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MONETARY GROWTH RULES IN AN EMERGING OPEN ECONOMY

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Monetary Growth Rules in an Emerging Open Economy

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Abstract

We develop a small open economy model interacting with a rest-of-the-world bloc, containing several emerging economies' features: Calvo-type nominal frictions in prices and wages, financial frictions in the form of limited asset markets participation (LAMP), as well as both formal and informal sectors. In addition, we introduce incomplete exchange rate pass-through via a combination of producer and local currency pricing for exports, as well commodity-dependence in the form of an oil export sector. We contrast the stability and determinacy properties of money growth and standard Taylor-type interest rate rules, showing that monetary rules are stable regardless of the level of asset market participation, i.e. they avoid the inversion of the Taylor principle. We estimate our 2-bloc model using data for Iran and the USA employing Bayesian methods and we study the empirical relevance of the frictions in our model. Our results reveal important propagation channels active in emerging economies and that taking these into account is essential for policymaking decisions. Indeed, shocks to the economy are amplified by the presence of LAMP, while trade autarky further intensifies the effects of financial frictions. On the other hand, the informal sector acts as buffer to several shocks, lowering the variability of aggregate and formal fluctuations.

Keywords: Emerging Economies; Informality; Limited Asset Market Participation; Producer and Local Currency Pricing; Money Growth Rule.

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1 Introduction

What is the nature of monetary policy transmission in emerging economies? Which type of policy rules can successfully stabilize this type of economy? Recent episodes of financial turmoil have highlighted the need to understand how large external shocks are propagated in small open economies. This is particularly relevant in emerging market countries, since these economies face additional vulnerabilities that make them very different from advanced economies. Indeed, these economies usually display weak fiscal, monetary and financial institutional frameworks, and have imperfect access to capital markets.

These are active and unsettled areas of research, with empirical evidence suggesting that monetary policy transmission, especially standard interest rate pass-through, is weak and incomplete in developing countries (see Mishra *et al.* 2012). Various features of emerging economies are relevant to understand their distinctive monetary policy transmission channels. For instance, these countries tend to be characterised by a substantial degree of openness, which makes them vulnerable to external shocks. Moreover, their capital, money and interbank markets are typically inefficient and underdeveloped, with financial frictions generating countercyclical costs of financing (see Bhattacharya *et al.* 2011; Mishra *et al.* 2012). In addition, the existence of a large proportion of credit-constrained consumers, as well as considerable informality in the goods and labour markets, imposes constraints on inflation management policies. All these elements imply that the reactions to structural shocks will differ from those of a developed economy, which has important implications for the conduct of monetary and stabilisation policies.

Thus, in this paper we develop a small open economy (SOE) model containing several of these features: we model explicitly a fully-fledged rest-of-the-world (ROW) bloc interacting with a SOE bloc with Calvo-type nominal frictions in prices and wages, financial frictions in the form of limited asset markets participation (LAMP), as well as both formal and informal sectors.[1] We maintain that informality and financial exclusion must feature in any model of emerging economies, but they must be considered separately when studying the effects of monetary policy. In addition, we introduce incomplete exchange rate pass-through via a combination of producer and local currency pricing for exports (PCP and LCP, respectively), as well commodity-dependence in the form of an oil export sector.[2] Finally, we also allow for fiscal deficits and government debt.

A novel aspect of our paper is the focus on the adequacy of a money growth rule to implement monetary policy actions in this environment. We do so for several reasons. First, the existence of LAMP has an impact on the efficacy of interest rate rule-based monetary policy, as constrained access to financial markets makes demand of such consumers insensitive to interest rate fluctuations (see Gabriel *et al.* 2012 and Anand *et al.* 2015). Second, and to compound the latter, Bilbiie (2008) finds that LAMP significantly distorts the saddle-path stability of traditional Taylor rules. This

[1] We will use LAMP interchangeably with 'credit-constrained', 'liquidity-constrained', 'non-Ricardian' or 'rule-of-thumb' agents, also sometimes referred to as 'hand-to-mouth', 'non-asset holder', or 'current-income' consumers.

[2] A possible model of currency pricing could assume both types with fixed proportions or ideally endogenous switching - Smets and Wouters (2002), Christiano *et al.* (2011) have models of the former and Gopinath *et al.* (2010) provide empirical evidence across countries for a low degree of pass-through. However, in this paper, while keeping the perfect exchange rate pass-through for imports, we have assumed that retailers set a fixed proportion of export prices following LCP.

flip of the relationship between aggregate output and the real interest rate is termed the inverted aggregate demand logic (IADL) and when it applies, the central bank has to adopt a passive policy rule to ensure equilibrium uniqueness. However, we show an important result that money growth rules avoid these problems altogether, ensuring both stability and determinacy. Finally, money growth rules overcome a perceived defect of conventional interest rate rules in the case of emerging economies that are keen on pursuing Islamic-compliant monetary actions.

Indeed, while most economies have transitioned to some form of inflation targeting (IT) or 'IT-lite', explicitly targeting monetary aggregates can still be a useful policy device, especially in emerging settings, where fiscal dominance tends to be is widespread, central bank independence is weaker or with underdeveloped financial markets, thus making the transmission mechanism hard to understand and leaving a relatively small role for interest rates (see Stone and Bhundia 2004). This usefulness is also grounded on evidence suggesting the importance of strict reserve money targeting in bringing inflation under control, with a survey by the International Monetary Fund (2008) indicating that in high-inflation countries reserve money growth is higher than expected, which leads to inflation, whereas in low inflation countries, greater money target flexibility is not associated with inflation surprises.

Furthermore, we also contribute to the literature on inflation versus monetary targeting by throwing light on the empirical relevance of the mechanisms identified above. We employ Bayesian methods to estimate our model, using data for Iran as the SOE and taking the US as the ROW. This paper is, to the best of our knowledge, the first to estimate a two-bloc model comprising an emerging and a developed economy. We focus on the case of Iran given that it is an emerging, oil-rich economy, dependent on exports of this commodity, with sizeable degrees of LAMP and informality. It also provides an interesting testing ground to study the effects of financial autarky, as well as mimicking the conduct of monetary policy in a quasi-Islamic setting (without the recourse of interest rate-based stabilisation), for which there is scant evidence in the literature.[3]

Having established the empirical pertinence of these features, we analyse and compare how they impact the transmission of shocks and business cycles. Our model is able to replicate the main characteristics of fluctuations in emerging economies, with posterior impulse responses indicating that shocks to the economy are amplified by the presence of LAMP. Moreover, trade autarky is found to further intensify the effects of financial frictions, while on the other hand the informal sector acts as buffer to several shocks, lowering the variability of aggregate and formal fluctuations.

A major contribution of our paper is to bring together different strands in the literature that have hitherto remain disparate. First, our setup is related to the work of Bilbiie (2008) and Boerma (2014), looking at the linkages between the financial markets and the real economy by incorporating financial frictions in the form of credit-constrained households.[4] The financial crisis revived interest in these linkages, as a disruption in financial markets propagated into a sharp contraction in the economy. It also highlights the vulnerability of emerging economies to shocks in foreign countries.

$<$ Table 1 here $>$

[3]Komijani and Tavakolian (2012) and Manzoor and Taghipour (2016) estimate a DSGE model for Iran, but without informality, financial frictions or exchange rate pass-through imperfections.

[4]We note, however, that these studies do not take into account informality or money growth rules as an alternative stabilization tool.

Indeed, Table 1 shows that the level of financial inclusion varies significantly across countries. In low-income countries, only 19% of the population has access to basic financial products, contrasting with a figure of 89% for high-income countries, which also tend to be more open, an important aspect to consider. Boerma (2014) shows that the 'inverted Taylor principle' is less likely to apply in a small open economy because the terms of trade channel of monetary policy is also contractionary for a rise in the real interest rate. But monetary authorities can still mistakenly adopt passive Taylor rules if they do not take into account the impact of openness on the monetary policy transmission mechanism, assuming they do take into account LAMP.

Second, the role of informality sector in a DSGE setting has been analysed by Castillo and Montoro (2010), Batini *et al.* (2011), Gabriel *et al.* (2012) and Khera (2016), *inter alia*, emphasising the importance of intra-sectoral reallocations as an additional friction that weakens conventional interest rate-based pass-through. In particular, informal labour markets generate a "buffer" effect that diminishes the pressure of demand shocks on aggregate wages and inflation; a significant degree of informality lowers the correlation between inflation and the output gap conditional on demand shocks, thus further weakening the interest rate channel of monetary policy. Nonetheless, these papers (with the exception of Khera 2016) are developed in a closed-economy setting, thus ignoring issues such as incomplete exchange rate pass-through and the impact of exchange rate fluctuations, in contrast with our SOE framework with LAMP and a monetary growth rule, applicable in a wide array of contexts.

Third, our paper complements existing studies on alternative monetary policy rules for shock-prone economies. As discussed in Peiris and Saxegaard (2007), interest rates rules are used as a monetary policy tool when financial markets are well developed and, therefore, monetary authorities in many emerging economies must resort to other instruments, such as exchange rate rate stabilization or open-market operations. Our results are consistent with the work of Berg *et al.* (2010), who formalize flexible money targeting in a New Keynesian setup and show that choosing a monetary target is consistent with a Taylor rule for the relevant interest rate, i.e. it becomes a signal extraction problem where money market information is used by the central bank to update its estimate of the state of the economy.

Finally, our research relates to the issue of monetary policy conduct under an Islamic setup, which still remains a challenge. As suggested by Khatat (2016), the central bank should gear its monetary policy to the generation of a growth in money supply that is adequate to finance potential output growth in the medium and long-terms, within the framework of stable prices and other relevant socio-economic goals. While we do not claim that our model provides a comprehensive framework for Islamic monetary policy, we believe we offer a useful platform to explore policy tools for emerging economies in general, and for Islamic countries in particular.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 describes the data and estimation methodology. Section 4 sets out the results, with an emphasis on the relative importance of the various frictions and how these impact the transmission and responses to exogenous shocks. Section 5 contrasts the stability and determinacy properties of interest rate-based and money growth rules, while Section 6 contains some concluding remarks.

2 Model Description

This section provides a brief sketch of the model (all the model details can be found in the Supplementary Appendix). The model economy is a two-bloc, two-sector money-in-utility-function SOE with complete financial autarky, but interacting with the rest of the world and, in particular, being dependent on the monetary policy of the ROW bloc.[5] The latter is modelled as a closed New Keynesian model, which is estimated separately. Figure 1 provides a schematic illustration of the interactions in our model.

< Figure 1 here >

In addition, we consider incomplete exchange rate pass-through for exports, an important feature distinguishing the informal sector, displaying PCP, and the tradable, formal sector, combining PCP and LCP. Strict regulations in the formal sector have been identified as one of the key drivers of a large informal sector, so these are modelled as rigidities in the labour and goods market. Moreover, taxes, government spending and investment operate solely in the formal sector. The model is further extended to include an oil sector, as well as important features of developing economies such as fiscal deficits, government debt and LAMP. We consider the main features in turn.

2.1 Households

The SOE is inhabited by a continuum of infinitely-lived households on the unit interval, each indexed by $h \in [0, 1]$. Households are divided in those who participate in the financial sector and can lend or borrow to each other, which we refer to as *Ricardian* consumers (R), while the remaining λ *rule-of-thumb* consumers (RoT) are credit-constrained and must consume out of wage income net of tax each period.[6]

Consider first the proportion $(1-\lambda)$ of Ricardian households. Their decision problem is to choose a path of aggregate consumption $\{C_t^R\}$ money holding $\{m_t^R\}$ and labour supply $\{H_t^R\}$ that maximizes

$$U_t^R = U(C_t^R, H_t^R, m_t^R) = \frac{\left(C_t^R - \chi C_{t-1}^R\right)^{(1-\varrho)(1-\sigma)} \left(1 - H_t^R\right)^{\varrho(1-\sigma)} - 1}{1-\sigma} + \Psi \left(\frac{m_t^{R\,1-\psi} - 1}{1-\psi} \right) \tag{1}$$

where the utility function is non-separable and is consistent with a balanced growth path when the inter-temporal elasticity of substitution, $1/\sigma$, is not unitary ($\sigma > 0$ is a risk aversion parameter). The parameter ϱ in the interval (0, 1) defines the relative weight households place on utility from leisure relative to consumption. The total time available is normalized to 1, so that $(1 - H_t^R)$ denotes leisure. Ψ is the relative weight assigned to real money balances, with ψ as the inverse elasticity of real money holdings, while χC_{t-1}^R is external habit taken as given by the individual household, where the parameter χ is in the interval [0, 1).

