimated SW model in case where the choice of response parameters $\rho \equiv [\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}]$ is delegated to a central bank with a 'modified' objective of the form (54) where U_t is household utility and the rule takes the form (53).

5.1 Stage 3 of the Delegation Game

At Stage 3 the central bank at time t is instructed to maximize Ω_t^{mod} with respect to the feedback coefficients $\rho \equiv [\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}]$ given the long-run inflation target Π and the

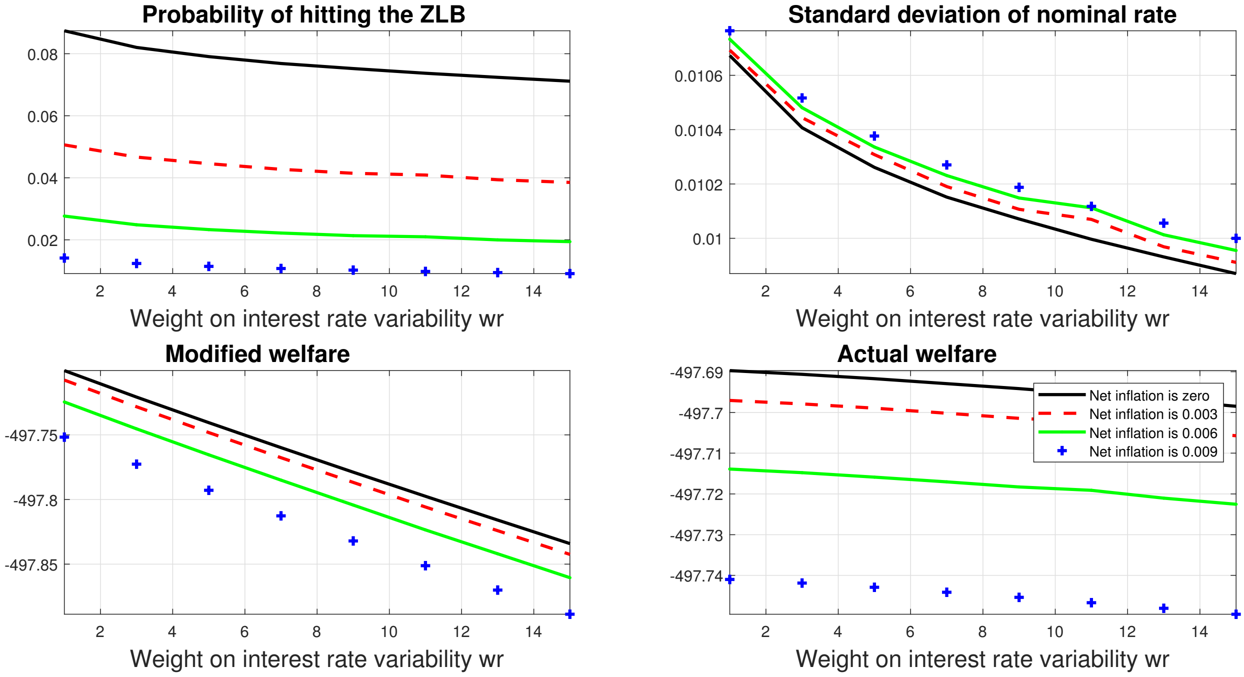

Figure 1: Plots time-varying weight on interest rate variability, w_r

weight w_r. The first subplot of figure 1 shows the relationship between the ZLB probability for each value of the gross steady state inflation rate. In particular, the probability of hitting the ZLB is a decreasing function in the level of the inflation target (see Coibion *et al.* (2012), Ngo (2017)).

However, increasing the inflation target has two opposite effects on the probability of hitting the ZLB: the first is on the first moment by shifting the density function to the right *reducing* the probability of hitting ZLB; the second effect is on the second moment making the shape of the density function more fat-tailed. Subplot 2 shows this second effect: namely that the standard deviation of the nominal rate is an *increasing* function of the inflation target thus *increasing* the probability of hitting ZLB.

From the fourth subplot of figure 1 we can see that actual welfare is a decreasing function in the inflation target. There are two remarks on this: first, if the ZLB is not taken into account, the optimal rate of net inflation is zero because there are only costs to inflation and no required shifting of the inflation rate; second, considering the set of all the steady state inflation levels which satisfies the ZLB (the orange arrow in figure 2), $\Pi^* \geq \Pi^{**} = 1.003$, the CB chooses the target inflation of $\Pi^{**} = 1.003$ with a given value of $w_r = 1$ and $\bar{p} = 0.051$. In other words, given a set of inflation target levels which satisfy the ZLB constraint, the lowest inflation target level is always chosen to maximize the actual welfare.

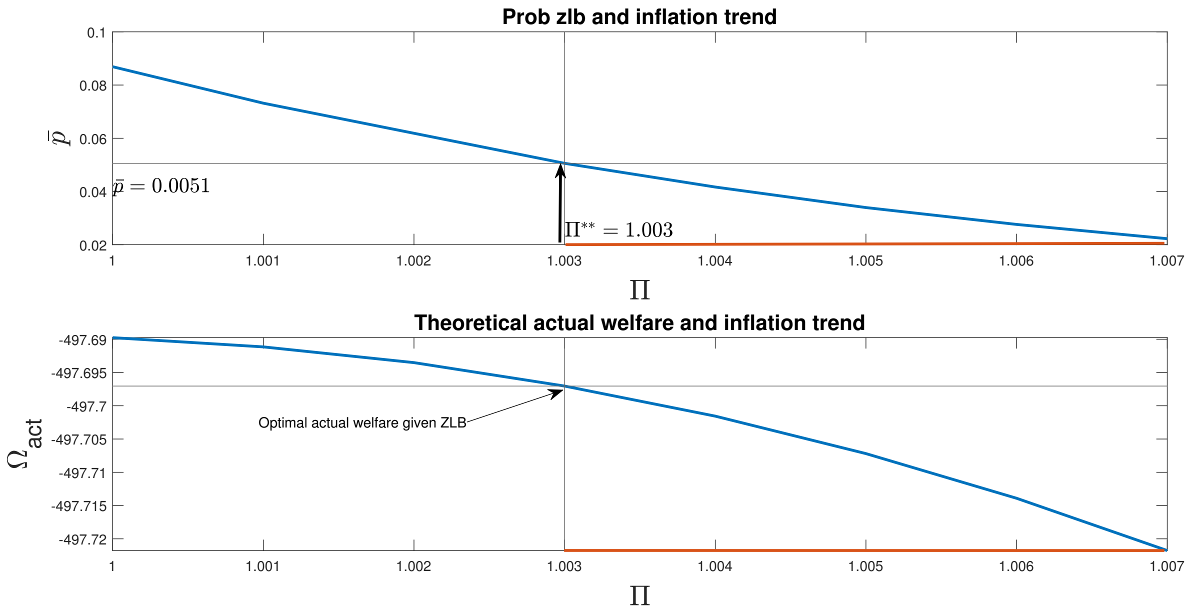

Figure 2: Figure illustrates the pick of optimal trend inflation from the set of steady state inflation satisfying the ZLB constraint.

5.2 Stages 2 and 1: Imposing the ZLB and Choice of w_r

In this section, we impose the ZLB constraint (58) where the optimal inflation target is chosen to maximize the welfare for each value of w_r as explained in the previous section.

In order to examine the model's behaviour under the binding ZLB constraint we set the value of $\bar{p} = 0.01$ quarterly. Other values of $\bar{p}$ will be examined later. Figure 3 shows the outcome for some variables under the binding ZLB constraint. The first plot of figure 3 shows the minimum values of steady state inflation rate, Π^*, which satisfies (57) in Stage 2 with equality. As we argued above, any value of Π which is larger than Π^* satisfies the ZLB constraint, but since welfare is a decreasing function of the inflation target values, the central bank will set the lowest inflation target satisfying the ZLB. In addition, with a higher level of weight attached on the variability of the nominal interest rate the central bank has is less aggressive in conducting its monetary policy in term of stabilizing the price; i.e.,we see a fall in feedback rule parameter on inflation. Finally, the equilibrium is represented by the red dotted point and is documented in table 2. Under the ZLB constraint, the optimal weight imposed on the penalty term of ZLB mandate is relatively high at $w_r^* = 14$.

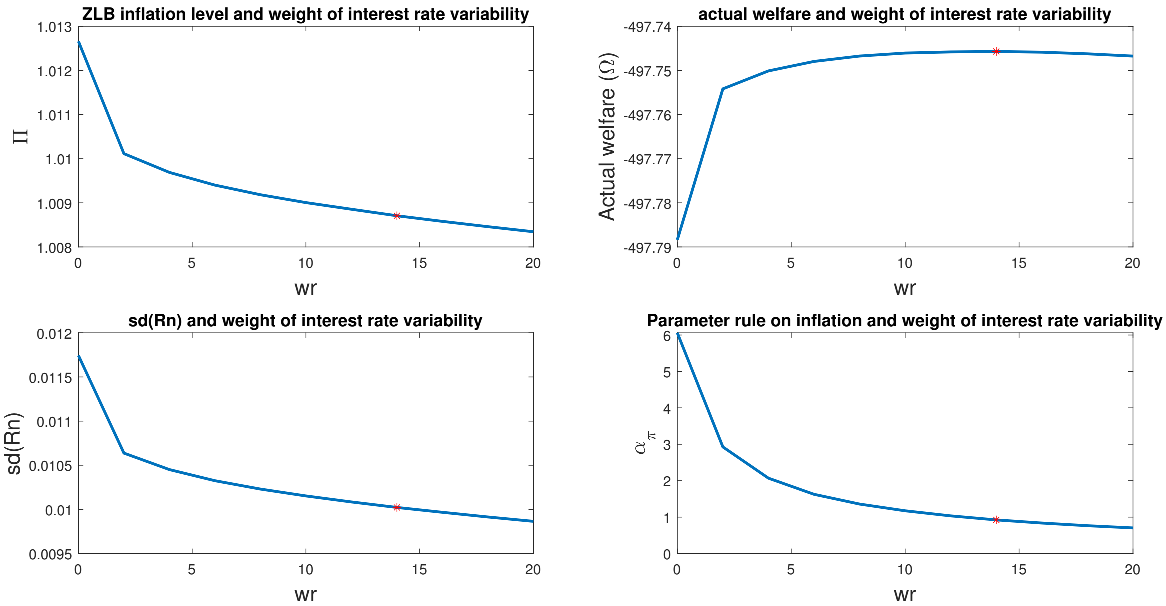

Figure 3: ZLB mandate

Given the quarterly probability of hitting the ZLB at 1% (once every 25 years), the optimal steady-state net inflation rate is then 3.5% annually. However if we relax the ZLB constraint to an allowed probability of hitting the bound of 2.5%, the optimal steady state net annual inflation is roughly 2.2%, a rate very close to the 2% inflation target of the Fed and other central banks. For a probability 5% we find a corresponding optimal inflation target of just over 1%. This optimal inflation rate is comparable to Coibion *et al.* (2012) who find that the optimal net annual inflation rate is around 1.5% in their benchmark model given their calibration of the unconditional probability of hitting the ZLB at 5% based on the post World War II US experience.

5.3 Welfare Gains from Optimization and Costs of the ZLB

In Table 2 we now assess the stabilization gains from optimized rules compared with the empirical estimated rule. By considering rules with and without the ZLB constraint its cost can also be quantified. We use the outcome of the optimized rule with zero net inflation and no ZLB constraint as the benchmark against which other regimes are measured.