[5] The variables in the ROW bloc follow the same notation as in the SOE, but are denoted with *.

[6] The non-Ricardian agents are consumers that are unable to borrow and save - these can alternatively be regarded as households that are myopic and face a no-borrowing constraint.

The Ricardian household solves

$$\max_{C_t^R, H_t^R, m_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} PS_t \, \beta^s \, U(C_{t+s}^R, H_{t+s}^R, m_{t+s}^R) \right] \tag{2}$$

subject to a nominal budget constraint given by

$$P_t{}^B B_{H,t} + P_t C_t^R + P_t m_t^R = B_{H,t-1} + W_{1,t}^{nh} \left(1 - \tau_t^w\right) H_{1,t}^R + W_{2,t}^{nh} H_{2,t}^R + P_{t-1} m_{t-1}^R + \Gamma_t \tag{3}$$

where subscripts 1 and 2 refer to the formal and informal sectors, respectively, superscript nh refers to nominal (n) homogeneous (h) labour, such that $W_{1,t}^{nh}$ and $W_{2,t}^{nh}$ are pre-tax nominal wage rates, nominal profits are given by Γ_t and τ_t^w is a proportional endogenous labour tax. Note that taxes are not paid for wages from employment in the informal sector. $B_{H,t}$ denotes nominal domestic bonds bought at nominal price $P_t^B = 1/R_t$ and denominated in the respective currency, where R_t is the gross nominal interest rate paid on assets held at the beginning of period t; P_t is the CPI index that includes an imported component, see (77) below, β is home discount factor and PS_t is preference shock.

Consider next the proportion λ of credit-constrained consumers. This group of households have no income from monopolistic retail firms, work in both formal and informal sectors and must consume out of wage income. Their consumption given by

$$C_t^{RoT} = W_{1,t}^h \left(1 - \tau_t^w\right) H_{1,t}^{RoT} + W_{2,t}^h H_{2,t}^{RoT} - m_t^{RoT} + \frac{m_{t-1}^{RoT}}{\Pi_t} \tag{4}$$

Liquidity-constrained consumers now choose C_t^{RoT} and H_t^{RoT} to maximize an analogous welfare function to (52) subject to (4), with analogous equilibrium conditions resulting.[7] Total labour supply by Ricardian and non-Ricardian households to the formal and informal sectors is then $\sum_{i=1}^{2} \lambda H_{i,t}^{ROT} + (1-\lambda) H_{i,t}^R$.

Consumption Demand for Domestic and Imported Goods

For given aggregate consumption $C_t = C_t^R, C_t^{RoT}$ for both Ricardian and RoT consumers, household demand for consumption goods from domestic retailers (C_H) and foreign retailers (C_F, i.e. imports) is chosen to maximise the Dixit-Stigitz quantity aggregator

$$C_t = \left[\mathrm{w}_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C - 1}{\mu_C}} + (1 - \mathrm{w}_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}} \tag{5}$$

The corresponding Dixit-Stigitz price index is given by

$$P_t = \left[\mathrm{w}_C (P_{H,t})^{1-\mu_C} + (1 - \mathrm{w}_C)(P_{F,t})^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{6}$$

[7] Households's conditions in the ROW are derived under the same assumptions, see the Supplementary Appendix for details.

Now define CPI, domestic and imported inflation rates over the time interval $[t-1,t]$ by $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ respectively. Then from (77) we have

$$\Pi_t = \left[\mathrm{w}_C \left(\Pi_{H,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-\mathrm{w}_C) \left(\Pi_{F,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{7}$$

Parameter μ_C is the elasticity of substitution between home and foreign goods, while parameter w_C is related to the degree of home-bias in preferences and plays a critical role in this paper. In turn, $1-\mathrm{w}_C$ is interpreted as an index of openness to international trade in final goods: when $\mathrm{w}_C = 1$, the share of foreign goods in the composite consumption index approaches zero. The degree of openness $1-\mathrm{w}_C$ is identical across economies and $\mathrm{w}_C = 1$ denotes an economy in autarky, i.e. a closed economy. In contrast, if $\mathrm{w}_C = 0$, there is no home-bias in consumption. Note also that there is no international trade in intermediate goods.

Maximizing total consumption (69) subject to a given aggregate expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ yields

$$C_{H,t} = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \quad \text{and} \quad C_{F,t} = (1-\mathrm{w}_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \tag{8}$$

In our general model set-up we assume that a fixed proportion of retail firms set prices in home currency (i.e. PCP), and the remaining proportion are local or destination pricers (i.e. LCP) - see Section 2.5.2. For now, however, we assume PCP. Define the real exchange rate as the relative aggregate consumption price $RER_t \equiv \frac{P_t^* S_t}{P_t}$, where S_t is the nominal exchange rate. With PCP, because the home country is small, the law of one price (LOP), i.e. perfect exchange rate pass-through for imports, implies that $P_t^* = P_{F,t}^*$, $S_t P_t^* = P_{F,t}$, so $RER_t = \frac{P_{F,t}}{P_t}$ and *terms of trade* for the home country are defined as $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, i.e. the price of imported goods relative to domestic ones, and

$$\frac{\mathcal{T}_t \ tot_t}{\mathcal{T}_{t-1} \ tot_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} \tag{9}$$

where tot_t is a terms of trade shock.

Analogous Dixit-Stiglitz quantity and price aggregators apply to goods sold by the formal and informal sectors:

$$C_{H,t} = \left[\mathrm{w}_S^{\frac{1}{\mu_S}} C_{1,t}^{\frac{\mu_S-1}{\mu_S}} + (1-\mathrm{w}_S)^{\frac{1}{\mu_S}} C_{2,t}^{\frac{\mu_S-1}{\mu_S}} \right]^{\frac{\mu_S}{\mu_S-1}} \tag{10}$$

$$P_{H,t} = \left[\mathrm{w}_S (P_{1,t})^{1-\mu_S} + (1-\mathrm{w}_S)(P_{2,t})^{1-\mu_S} \right]^{\frac{1}{1-\mu_S}} \tag{11}$$

where w_S and $1-\mathrm{w}_S$ are sector shares and μ_S is the elasticity of substitution between formal and informal goods. The corresponding CPI inflation corresponding to (7) is then given by

$$\Pi_{H,t} = \left[\mathrm{w}_S \left(\Pi_{1,t} \frac{P_{1,t-1}}{P_{H,t-1}} \right)^{1-\mu_S} + (1 - \mathrm{w}_S) \left(\Pi_{2,t} \frac{P_{2,t-1}}{P_{H,t-1}} \right)^{1-\mu_S} \right]^{\frac{1}{1-\mu_S}} \tag{12}$$

where $\Pi_{H,t}$, $\Pi_{1,t}$ and $\Pi_{2,t}$ are home, formal and informal CPI inflation, respectively. Then, optimal demand for formal and informal goods is obtained as for home-produced and imported goods.[8]

2.2 Capital Producers

Capital producers accumulate the capital stock and rent it to firms. They convert investment goods (I_t) into $[1 - \mathcal{S}(X_t)] I_t$ of new capital sold at real price Q_t at a cost of $\mathcal{S}(X_t)$ to maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k}(1 - \mathcal{S}(I_{t+k}/I_{t+k-1}))I_{t+k} - I_{t+k} \right]$$

where total capital accumulates according to

$$K_t = (1-\delta)K_{t-1} + [1 - \mathcal{S}(X_t)]\, I_t\, IS_t \tag{13}$$

where beginning of period capital stock $K_t = K_{1,t} + K_{2,t}$ is summed over formal and informal sectors, and IS_t is an investment shock.

Dixit-Stiglitz aggregators over home and imported investment are:

$$I_t = \left[\mathrm{w}_I^{\frac{1}{\mu_I}} I_{H,t}^{\frac{\mu_I - 1}{\mu_I}} + (1 - \mathrm{w}_I)^{\frac{1}{\mu_I}} I_{F,t}^{\frac{\mu_I - 1}{\mu_I}} \right]^{\frac{\mu_I}{\mu_I - 1}} \tag{14}$$

$$P_{I,t} = \left[\mathrm{w}_I (P_{H,t})^{1-\mu_I} + (1 - \mathrm{w}_I)(P_{F,t})^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}} \tag{15}$$

and analogous demand for home and imported investment goods to (8) are

$$I_{H,t} = \mathrm{w}_I \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_H} I_t \quad \text{and} \quad I_{F,t} = (1 - \mathrm{w}_I) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t \tag{16}$$

For the FOCs, we define the gross real return on capital R_t^K as

$$R_t^K = \frac{r_t^K + (1-\delta)Q_t}{Q_{t-1}} \tag{17}$$

such that the right-hand-side is the gross return to holding a unit of capital from $t-1$ to t, while the left-hand-side is the gross return from holding bonds and the opportunity cost of capital. We

[8] In the full model set out in the Appendix we define the terms of trade as the relative price of *formal* home-produced to imported goods.

further define investment adjustment costs and the rate of change of investment as

$$\mathcal{S}(X_t) \equiv \phi_X (X_t - X)^2 \tag{18}$$

$$X_t \equiv \frac{I_t}{I_{t-1}}\,;\;\; S',\, S'' \geq 0\,;\;\; S(1) = S'(1) = 0 \tag{19}$$

where ϕ_X is the elasticity of investment adjustment costs.

2.3 ROW and Exports

Now consider the ROW bloc. In each bloc, domestically produced and imported consumption goods are consumed with prices denominated in the country's currency with notation summarized in Table 2. An analogous table applies to investment goods.

< Table 2 here >

Continuing with the PCP assumption (relaxed in the full model), total exports of consumption ($EX_{C,t}$) and investment ($EX_{I,t}$) goods from the home country are imports into the ROW and by analogy with (8) and (16) are given by

$$EX_t \equiv C^*_{H,t} + I^*_{H,t} \;\; = \;\; (1 - \mathrm{w}^*_C)\mathcal{T}_t^{\mu^*_C} C^*_t + (1 - \mathrm{w}^*_I)\mathcal{T}_t^{\mu^*_I} I^*_t \equiv EX_{C,t} + EX_{I,t} \tag{20}$$

We now consider the limit as the SOE becomes very small relative to ROW. The latter then becomes closed from its own viewpoint, and $\mathrm{w}^*_C \to 1$. However, C^* becomes very large relative to the SOE so that exports EX_t remain finite.

In what follows, we consider a steady state where all prices and the terms of trade are normalized at unity. Then, in this steady state exports and consumption and investment goods are given respectively by

$$EX_C = cs_{exp}EX \quad \text{and} \quad EX_I = is_{exp}EX$$

where export shares cs_{exp} and is_{exp} are the limits of $(1 - \mathrm{w}^*_C)C^*$ and $(1 - \mathrm{w}^*_I)I^*$ respectively described above. These are calibrated from trade data. Then, defining an equilibrium that pins down EX in this steady state, we write total exports as

$$\frac{EX_t}{EX} = cs_{exp}\mathcal{T}_t^{\mu^*_C}\frac{C^*_t}{C^*} + is_{exp}\mathcal{T}_t^{\mu^*_I}\frac{I^*_t}{I^*}$$

Thus, an improvement in the terms of trade in the home SOE increases exports of consumption and investment goods. Aggregate consumption and investment in the ROW, C^*_t and I^*_t respectively, is modelled and estimated independently of the SOE.

2.4 Labor market and wage setting

We model a formal and an informal sector in the labour market, in which we assume that each household supplies homogeneous labour at a nominal wage rate $W^{nh}_{i,t}$ to a monopolistic trade union, who differentiates the labour and sells type $H_{it}(j)$ at a nominal wage $W^n_{i,t}(j) > W^{nh}_{i,t}$ to a labour packer in a sequence of Calvo staggered nominal wage contracts, as in Smets and Wouters (2007). The real wage is then defined as $W_{i,t} \equiv \frac{W^n_{i,t}}{P_{i,t}}$. We now have to distinguish between *price inflation*, which uses the notation $\Pi_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}}$, and *wage inflation* $\Pi^W_{i,t} \equiv \frac{W^n_{i,t}}{W^n_{i,t-1}}$. Each sector is equivalent and so, for neatness, we describe it once for sector $i \in \{1,2\}$

As with price contracts, we employ Dixit-Stiglitz quantity and price aggregators. Calvo probabilities are now ξ_i and $\xi_{w,i}$ for price and wage contracts, respectively. The competitive labour packer forms a composite labour service according to $H^d_{i,t} = \left(\int_0^1 H_{i,t}(j)^{(\zeta_{w,i}-1)/\zeta_{w,i}} dj\right)^{\zeta_{w,i}/(\zeta_{w,i}-1)}$ and sells onto the intermediate firm, where $\zeta_{i,w}$ is the elasticity of substitution across labour varieties in sector i. For each i, j, the labour packer chooses $H_{i,t}(j)$ at a wage $W^n_{i,t}(j)$ to maximize $H^d_{i,t}$ given total expenditure $\int_0^1 W^n_{i,t}(j)H_{i,t}(j)dj$. This results in a set of labour demand equations for each differentiated labour type j with wage $W^n_{i,t}(j)$ of the form

$$H_{i,t}(j) = \left(\frac{W^n_{i,t}(j)}{W^n_{i,t}}\right)^{-\zeta_{w,i}} H^d_{i,t} \quad i = 1,2 \tag{21}$$

where $H_{i,t}(j)$ is the quantity of household homogeneous labour provided by both Ricardian and RoT households needed to produce a differentiated labour service j in sector i and $W^n_{i,t} = \left[\int_0^1 W^n_{i,t}(j)^{1-\zeta_{w,i}} dj\right]^{\frac{1}{1-\zeta_{w,i}}}$, $i = 1,2$, is the aggregate wage index in each sector.

Wage setting by the trade union follows the standard Calvo framework supplemented with indexation. At each period there is a probability $1 - \xi_{i,w}$ that the wage is set optimally. The optimal wage derives from maximizing discounted profits. For those trade unions unable to reset, wages are indexed to last period's aggregate inflation, with wage indexation parameter $\gamma_{w,i}$. Then, as with price contracts, the wage rate trajectory with no re-optimization is given by $(W^n_{i,t})^O(j)$, $(W^n_{i,t})^O(j)\left(\frac{P_{i,t}}{P_{i,t-1}}\right)^{\gamma_{w,i}}$, $(W^n_{i,t})^O(j)\left(\frac{P_{i,t+1}}{P_{i,t-1}}\right)^{\gamma_{w,i}}$, etc. The trade union buys homogeneous labour at a nominal price $W^n_{i,t}$ and converts it into a differentiated labour service of type j. The trade union at time t then chooses $(W^n_{i,t})^O(j)$ to maximize real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k_{w,i} \frac{\Lambda_{t,t+k}}{P_{i,t+k}} H_{i,t+k}(j) \left[(W^n_{i,t})^O(j)\left(\frac{P_{i,t+k-1}}{P_{i,t-1}}\right)^{\gamma_{w,i}} - W^{nh}_{i,t+k} MS_{W_{i,t}}\right], i = 1,2 \tag{22}$$

subject to

$$H_{i,t+k}(j) = \left(\frac{(W^n_{i,t})^O(j)}{W^n_{i,t+k}}\left(\frac{P_{i,t+k-1}}{P_{i,t-1}}\right)^{\gamma_{w,i}}\right)^{-\zeta_{w,i}} H^d_{i,t+k} \tag{23}$$

where $MS_{W_{i,t}}$ is a wage markup shock.

2.5 Firms

Within each sector (formal and informal) there are wholesale and retail sectors. The former acts in perfect competition producing a homogeneous intermediate good, the latter in monopolistic competition producing differentiated final goods. An important distinction between the formal and informal sectors, indexed by $i \in \{1, 2\}$, is that in the former wages are subject to a proportional tax rate, τ_t^w, whereas the informal sector is untaxed. Parameters describing price and wage stickiness and elasticities of substitution for output and labour markets differ, but otherwise each sector is equivalent. In addition to these sectors, for modelling convenience, we have capital producers.

As this is a model with LCP in the formal retail sector and PCP in the informal one, respectively, we have perfect exchange rate pass-through in the informal sector and therefore the LOP applies to each informal differentiated good, but we have the issue of incomplete exchange rate pass-through for exports and keep the assumption of perfect exchange rate pass-through for imports.

2.5.1 Wholesale Sector

We assume a Cobb-Douglas production function in which $A_{i,t}$ is the technology shock in sector i

$$Y_{i,t}^W = F(A_{i,t}, H_{i,t}^d, K_i) = (A_{i,t} H_{i,t}^d)^{\alpha_i} K_i^{1-\alpha_i} \,; i = 1, 2 \tag{24}$$

noting that hours demanded $H_{i,t}^d$ are hours worked. Wholesale firms sell at nominal price $P_{i,t}^W$ to retailers, so profit maximisation implies labour demand and capital demand in each sector respectively as follows

$$F_{H_{i,t}} = \alpha_i \frac{Y_{i,t}^W}{H_{i,t}^d} MC_{i,t} = W_{i,t} \,, i = 1, 2 \tag{25}$$

$$F_{K_{1,t}} = (1 - \alpha_1) \frac{Y_{1,t}^W}{K_{1,t}} MC_{1,t} (1 - \tau_t^K) \frac{P_{1,t}}{P_t} = r_t^K \tag{26}$$

$$F_{K_{2,t}} = (1 - \alpha_1) \frac{Y_{2,t}^W}{K_{2,t}} MC_{2,t} \frac{P_{2,t}}{P_t} = r_t^K \tag{27}$$

where τ_t^K is a tax on corporate profits in the formal sector and r_t^K is the rental rate of capital.

Note that in the informal sector there is no tax on profits, and $P_{1,t}$ and P_t are price indexes of formal and final consumption goods. In (25), $F_{H_{i,t}}$ equates the marginal product of labour with the real wage in each sector, while in (26) $F_{K_{i,t}}$ equates the marginal product of capital with the rental rate in each sector. Also, note that, owing to the friction introduced by wage-setters, there is under-employment in the model, i.e. $\sum_{i=1}^2 H_{i,t}^d < \sum_{i=1}^2 H_{i,t}$.

2.5.2 Retail Sector and Incomplete Exchange Rate Pass-through For Exports

Following the empirical literature on emerging economies, we introduce incomplete exchange rate pass-through from exports to prices through a general set-up in which a fixed proportion θ of formal retailers set export prices $P_{1,t}^{*\,p}$ in the home currency (producer currency pricing) and a proportion $1-\theta$ of them set export prices $P_{1,t}^{*\,\ell}$ in, say, US dollars (local or destination currency pricing). Then, the price of exports in foreign currency is given by

$$P_{1,t}^* = \theta P_{1,t}^{*\,p} + (1 - \theta) P_{1,t}^{*\,\ell} \tag{28}$$

where

$$S_t\, P^{*\,p}_{1,t} = P_{1,t} \tag{29}$$

Putting $\theta = 1$ gets us back to the model with complete exchange rate pass-through, so we have as following: each retailer $m \in (0,1)$ in sector $i = 1,2$ purchases output from the intermediate good sector at price $P^W_{i,t}$ and converts into a differentiated home goods sold at price $P_{i,t}$ to households, capital good producers and governments, who use the technology

$$C_{i,t} = \left(\int_0^1 C_{i,t}(m)^{(\zeta_{p,i}-1)/\zeta_{p,i}} dm\right)^{\zeta_{p,i}/(\zeta_{p,i}-1)}, i = 1,2 \tag{30}$$

to combine into baskets, where $\zeta_{p,i}$ is the elasticity of substitution between the goods in sector i. Maximising (30) subject to $P_{i,t}C_{i,t} = \int_0^1 P_{i,t}(m)C_{i,t}(m)\mathrm{d}m$ implies a set of demand equations for each intermediate good m with price $P_{i,t}(m)$ of the form

$$C_{i,t}(m) = \left(\frac{P_{i,t}(m)}{P_{i,t}}\right)^{-\zeta_{p,i}} C_{i,t}\,, i = 1,2 \tag{31}$$

where $P_{i,t} = \left[\int_0^1 P_{i,t}(m)^{1-\zeta_{p,i}} dm\right]^{\frac{1}{1-\zeta_{p,i}}}$. P_t is the aggregate price index of home produced goods. There are equivalent demand schedules for investment goods, government consumption and for foreign demand. Summing the demand schedules from each buyer implies a total demand for home produced good m given by

$$Y_{i,t}\,(m) = \left(\frac{P_{i,t}(m)}{P_{i,t}}\right)^{-\zeta_{p,i}} Y_{i,t}\,, i = 1,2 \tag{32}$$

Every period, each firm faces a fixed probability $1-\xi_{p,i}$ that they will be able to update their prices.[9] Denoting the optimal price at time t for good m as $P^O_{i,t}\,(m)$, the firms allowed to re-optimize prices maximise expected discounted profits by solving

$$\max_{P^O_{i,t}(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k_{p,i} \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{i,t+k}\,(m)\left[P^O_{i,t}\,(m) - P^W_{i,t+k}\, MS_{i,t+k}\right], i = 1,2 \tag{33}$$

with $MS_{i,t}$ as a markup shock in each sector and real marginal cost is given by $MC_{i,t} \equiv \frac{P^W_{i,t}}{P_{i,t}}$.

Price setting in export markets by domestic LCP exporters follows in a very similar fashion to domestic pricing. Recalling that S_t is the nominal exchange rate, the optimal price in units of domestic currency is $\hat{P}^{*\,\ell}_{1,t} S_t$, costs are as for domestically marketed goods, so (33) becomes

$$\max_{(P^{*\,\ell}_{1,t})^O(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k_{p,1} \frac{\Lambda_{t,t+k}}{P^{*\,\ell}_{1,t+k}} Y^{*\,\ell}_{1,t+k}\,(m)\left[(P^{*\,\ell}_{1,t})^O\,(m)\, S_{t+k} - P^W_{1,t+k} MS^{*\,\ell}_{1,t+k}\right] \tag{34}$$

[9] We also allow for prices to be indexed to last period's aggregate inflation, with a price indexation parameter $\gamma_i \in [0,1]$.

with

$$MC^{*\,\ell}_{1,t} \equiv \frac{MC_{1,t}\,P_{1,t}}{S_t\,P^{*\,\ell}_{1,t}}. \tag{35}$$

The system is completed with

$$\Pi^*_{1,t} = \theta \Pi^{*\,p}_{1,t} + (1-\theta)\Pi^{*\,\ell}_{1,t} \tag{36}$$

where $\Pi^*_{1,t,t+1}$ is the aggregate export inflation in foreign currency. Thus, PCP inflation ($\Pi^{*\,p}_{1,t}$) is the part of export inflation set according to the LOP, while LCP inflation ($\Pi^{*\,\ell}_{1,t}$) is the part of export inflation arising from imperfect exchange rate pass-through, such that θ is the share of price setting in the export sector holding the LOP.

As $P^{*\,p}_{1,t}$ is set on producer currency prices, from the LOP, $S_t P^{*\,p}_{1,t} = P_{1,t}$ and $RER_t \equiv \frac{S_t P^*_t}{P_t}$, we have that PCP inflation

$$\Pi^{*\,p}_{1,t} = \frac{RER_{t-1}}{RER_t}\frac{\Pi_{1,t}}{\Pi_t}\Pi^*_t \tag{37}$$

Exporters from the foreign bloc are PCPers, so $S_t P^*_{F,t} = P_{F,t}$. Therefore, by analogy with (37) we have imported inflation as

$$\Pi_{F,t} = \frac{RER_t}{RER_{t-1}}\frac{\Pi_t}{\Pi^*_t}\Pi^*_{F,t} \tag{38}$$

Now, we have that

$$\mathcal{T}^*_t \equiv \frac{P^*_{1,t}}{P^*_{F,t}} = \frac{\theta P^{*\,p}_{1,t} + (1-\theta)P^{*\,\ell}_{1,t}}{P^*_{F,t}} = \frac{\theta \frac{P_{1,t}}{S_t} + (1-\theta)P^{*\,\ell}_{1,t}}{\frac{P_{F,t}}{S_t}}$$

It follows that

$$\mathcal{T}_t\mathcal{T}^*_t = \theta + (1-\theta)\frac{S_t P^{*\,\ell}_{1,t}}{P_{1,t}} = \theta + (1-\theta)\frac{RER_t \frac{P^{*\,\ell}_{1,t}}{P^*}}{\frac{P_{1,t}}{P_t}} \tag{39}$$

and hence from (110) and (39)

$$MC^{*\ell}_{1,t} = \frac{(1-\theta)\ MC_{1,t}}{\mathcal{T}_t\mathcal{T}^*_t - \theta} \tag{40}$$

As $\theta \to 1$, we get back to the previous model with complete exchange rate pass-through. Thus, when we come to estimation, we can draw a distinction between exchange rate deflation $\Pi_{S,t} = \frac{S_t}{S_{t-1}}$ that would occur under perfect exchange rate pass-through and the actual $\Pi^{obs}_{S,t}$ observed in the data.