Sub-table (1): Estimated Model. Given the estimated monetary policy shock of 0.33% quarterly, welfare cost of the estimated inflation trend of 0.87% (3.5% annually) is approximately a 0.15% permanent reduction in consumption per quarter.[12] However, the higher empirical inflation trend results in a smaller probability of the nominal interest hitting the ZLB, i.e. a zero steady-state inflation results in a per quarter probability of the ZLB incidence at 0.18 (approx 10 years in every 60 years), while a 3.5% of inflation trend induces a probability to 0.07 (or 4 years in every 60 years).

The **business cycle cost** is relatively significant in our NK framework with a utility function that has habit and labour supply (see sub-table **No business cycle (**$\Pi = 1.0087$**)** of Table 2).[13] Comparing the welfare outcome in the absence of shocks and with zero net inflation we see that our benchmark optimal policy without a ZLB constraint results in a CEV cost of 0.24%. This is a much higher cost of the business cycle that found in Lucas (1987) and Lucas (2003).[14]

Sub-table (2): Optimized simple rule without the ZLB constraint. Putting $w_r = 0$ for these reults, there is a significant welfare gain of the optimal rule compared to the estimated rule with the same steady-state inflation; i.e. given a zero steady-state inflation welfare increases by 0.083 CEV%. Moreover, the frequency of the nominal interest rate hitting the ZLB under optimal rule is significantly smaller than that under the estimated one. The reason for this is that the optimal rule induces a lower volatility of the model compared to the estimated one, which makes the nominal interest rate also become less volatile (hence a smaller standard deviation0. As a result, probability of the nominal interest rate hitting the ZLB becomes smaller.

We also examine the performance of the **original Taylor rule** with its parameters

[12] The consumption equivalent variations (CEV) is calculated from the table as follows:

$$CEV(w_r) = \frac{\Omega^{act}(regime_i) - \Omega^{act}(OSRw/oZLB)}{CE} \tag{60}$$

where CE is the consumption equivalent at the steady state, which represents the utility gain when consumption increases by 1 %, this value at the optimal simple rule without the ZLB constraint is equal to 1.094. Hence, the CEV is the welfare gain(loss) with different monetary regimes the society's welfare compared to when the central bank pursues an optimal simple rule without ZLB consideration.

[13] To compute this cost, we eliminate all the estimated shocks from the model, or equivalently, we calculate the steady state level of actual welfare.

[14] The higher business cycle cost for a given volatility of variables entering into utility arises from labour supply and habit in the utility function.

calibrated from Taylor (1993a). Overall, there is a large welfare loss associated with the original Taylor rule of 4–5 CEV% for the same target inflation rate and a very high probability of the nominal interest rate hitting the ZLB. This indicates that the inertia term on nominal interest rate, which is absent in the original Taylor rule, plays a crucial role in stabilizing the economy and lowering the possibility of the ZLB episode.

Sub-table (3): optimized simple rule with the ZLB constraint. To examine the welfare cost of the ZLB constraint, we now consider how changes in the frequency of nominal interest rate hitting the ZLB affect welfare by examining different values of $\bar{p}$. We consider three different levels of $\bar{p}$: 0.01 (one quarter every 25 years), 0.025 (one quarter every 10 years), and 0.05 (one quarter every 5 years). The latter corresponds to the post-WWII experience of the US used the calibration of Coibion *et al.* (2012). Proceeding from the most to the least frequent of ZLB episodes we then see the CEV cost rising from 0.007% to 0.052%.

Our framework results in this welfare loss of ZLB episodes from two sources. First, the optimal steady-state inflation rate rises which, as discussed in Section 1.2, directly generates a welfare loss in the New Keynesian model through a higher price and wage dispersion. Second, the optimal mandate (the optimal determination of the penalty term in the delegated mandate, w_r^*) generates a sub-optimal result from the social welfare point by constraining the use of the nominal interest rate for stabilization. It is worth noting that as the constraint becomes tighter, the optimized simple rule converges to a Taylor-type rule with a very high persistence.

Sub-table (4): Optimized simple rule with an asymmetric ZLB constraint. We next investigate the framework under an asymmetric formation of the ZLB mandate in the spirit of Den Haan and Wind (2012), which only penalizes the zero net interest rate in an asymmetric fashion. This type of central bank's objective function implies that there is asymmetry under various economic structures which is fully taken into account by the central bank in conducting it monetary policy. The results in the table (2) are very similar in the symmetric and asymmetric cases suggesting that our ZLB mandate result is robust across these different forms of the ZLB mandate.

Sub-table (5): Optimized price level rule. We next consider a special case of the Taylor-type simple rule (72):

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \tag{61}$$

Integrating the rule above gives:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \alpha_\pi \log\left(\frac{P_t}{\bar{P}_t}\right) \tag{62}$$

which is a *price level rule* with the trend price-level given by $\frac{\bar{P}_t}{\bar{P}_{t-1}} = \Pi$.

Giannoni (2014) argues that such simple price-level rule, comparing to the simple rule that responds to inflation, delivers superior results in several aspects of optimal monetary policy in the context of a NK model. First, price-level stabilization delivers outcomes that are closest to optimal. Second, such a policy has a robustness feature that delivers desirable outcomes even in the face of key types of model misspecification. Finally, price-level stabilization is more likely to result in a unique bounded equilibrium. Our results add a further advantage: the price-level rule closely mimics the general optimized rule with potential output feedbacks so price level targeting helps to escape the ZLB. The intuition for the benefits of price- targeting is as follows: faced with of an unexpected temporary rise in inflation price-level stabilization commits the policymaker to bring inflation below the target in subsequent periods. In contrast, with inflation targeting, the drift in the price level is accepted.

Sub-table 5 of the Table 2 represents the results of our mandate framework when the central bank is committed to price-level targeting rule. Overall, the price-level targeting rule replicates the mandate equilibrium. However, without a small trade-off between inflation and output activities there is a higher welfare loss under the price-level targeting rule compared to the optimized simple rule.

5.4 A Case of a 4% Annual Trend Inflation Target

In light of the recent consistent effective lower bound to central bank interest rates, the literature has also suggested that it is desirable to raise the inflation target to 4% to combat the ZLB on nominal interest rates (see for example Ball (2013)). In this section, we examines the impact of a 4% annual inflation trend scenario on the probability of the nominal interest rate hitting the ZLB in via our framework.

Table 2: Welfare stabilization from optimized rules compared with the estimated and original Taylor rules

	(1) Estimated model									
Steady State Est. model	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	Π	Act welfare	CEV (%)	p_zlb	w_r	MPS
No business cycle ($\Pi = 1.0087$)	0.7509	2.6360	-0.0418	0.1967	1.0087	-497.4721	0.1992	-	-	-
No business cycle ($\Pi = 1.00$)	0.7509	2.6360	-0.0418	0.1967	1.00	-497.4250	0.2424	-	-	-
Regimes	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	Π	Act welfare	CEV (%)	p_zlb	w_r	MPS
Estimated rule ($\Pi = 1$)	0.7509	2.6360	-0.0418	0.1967	1	-497.7416	-0.0479	0.1758	-	0.0
Estimated rule ($\Pi = 1$)	0.7509	2.6360	-0.0418	0.1967	1	-497.7796	-0.0826	0.1765	-	0.0033
Estimated rule ($\Pi = 1.0087$)	0.7509	2.6360	-0.0418	0.1967	1.0087	-497.8434	-0.1410	0.0693	-	0.0033
	(2) Original Taylor and optimized simple rule without ZLB									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π	Act welfare	CEV	p_zlb	w_r	MPS
Taylor ($\Pi = 1.005$)	0	1.50	0.5	0	1.005	-503.7593	-5.5485	0.41	n.a.	0.0
Taylor ($\Pi = 1.00$)	0	1.50	0.5	0	1.00	-502.4471	-4.3491	0.4311	n.a.	0.0
OSR w/o ZLB ($\Pi = 1.0$)	0.6046	5.9990	0.0306	0.9344	1.0	-497.6892	0	0.1107	0.	0.0
OSR w/o ZLB ($\Pi = 1.0087$)	0.6828	6.0338	0.0477	0.8848	1.0087	-497.7369	-0.0436	0.0693	0	0.0
	(3) Optimized simple rule with ZLB Mandate									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.923	0.0067	0.0931	1.0087	-497.7457	-0.0516	0.01	14	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9999	1.3055	0.0085	0.1501	1.0054	-497.7134	-0.0221	0.025	8	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	0.9999	1.9043	0.0104	0.2482	1.0025	-497.6967	-0.0069	0.05	4	0.0
	(4) Optimized simple rule with asymmetric ZLB Mandate									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.9757	0.0071	0.0999	1.0088	-497.7457	-0.0516	0.01	13	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9999	1.4243	0.0092	0.1679	1.0055	-497.7134	-0.0221	0.025	7	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	0.9999	2.2582	0.0	0.3643	1.0027	-497.6969	-0.0070	0.05	5	0.0
	(5) Optimized simple rule with ZLB Mandate and price-level rule									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.8345	0	0	1.0087	-497.7494	-0.0550	0.01	14	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0	1.0712	0	0	1.0053	-497.7174	-0.0258	0.025	8	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.3624	0	0	1.0023	-497.7011	-0.0109	0.05	4	0.0

Result from the Figure 4 suggests that if the inflation target is set at 4% annually (or 1.01 gross inflation quarterly), the ZLB episode would happen at a very low frequency of 7 quarters every 250 years (0.7 %). In addition, the optimized simple rule at the 4% inflation trend also converges to a price level target rule[15], which ensures the determinacy property of the simple rule. Kara and Yates (2017) Ascari and Ropele (2007) suggest that a case of 4% inflation trend would indeed give a leeway for the central bank under the effective lower bound on nominal rates episode, but will also significantly narrows the determinary

[15] The optimized Taylor rule's parameters are as follows: $[\rho_r^* \quad \alpha_\pi^* \quad \alpha_y^* \quad \alpha_{dy}^*] = [1.0 \quad 0.8509 \quad 0.0064 \quad 0.0828]$ which is close to a price-level rule.

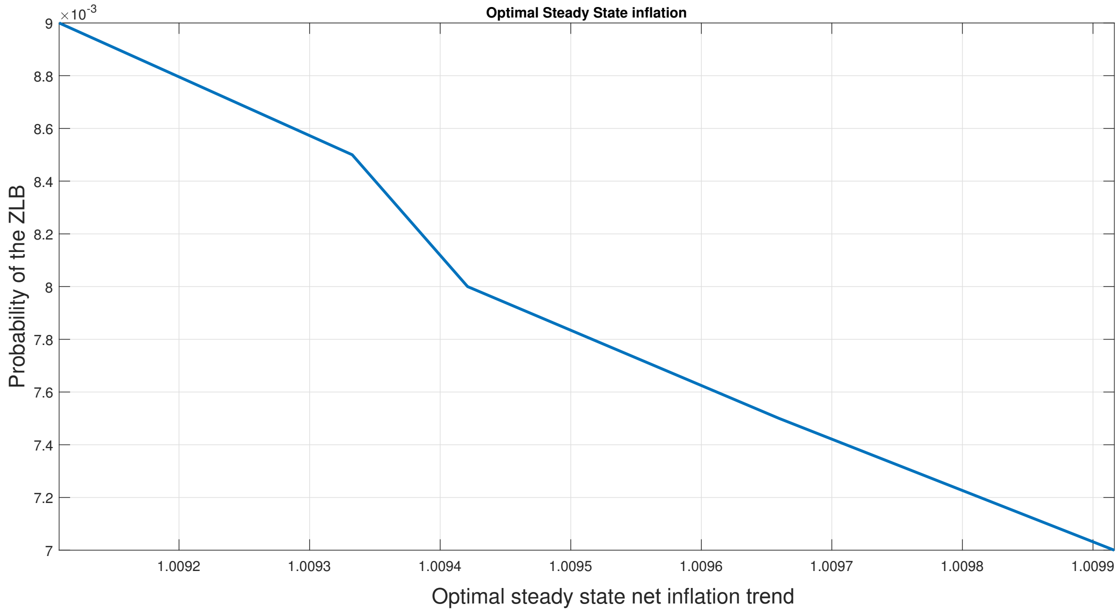

Figure 4: Plot the probability of ZLB on optimal net inflation trend

region for monetary policy rules. Our results suggest that a 4% inflation trend can combat the ZLB while minimizing the impacts on the determinary region of the rules if the central bank operates under an optimized simple rule in form of a price-level target rule.

6 Quadratic Loss Function Mandates

We now turn to simple quadratic loss function mandates designed to increase transparency in the conduct of monetary policy. In the five mandates below the particular loss function is matched with a simple rule with the same targets. Thus the transparency of the loss function is reinforced with that of the interest rate rule. We first set out the details of the mandates followed by numerical results. A final sub-section compares and discusses these results across mandates.

6.1 Mandates I - V

Our first transparent delegated **mandate I** is expressed in terms of the *level* of output Y_t and gross inflation Π_t relative to their steady state levels (Y, Π). The delegated mandate consists of the objective:

$$\Omega_t^{mod} \equiv -\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left((\Pi_t - \Pi)^2 + w_y (Y_t - Y)^2 + w_r (R_{n,t} - R_n)^2 \right) \right] \tag{63}$$

which includes, as before, a term penalizing nominal interest rate volatility that along with the choice of target gross inflation Π that enforces the ZLB constraint. The corresponding simple rule also targets (Y_t, Π_t and is given by

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) \tag{64}$$

Our second transparent delegated mandate is expressed in terms of the *growth* of output $\frac{Y_t}{Y_{t-1}}$ and gross inflation Π_t relative to their steady states. The delegated **mandate II** now consists of:

$$\Omega_t^{mod} \equiv -\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left((\Pi_t - \Pi)^2 + w_{dy} \left(\frac{Y_t}{Y_{t-1}} - (1+g) \right)^2 + w_r (R_{n,t} - R_n)^2 \right) \right] \tag{65}$$

with a corresponding simple rule:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) \tag{66}$$

Our next **mandate III** is a special form of the price-level rule corresponding to strict inflation targeting mandate:

$$\Omega_t^{mod} \equiv -\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left((\Pi_t - \Pi)^2 + w_r (R_{n,t} - R_n)^2 \right) \right] \tag{67}$$

with a corresponding simple rule

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \tag{68}$$

Mandate IV is expressed in terms of real wage growth $\frac{W_t}{W_{t-1}}$ and gross inflation Π_t relative to their steady states. The delegated mandate now consists of:

$$\Omega_t^{mod} \equiv -\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left((\Pi_t - \Pi)^2 + w_{dw} \left(\frac{W_t}{W_{t-1}} - (1+g) \right)^2 + w_r (R_{n,t} - R_n)^2 \right) \right] \tag{69}$$

with a corresponding simple rule

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_{dw} \log\left(\frac{W_t}{W_{t-1}}\right) \tag{70}$$

Our final transparent delegated **mandate V** is expressed in terms of employment growth $\frac{H_t^d}{H_{t-1}^d}$ and gross inflation Π_t relative to their steady states. The delegated mandate now consists of:

$$\Omega_t^{mod} \equiv -\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^\tau\left((\Pi_t-\Pi)^2 + w_{dh}\left(\frac{H_t^d}{H_{t-1}^d}-1\right)^2 + w_r\left(R_{n,t}-R_n\right)^2\right)\right] \tag{71}$$

Corresponding simple rule:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_{dh} \log\left(\frac{H_t^d}{H_{t-1}^d}\right) \tag{72}$$

6.2 Numerical Results

Table 3: Results for Mandate I

	Optimized simple rule with Quadratic ZLB Mandate ($w_y = 0$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.6309	0.0016	0.0	1.0082	-497.7502	-0.0558	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9846	0.9791	0.0132	0.0	1.0055	-497.7188	-0.0271	0.025	3	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	0.9847	0.9612	0.0130	0.0	1.0023	-497.7026	-0.0122	0.05	3	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_y = 0.1$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	0.9999	0.6752	0.0029	0.0	1.0083	-497.7492	-0.0548	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9999	0.6375	0.0067	0.0	1.0047	-497.7203	-0.0284	0.025	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	0.8587	1.5699	0.0113	0.0	1.0042	-497.7075	-0.0167	0.05	0	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_y^* = 0.2$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	0.9997	0.6827	0.0042	0.0	1.0084	-497.7491	-0.0548	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9997	0.5823	0.0114	0.0	1.0047	-497.7229	-0.0308	0.025	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.5143	0.0154	0.0	1.0017	-497.7149	-0.0235	0.05	0	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_y = 0.5$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	0.9996	0.6308	0.0084	0.0	1.0083	-497.7498	-0.0554	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.000	0.2621	0.0125	0.0	1.0044	-497.7747	-0.0782	0.025	3	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.1966	0.0131	0.0	1.0018	-497.8113	-0.1116	0.05	3	0.0

Table 4: Results for Mandate II

	Optimized simple rule with Quadratic ZLB Mandate ($w_{dy}=0$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	0.6331	0.0	0.0	1.0083	-497.7508	-0.0563	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	0.6328	0.0	0.0001	1.0046	-497.7215	-0.0295	0.025	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	0.8397	6.1637	0.0	1.3771	1.0041	-497.7028	-0.0124	0.05	2	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dy}^*=0.2$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.00	0.6831	0.0	0.0649	1.0083	-497.7479	-0.0537	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	0.6821	0.0	0.0647	1.0047	-497.7183	-0.0266	0.025	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	0.6812	0.0	0.0645	1.0016	-497.7046	-0.0141	0.05	1	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dy}=0.5$)									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	0.7757	0.0	0.1792	1.0086	-497.7506	-0.0561	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	0.7744	0.0	0.1795	1.0049	-497.7198	-0.0280	0.025	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.000	0.7732	0.0	0.1798	1.0018	-497.7054	-0.0148	0.05	1	0.0

Table 5: Results for Mandate III

	Optimized simple rule with Quadratic ZLB Mandate									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	0.9323	0.0	0.0	1.0088	-497.7495	-0.0551	0.01	0.6	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	1.2392	0.0	0.0	1.0055	-497.7174	-0.0258	0.025	0.4	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.2392	0.0	0.0	1.0022	-497.7012	-0.0110	0.05	0.40	0.0

Tables 3-7 set out the numerical results for our 5 quadratic mandates. The optimal mandate is described by the choice of weights (w_r^*, w_y^*) for mandate I, (w_r^*, w_{dy}^*) for mandate II, w_r^* for mandate III, (w_r^*, w_{dw}^*) for mandate IV and (w_r^*, w_{dh}^*) for mandate V. This choice depends on the tightness of the ZLB constraint, $\bar{p}_{zlb}$. In the tables we report the welfare-optimal mandate for the case $\bar{p}_{zlb}=0.01$.

For these optimal mandates we find that the weights attached to real economic activities (i.e. output or labour demand) are relatively small compared with the weight attached to price inflation. The exception is for mandate IV, targeting directly both nominal price and real wage growth where we find the relative optimal weights attached on nominal price and wage inflation volatility are close to equality. This result is in contrast with Debortoli *et al.* (2019) who find that simple loss functions should feature a high weight on measures of economic activity, sometimes even larger than the weight on inflation. The source of this difference lies in the presence of the ZLB constraint coupled with the optimized

Table 6: **Results for Mandate IV**

	Optimized simple rule with Quadratic ZLB Mandate ($w_{dw}=0.2$)									
Regimes	ρ_r^*	α_π^*	–	α_{dw}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	1.0659	0.0	0.0941	1.0089	-497.7429	-0.0491	0.01	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	1.0621	0.0	0.0919	1.0052	-497.7110	-0.0199	0.025	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.0597	0.0	0.0902	1.0020	-497.6960	-0.0062	0.05	0.0	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dw}^*=0.8$)									
Regimes	ρ_r^*	α_π^*	–	α_{dw}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.00	1.1641	0.0	0.3764	1.0089	-497.7369	-0.0436	0.01	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.00	1.1559	0.0	0.3711	1.0052	-497.7047	-0.0142	0.025	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.1493	0.0	0.3667	1.0026	-497.6895	-0.0003	0.05	0.0	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dw}=1.5$)									
Regimes	ρ_r^*	α_π^*	–	α_{dw}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.00	1.2842	0.0	0.6682	1.0091	-497.7428	-0.0490	0.01	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.00	1.9546	0.0	1.000	1.0057	-497.7082	-0.0174	0.025	0.0	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	0.9999	1.9579	0.0	0.9999	1.0024	-497.6906	-0.0013	0.05	0.0	0.0

Table 7: Results for Mandate V

	Optimized simple rule with Quadratic ZLB Mandate ($w_{dh}=0.05$)									
Regimes	ρ_r^*	α_π^*	–	α_{dh}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	1.0753	0.0	0.0002	1.0090	-497.7500	-0.0556	0.01	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.0	1.0729	0.0	0.0002	1.0053	-497.7174	-0.0258	0.025	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.0711	0.0	0.0002	1.0021	-497.7018	-0.0115	0.05	0.0	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dh}=0.1$)									
Regimes	ρ_r^*	α_π^*	–	α_{dh}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.00	1.0977	0.0	0.0156	1.0090	-497.7499	-0.0555	0.01	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.00	1.0931	0.0	0.0143	1.0053	-497.7170	-0.0254	0.025	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.0898	0.0	0.0135	1.0021	-497.7014	-0.0112	0.05	0.5	0.0
	Optimized simple rule with Quadratic ZLB Mandate ($w_{dh}^*=0.2$)									
Regimes	ρ_r^*	α_π^*	–	α_{dh}^*	Π^*	Act welfare	CEV	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.00	0.7022	0.0	0.0252	1.0084	-497.7496	-0.0552	0.01	1	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.025$)	1.00	1.1708	0.0	0.0888	1.0054	-497.7163	-0.0248	0.025	0.5	0.0
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.00	1.1639	0.0	0.0866	1.0022	-497.7002	-0.0101	0.05	0.5	0.0

appropriate interest rate rule in our framework. While the latter paper studies a Ramsey problem of the simple loss function as parsimonious approximations to social welfare, our paper investigates the simple rule regime where the central bank has delegated a loss function under a presence of the ZLB constraint. For example, under a strict inflation targeting mandate I, $w_y^* = w_{dy}^* = 0.2$ which gives a regime close to a strict inflation targeting. The central bank then reacts more aggressively to the inflation component of its optimized simple rule; by concentrating aggressively on the inflation volatility the central bank mitigates the welfare cost of price dispersion due to the high level of trend inflation and at the same time reduces the frequency of hitting the ZLB constraint. For the best mandate IV, which involve real wage growth targeting, the optimized interest rate rule responds strongly to this component. In doing so it also reduces nominal wage dispersion resulting in a further welfare benefit.[16] The associated optimal mandate then has a weight on wage growth close to that on inflation.