2.6 The Commodity Sector and Trade Balance

We introduce a commodity sector, more specifically oil, treating its output as an exogenous constant endowment, which is given by

$$Y_t^O = \kappa Y \tag{41}$$

where Y is the steady state trend path of output Y_t. Revenues are then driven only by the price of the commodity $P_{O,t}^*$ denominated in foreign currency, which is an exogenous process, subject to a shock $\epsilon_{P_O^*,t}$

$$\log \frac{P_{O,t}^*}{P_t^*} - \log \frac{P_O^*}{P^*} = \rho_{P_O^*} \left(\log \frac{P_{O,t-1}^*}{P_{t-1}^*} - \log \frac{P_O^*}{P^*} \right) + \epsilon_{P_O^*,t} \tag{42}$$

The commodity is entirely exported and the only channel through which oil production and price affects the model is via the trade balance and the government budget constraint.[10]

The nominal trade balance

$$P_t TB_t = S_t P_{O,t}^* Y_t^O + P_{H,t} Y_t - P_t C_t - P_{I,t} I_t - P_{H,t} G_t \tag{43}$$

is the difference between output, commodity revenue, private and public consumption, and investment.

2.7 Monetary and Fiscal Policy

Consider government borrowing as the domestic nominal bonds $B_{H,t}$ held by domestic households and define the total stock of government bonds held in home country consumption units as

$$B_{G,t} \equiv \frac{B_{H,t}}{P_t} \equiv B_{GH,t} \tag{44}$$

Then, by analogy with the national budget constraint (53), the government budget constraint is

$$P_t^B B_{GH,t} = \frac{1}{\Pi_{t-1,t}} B_{GH,t-1} + D_t \tag{45}$$

where, recalling that $MC_{1,t} \equiv \frac{P_{1,t}^W}{P_{1,t}}$, the nominal government deficit is given by

$$P_t D_t = P_{1,t} G_t - P_t \, W_{1,t}^{nh} H_{1,t} \tau_t^w - (1-\alpha_1) \, Y_{1,t}^W \, P_{1,t} \, MC_{1,t} \, \tau_t^K - P_t (m_t - \frac{m_{t-1}}{\Pi_t}) - \tau_t^o S_t P_{O,t}^* Y_t^O \tag{46}$$

and τ_t^o is a tax on oil revenue.

We assume that the tax rate τ_t^K is held fixed at the steady state value of τ^K. The fiscal

[10] Given the empirical case we study below, we focus on an oil sector, but this setup can easily be applied to capture commodity dependence on energy, minerals or agricultural exports.

stabilization instrument G_t follows a Taylor-type rule

$$\log\left(\frac{G_t}{G}\right) = \rho_G \log\left(\frac{G_{t-1}}{G}\right) + (1-\rho_G)\Big(\log(\bar{G}_t) - \theta_{bg}\log\left(\frac{B_{G,t-1}}{B_G}\right)\Big) + \epsilon_{G,t} \tag{47}$$

where θ_{bg} is the long run elasticity of domestic bonds with respect to government expenditures and $\epsilon_{G,t}$ is a fiscal policy shock process.

A central feature of our paper is the monetary policy rule. We take money growth rate (μ_t) as the monetary policy instrument, defined as

$$\mu_t = \frac{M_t}{M_{t-1}} = \frac{m_t}{m_{t-1}}\Pi_{1,t} \tag{48}$$

The monetary authority sets the money growth rate to stabilize business cycle fluctuations, based on an inertial Taylor-type feedback rule, responding to deviations in inflation, GDP, GDP growth and exchange rate depreciation, with their respective long run elasticities θ_π, θ_s, θ_y and θ_{dy} are the long-run elasticities of the inflation, depreciation rate, output and output growth.[11] Moreover, $\epsilon_{\mu,t}$ is a monetary policy shock process

$$\begin{aligned}\log\left(\frac{\mu_t}{\mu}\right) &= \rho_\mu \log\left(\frac{\mu_{t-1}}{\mu}\right) - (1-\rho_\mu)\Big(\theta_\pi \log\left(\frac{\Pi_{1,t}}{\Pi_1}\right) + \theta_s \log\left(\frac{\Pi_{S,t}}{\Pi_S}\right) \\ &+ \theta_y \log\left(\frac{Y_{1,t}}{Y_1}\right) + \theta_{dy}\log\left(\frac{Y_{1,t}}{Y_{1,t-1}}\right)\Big) + \epsilon_{\mu,t}\end{aligned} \tag{49}$$

where $\Pi_{S,t}$ is the rate of nominal exchange rate depreciation.

Up to now the nominal interest rate is endogenously determined given the rule (49). We later compare this monetary growth rule with the standard nominal interest rate rule analogous to (49) of the form

$$\begin{aligned}\log\left(\frac{R_t}{R}\right) &= \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\Big(\theta_\pi \mathbb{E}_t \log\left(\frac{\Pi_{1,t+j}}{\Pi_1}\right) + \theta_s \log\left(\frac{\Pi_{S,t}}{\Pi_S}\right) \\ &+ \theta_y \log\left(\frac{Y_{1,t}}{Y_1}\right) + \theta_{dy}\log\left(\frac{Y_{1,t}}{Y_{1,t-1}}\right)\Big) + \epsilon_{R,t}\,; \quad j = 0, 1, 4\end{aligned} \tag{50}$$

and make μ_t endogenous. (50) rule has been generalized to j-period forward-looking expected inflation rate targeting rule and in our comparison with the monetary rule (49) we introduce the same forward-looking inflation-targeting feature. The 'Taylor Principle' for these rules is that $\theta_\pi > 1$ so that in the long run the instrument is responding more than one-to-one to changes in the possibly forward-looking inflation target, which in (49) and (50) is formal sector inflation $\Pi_{1,t}$.

[11] The Bank of Iran, amongst other EE monetary authorities, uses a 'multiple indicator approach', i.e. it considers a range of economic and financial variables as policy indicators, with the control of money supply as an explicit instrument. Hence, we specify a general Taylor-type rule, where the central bank adjusts money growth rate in response to deviations in inflation, output, output growth and the exchange rate depreciation.

3 Method of Estimation

3.1 Data

To estimate the model, we use quarterly information on eight key variables for Iran: GDP, consumption, investment, consumer price index (CPI), broad money (M2), oil price, nominal exchange rate and wages for the sample period 1993:Q1 to 2009:Q4.[12] Crude oil prices are taken from FRED Economic Data, annual wages are compensation of employees from the World Bank and converted to quarterly data by as in Gabriel *et al.* (2012), all other variables are from IMF's International Financial Statistics. All variables are in log-differences, with real variables obtained by deflating with the CPI and seasonally adjusted with the exception of the exchange rate. The nominal exchange rate displays a sharp structural break around 2001, so to account for it we define a dummy variable (EXW) in the form of AR(1) process as following and estimate its standard deviation.

$$\log EXW_t - \log EXW = \rho_{EXW}(\log EXW_{t-1} - \log EXW) + \epsilon_{EXW,t} \tag{51}$$

3.2 Calibrated Parameters

In order to evaluate the performance of the model, we use a combination of estimated and calibrated parameters, mainly because the data set is not rich enough to identify all of them, particularly the informal sector ones. Table 3 summarizes the calibration of parameters and the steady state values of selected endogenous variables, matching, as accurately as possible, the empirical evidence and available (quarterly) data on key statistics of formal and informal sector in emerging economies.

< Table 3 here >

As in much of the literature, the depreciation rate of capital, δ, is set at 10 per cent per annum, implying a quarterly value of 0.025. The home discount rate is set at $\beta = 0.99$, consistent with estimates for emerging economies (see Gabriel *et al.* 2012, Khera 2016 and Anand and Khera 2016, for example). For the risk aversion parameter, we set σ_R to 2, given the estimates of Tabova (2011) for middle-income countries and Gabriel *et al.* (2012) for India.

Estimates of the substitution elasticity between imported and home goods (μ_C) in the literature range from 1.07 to 2.50, so we calibrate it at 1.50.[13] Following Medina and Soto (2005), Chang *et al.* (2015) and Adler *et al.* (2016), the export elasticity demand μ_C^* is set to 1.50. The inverse elasticity of real money holdings, ψ, is calibrated at 1.4 based on empirical evidence for Iran (see Komijani and Tavakolian 2012 and Manzoor and Taghipour 2016). Using IMF and World Bank data on broad aggregates, we calibrate the government share of production (g_y), export share of production (θ_1) and the relative weight of oil export with respect to production at 23%, 30% and 20%, respectively. Moreover, using data provided by the Tehran Securities Exchange Technology Management Co., the government bonds share of production b_g is set at 10%. Finally, we assume an optional share of LCP setting in the export sector $(1-\theta)$ at 0.80.

[12] Wage data limits our sample period, up-to-date data for other variables is available.

[13] For example, Castillo and Montoro (2010) 2.5 for Peru, 0.6 in Medina and Soto (2005) for Chile, 1.45 in Gabriel *et al.* (2012) for India, 1.15 and 1.20 in Khera (2016) for South Africa and India, respectively, and 1.50 in Batini *et al.* (2011) for emerging economies.

3.3 Matching Informality Statistics

We now turn to the differences between the formal and informal sectors in terms of labour intensity (α_i), the degree of price and wage stickiness ($\xi_{p,i}$, $\xi_{w,i}$), market power ($\zeta_{p,i}$, $\zeta_{w,i}$), the elasticity of substitution between goods in the formal and informal sectors (μ_S), the elasticity of substitution between labours in the formal and informal sectors (μ_H), the size of the informal sector in the goods and labour markets ($1 - \mathrm{w}_S$, $1 - \mathrm{w}_H$) and, finally, the price indexation parameters in each sector ($\gamma_{p,i}$).

The parameters relating to labour and goods market rigidities are all calibrated to be higher in the formal sector, capturing the existence of regulations in this sector, as opposed to the unregulated informal sector. Moreover, labour shares in formal and informal production are set at $\alpha_1 = 0.70$ and $\alpha_2 = 0.80$, given the estimates of 0.68-0.74 in Gabriel *et al.* (2012) and 0.66-0.66 in Anand and Khera (2016). Following literature on the size of the informal sector, w_S is set at 0.80.

Regarding price stickiness, estimates for developing countries tend to be lower than those of developed countries (e.g. Smets and Wouters 2007). Castillo and Montoro (2010) find values ranging from 0.34 to 0.52 for domestic price rigidity. Furthermore, Gabriel *et al.* (2012) find significantly higher rigidities at 0.75 in 'formal' prices, compared to 0.30 for 'informal' prices, while Komijani and Tavakolian (2012) and Manzoor and Taghipour (2016) estimate those at 0.58 and 0.25. Thus, given the lack of evidence for the informal sector in Iran, we set values of 0.60 and 0.30 for formal price stickiness (ξ_1) and informal price stickiness (ξ_2), respectively. On the other hand, concerning the elasticity of substitution among different retail varieties in the formal and informal sector, informed by the work of Khera (2016) and Gabriel *et al.* (2012) and the view that we expect the informal sector to have less market power that the formal sector, we adopt a mean of 7 and 9 for (ζ_1) and (ζ_2), respectively.