6.3 General Discussion

We have examined several forms of transparent loss function mandates that impose the ZLB constraint by penalizing the volatility of nominal interest rate. The results are striking in several respects. First, from the society welfare criterion, the integrated-ZLB conventional inflation-output optimal mandate I results in the highest welfare loss, whilst the nominal price and wage inflation targeting mandate IV leads to the lowest. The consumption equivalent differences are quite small and of the order 0.01%. But for all optimal mandates there is a substantial welfare stabilization gain of the order of 0.1% compared with the outcome of the estimated rule. Generally in our NK model with labour supply and habit in the utility function and with price and wage stickiness both the welfare costs of the business cycle and the potential gains from stabilization policy are substantially greater than those found by Lucas (2003) and in the RBC model without such distortions.

Second, in the quadratic mandates, the optimal weights attached to real economic activities (i.e. output or labour demand) are relatively small compared with the weight attached to price inflation. Only for mandate IV targeting directly both nominal price and real wage growth do we find that the relative optimal weights attached to these targets are close to unity. This result is in contrast with Debortoli *et al.* (2019) who find that simple loss functions should feature a high weight on measures of economic activity, sometimes even larger than the weight on inflation. The source of this difference has been discussed in sub-section 6.2.

Third, the associating optimized simple rules all converge to a price-level targeting rule, or at least to an interest-rate rule with the smoothing component on the nominal interest rate close to unity. Overall, the central bank reacts positively to output and labour

[16] This result is consistent with that of Levine *et al.* (2008) who find that an optimized simple rule that targets the real wage closely mimics the Ramsey solution.

demand, so a looser monetary policy corrects for a decline in these real economic variables. However, because the optimized weights in the loss function are relatively small, these real economic activity components also carry a small weight in the optimized simple rules.

Fourth, the formation of the optimal mandates has a little impact on the optimal steady state inflation level. Overall, the optimal steady state inflation does not vary across different formations of the mandate and the tightness level of the ZLB (the allowed probability of the nominal interest rate hitting the ZLB) is the main driving force on the optimal steady state inflation. The allowed highest quarterly probability of 5% (once every 5 years) corresponds to the calibration employed by Coibion *et al.* (2012) which they note is consistent with the historical experience of the ZLB frequency for the US since 1945. They find an optimal inflation trend rate in the region 1-2%, a result rather higher that our finding of close to 1%. There are two possible reasons for this discrepancy: first, our model is estimated by Bayesian methods with an empirical inflation trend and could claim to be more empirical; second, our choice of welfare function to design the mandate avoids any quadratic steady-state small-distortions approximation. We use a second-order perturbation approach that only assumes the variance of shocks are small which is confirmed in the estimation. We suspect that the first of these two reasons is the more important.

Finally in the 2% versus 4% debate over the optimal inflation target (see Section 5.4) our results favour the latter. The higher inflation target drives the frequency of ZLB episodes down to $\bar{p}_{zbb} = 0.007$ to (7 quarters every 250 years) with an optimized interest rate rule close to a price-level rule. As we have noted, the latter has good determinacy properties even at the higher inflation trend.

7 Conclusions

Our paper has presented a general framework for monetary policy delegated to an instrument-independent but goal-dependent central bank. The latter is required to optimize a particular form of interest rate rule with a steady state inflation target. The goal mandate includes a penalty on the interest rate variance chosen to be welfare-optimal given the constraint of hitting the ZLB with a particular frequency. Different goal mandates are considered all associated with Taylor-type simple rules with the same target variables. Moreover, the optimal inflation target found from this paper supports the view that raising the inflation target above the standard 2% per year to 4% is a feasible welfare-enhancing mandate for the central banks to deal very effectively with the ZLB in that the quarterly frequency of violating the constraint falls to well under 1%. The price-level form of the optimized rule avoids the indeterminacy problem highlighted by proponents of the 2% target.

Future work will explore a number of directions: first, to develop a general fiscal-monetary mandate (sketched in Appendix E); second to follow Coibion *et al.* (2012) and examine state-dependent wage and price contracts with endogenous durations; and third to study our mandate framework in a behavioural NK model that relaxes the rational expectations assumption in different ways.[17]

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Appendix

A Stationary equilibrium

To stationarize the model labour-augmenting technical progress parameter is decomposed into a cyclical component, stationary A_t^c, and a deterministic trend $\bar{A}_t$:

$$
\begin{aligned}
A_t &= \bar{A}_t A_t^c \\
\bar{A}_t &= (1+g)\bar{A}_{t-1}
\end{aligned}
$$

Then we can define stationarized variables by

$$
\begin{aligned}
\frac{\Omega_t}{\bar{A}_t^{1-\sigma}} &= \frac{U_t}{\bar{A}_t^{1-\sigma}} + \beta E_t \frac{\Omega_{t+1}}{\bar{A}_{t+1}^{1-\sigma}} \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{1-\sigma} \\
\frac{U_t}{\bar{A}_t^{1-\sigma}} &= \frac{\left[\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \\
\Lambda_{t,t+1} &= \beta \frac{U_{C,t+1}}{U_{C,t}} = \beta(1+g)^{(1-\varrho)(1-\sigma)-1} \frac{U_{C,t+1}^c}{U_{C,t}^c} \equiv \beta_g \frac{U_{C,t+1}^c}{U_{C,t}^c}
\end{aligned}
$$

where the growth-adjusted discount rate is defined as

$$
\beta_g \equiv \beta(1+g)^{1-\sigma},
$$

the Euler equation is still

$$
E_t\left[\Lambda_{t,t+1} R_{t+1}\right]
$$

Now stationarize remaining variables by defining cyclical components:

$$
\begin{aligned}
\frac{U_{C,t}}{\bar{A}_t^{-\sigma}} &= \frac{(1-\sigma)\frac{U_t}{\bar{A}_t^{1-\sigma}}}{\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}} - \beta\chi \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma} \frac{(1-\sigma)\frac{U_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{A}_{t+1}} - \chi \frac{C_t}{\bar{A}_t} \frac{\bar{A}_t}{\bar{A}_{t+1}}} \\
Y_t^c &\equiv \frac{Y_t}{\bar{A}_t} = \frac{(A_t H_t^d)^\alpha \left(\frac{K_{t-1}}{\bar{A}_t}\right)^{1-\alpha} - \frac{F_t}{\bar{A}_t}}{\Delta_t^p} = \frac{(A_t H_t^d)^\alpha \left(\frac{K_{t-1}^c}{(1+g_t)}\right)^{1-\alpha} - F}{\Delta_t^p} \\
K_t^c &\equiv \frac{K_t}{\bar{A}_t}
\end{aligned}
$$

$$
\begin{aligned}
K_t^c &= (1-\delta)\frac{K_{t-1}^c}{1+g_t} + (1-S(X_t^c))I_t^c \\
X_t^c &= (1+g_t)\frac{I_t^c}{I_{t-1}^c} \\
S(X_t^c) &= \phi_X(X_t^c - 1 - g_t)^2 \\
S'(X_t^c) &= 2\phi_X(X_t^c - 1 - g_t) \\
C_t^c &\equiv \frac{C_t}{\bar{A}_t} \\
I_t^c &\equiv \frac{I_t}{\bar{A}_t} \\
W_t^c &\equiv \frac{W_t}{\bar{A}_t}
\end{aligned}
$$

Rewrite the equilibrium conditions as

Household:

$$\frac{\Omega_t}{\bar{A}_t^{1-\sigma}} = \frac{U_t}{\bar{A}_t^{1-\sigma}} + \beta E_t \frac{\Omega_{t+1}}{\bar{A}_{t+1}^{1-\sigma}} \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{1-\sigma} \tag{A.1}$$

$$\frac{U_t}{\bar{A}_t^{1-\sigma}} = \frac{\left[\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \tag{A.2}$$

$$\frac{K_t}{\bar{A}_t} = (1-\delta)\frac{K_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t} + (1-S(X_t))\frac{I_t}{\bar{A}_t} IS_t \tag{A.3}$$

$$X_t = \frac{\frac{I_t}{\bar{A}_t}\bar{A}_t}{\frac{I_{t-1}}{\bar{A}_{t-1}}\bar{A}_{t-1}} \tag{A.4}$$

$$S(X_t) = \phi_X(X_t - 1 - g)^2 \tag{A.5}$$

$$S'(X_t) = 2\phi_X(X_t - 1 - g) \tag{A.6}$$

$$\frac{\lambda_t}{\bar{A}_t^{-\sigma}} = \frac{(1-\sigma)\frac{U_t}{\bar{A}_t^{1-\sigma}}}{\frac{C_t}{\bar{A}_t} - \chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}} - \beta\chi\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma} \frac{(1-\sigma)\frac{U_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{A}_{t+1}} - \chi\frac{C_t}{\bar{A}_t}\frac{\bar{A}_t}{\bar{A}_{t+1}}} \tag{A.7}$$

$$\frac{W_{h,t}}{\bar{A}_t} = \frac{\left[\frac{C_t}{\bar{A}_t} - \chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}\right] H_t^{\psi}}{1 - \beta\chi\frac{U_{t+1}/\bar{A}_{t+1}^{1-\sigma}}{U_t/\bar{A}_t^{1-\sigma}}\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma}\frac{\frac{C_t}{\bar{A}_t}-\chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}}{\frac{C_{t+1}}{\bar{A}_{t+1}}-\chi\frac{C_t}{\bar{A}_t}\frac{\bar{A}_t}{\bar{A}_{t+1}}}} \tag{A.8}$$

$$r_t^K = a'(u_t) \tag{A.9}$$

$$1 = RPS_t \mathbb{E}_t\left[\Lambda_{t,t+1}R_{t+1}\right] \tag{A.10}$$

$$Q_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[r^K_{t+1} u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta) \right] \right\} \tag{A.11}$$

$$\begin{aligned} 1 &= Q_t \left[1 - S(X_t) - S'(X_t)X_t\right] IS_t \\ &+ \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2 IS_{t+1} \right] \end{aligned} \tag{A.12}$$

$$\Lambda_{t,t+1} = \beta \frac{\frac{\lambda_{t+1}}{\bar{A}_{t+1}^{-\sigma}} \bar{A}_{t+1}^{-\sigma}}{\frac{\lambda_t}{\bar{A}_t^{-\sigma}} \bar{A}_t^{-\sigma}} \tag{A.13}$$

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t} \right] \tag{A.14}$$

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{1-\gamma_2} \frac{\gamma_1}{2} (u_t - 1)^2 \tag{A.15}$$

$$a'(u_t) = \gamma_1 + \frac{\gamma_2}{1-\gamma_2} \gamma_1 (u_t - 1) \tag{A.16}$$

Wage setting:

$$\Pi_t^w = \frac{\frac{W_t}{\bar{A}_t}}{\frac{W_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}} \Pi_t \tag{A.17}$$

$$\begin{aligned} \frac{J_t^w}{\bar{A}_t} &= \frac{1}{1 - \frac{1}{\zeta_w}} \frac{W_{h,t}}{\bar{A}_t} H_t^d MRSS_t \\ &+ \xi_w \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w \zeta_w}} \frac{J_{t+1}^w}{\bar{A}_{t+1}} \frac{\bar{A}_{t+1}}{\bar{A}_t} \end{aligned} \tag{A.18}$$

$$JJ_t^w = H_t^d + \xi_w \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w(\zeta_w - 1)} \Pi_{t,t+1}} JJ_{t+1}^w \tag{A.19}$$

$$\frac{W_{n,t}^O}{W_{n,t}} = \frac{\frac{J_t^w}{\bar{A}_t}}{\frac{W_t}{\bar{A}_t} JJ_t^w} \tag{A.20}$$

$$1 = \xi_w \left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t^w} \right)^{1-\zeta_w} + (1-\xi_w) \left(\frac{W_{n,t}^O}{W_{n,t}} \right)^{1-\zeta_w} \tag{A.21}$$

$$\Delta_{w,t} = \xi_w \frac{(\Pi_t^w)^{\zeta_w}}{\Pi_{t-1}^{\zeta_w \gamma_w}} \Delta_{w,t-1} + (1-\xi_w) \left(\frac{W_{n,t}^O}{W_{n,t}} \right)^{-\zeta_w} \tag{A.22}$$

Retail firm:

$$\frac{Y_t^W}{\bar{A}_t} = \left(\frac{A_t}{\bar{A}_t} H_t^d \right)^{\alpha} \left(u_t \frac{K_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t} \right)^{1-\alpha} - \tilde{F} \frac{Y_t^W}{\bar{A}_t} \tag{A.23}$$

$$\frac{W_t}{\bar{A}_t} = \alpha \frac{P_t^W}{P_t} \frac{\frac{Y_t^W}{\bar{A}_t} + \tilde{F} \frac{Y_t^W}{\bar{A}_t}}{H_t^d} \tag{A.24}$$

$$r_t^K = (1-\alpha)\frac{P_t^W}{P_t}\frac{\frac{Y_t^W}{\bar{A}_t} + \tilde{F}\frac{Y_t^W}{\bar{A}_t}}{u_t\frac{K_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}} \tag{A.25}$$

Price setting:

$$MC_t = \frac{P_t^W}{P_t} \tag{A.26}$$

$$\frac{J_t^p}{\bar{A}_t} = \frac{1}{1-\frac{1}{\zeta_p}}\frac{Y_t}{\bar{A}_t}MC_t MCS_t + \xi_p \mathbb{E}_t \Lambda_{t,t+1}\frac{(\Pi_{t,t+1})^{\zeta_p}}{(\Pi_{t-1,t})^{\gamma_p\zeta_p}}\frac{J_{t+1}^p}{\bar{A}_{t+1}}\frac{\bar{A}_{t+1}}{\bar{A}_t} \tag{A.27}$$

$$\frac{JJ_t^p}{\bar{A}_t} = \frac{Y_t}{\bar{A}_t} + \xi_p \mathbb{E}_t \Lambda_{t,t+1}\frac{(\Pi_{t,t+1})^{\zeta_p-1}}{(\Pi_{t-1,t})^{\gamma_p(\zeta_p-1)}}\frac{JJ_{t+1}^p}{\bar{A}_{t+1}}\frac{\bar{A}_{t+1}}{\bar{A}_t} \tag{A.28}$$

$$\frac{P_t^0}{P_t} = \frac{\frac{J_t^p}{\bar{A}_t}}{\frac{JJ_t^p}{\bar{A}_t}} \tag{A.29}$$

$$1 = \xi_p\left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{1-\zeta_p} \tag{A.30}$$

$$\Delta_{p,t} = \xi_p\frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p\gamma_p}}\Delta_{p,t-1} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{-\zeta_p} \tag{A.31}$$

Monetary policy:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r\log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\pi\log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y\log\left(\frac{Y_t}{Y}\right) + \theta_{dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \log MPS_t \tag{A.32}$$

Aggregation:

$$\frac{Y_t}{\bar{A}_t} = \frac{C_t}{\bar{A}_t} + \frac{G_t}{\bar{A}_t} + \frac{I_t}{\bar{A}_t} + \frac{a(u_t)}{IS_t}\frac{K_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t} \tag{A.33}$$

$$H_t = \Delta_{w,t}H_t^d \tag{A.34}$$

$$\frac{Y_t^W}{\bar{A}_t} = \Delta_{p,t}\frac{Y_t}{\bar{A}_t} \tag{A.35}$$

$$R_t^K = \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}} \tag{A.36}$$

Shock processes:

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{A.37}$$

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{A.38}$$

$$\log MCS_t - \log MCS = \rho_{MCS}(\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t} \tag{A.39}$$

$$\log MRSS_t - \log MRSS = \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \tag{A.40}$$

$$\log IS_t - \log IS = \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{A.41}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{A.42}$$

$$\log RPS_t - \log RPS = \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \tag{A.43}$$

Use change of variables to arrive to the following equilibrium conditions:

Household:

$$\Omega_t^c = U_t^c + \beta(1+g)^{1-\sigma} E_t \Omega_{t+1}^c \tag{A.44}$$

$$U_t^c = \frac{\left[C_t^c - \chi \frac{C_{t-1}^c}{1+g}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \tag{A.45}$$

$$K_t^c = (1-\delta)\frac{K_{t-1}^c}{1+g} + (1-S(X_t))I_t^c IS_t \tag{A.46}$$

$$X_t = \frac{I_t^c}{I_{t-1}^c}(1+g) \tag{A.47}$$

$$S(X_t) = \phi_X(X_t - 1 - g)^2 \tag{A.48}$$

$$S'(X_t) = 2\phi_X(X_t - 1 - g) \tag{A.49}$$

$$\lambda_t^c = \frac{(1-\sigma)U_t^c}{C_t^c - \chi\frac{C_{t-1}^c}{1+g}} - \beta\chi(1+g)^{-\sigma}\frac{(1-\sigma)U_{t+1}^c}{C_{t+1}^c - \chi\frac{C_t^c}{1+g}} \tag{A.50}$$

$$W_{h,t}^c = \frac{\left[C_t^c - \chi\frac{C_{t-1}^c}{1+g}\right] H_t^\psi}{1 - \beta\chi(1+g)^{-\sigma}\frac{U_{t+1}^c}{U_t^c}\frac{C_t^c - \chi\frac{C_{t-1}^c}{1+g}}{C_{t+1}^c - \chi\frac{C_t^c}{1+g}}} \tag{A.51}$$

$$r_t^K = a'(u_t) \tag{A.52}$$

$$1 = RPS_t \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}\right] \tag{A.53}$$

$$Q_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[r^K_{t+1} u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta) \right] \right\} \tag{A.54}$$

$$1 = Q_t \left[1 - S(X_t) - S'(X_t) X_t \right] IS_t + \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X^2_{t+1} IS_{t+1} \right] \tag{A.55}$$

$$\Lambda_{t,t+1} = \beta(1+g)^{-\sigma} \frac{\lambda^c_{t+1}}{\lambda^c_t} \tag{A.56}$$

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t} \right] \tag{A.57}$$

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{1-\gamma_2} \frac{\gamma_1}{2} (u_t - 1)^2 \tag{A.58}$$

$$a'(u_t) = \gamma_1 + \frac{\gamma_2}{1-\gamma_2} \gamma_1 (u_t - 1) \tag{A.59}$$

Wage setting:

$$\Pi^w_t = (1+g) \frac{W^c_t}{W^c_{t-1}} \Pi_t \tag{A.60}$$

$$J^{w,c}_t = \frac{1}{1-\frac{1}{\zeta_w}} W^c_{h,t} H^d_t MRSS_t + \xi_w (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi^w_{t,t+1}\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w \zeta_w}} J^{w,c}_{t+1} \tag{A.61}$$

$$JJ^w_t = H^d_t + \xi_w \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi^w_{t,t+1}\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w(\zeta_w - 1)} \Pi_{t,t+1}} JJ^w_{t+1} \tag{A.62}$$

$$\frac{W^O_{n,t}}{W_{n,t}} = \frac{J^{w,c}_t}{W^c_t JJ^w_t} \tag{A.63}$$

$$1 = \xi_w \left(\frac{\Pi^{\gamma_w}_{t-1}}{\Pi^w_t} \right)^{1-\zeta_w} + (1-\xi_w) \left(\frac{W^O_{n,t}}{W_{n,t}} \right)^{1-\zeta_w} \tag{A.64}$$

$$\Delta_{w,t} = \xi_w \frac{(\Pi^w_t)^{\zeta_w}}{\Pi^{\zeta_w \gamma_w}_{t-1}} \Delta_{w,t-1} + (1-\xi_w) \left(\frac{W^O_{n,t}}{W_{n,t}} \right)^{-\zeta_w} \tag{A.65}$$

Retail firm:

$$Y^{W,c}_t = \left(A^c_t H^d_t \right)^{\alpha} \left(u_t \frac{K^c_{t-1}}{1+g} \right)^{1-\alpha} - \tilde{F} Y^{W,c} \tag{A.66}$$

$$W^c_t = \alpha \frac{P^W_t}{P_t} \frac{Y^{W,c}_t + \tilde{F} Y^{W,c}}{H^d_t} \tag{A.67}$$

$$r^K_t = (1-\alpha) \frac{P^W_t}{P_t} \frac{Y^{W,c}_t + \tilde{F} Y^{W,c}}{u_t \frac{K^c_{t-1}}{1+g}} \tag{A.68}$$

Price setting:

$$MC_t = \frac{P_t^W}{P_t} \tag{A.69}$$

$$\begin{aligned} J_t^{p,c} &= \frac{1}{1-\frac{1}{\zeta_p}} Y_t^c MC_t MCS_t \\ &+ \xi_p (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p}}{(\Pi_{t-1,t})^{\gamma_p \zeta_p}} J_{t+1}^{p,c} \end{aligned} \tag{A.70}$$

$$JJ_t^{p,c} = Y_t^c + \xi_p (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p - 1}}{(\Pi_{t-1,t})^{\gamma_p(\zeta_p - 1)}} JJ_{t+1}^{p,c} \tag{A.71}$$

$$\frac{P_t^0}{P_t} = \frac{J_t^{p,c}}{JJ_t^{p,c}} \tag{A.72}$$

$$1 = \xi_p \left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t} \right)^{1-\zeta_p} + (1-\xi_p) \left(\frac{P_t^0}{P_t} \right)^{1-\zeta_p} \tag{A.73}$$

$$\Delta_{p,t} = \xi_p \frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p \gamma_p}} \Delta_{p,t-1} + (1-\xi_p) \left(\frac{P_t^0}{P_t} \right)^{-\zeta_p} \tag{A.74}$$

Monetary policy:

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &+ (1-\rho_r) \left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t^c}{Y^c}\right) + \theta_{dy} \log\left(\frac{Y_t^c}{Y_{t-1}^c}\right) \right) \\ &+ \log MPS_t \end{aligned} \tag{A.75}$$

Aggregation:

$$Y_t^c = C_t^c + G_t^c + I_t^c + a(u_t) \frac{K_{t-1}^c}{1+g} \tag{A.76}$$

$$H_t = \Delta_{w,t} H_t^d \tag{A.77}$$

$$Y_t^{W,c} = \Delta_{p,t} Y_t^c \tag{A.78}$$

$$R_t^K = \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}} \tag{A.79}$$

Shock processes:

$$\log A_t^c - \log A^c = \rho_A (\log A_{t-1}^c - \log A^c) + \epsilon_{A,t} \tag{A.80}$$

$$\log G_t^c - \log G^c = \rho_G(\log G_{t-1}^c - \log G^c) + \epsilon_{G,t} \tag{A.81}$$

$$\log MCS_t - \log MCS = \rho_{MCS}(\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t} \tag{A.82}$$

$$\log MRSS_t - \log MRSS = \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \tag{A.83}$$

$$\log IS_t - \log IS = \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{A.84}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{A.85}$$

$$\log RPS_t - \log RPS = \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \tag{A.86}$$

This is a system of 44 equation in the following 44 "variables" (in order of appearance): V^c, U^c, C^c, H, K^c, $S(X)$, X, I^c, IS, $S'(X)$, λ^c, W_h^c, r^K, $a'(u)$, RPS, Λ, R, Q, u, $a(u)$, R_n, Π, Π^w, W^c, $J^{w,c}$, H^d, $MRSS$, JJ^w, $\frac{W_n^O}{W_n}$, Δ_w, $Y^{W,c}$, A^c, $\frac{P^W}{P}$, MC, $J^{p,c}$, Y^c, MCS, $JJ^{p,c}$, $\frac{P^0}{P}$, Δ_p, MPS, G^c, R^K.