As for the parameters that characterize the interactions between the formal and informal sectors, the elasticity of substitution between formal and informal labour supply (μ_H) is set at 2.00 following Khera (2016) and Anand and Khera (2016); the fraction of households providing labour to formal sector entrepreneurs (w_H) is calibrated at 55%, consistent with Batini *et al.* (2011) and Gabriel *et al.* (2012); regarding the elasticity of substitution among different skilled labour in the formal sector, we calibrate $\zeta_{w,1}$ at 2.50, which corresponds to a formal wage premium of $\frac{\zeta_{w,1}}{\zeta_{w,1}-1} = 1.67$, and $\zeta_{w,2}$ at 6.00, resulting in an informal wage premium of $\frac{\zeta_{w,2}}{\zeta_{w,2}-1} = 1.20$;[14] finally, we adopt a mean value of 0.60 and 0.30 for formal wage stickiness ($\xi_{1,w}$) and informal wage stickiness ($\xi_{2,w}$).

3.4 Bayesian Estimation

We estimate the model by Bayesian methods, which entails retrieving the posterior distribution of the model's parameters, say Θ, conditional on the data. Using the Bayes' theorem, the posterior distribution is obtained as

$$p(\Theta|Y^T) = \frac{L(Y^T|\Theta)p(\Theta)}{\int L(Y^T|\Theta)p(\Theta)d\Theta}$$

[14] For instance, Gabriel *et al.* (2012) and Anand and Khera (2016) estimate a value of 1.78 and 1.75, respectively, for this wage premium in India, while Adler *et al.* (2016) set it for 1.87 for a small open economy.

where $p(\Theta)$ denotes the prior density of the parameter vector Θ, $L(Y^T|\Theta)$ is the likelihood of the sample Y^T with T observations (evaluated with the Kalman filter) and $\int L(Y^T|\Theta)p(\Theta)d\Theta$ is the marginal likelihood. Since there is no closed form analytical expression for the posterior, this must be simulated.[15]

In our two-bloc setup, the ROW bloc does not depend on interactions with the SOE, so it can be estimated separately, which is carried out using US data.[16] We estimate and compare some model variants using the techniques described above. Each variant has an associated set of unknown parameters $\Theta \in \bar{\Theta}$ for which we want to characterise the posterior distribution. As discussed in section 2, the model is augmented with a set of orthogonal log-linearised structural shocks that follow a stationary AR(1), such that $\log \vartheta_t - \log \vartheta = \rho_\vartheta(\log \vartheta_{t-1} - \log \vartheta) + \epsilon_{\vartheta,t}$, with $\vartheta_t = \{A_{1,t}, A_{2,t}, MS_{1,t}, MS_{2,t},\ MS_{w1,t}, MS_{w2,t},\ I_t,\ PS_t,\ tot_t, MS^*_{1,t}, M, A^*_t,\ G^*_t, MS^*_t, IS^*_t, PS^*_t\}$ (with ϑ containing the corresponding steady state values of 1) and two i.i.d. shocks affecting monetary and fiscal policy. Recall that the price of oil ($P^*_{O,t}$) has an exogenous process in the model, so we have estimated the standard deviation of this shock separately by fitting an AR(1) process and then use the estimated coefficient and standard deviation in the SOE model.

3.5 Prior distributions

In order to implement Bayesian estimation, prior distributions must be defined for the parameters Θ and the structural shocks in ϑ_t. This choice is usually guided by inherent theoretical restrictions and evidence from previous studies. We use normal distributions as priors for unbounded parameters when more informative priors seem to be necessary, while beta distributions are used for all parameters bounded between 0 and 1, i.e., fractions or probabilities. We use inverse gamma distributions as priors when non-negativity constraints are necessary. All priors are assumed to be the same across specifications. However, since estimated DSGE models for emerging economies are less abundant, especially for Iran, we will work with relatively diffuse priors, thus reducing the importance of the mean of the prior distribution on the outcome of the estimation. Prior means and standard deviations are shown below in Table 4.

We assume a normal distribution centred at 2 and a standard deviation of 0.25 for the risk aversion parameter of non-Ricardian households (σ_C), in line with the literature on (Ricardian) risk aversion. Moreover, following Gabriel *et al.* (2012), the prior for the proportion λ of RoT consumers is a beta distribution with mean of 0.5 and standard deviation of 0.1. To ensure that the consumption habit persistence, χ, is bounded between 0 and 1, we assign a beta distribution (mean of 0.5, standard deviation of 0.1), as previous studies show mixed evidence regarding its value for developing countries.[17] Likewise, the prior for price indexation parameters are beta distributions with mean of 0.5 and standard deviation of 0.1, consistent with estimates of Manzoor and Taghipour (2016) for Iran. For trade openness (w_C and w_I), we follow Boerma (2014) in

[15] In a first step, the posterior mode and corresponding Hessian are obtained and then the posterior density is approximated by using the Monte-Carlo Markov Chain Metropolis-Hastings (MCMC-MH) algorithm, with two parallel chains with 100,000 draws, sufficient to ensure convergence according to the the indicators recommended by Brooks and Gelman (1998) and Gelman *et al.* (2004).

[16] Full results are available in the Supplementary Appendix.

[17] Castillo and Montoro (2010) estimate a large value in the range of 0.7 to 0.9 for Peru, whereas smaller values of 0.24 are estimated for India in Khera (2016), and a value of 0.3 for Iran in Manzoor and Taghipour (2016).

defining a beta $(0.5, 0.1)$, while we assume a $N(3, 1.5)$ for investment adjustment costs (ϕ_I) based on the calibrated values in Khera (2016), Gabriel *et al.* (2012) and the estimation for Iran in Manzoor and Taghipour (2016). As for the substitution elasticity between formal and informal goods (μ_S), since formal goods are traded and informal goods our non-traded, we match this to values commonly used in the literature for the substitution elasticity between traded and non-traded goods, namely Mendoza (1995) and Castillo and Montoro (2010), as well as Khera (2016), so we choose a $N(1.5, 0.2)$ prior.

Regarding the monetary policy rule, previous estimates for Iran for the inflation feedback parameter range from 1.07 in Komijani and Tavakolian (2012) to 1.54 in Manzoor and Taghipour (2016). For other developing countries, it ranges from 1.27 in Carlstrom and Fuerst (2001) to 2.5 and 1.5 in Gabriel *et al.* (2012) and Khera (2016), respectively, so with these values in mind, we assign a $N(1.5, 0.25)$ prior to θ_Π. For θ_y, evidence suggests that the policy responses to GDP movements in emerging economies are not conducted in a systematic fashion, with both Saxegaard *et al.* (2010) and Castillo and Montoro (2010) finding estimates close to zero, whereas Manzoor and Taghipour (2016) and Komijani and Tavakolian (2012) estimate values of around 2.50 and 1.5 for Iran. Thus, we set a diffuse $N(1.5, 0.25)$ prior, letting the data guide its estimation. Regarding θ_{dy}, we assume a $N(0.1, 0.05)$ prior, while for the response to exchange rate movements(θ_{ds}) we follow Manzoor and Taghipour (2016) in setting a $N(0.5, 0.25)$ prior. Finally, the policy smoothing coefficient (ρ_μ) is assigned a beta prior with mean of 0.75 and standard deviation of 0.1, given the mixed results in the literature: Manzoor and Taghipour (2016) and Castillo and Montoro (2010) find it to be 0.4 Peru, whereas Komijani and Tavakolian (2012) and Gabriel *et al.* (2012) estimate significantly higher values of 0.8 for Iran and India, respectively.

Regarding shock processes, we use a beta distribution for the persistence of all shocks with a mean of 0.75 and a standard deviation of 0.10. Given the uncertainty regarding the sources of business cycle fluctuations, we adopt uninformative gamma distributions for the standard deviations of all shocks, with a prior mean of 3.00 along with a standard deviation of 3.00. For the correlations between sectoral wage-price markups and technology shocks, we assume a beta distribution with mean of 0.50 and standard deviation of 0.20.

4 Empirical Results

4.1 Posterior Estimates

Table 4 (column 5) displays the posterior means of the Bayesian estimation along with 95% confidence intervals. We first note that the posterior estimates are much more sharply peaked than the prior distribution for most parameters, implying that the data is reasonably informative, away from the priors.[18] However, estimation is somewhat less accurate for parameters such as the informal wage-price index, the exchange rate depreciation feedback, the persistence of the wage-price markup and money demand shocks.

< Table 4 here >

[18]See Supplementary Appendix; the procedures of Ratto and Iskrev (2011) provide more formal checks and indicate that all parameters are reasonably well identified.

Overall, the parameter estimates are quite plausible. For instance, habit persistence χ is estimated to be somewhat lower than Castillo *et al.* (2006), but close to the estimates of Manzoor and Taghipour (2016) and Saxegaard *et al.* (2010). Given the high share of credit-constrained consumers estimated ($\lambda = 0.31$), this result seems reasonable, as a large share of households in Iran is unable to smooth consumption. The posterior mean estimate for investment adjustment cost ($\phi_I = 1.44$) is consistent with the calibrated value in Gabriel *et al.* (2012), although lower than that of Manzoor and Taghipour (2016).

On the other hand, our estimations indicate that price indexation is higher in the informal sector relative to formal sector ($\gamma_2 = 0.48$ compared to $\gamma_1 = 0.35$). Turning to the estimates for home shares, these are $\mathrm{w}_C = 0.56$ and $\mathrm{w}_I = 0.54$, which correspond to an upper-middle income in the country classification of openness in Boerma (2014). Moreover, our estimate for the substitution elasticity between traded and non-traded goods ($\mu_S = 1.21$) concurs with Khera (2016) and Mendoza (1995).

Regarding the policy parameters, we observe a moderately high degree of policy inertia ($\rho_\mu = 0.71$) and, consistent with other estimates in the literature (Manzoor and Taghipour, 2016) that the central bank responds strongly to inflationary pressures ($\theta_\Pi = 1.48$), but only modestly to output fluctuations $\theta_y = 0.099$ - a result also found in Saxegaard *et al.* (2010), but contrasting with Manzoor and Taghipour (2016). Moreover, we find a relatively low response of money growth to movements in output growth ($\theta_{dy} = 0.11$) and similarly with regards to exchange rate depreciation ($\theta_{ds} = 0.33$), thus indicating that the central bank is likely to choose a relatively less flexible exchange rate regime.

4.2 Shocks and Variance Decomposition of Business Cycle Fluctuations

Our estimation exercise returns high values for the volatility of shocks hitting the Iranian economy, in particular for government spending, investment, oil price and formal wage-price markup shocks. This is consistent with existing empirical evidence on emerging economies (Uribe and Schmitt-Grohé, 2017), but also reflects the vulnerability of the Iranian economy to shocks from a range of sources. Overall, the persistence of all shocks is relatively high, but not unduly so. As shown in Table 5, the model does reasonably well in matching second moments in the data (volatilities, persistence and cross-correlations with output).

< Table 5 here >

Further analysis of the estimated shocks helps us to disentangle their contribution in driving business cycle fluctuations in the economy of interest. Indeed, a variance decomposition of the shocks reveals that our results are in line with existing evidence for emerging economies. The disturbances from the 'formal' price markup (ϵ_{MS_1}), 'formal' wage markup ($\epsilon_{MS_{1,w}}$) and 'formal' technology (ϵ_{A_1}) appear to be the most relevant in explaining the dynamics of the endogenous variables, with relatively smaller contributions originating from the remaining shocks. Most notably, price markup shocks account for 25 to 35% of fluctuations in output, consumption, inflation inflation, money, labour and wages.

< Table 6 here >

On the other hand, monetary policy shocks explain most of the variation in domestic and formal inflation, as well as wage inflation, government deficit and opportunity cost of capital. Interestingly, these shocks contribute more to fluctuations in the formal than in the informal sector. Foreign shocks are also important, with foreign productivity appearing to have the largest impact, in particular for imported consumption, trade balance, real exchange rate and terms of trade.

Looking at specific variables, it is interesting to note that output and consumption fluctuations are mainly explained by price markup and technology shocks, with little differences between the formal and informal sectors. Turning to the differences between Ricardian and non-Ricardian agents, changes in consumption and money demand for the latter are mostly explained by wage-price markup shocks, whilst for the former they are chiefly driven by technology shocks. However, for their labour supplies, fluctuations for Ricardian agents are due to the markup shocks, whereas monetary policy accounts for those of non-Ricardian workers.