Finally we define a consumption equivalent welfare measure CE_t as the inter-temporal increase in welfare resulting from a permanent 1% increase in the equilibrium path of consumption as

$$\begin{aligned} CE_t &= \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(1.01C_{t+s}, 1.01C_{t-1+s}, H_{t+s})\right] \\ &- \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(C_{t+s}, C_{t-1+s}, H_{t+s})\right] \\ &= \frac{[1.01C_t - \chi 1.01C_{t-1}]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] - U(C_t, C_{t-1}, H_t) \\ &+ \beta\mathbb{E}_t CE_{t+1} \\ &= (1.01^{1-\sigma} - 1)U_t + \beta\mathbb{E}_t CE_{t+1} \end{aligned} \tag{A.87}$$

The stationary version is then

$$CE_t^c = (1.01^{1-\sigma} - 1)U_t^c + \beta(1+g)^{1-\sigma}\mathbb{E}_t CE_{t+1}^c \tag{A.88}$$

In our results we compute consumption equivalent differences using the stationary steady state CE^c.

A.1 Balanced-Growth Steady state

Having stationarized the model we now drop the superscript c. The exogenous variables have steady states $A^c = MCS = MRSS = IS = MPS = RPS = 1$, $G = g_y Y$. Moreover,

$u = 1$ in steady state. Given the steady state inflation rate Π and hours H, the steady state values of the other variables can be computed in stationary form as

$$
\begin{aligned}
S(X) &= 0 \\
S'(X) &= 0 \\
\Pi^w &= (1+g)\Pi \\
Q &= 1 \\
\Lambda &= \beta(1+g)^{-\sigma} \\
r^K &= \frac{1}{\Lambda} - (1-\delta) \\
a(u) &= 0 \\
a'(u) &= \gamma_1 \\
r^K &= \gamma_1 \Rightarrow \gamma_1 = \frac{1}{\beta(1+g)^{-\sigma}} - (1-\delta) \\
\frac{P^0}{P} &= \left(\frac{1-\xi_p \Pi^{(1-\gamma_p)(\zeta_p-1)}}{1-\xi_p}\right)^{\frac{1}{1-\zeta_p}} \\
\Delta_p &= \frac{1-\xi_p}{1-\xi_p \Pi^{\zeta_p(1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p} \\
MC &= \left(1-\frac{1}{\zeta_p}\right) \frac{1-\xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}}{1-\xi_p(1+g)\Lambda\Pi^{(\zeta_p-1)(1-\gamma_p)}} \frac{P^0}{P} \\
\frac{P^W}{P} &= MC \\
\frac{W_n^O}{W_n} &= \left(\frac{1-\xi_w \Pi^{\gamma_w(1-\zeta_w)}(\Pi^w)^{\zeta_w-1}}{1-\xi_w}\right)^{\frac{1}{1-\zeta_w}} \\
\Delta_w &= \frac{1-\xi_w}{1-\xi_w \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\zeta_w\gamma_w}}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w} \\
H^d &= \frac{H}{\Delta_w} \\
\frac{K}{Y^W} &= \frac{(1-\alpha)(1+g)(1+\tilde{F})}{ur^K} \frac{P^W}{P} \\
Y^W &= \frac{H^d}{(1+\tilde{F})^{\frac{1}{\alpha}}} \left(\frac{\frac{K}{Y^W}}{1+g}\right)^{\frac{1-\alpha}{\alpha}} \\
K &= Y^W \frac{K}{Y^W} \\
Y &= \frac{Y^W}{\Delta_p}
\end{aligned}
$$

$$
\begin{aligned}
I &= \frac{K^c}{1}\frac{g+\delta}{1+g} \\
G &= g_y Y \\
C &= Y - G - I \\
JJ^w &= \frac{H^d}{1-\xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w(1-\zeta_w)-1}} \\
W &= \alpha \frac{P^W}{P}\frac{Y^{W,c}+F}{H^d} \\
J^w &= \frac{W_n^O}{W_n} W JJ^w \\
\frac{W_h}{W} &= \frac{\left(1-\xi_w(1+g)\Lambda\frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1-\frac{1}{\zeta_w}\right)J^w}{WH^d} \\
&= \frac{\left(1-\xi_w(1+g)\Lambda\frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1-\frac{1}{\zeta_w}\right)\frac{W_n^O}{W_n}}{1-\xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w(1-\zeta_w)-1}}
\end{aligned}
$$

To examine the impact of trend inflation Π on the steady state further we consider the zero growth case $g = 0$ for which wage and price inflation are equal ($\Pi^w = \Pi$). Then we have for price-setting:

$$
\begin{aligned}
\frac{P^0}{P} &= \left(\frac{1-\xi_p \Pi^{(1-\gamma_p)(\zeta_p-1)}}{1-\xi_p}\right)^{\frac{1}{1-\zeta_p}} \\
\Delta_p &= \frac{1-\xi_p}{1-\xi_p \Pi^{\zeta_p(1-\gamma_p)}}\left(\frac{P^0}{P}\right)^{-\zeta_p} \\
MC &= \left(1-\frac{1}{\zeta_p}\right)\frac{1-\xi_p \Lambda \Pi^{\zeta_p(1-\gamma_p)}}{1-\xi_p(1+g)\Lambda \Pi^{(\zeta_p-1)(1-\gamma_p)}}\frac{P^0}{P}
\end{aligned}
$$

and for wage-setting:

$$
\begin{aligned}
\frac{W_n^O}{W_n} &= \left(\frac{1-\xi_w \Pi^{(1-\gamma_w)(\zeta_w-1)}}{1-\xi_w}\right)^{\frac{1}{1-\zeta_w}} \\
\Delta_w &= \frac{1-\xi_w}{1-\xi_w \Pi^{(1-\gamma_w)\zeta_w}}\left(\frac{W_n^O}{W_n}\right)^{-\zeta_w} \\
\frac{W_h}{W} &= \frac{\left(1-\xi_w \Lambda \Pi^{(1-\gamma_w)\zeta_w}\right)\left(1-\frac{1}{\zeta_w}\right)\frac{W_n^O}{W_n}}{1-\xi_w \Lambda \Pi^{(1-\gamma_w)(\zeta_w-1)}}.
\end{aligned}
$$

Thus for $\zeta_p > 1$, both the optimized price $\frac{P^0}{P}$ and price dispersion Δ_p *increase* with the trend inflation rate Π. However noting that the price mark-up is the inverse of the real marginal cost, i.e., equal to $= 1/MC$, we can see that the price response to the re-optimized price *decreases* with Π. Analogous results for $\zeta_w > 1$ hold for the optimized nominal wage, wage dispersion and the wage mark-up which is the inverse of $\frac{W_h}{W}$.

A.2 Solution of the Steady State

We solve for the steady state as follows:

1. We guess the value of H.

2. We solve for the steady state of the model given our guess.

3. We use the foc on hours

$$W^c_{h,t} = \frac{\left[C^c_t - \chi \frac{C^c_{t-1}}{1+g}\right] H^\psi_t}{1 - \beta\chi(1+g)^{-\sigma} \frac{U^c_{t+1}}{U^c_t} \frac{C^c_t - \chi\frac{C^c_{t-1}}{1+g}}{C^c_{t+1} - \chi\frac{C^c_t}{1+g}}} \tag{A.89}$$

   to evaluate our guess. Note that the above equation in steady state simplifies to

$$W^c_h = \frac{\left[C^c - \chi \frac{C^c}{1+g}\right] H^\psi}{1 - \beta\chi(1+g)^{-\sigma}} \tag{A.90}$$

   which elimantes the need to compute the steady state value for utility.

The rest of the variables can be computed as

$$U^c = \frac{\left[C^c - \chi \frac{C^c}{1+g}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma - 1)\frac{H^{1+\psi}}{1+\psi}\right] \tag{A.91}$$

$$V^c = \frac{U^c}{1 - \beta(1+g)^{1-\sigma}} labelVss \tag{A.92}$$

$$X = 1 + g \tag{A.93}$$

$$\lambda^c = \frac{(1-\sigma)U^c}{C^c - \chi\frac{C^c}{1+g}} - \beta\chi(1+g)^{-\sigma}\frac{(1-\sigma)U^c}{C^c - \chi\frac{C^c}{1+g}} \tag{A.94}$$

$$R = \frac{1}{\Lambda} \tag{A.95}$$

$$R_n = R\Pi \tag{A.96}$$

$$J^{p,c} = \frac{YMCMCS}{\left(1 - \frac{1}{\zeta_p}\right)\left(1 - \xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}\right)} \tag{A.97}$$

$$JJ^{p,c} = \frac{J^{p,c}}{\frac{P^0}{P}} \tag{A.98}$$

$$R^K = r^K + 1 - \delta \tag{A.99}$$

$$CEquiv^c = \frac{(1.01^{1-\sigma} - 1)U}{1 - \beta(1+g)^{1-\sigma}} \tag{A.100}$$

B Calibrated and Estimated Parameters

The parameters $\bar{\Pi}$, $\bar{Rn}$ and $\bar{g}$ are related to the steady state variables of our model by

$$\begin{aligned} \Pi &= \frac{\bar{\Pi}}{100} + 1 \\ R_n &= \frac{\bar{Rn}}{100} + 1 \\ g &= \frac{\bar{g}}{100} \end{aligned}$$

From our non-zero-inflation-growth steady state this implies that we can impose the restrictions

$$R_n = \frac{\Pi}{\beta(1+g)^{-\sigma}} = \frac{\bar{Rn}}{100} + 1 \tag{B.101}$$

on β. This implies that β **can** be calibrated as

$$\beta = \frac{\frac{\bar{\Pi}}{100} + 1}{\left(\frac{\bar{Rn}}{100} + 1\right)\left(1 + \frac{\bar{g}}{100}\right)^{-\sigma}} \tag{B.102}$$

However, in order to evaluate welfare ranking with a consistent form of the objective function, we still impose the standard quarterly value in the literature, $\beta = 0.99$, and impose R_n accordingly.

The first-order condition for capital utilisation is

$$r_t^K = a'(u_t) \tag{B.103}$$

which has the linear approximation

$$\hat{r}_t^K = \frac{\gamma_2}{\gamma_1}\hat{u}_t \tag{B.104}$$

Smets and Wouters write the above equation as (see equation (6) in their paper)

$$z_t = z_1 r_t^k \tag{B.105}$$

where $z_1 = \frac{1-\psi}{\psi}$ and they estimate ψ. Consequently, $z_1 = \frac{\gamma_1}{\gamma_2}$.