It is also instructive to consider the sources of fluctuations in the informal sector. Unsurprisingly, the informal price markup and informal technology shocks govern the dynamics of this sector. Evidence in the literature points toward the idea that the informal sector is relatively more competitive than the formal sector (see Charlot *et al.* 2011, for example). Thus, this suggests that the effects of informal shocks are dampened due to its more competitive environment, with 'formal' shocks appearing to have considerable effects on the informal sector, namely formal markups and formal technology shocks.

Regarding oil price shocks, they are quite significant in explaining variations in imported consumption and inflation, real exchange rate and aggregate inflation, as well as on imported investment and non-Ricardian labour supply. We also observe that fluctuations in domestic and formal inflation are, as expected, driven by monetary policy, formal price markup and formal technology shocks. In comparison, monetary policy contributes much less to variations in informal inflation, which is mainly explained by sectoral shocks.

In an additional meaningful exercise, and relative to our baseline model, we contrast in turn variance decompositions for the specifications with higher values for the share of RoT consumers (λ), level of trade openness ($1 - \mathrm{w}_C$ and $1 - \mathrm{w}_I$), market power, share of informal sector (w_S), common price stickiness formal and informal sectors in both markets (ζ_2 and ξ_2, $\zeta_{2,w}$ and $\xi_{2,w}$) and no LCP setting in the export market (θ). When trade openness is higher, that naturally increases the role of foreign shocks, as well as terms of trade and oil shocks, in overall fluctuations, including the informal sector, while monetary policy shocks are less pertinent. The picture of a more open economy contrasts starkly with that of the model of trade autarky, in which oil price shocks explains 90% of the fluctuations in the model.

Likewise, a higher share of RoT consumers dampens the role of informal shocks in this sector, while monetary policy is also less prominent in explaining overall variations. Conversely, a higher share of the informal sector significantly amplifies informal shocks' contribution to both formal and informal sector fluctuations, in particular for aggregate output, aggregate consumption and the real exchange rate, whilst the role of monetary and government spending policies decreases. In its turn, increasing informal stickiness and market power to the same level of the formal sector foreseeably magnifies the role of informal shocks in driving informal fluctuations, while monetary, government spending and formal shocks increase their relevance for the dynamics of aggregate variables - the more the two sectors are alike, the less important the intersectoral interactions in amplifying cyclical variations. Finally, if we shut down the LCP setting, formal shocks are dampened, with a larger contribution of the terms of trade shock, i.e. if we ignore this mechanism, the misspecification will inflate the role of terms of trade.

The above comparative analysis reinforces our preference for the baseline specification. It also highlights that, in order to understand the relative importance of the different driving forces in emerging economies, one should do so by suppressing, in succession, the distinct channels in the model, otherwise one runs the risk of over or understating the role of particular shocks - indeed, they can no longer be interpreted as 'structural'.

4.3 Posterior Impulse Response Analysis

The previous exercise allowed us to gauge the contributions of each structural shock in driving business cycle fluctuations in Iran. We now briefly turn to the analysis of selected (posterior) impulse response functions (IRFs), which gives us a more nuanced view of the dynamics of the shocks' effects under different specifications. We focus on responses to shocks to monetary policy (ϵ_μ), government spending (ϵ_G), formal productivity (ϵ_{A_1}), formal price markups (ϵ_{MS_1}) and oil price shocks ($\epsilon_{P_O^*}$), obtained from a positive one standard deviation of each shock's innovation, showing the quarterly percentage changes to the relevant variables about their steady-state values. We compute IRFs for our baseline model (black line), a specification with a larger informal sector size (red line), a model with informal stickiness and market power the same as the formal ones (blue line), a specification with higher proportion of credit-constrained agents (magenta line), a model with trade autarky (green line) and, finally, a model without LCP (indigo line).[19] These will allow us to evaluate the effects of the main features of our model, i.e. the impact of informality and financial frictions, as well as the effect of trade and imperfect exchange rate pass-through.

4.3.1 Impulse Responses to a Technology Shock

From Figure 2, we observe that a positive technology shock in the formal sector (ϵ_{A_1}) improves the returns to capital, leading to an increase in aggregate output, investment, Tobin's Q and Ricardian consumption. However, as the marginal rate of substitution between labour supply and consumption falls, which is equal to the reduction in wage rates, and given that the RoT households consume out of their wages, their consumption reduces - thus, the aggregate consumption effect depends on the proportion of RoT consumers. Inflation falls as supply initially exceeds demand, leading to a fall in formal tradable goods prices, in turn leading to a depreciation, which improves the trade balance. Consequently, the central bank increases the money growth rate in response to the fall in inflation.

< Figure 2 here >

[19] Note that in these figures output, hours worked and wage rate refer to the formal sector, whereas consumption is the aggregate consumption of the households.

Consider the formal labour market and the behaviour of the formal real wage. Whilst falling inflation pushes real wages up, the increase in demand is smaller than the technology improvement, so labour demand falls, shifting the demand curve inward, which pushes the real wage down. The former effect dominates and we see that the real wage rises, but in the informal labour market the latter effect dominates and thus wages decrease. After the initial impact of the shock, the increase in formal real wages puts pressure on marginal costs and consequently on inflation, which leads to a reversion of some of the previous effects (marginal rate of substitution, RoT consumption and return to capital) and an intensification of others (further increase in output, investment, real interest rate).

The informal sector, on the other hand, marginally reduces the impact on output, consumption and investment in the economy, as seen by comparing the responses of the baseline model with the specification with a larger sector share w_S (red line). An increase in formal sector productivity increases the relative price of informal goods, so the higher the w_S, the more dampened this shock will be. As consumption falls in both sectors, domestic consumption reduces and import demand increases more. This terms of trade effect between the two sectors also lowers the cyclical impact in the labour market, leading to a lower increase in real wages. Overall, the effect between the formal and the informal sector acts like a 'stabilizer' over the economy's response to an exogenous shock. In contrast to the above, when frictions are similar in both sectors (blue line), the discrepancies noted above become more muted.

The exchange rate channel comes to the fore when we compare the trade autarky specification (green line) with the baseline model. The reduction in inflation is less stark, aggregate output and consumption increase by more, with sharper movements in the formal and informal sectors. In turn, shutting down the LCP mechanism dampens the reduction in exports, with larger imports falls and an improvement of the trade balance. Moreover, in the autarky case, the effect of productivity shocks is amplified in the informal sector, with larger decreases in informal output, consumption and labour demand - in other words, the more open the economy, the more salient the informal sector mechanisms. Finally, considering higher levels of financial exclusion (magenta line), a larger λ leads to an amplification in the effects of financial frictions.

4.3.2 Impulse Responses to a Formal Price Markup Shock

Figure 3 shows that a positive price markup shock (ϵ_{MS_1}) lowers demand for output, shifting labour demand inwards, such that the real wage falls and with it marginal cost, offsetting the original shock.[20] Monetary policy is contractionary in response to the rise in the inflation rate, so that the interest rate increases, pushing the consumption of Ricardian and non-Ricardian households down, reducing investment and causing Tobin's Q to fall. The marginal rate of substitution falls, which results in a drop in non-Ricardian households' consumption, reducing demand for labour further and adding to the output fall. The marginal cost fall offsets the original inflation shock, so now inflation falls resulting in an expansionary monetary response, reversing the movements in real interest, investment and Tobin's Q.

[20] IRFs for a wage markup shock are qualitatively very similar.

< Figure 3 here >

We also observe that the informal sector acts as a buffer, absorbing workers who are no longer employed in the formal sector, which raises informal output, consumption and hours worked. Recall that CPI inflation rate includes informal sector inflation, which is directly affected by the shock and so aggregate inflation increases less compared to formal inflation alone. Regarding the reduction in informal wages, the effect of informal labour supply dominates the outward shift of informal labour demand and the reduction of informal inflation, so, overall, we have a reduction in all wage rates.

Looking at the open economy channels, the real exchange rate appreciates and so exports and the trade balance fall. The increase in inflation reduces the competitiveness of goods and labour in the formal sectors and, consequently, this has a negative impact on aggregate and formal variables. Furthermore, comparing the baseline and the trade autarky specifications, the latter amplifies aggregate and formal output declines, while boosting the informal sector variables, suggesting that openness lessens nominal frictions. On the other hand, when the LCP channel is shut, increases in price markups lead to higher export prices and less pronounced real exchange rate appreciation and macro aggregates declines.

4.3.3 Impulse Response to a Monetary Policy Shock

Considering now an expansionary monetary policy shock (ϵ_μ), we observe from Figure 4 that, unsurprisingly, aggregate activity is boosted, but this also generates inflationary pressures and a reduction in the real interest rate. Thus, Ricardian households invest less in domestic bonds (a substitution effect) and increase their consumption, as the marginal utility of savings compared to consumption decreases. The consumption of RoT consumers follows the same path due to the increase in the marginal rate of substitution (or the real wage rate they offer to trade union) and, therefore, aggregate consumption increases further the larger the proportion of RoT consumers. Since investment is formed entirely from the output of the formal sector, investment increases. Because of the fall in real interest rate and the rise in money supply, demand for money also expands for both types of households.

< Figure 4 here >

Higher domestic demand leads to expansion in formal output, also aided by increased competitiveness of formal exports due to a real exchange rate depreciation brought about by increased liquidity. Informal output actually shrinks, again due to the terms of trade effect between the formal and the informal sector, as relative prices of informal goods increase (driven by the outward shift in informal supply dominating an inward shift in informal demand).

Regarding the labour market, two opposing forces determine formal real wage rate dynamics: inflation pushes it down, higher demand for formal labour drives it up, with the latter effect dominating. In the informal labour market, due to the fall in supply, demand for labour decreases, but the higher demand in the formal sector absorbs informal workers. Thus, in spite of the rise in the informal inflation, the buffer effect of formal labour market dominates the downward pressures of lower labour demand and higher informal inflation and, therefore, the informal real wage rises.

As above, the 'stabilizer' effect of the informal sector in the baseline model is again evident when we compare it to the higher informal size specification, i.e. it dampens the expansionary nature of the shock in aggregate terms, while amplifying the negative response in the informal sector, due to the terms of trade effect between the two sectors. Removing the LCP setting from the tradable sector leads to a larger increase of exports and the trade balance, and a decrease in export prices is seen.

4.3.4 Impulse Response to a Government Spending Shock and an Oil Price Shock

Turning to the responses to a government spending shock (Figure 5), the striking feature is the presence of a 'classical' crowding-out effect. Indeed, the initial fiscal stimulus increases inflation leading to an aggressive contraction in money growth by the central bank, a rise in the interest rate, which provokes a fall in investment and money demand. The marginal rate of substitution and the consumption of RoT households increase, which further boosts aggregate output and consumption. Given that the demand shock is concentrated in the formal sector, this shifts informal demand (and eventually supply as well) to the formal sector, putting an upward pressure on formal real wages, counteracting the downward pressure from formal inflation.

< Figure 5 here >

Another interesting exercise is to consider the simultaneous impact of oil price shocks ($\epsilon_{P_O^*}$) and economic sanctions in Iran, which can be proxied, albeit in a crudely manner, by the trade autarky variant of the model. Figure 6 plots the corresponding IRFs and it is quite clear that, compared to all the other scenarios, trade autarky (green line) greatly exacerbates the dependence on (exogenous) fluctuations in the price of the commodity. Foreign exchange revenue increases, leading to a rise in the monetary base and a real exchange rate appreciation. Higher import consumption and investment leads to an increase in aggregate consumption and investment, but the corresponding domestic aggregates fall, which eventually leads to output and inflation falling. The central bank then increases money growth, the rise in real interest rate leading to higher money demand.

< Figure 6 here >

5 Money Growth vs Nominal Interest Rate Rules

It is well known in the New Keynesian model literature that the 'Taylor Principle' needs to be satisfied in order for interest rate rules to be stable, while forward-looking rules may give rise to indeterminacy, i.e. they may be unable to uniquely pin down the behaviour of one or more real and/or nominal variables, making many different paths compatible with equilibrium (see Chari *et al.* 1998; Carlstrom and Fuerst 2001 and Carlstrom and Fuerst 2000; Benhabib *et al.* 2001 and Woodford 2003). Sunspot equilibria are of interest because they are typically welfare-reducing and these losses can potentially be quite large.

< Figures 7 and 8 here >

Figure 7 and figure 8 show the indeterminacy and instability analysis of interest rate rules and money growth rules,(50) and (49) respectively, by exploring a simple grid for the inflation feedback parameter (θ_π), the degree of policy inertia (ρ_r) and the proportion of RoT consumers λ. All other parameter values are kept at their estimated values.

As we can see in panels 7a to 7c, increasing the degree with which the interest rate rule is forward-looking increases the region of indeterminacy. However, when we turn to a similar analysis for the money growth rule, panels 8a to 8c show that, in all scenarios, we have complete determinacy and stability.

Another substantial advantage of monetary rules is the avoidance of the Taylor Principle inversion as the degree of LAMP increases, while not imposing any limitation on the model in order to be determinate and stable. Indeed, Galí *et al.* (2004) show that the presence of credit-constrained households can change the properties of interest rate rules, while Bilbiie (2008) highlights that when the fraction of non-Ricardian agents is high, LAMP may overturn the contractionary effect of an interest rate increase - the 'inverted aggregate demand logic' (IADL) - leading to the 'inverted Taylor Principle'. However, Boerma (2014) shows that the IADL is attenuated in a small open economy because the terms of trade channel of monetary policy is also contractionary for a rise in the real interest rate.

$<$ Figures 9 and 10 here $>$

However, under a monetary rule, and unlike an interest rate rule, the effects of monetary policy are not exclusively imposed on Ricardian consumers and thus the transmission channels of the money growth rule do not depend neither on the level of LAMP nor on the degree of openness. Therefore, the IADL does not apply in such an economy and a money growth policy delivers equilibrium determinacy. Figures 9 and 10 examine the saddle-path stability and determinacy properties of a Taylor-type rule and a monetary rule by analysing a grid over the inflation feedback parameter (θ_π) and the share of credit-constrained households (λ). Panels 9a to 9c show that increasing the degree with which agents are forward-looking increases the indeterminacy region (albeit improving stability), while for the money growth rule panels 10a to 10c display full determinacy and stability over any grid-point. This confirms the important policy implication that money growth rules deliver saddle-path stability for any value of the inflation feedback parameter and for the full range $\lambda \in [0, 1)$ of RoT households.

The case for monetary supply as opposed to interest rate rules is an old one, famously set out in Sargent and Wallace (1975). They show that in a simple rational expectations IS-LM model an exogenous path for the nominal interest rate leads to indeterminacy, whereas the equivalent policy for the money supply results in determinacy. Hence, they argue that the optimal monetary policy problem reduces to a choice of the best money supply rule. However, in our paper we are considering rules for the two instruments that are functions of endogenous variables specified in the Taylor-type rules. The advantage of monetary rules is then not so straightforward as interest rate rules can, as we have seen, be designed to avoid indeterminacy. With LAMP, however, this becomes more difficult because of the IADL and the need to switch from (to) a rule obeying the Taylor Principle to (from) one obeying the inverted form as the proportion of RoT consumers increases (decreases).

That said, the well-known advantages interest rate rules in a world of increasingly efficient financial markets remain, but are of less relevance in emerging economies with LAMP and financial autarky as modelled in this paper.[21]

6 Conclusions

In this study, we propose a framework to implement money growth rules in a medium-sized two-bloc model of a SOE emerging economy interacting with the rest of the world. We incorporate several desirable and realistic features typical of these economies in a unified framework, namely nominal frictions, informality, limited asset market participation, imperfect exchange rate pass-through, trade openness, commodity exports dependence and financial autarky. In particular, our framework includes a monetary growth Taylor-type rule and examines whether or not such a rule can successfully stabilize the economy. By taking the model to the data, we are then able to study the main drivers of business cycle fluctuations and empirically gauge the importance of the different mechanisms in our model.

Our findings can be summarised as follows. First, we show that, with high levels of credit-constrained agents and low trade openness, a money growth rule displays full stability and determinacy, even under forward-lookingness of up to four quarters, thus precluding the inverted aggregate demand logic of Bilbiie (2008). Second, our empirical results reveal the dominant role of shocks emanating from the formal sector (such as price-wage markup and productivity shocks) in overall fluctuations. Third, and relative to our baseline model, the different mechanisms play different roles in amplifying or moderating the effects of different shocks: the more significant the informal sector, the more dampened fluctuations become; the higher the trade openness, the more significant the role of 'foreign' shocks (e.g. oil price, technology, terms of trade) in overall variations, including in the informal sector; the higher the share of rule-of-thumb consumers the larger the effects of several shocks, but the more attenuated the effects of monetary policy; ignoring local currency pricing inflates the effects of other shocks, namely terms of trade shocks. These results reinforce the importance of appropriately modelling the different propagation channels that characterise business cycles in emerging economies.

Our research has its limitations, mainly on the empirical front, something that is not unusual in this literature. Indeed, some second moments in the data tend to be somewhat overestimated, while we also find that the data is not very informative about some parameters, in particular the volatility of shocks. Naturally, better and more data would help, but research on emerging economies is held back by a combination of short spans of data, quarterly data of low quality for some variables, structural breaks and changes in policy regimes. All these features make estimation a challenging undertaking.

Nevertheless, there are some promising techniques being proposed that might alleviate these issues, namely the use of endogenous priors as in Del Negro and Schorfheide (2008) and Christiano *et al.* (2011), which may lead to a better matching between model and data moments. The use of data of different frequencies along the lines of Schorfheide *et al.* (2018) can allow for the use

[21] See Woodford (2003), Chapter 1 for a general discussion of these issues.

of more variables and avoiding ad-hoc 'quarterization' of annual observations. On the modelling front, an interesting avenue to explore would be the explicit modelling of the degree of financial openness through the introduction of a banking sector. This would generalize the financial autarky assumption in the model of this paper.

Finally, by not relying on interest rate-based rules, we offer a relatively simple, yet rich enough, platform towards a framework that fully embeds further aspects of Islamic monetary policy. This could include a prominent role that equity and equity-like financial instruments have in Islamic finance which seem to be less volatile than non-Islamic bonds (see Buiter and Rahbari 2015, Khatat 2016 and Akhtar *et al.* 2017, for instance). These and other developments will feature in our future research.

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Appendices

A Tables

Table 1: Cross-Country Data on Financial Inclusion, and Openness

Country Classification	No. of Countries	Financial Exclusion	Openness
Low Income	25/36	0.81	0.40
Lower Middle Income	31/48	0.72	0.45
Upper Middle Income	33/55	0.50	0.46
High Income	40/78	0.11	0.55

Notes: Financial inclusion is measured by the percentage of adults with a bank account at a formal financial institution. This 'narrow' definition of asset market participation precludes the use of LAMP as a free parameter to capture the impact of financial frictions, uncertainty, and suboptimal decision-making on the aggregate marginal propensity to consume; the degree of openness is approximated by domestic imports over domestic spending.
Source: World Bank Development Indicators, World Bank Global Financial Inclusion Database, and author's own calculations.

Table 2: Home and ROW Notation

	Domestic Production	Imported Good	Aggregate
Home Country Quantity	$C_{H,t}$	$C_{F,t}$	C_t
Home Country Price	$P_{H,t}$	$P_{F,t}$	P_t
Foreign Country Quantity	$C^*_{F,t}$	$C^*_{H,t}$	C^*_t
Foreign Country Price	$P^*_{F,t}$	$P^*_{H,t}$	P^*_t

Table 3: Calibrated Parameters

Calibrated parameters	Symbol	Value
Home Discount factor	β	0.99
Depreciation rate	δ	0.025
Ricardian Risk aversion	σ_R	2.00
Inverse elasticity of real money holding	ψ	1.40
Hours worked	H^W	1/3
Preference parameter	ϱ	calibrated so H^W=1/3
Preference parameter on money holding	Ψ	calibrated som/Y_1^T=1
Government spending	g_y	0.23
Capital taxation rate	τ_t^K	0.20
Oil export relative to output	κ	0.20
Share of government bonds	b_g	0.10
Export share of production	θ_1	0.30
Share of Local Currency Pricing	$1-\theta$	0.80
Share of formal sector	w_S	0.80
Share of formal labour market	w_H	0.55
Labour share in the formal sector	α_1	0.70
Labour share in the informal sector	α_2	0.80
Substitution elasticity (Formal/Informal labours)	μ_H	2.0
Substitution elasticity (Home/Foreign goods)	$\mu_C = \mu_I$	1.50
Substitution elasticity (Export/Foreign goods)	$\mu_C^* = \mu_I^*$	1.50
Share of Investment in exports	is_{exp}	0.001
Share of consumption in exports	cs_{exp}	0.999
Formal Calvo prices	ξ_1	0.60
Informal Calvo prices	ξ_2	0.30
Formal Calvo wages	$\xi_{1,w}$	0.60
Informal Calvo wages	$\xi_{2,w}$	0.30
Formal substitution elasticity	ζ_1	7.00
Informal substitution elasticity	ζ_2	9.00
Formal labour substitution elasticity	$\zeta_{1,w}$	2.50
Informal labour substitution elasticity	$\zeta_{2,w}$	6.00

Table 4: SOE Posterior estimation results

Estimated Parameter Values	Symbol	Prior Dist.	Prior (Mean, Std Dev)	Posterior Mean	90% HPD Interval
Formal technology shock persistence	ρ_{A1}	β	0.75, 0.10	0.7989	$[0.6552, 0.9455]$
Informal technology shock persistence	ρ_{A2}	β	0.75, 0.10	0.7510	$[0.5993, 0.9062]$
Formal markup shock persistence	ρ_{MS1}	β	0.75, 0.10	0.7255	$[0.6102, 0.8598]$
Informal markup shock persistence	ρ_{MS2}	β	0.75, 0.10	0.7448	$[0.5903, 0.9039]$
Formal wage markup shock persistence	ρ_{MSw1}	β	0.75, 0.10	0.6953	$[0.5715, 0.8108]$
Informal wage markup shock persistence	ρ_{MSw2}	β	0.75, 0.10	0.7500	$[0.5821, 0.9027]$
Investment shock persistence	ρ_{IS}	β	0.75, 0.10	0.5165	$[0.3350, 0.6958]$
Preference shock persistence	ρ_{PS}	β	0.75, 0.10	0.7414	$[0.5887, 0.8982]$
Terms of trade shock persistence	ρ_{tot}	β	0.75, 0.10	0.7254	$[0.5523, 0.9005]$
Money demand shock persistence	ρ_{M}	β	0.75, 0.10	0.7510	$[0.5983, 0.9155]$
LCP markup shock persistence	$\rho_{MS_1^{*\ell}}$	β	0.75, 0.10	0.8419	$[0.7122, 0.9748]$
Government shock persistence	ρ_{G}	β	0.75, 0.10	0.5724	$[0.4094, 0.7289]$
Monetary Policy shock persistence	ρ_{μ}	β	0.75, 0.10	0.7009	$[0.5961, 0.8040]$
Dummy AR(1) process persistence	ρ_{EXW}	β	0.75, 0.10	0.6435	$[0.4974, 0.7870]$
Share of non-Ricardian consumers	λ	β	0.50, 0.10	0.3057	$[0.2027, 0.4095]$
Consumption habit formation	χ	β	0.50, 0.10	0.4113	$[0.2595, 0.5585]$
Formal price and wage index	γ_1	β	0.50, 0.10	0.3532	$[0.2115, 0.4933]$
Informal price and wage index	γ_2	β	0.50, 0.10	0.4833	$[0.3168, 0.6359]$
Elasticity of Investment adjustment cost	ϕ_x	N	3.00, 1.50	1.4438	$[0.4668, 2.4255]$
Non-Ricardian risk aversion	σ_c	N	2.00, 0.25	1.9046	$[1.4990, 2.3131]$
Feedback from inflation	θ_{π}	N	1.50, 0.25	1.4851	$[1.0973, 1.9003]$
Feedback from output	θ_y	N	1.50, 0.25	0.0999	$[-0.0090, 0.2023]$
Feedback from output growth	θ_{dy}	N	0.10, 0.05	0.1051	$[0.0296, 0.1902]$
Feedback from exchange rate depreciation	θ_{ds}	N	0.50, 0.25	0.3351	$[-0.0138, 0.6563]$
Home Share of consumption	w_C	β	0.50, 0.10	0.5566	$[0.4541, 0.6648]$
Home Share of investment	w_I	β	0.50, 0.10	0.5395	$[0.3820, 0.6942]$
Substitution elasticity (Formal/Informal goods)	μ_S	N	1.50, 0.20	1.2009	$[0.8886, 1.5303]$
Formal technology shock	ϵ_{A1}	IG	$3.00, 3.00$	2.0396	$[1.0945, 2.9718]$
Informal technology shock	ϵ_{A2}	IG	$3.00, 3.00$	2.4307	$[0.9799, 3.9724]$
Formal markup shock	ϵ_{MS1}	IG	$3.00, 3.00$	7.6984	$[6.0280, 9.5451]$
Informal markup shock	ϵ_{MS2}	IG	$3.00, 3.00$	2.6844	$[0.9783, 4.6379]$
Formal wage markup shock	ϵ_{MSw1}	IG	$3.00, 3.00$	11.597	$[9.2047, 13.898]$
Informal wage markup shock	ϵ_{MSw2}	IG	$3.00, 3.00$	2.5144	$[0.9686, 4.1623]$
Investment shock	ϵ_{IS}	IG	$3.00, 3.00$	10.105	$[4.2953, 16.0312]$
Preference shock	ϵ_{PS}	IG	$3.00, 3.00$	1.9255	$[0.9352, 2.8045]$
Terms of trade shock	ϵ_{tot}	IG	$3.00, 3.00$	6.7394	$[4.2733, 9.4597]$
Money demand shock	ϵ_{M}	IG	$3.00, 3.00$	3.0379	$[0.9014, 5.8718]$
LCP markup shock	$\epsilon_{MS_{1,t}^{*\ell}}$	IG	$3.00, 3.00$	16.352	$[4.8382, 28.268]$
Government shock	ϵ_{G}	IG	$3.00, 3.00$	11.386	$[8.2918, 14.852]$
Monetary policy shock	ϵ_{μ}	IG	$3.00, 3.00$	2.6780	$[2.2266, 3.11470]$
Dummy AR(1) process standard deviation	ϵ_{μ}	IG	$3.00, 3.00$	9.9035	$[6.9958, 12.5395]$