Recall that the capital utilisation adjustment function is

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 \tag{B.106}$$

which can be rewritten as

$$\begin{aligned} a(u_t) &= \gamma_1(u_t - 1) + \frac{\gamma_2}{\gamma_1}\frac{\gamma_1}{2}(u_t - 1)^2 \\ &= \gamma_1(u_t - 1) + \frac{1}{z_1}\frac{\gamma_1}{2}(u_t - 1)^2 \\ &= \gamma_1(u_t - 1) + \frac{\psi}{1-\psi}\frac{\gamma_1}{2}(u_t - 1)^2 \end{aligned} \tag{B.107}$$

Its derivative is

$$a'(u_t) = \gamma_1 + \frac{\psi}{1-\psi}\gamma_1(u_t - 1) \tag{B.108}$$

The production function (equation (5) in the paper) is given by

$$y_t = \phi_p(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a) \tag{B.109}$$

where $\phi_p = \frac{y_* + \Phi}{y_*}$ is one plus the share of fixed costs in production.[18] They use the prior $\phi_p \sim \mathcal{N}(1.25, 0.25)$ for the parameter (may be missing from the paper altogether), which implies that $\frac{\Phi}{y^*} \sim \mathcal{N}(0.25, 0.25)$. Hence we need to rewrite the equilibrium condition (17) as

$$Y_t^W = \left(A_t H_t^d\right)^\alpha (u_t K_{t-1})^{1-\alpha} - \tilde{F} Y^W \tag{B.112}$$

[18] In the technical appendix the production function is given by

$$y_t(i) = Z_t k_t(i)^\alpha L_t(i)^{1-\alpha} - \Phi \tag{B.110}$$

which becomes

$$\hat{y}_t = \alpha \frac{y_* + \Phi}{y_*}\hat{k}_t + (1-\alpha)\frac{y_* + \Phi}{y_*}\hat{L}_t + \frac{y_* + \Phi}{y_*}\hat{Z}_t \tag{B.111}$$

when loglinearized.

and define the prior on $\tilde{F} = \frac{F}{\frac{Y^W}{A_t}}$.

C Computation of the Likelihood Density using the Kalman Filter

- **Prediction**
  Predicted (a priori) state estimate: $s_{t|t-1} = A(\theta)s_{t-1|t-1}$.
  Predicted (a priori) estimate covariance: $P_{t|t-1} = A(\theta)P_{t-1|t-1}A^T(\theta) + B(\theta)\Sigma_{u_t}B^T(\theta)$.
  Where $P_{t|t}$ is the posteriori estimate covariance matrix (a measure of the estimated accuracy of the state estimate), $s_{t|t}$ is the posteriori state estimate at time k given observations up to and including at time t. Σ_{u_t} is the covariance of the transition noise which is assumed with zero-mean normal distribution.

- **Updating**
  Innovation or measurement pre-fit residual: $\hat{x}_t = y_t - C(\theta)s_{t|t-1}$
  Innovation (or pre-fit residual) covariance: $H_t = C(\theta)P_{t-1|t-1}C^T(\theta) + D(\theta)\Sigma_{\omega_t}D^T(\theta)$.
  Optimal Kalman gain: $K_t = P_{t|t-1}C^T(\theta)H_t^{-1}$ [19].
  Updated (a posteriori) state estimate: $s_{t|t} = s_{t|t-1} + K_t\hat{x}_t$.
  Updated (a posteriori) estimate covariance: $P_{t|t} = (I - K_tC(\theta))P_{t|t-1}$.
  Measurement post-fit residual: $\hat{x}_{t|t} = y_t - C(\theta)s_{t|t}$

Hence, the likelihood density, $L(p(y|\theta))$, is computed from the distribution of the measurement post-fit residual, $\hat{x}_{t|t}$.

D Identification and Estimation

Following Iskrev and Ratto (2010), we provide the identification (locally) analysis of the our tool model here. In the upper panel of the figure the bars depict the identification strength of the parameters based in the Fisher information matrix normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). Intuitively, the bars represent the normalized curvature of the log likelihood function at the prior mean in the direction of the parameter. If the strength is 0 (for both bars) the parameter is not identified as the likelihood function is flat in this direction. The

[19] The optimal Kalman gain minimizes the residual error (Miao ch. 10).

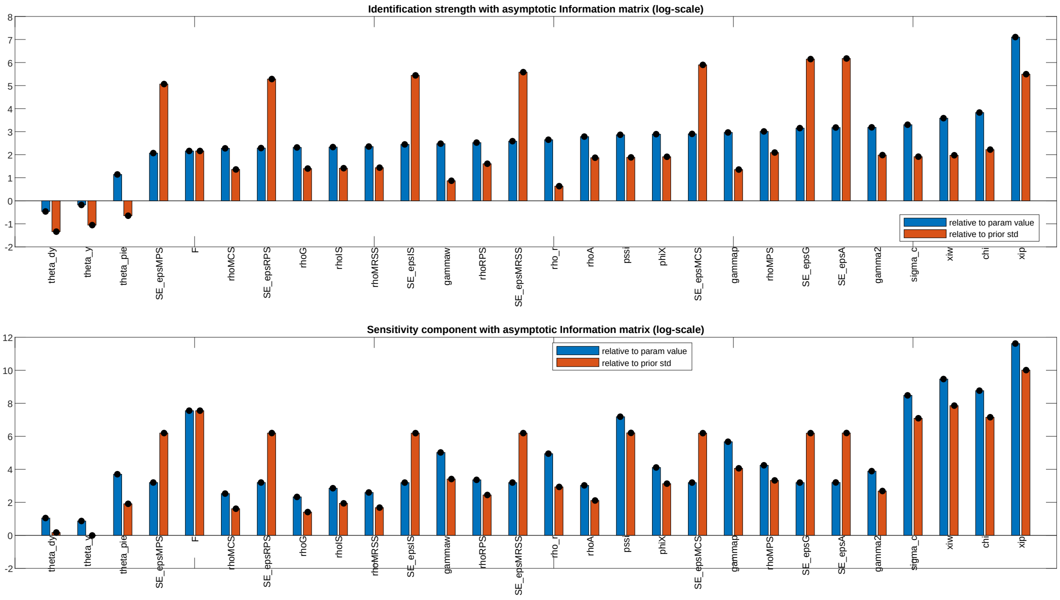

Figure 5: Identification Strength in the tool Model

larger the absolute value if the bars, the stronger the identification. Hence, it is clear that all parameters are identified in the model.

The convergence property is represented in figure (6). The appended (Interval) shows the Brooks and Gelman's convergence diagnostics for the 80% interval. The blue line shows the 80% interval/quantile range based on the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. The appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments, i.e. the squared and cubed absolute deviations from the pooled and the within-sample mean, respectively. All statistics are based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters. Convergence is indicated by the two lines stabilizing and being close to each other.

The figures from (7) to (10) indicate the prior-posterior plots. The grey line shows the prior density, while the black line shows the density of the posterior distribution. The green horizontal line indicates the posterior mode. If the posterior looks like the prior, either your prior was a very accurate reflection of the information in the data or the parameter under consideration is only weakly identified and the data does not provide much information to update the prior.

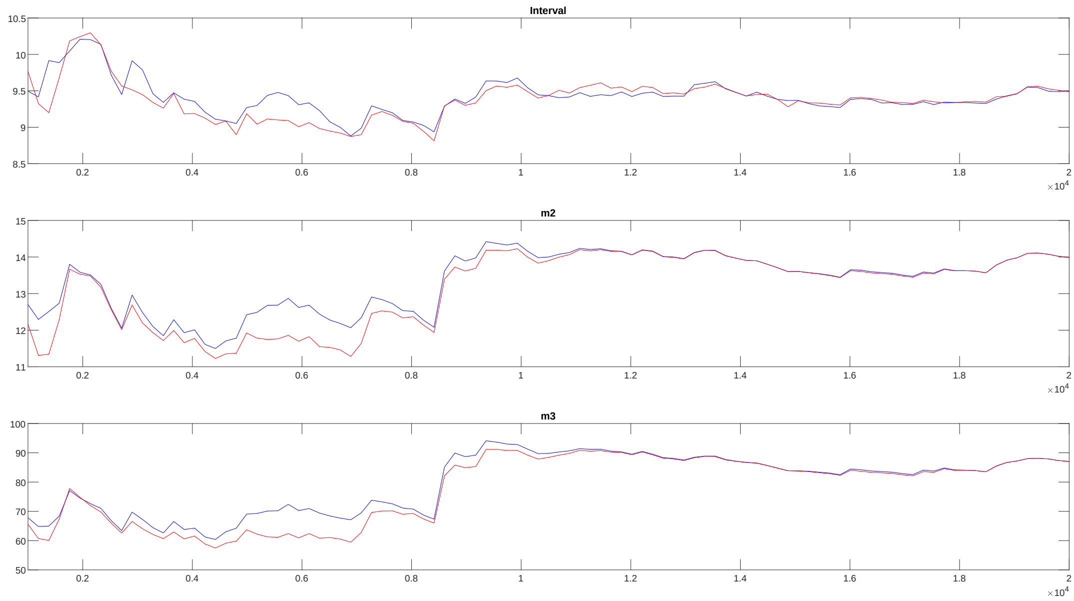

Figure 6: Multivariate convergence diagnostic

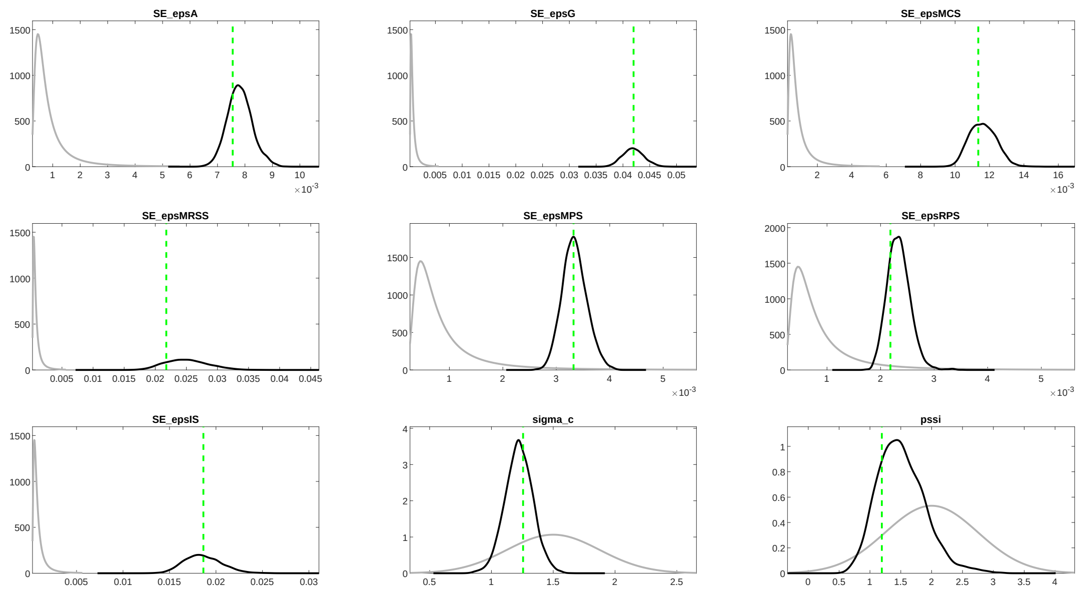

Figure 7: Priors and Posteriors for 20000 MCMC draws

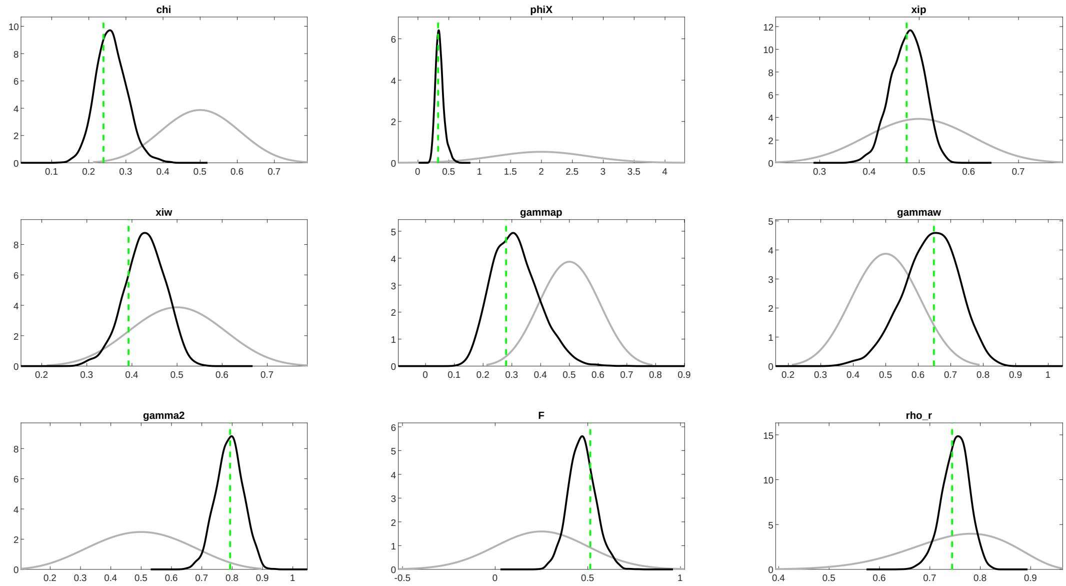

Figure 8: Priors and Posteriors for 20000 MCMC draws

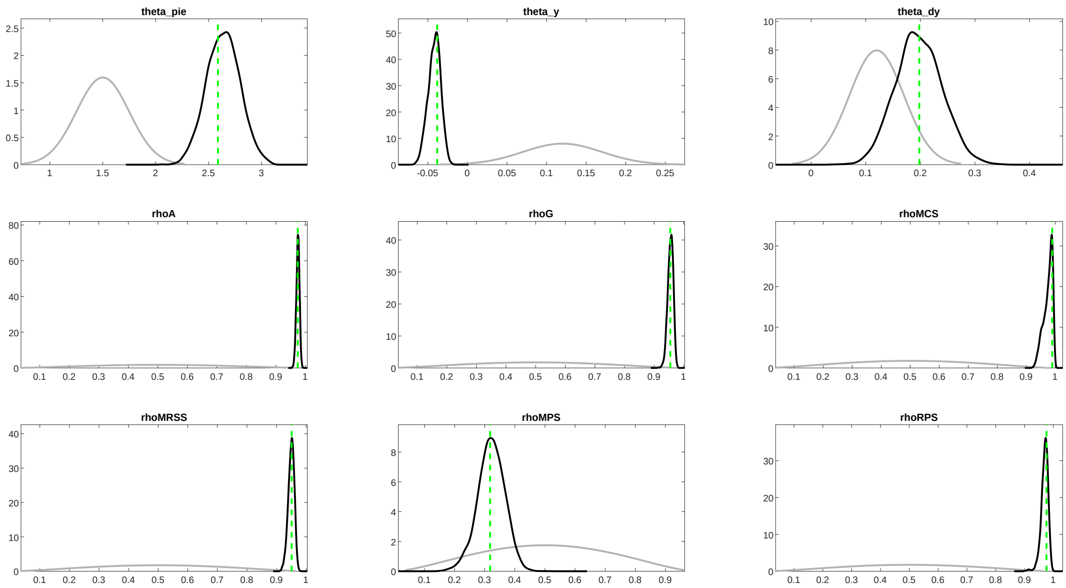

Figure 9: Priors and Posteriors for 20000 MCMC draws

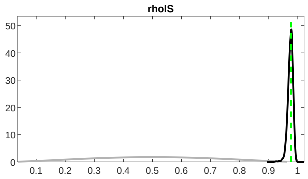

Figure 10: Priors and Posteriors for 20000 MCMC draws

E A General Mandate for Monetary-Fiscal Policy Interactions

We now sketch out a generalization of our mandate framework to include a fiscal dimension. First we describe the fiscal Taylor-type rules. Then we set out the fiscal mandate with a constraint on the upper bound for the debt-income ratio. Finally we propose a closed-loop Nash equilibrium for independent monetary and fiscal authorities with delegated mandate.

E.1 Fiscal Policy Rules

Let $\tau_{w,t} = \tau_t \tau_w$ and $\tau_{k,t} = \tau_t \tau_k$ so relative tax rates on labour and capital remain constant. We examine general fiscal rules suitable for non-flexi price NK models of the form: for governnment spending:

$$\log\left(\frac{G_t}{G}\right) = \rho_g \log\left(\frac{G_{t-1}}{G}\right) + (1-\rho_g)\left(-\theta_{g,B}\log\left(\frac{B^r_{t-1}}{B^r}\right) - \theta_{g,y}\log\left(\frac{Y_t}{Y}\right) - \theta_{g,dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \tag{E.113}$$

and for taxes:

$$\log\left(\frac{\tau_t}{\tau}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\tau}\right) + (1-\rho_\tau)\left(\theta_{\tau,B}\log\left(\frac{B^r_{t-1}}{B^r}\right)\theta_{\tau,y}\log\left(\frac{Y_t}{Y}\right) + \theta_{\tau,dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \tag{E.114}$$

In (E.113) and (E.114) $\theta_{g,B}, \theta_{\tau B} > 0$ to ensure a stable debt profile and we normally expect $\theta_{g,y}, \theta_{g,dy} > 0$ and $\theta_{\tau,y}, \theta_{\tau,dy} > 0$ as well. We also require $B^r \neq 0$.

The steady state choice of the debt-income ratio $D \equiv \frac{B^r}{Y}$ pins down the common tax rate. From

$$B^r = \frac{G - WH\tau\tau_w - r^K K\tau\tau_k}{1-R} \tag{E.115}$$

we have

$$\tau = \frac{g_y + (R-1)D}{\alpha\tau_w + (1-\alpha)\tau_k} > 0 \tag{E.116}$$

where $g_y \equiv \frac{G}{Y}$. This completes the fiscal additions to the model.

E.2 A Debt Upper Bound Mandate

As in the monetary policy ZLB mandate in we can design a mandate for a fiscal authority in the form of a tax or government spending rule, separately or together, that keeps the debt-to-income ratio below a target upper bound with a low probability, $\bar{p}$.

Consider the tax rule keeping G_t as an exogenous AR1 process as before. Write it in the form:

$$\log\left(\frac{\tau_t}{\tau}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\tau}\right) + \left(\alpha_{\tau,B}\log\left(\frac{B^r_{t-1}}{B^r}\right) + \alpha_{\tau,y}\log\left(\frac{Y_t}{Y}\right) + \alpha_{\tau,dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \tag{E.117}$$

Let $D_t \equiv \frac{B^r_t}{Y_t}$. We then have a 3-stage delegation game as follows:

Stage 3: Choice of the Fiscal Rule:

Given an initial debt-income rate ratio, D_0, a target steady state $D < D_0$ and transition rate $\rho_d \in (0,1)$ with a *transition path* D^*_t for $t \geq 1$

$$D^*_t = D^{1-\rho_d}(D^*_{t-1})^{\rho_d} \to D \text{ as } t \to \infty \tag{E.118}$$

Given the target steady state D, (E.116) then determines the tax rate τ that will achieve the target given g_y. Alternatively any combination τ and g_y consistent with (E.116) will result in the target D. For the transition to the new target (the deterministic component of policy), (E.118) replaces (E.117).[20]

The fiscal authority, separate from the Central Bank, then receives a mandate to implement this transition path alongside the stochastic stabilization rule (E.117) about the new steady state defined by D, τ and g_y. The authority then maximizes with respect to $\rho_f = [\rho_\tau,\, \alpha_{\tau,B},\, \alpha_{\tau,y},\, \alpha_{\tau,dy}]$ a modified welfare criterion

$$\begin{aligned} \Omega_t^{mod} &\equiv \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(U_{t+s} - w_d\left(D_{t+s} - D\right)^2\right) \\ &= U_t - w_d\left(D_t - D\right)^2 + \beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{mod}\right]\Big] \end{aligned} \tag{E.119}$$

In designing the rule we now decompose the expected welfare into deterministic and stochastic steady state components.[21] Then the fiscal authority takes the former as given and maximizes the latter as for the monetary delegation game.

Stage 2: Choice of the Steady State Debt-Income Ratio D Given a target low probability $\bar{p}$ and given w_d, $D = D^*$ in equilibrium is chosen so satisfy

$$p(D_t \geq D^{target}) \equiv p(D^*, \rho_d^*(D^*, w_d)) \geq \bar{p} \tag{E.120}$$

This then achieves the upper bound constraint

$$D_t \leq D^{target} \text{ with high probability } 1 - \bar{p} \tag{E.121}$$

Stage 1: **Design of the Mandate**

The policymaker first chooses a per period probability $\bar{p}$ of the debt-income rate hitting the UB (which defines the tightness of the UB constraint). Then it maximizes the **actual** household inter-temporal welfare

$$\Omega_t = \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s U_{t+s}\right] = U_t + \beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}\right] \tag{E.122}$$

[20] This deterministic component of policy is solved as a perfect foresight solution.

[21] In what follows we consider these components separately and use a second-order stochastic solution around the new steady-state. However this is an approximation which Ajevskis (2017) avoids by providing a perturbation solution about a deterministic steady state. The latter can be the accurate solution using the perfect foresight solver.

with respect to w_d and the speed of transition ρ_d.

This three-stage delegation game defines an equilibrium in choice variables w_d^*, ρ_f^*, D^* and ρ_d^* that maximizes the true household welfare subject to the UB constraint (E.121).

E.3 Closed-loop Nash Equilibrium

In the NK model with both fiscal and monetary policy conducted independently, in Stage 3 we need a Closed-loop Nash Equilibrium (CLNE) in the optimized feedback coefficients. First, we redefine the objective functions of the fiscal and monetary authorities in stage 3 here, respectively:

$$\begin{aligned}\Omega_t^{fiscal,mod} &\equiv \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(U_{t+s}-w_d\left(D_{t+s}-D\right)^2\right) \\ &= U_t - w_d\left(D_t-D\right)^2+\beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{fiscal,mod}\right]\Big]\end{aligned} \tag{E.123}$$

$$\begin{aligned}\Omega_t^{monetary,mod} &\equiv \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(U_{t+s}-w_r\left(Rn_{t+s}-Rn\right)^2\right) \\ &= U_t - w_r\left(Rn_t-Rn\right)^2+\beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{monetary,mod}\right]\Big]\end{aligned} \tag{E.124}$$

Hence, we denote $\Omega_t^{j,mod}$, where $j=[fiscal, \quad monetary]$, are the modified welfare function for each policy maker. Each policy maker has her own policy instrument, $int_{j,t}$. For instance, monetary policy maker uses nominal interest rule (53) and fiscal authority employs the rule (E.117), τ_t.