Table 5: Selected Second Moments Comparison

	Output	Consumption	Investment	Money	Inflation	Exchange Rate	Wage Rate
			Standard Deviation				
Data	4.3823	4.3158	7.3557	3.1009	2.3911	13.0836	3.8795
Model	5.7662	6.6679	9.9067	5.3847	3.5351	16.4737	3.1289
			Cross-Correlation with Output				
Data	1.00	0.2827	0.3229	0.3358	-0.3566	0.2203	0.1710
Model	1.00	0.7447	0.3085	0.3134	-0.2264	0.3409	0.0972
			Auto-correlations (Order=1)				
Data	0.0771	-0.1166	-0.2073	0.4569	0.6096	0.4773	0.4405
Model	0.0894	0.0491	0.3605	0.6093	0.5930	0.3116	0.2886
			Auto-correlations (Order=4)				
Data	-0.2406	-0.1482	-0.0143	0.0873	0.2618	-0.0489	-0.5246
Model	-0.1153	-0.1056	-0.1509	-0.2363	-0.1995	0.0992	-0.1395

Table 6: Variance Decomposition of Estimated Model (%)

	ϵ_{μ}	ϵ_{MS2}	ϵ_{MS1}	ϵ_{A2}	ϵ_{A1}	ϵ_{MSw1}	ϵ_{MSw2}	$\epsilon_{P^*_O,t}$	ϵ_G	$\epsilon_{MS^{*\ell}_{1,t}}$	ϵ_M	ϵ_{tot}	ϵ_{PS}	ϵ_{IS}	ϵ_{PS^*}	ϵ_{μ^*}	ϵ_{A^*}	ϵ_{G^*}	ϵ_{IS^*}	ϵ_{MS^*}
Total Output	3.73	3.65	15.70	3.60	10.67	1.56	7.61	16.87	12.17	7.76	0.04	3.53	1.11	11.82	0.00	0.00	0.19	0.00	0.01	0.00
Formal Output	3.34	0.55	30.69	0.25	14.42	0.23	28.05	3.55	7.51	2.73	0.02	0.53	0.73	7.34	0.01	0.00	0.05	0.00	0.00	0.00
Informal Output	1.40	13.86	35.70	11.10	3.78	5.92	24.24	0.77	0.49	1.76	0.01	0.25	0.12	0.25	0.01	0.00	0.34	0.01	0.01	0.00
Total Consumption	4.37	2.63	23.04	2.00	9.51	1.10	25.67	2.71	1.56	0.71	0.17	6.44	6.38	10.16	0.03	0.00	3.41	0.06	0.05	0.00
Domestic Consumption	3.79	4.74	18.37	3.95	7.65	2.01	12.06	14.97	1.09	9.29	0.11	10.78	3.25	6.41	0.01	0.00	1.45	0.03	0.05	0.00
Imported Consumption	0.26	0.53	0.97	0.57	0.45	0.23	5.33	52.62	0.39	12.52	0.02	1.00	1.51	8.48	0.09	0.01	14.46	0.27	0.28	0.00
Formal Consumption	3.24	0.40	40.60	0.28	8.90	0.17	27.01	5.10	0.84	2.34	0.05	5.13	1.89	3.70	0.00	0.00	0.33	0.01	0.01	0.00
Informal Consumption	1.40	13.86	35.70	11.10	3.78	5.92	24.24	0.77	0.49	1.76	0.01	0.25	0.12	0.25	0.01	0.00	0.34	0.01	0.01	0.00
Ricardian Consumption	5.92	1.71	18.81	1.74	12.14	0.72	18.35	5.16	0.97	1.55	0.04	5.74	6.56	15.41	0.03	0.00	5.00	0.08	0.07	0.00
Non-Ricardian Consumption	1.68	3.43	28.09	1.70	7.08	1.42	32.32	1.21	5.51	1.88	1.38	5.12	4.05	4.43	0.02	0.00	0.64	0.01	0.01	0.00
Total Investment	1.16	0.56	4.94	1.08	10.37	0.25	1.62	2.89	2.31	2.57	0.04	1.06	6.41	64.15	0.01	0.00	0.49	0.02	0.05	0.00
Domestic Investment	1.38	0.21	12.66	0.55	11.84	0.10	4.44	1.51	1.64	4.87	0.03	2.52	4.33	53.86	0.00	0.00	0.04	0.00	0.02	0.00
Imported Investment	1.16	1.07	2.18	1.63	5.68	0.47	2.14	15.47	2.69	0.36	0.06	0.23	7.62	56.06	0.04	0.00	2.92	0.09	0.14	0.00
Total Inflation	17.33	0.49	3.78	0.74	4.71	0.15	1.53	43.38	3.59	0.44	0.08	16.50	2.16	4.75	0.01	0.00	0.35	0.00	0.00	0.00
Domestic Inflation	5.88	0.06	0.67	0.08	1.30	0.02	0.38	57.90	1.25	0.33	0.05	27.70	1.05	3.11	0.01	0.00	0.21	0.00	0.00	0.00
Imported Inflation	35.67	3.23	13.00	3.83	11.44	1.07	3.87	1.30	7.24	5.58	0.08	5.74	3.24	4.27	0.05	0.00	0.38	0.00	0.00	0.00
Formal Inflation	18.35	0.70	43.27	0.28	11.43	0.26	17.21	0.39	1.64	1.96	0.02	1.88	0.78	1.55	0.02	0.00	0.25	0.00	0.00	0.00
Informa Inflation	10.97	12.16	27.34	11.01	3.15	4.23	15.30	0.70	4.96	2.66	0.05	3.09	2.26	2.00	0.02	0.00	0.08	0.00	0.00	0.00
Formal Marginal Cost	2.21	0.23	68.24	0.07	11.75	0.06	9.68	1.15	3.37	0.87	0.02	0.66	0.63	1.03	0.01	0.00	0.02	0.00	0.00	0.00
Informa Marginal Cost	3.04	64.15	2.86	9.79	3.10	1.84	1.44	1.35	7.62	0.79	0.08	1.19	1.78	0.92	0.01	0.00	0.04	0.00	0.00	0.00
Total Capital	0.52	0.35	2.09	0.72	8.98	0.16	0.58	0.41	0.39	1.75	0.01	0.34	5.06	74.95	0.01	0.00	3.51	0.09	0.08	0.00
Formal Capital	0.55	0.60	4.65	0.65	8.90	0.14	0.76	0.56	0.61	1.57	0.01	0.47	4.91	71.71	0.01	0.00	3.71	0.09	0.08	0.00
Informa Capital	1.92	14.69	34.40	1.65	3.84	0.71	2.64	0.44	4.67	11.78	0.02	3.36	1.42	18.06	0.02	0.00	0.36	0.01	0.03	0.00
Formal Wage	0.26	0.68	43.42	0.49	7.55	0.37	41.48	0.02	0.15	0.16	0.01	0.31	0.50	4.05	0.00	0.00	0.53	0.01	0.01	0.00
Informal Wage	2.69	6.59	26.38	4.23	5.49	9.12	34.93	0.06	1.61	0.57	0.04	1.69	1.66	4.31	0.01	0.00	0.61	0.01	0.01	0.00
Relative Informal to Formal Prices	1.87	8.62	41.98	6.85	5.69	3.68	28.30	0.07	0.55	0.47	0.01	0.62	0.40	0.78	0.00	0.00	0.10	0.00	0.00	0.00
Relative Imported to Domestic Prices	2.04	0.28	29.98	0.16	6.05	0.12	14.11	24.89	0.43	6.96	0.02	5.12	0.67	5.00	0.02	0.00	4.00	0.08	0.09	0.00
Formal Labour Supply	1.39	13.84	35.99	5.77	4.35	6.70	28.17	0.82	0.40	1.35	0.01	0.22	0.17	0.41	0.01	0.00	0.41	0.01	0.01	0.00
Informal Labour Supply	4.36	0.54	33.46	0.31	5.61	0.28	32.94	4.31	8.85	3.14	0.02	0.60	0.64	4.86	0.01	0.00	0.07	0.00	0.00	0.00
Ricardian Labour Supply	3.81	1.59	21.96	0.45	8.60	0.66	25.02	8.01	15.00	3.98	0.46	0.79	1.13	8.08	0.01	0.00	0.41	0.01	0.01	0.00
Non-Ricardian Labour Supply	11.23	1.03	24.39	0.74	6.74	0.40	12.08	20.87	4.59	1.11	8.42	5.17	0.74	2.42	0.00	0.00	0.06	0.00	0.00	0.00
Nominal Interest Rate	0.47	2.00	35.47	0.98	14.50	0.85	20.64	0.73	1.00	3.13	5.77	2.41	1.08	3.30	0.04	0.00	7.55	0.01	0.06	0.00
Money Growth	36.20	0.62	23.63	0.25	8.75	0.24	11.53	5.86	3.09	1.99	0.05	2.17	1.42	3.86	0.00	0.00	0.32	0.00	0.01	0.00
Total Money Demand	0.82	1.15	35.07	0.46	16.66	0.49	21.87	0.62	1.04	3.48	0.07	1.38	2.18	5.96	0.03	0.00	8.64	0.01	0.07	0.00
Ricardian Money Demand	0.93	1.34	30.92	0.51	16.89	0.57	17.96	1.00	1.67	4.03	4.89	1.42	2.19	6.74	0.03	0.00	8.81	0.01	0.07	0.00
Non-Ricardian Money Demand	0.50	0.64	39.70	0.28	12.61	0.25	29.53	0.15	0.59	1.64	2.49	1.18	1.70	3.04	0.02	0.00	5.65	0.01	0.04	0.00
Export	0.24	0.36	1.06	0.17	7.39	0.15	16.25	4.14	0.14	55.87	0.00	6.12	0.18	2.61	0.06	0.01	5.05	0.08	0.10	0.00
Government Expenditures	7.17	0.74	19.62	0.43	4.80	0.31	7.53	5.21	46.85	3.70	0.00	0.55	0.20	0.99	0.01	0.00	1.81	0.02	0.04	0.00

B Figures

Figure 1: Graphical illustration of the model structure

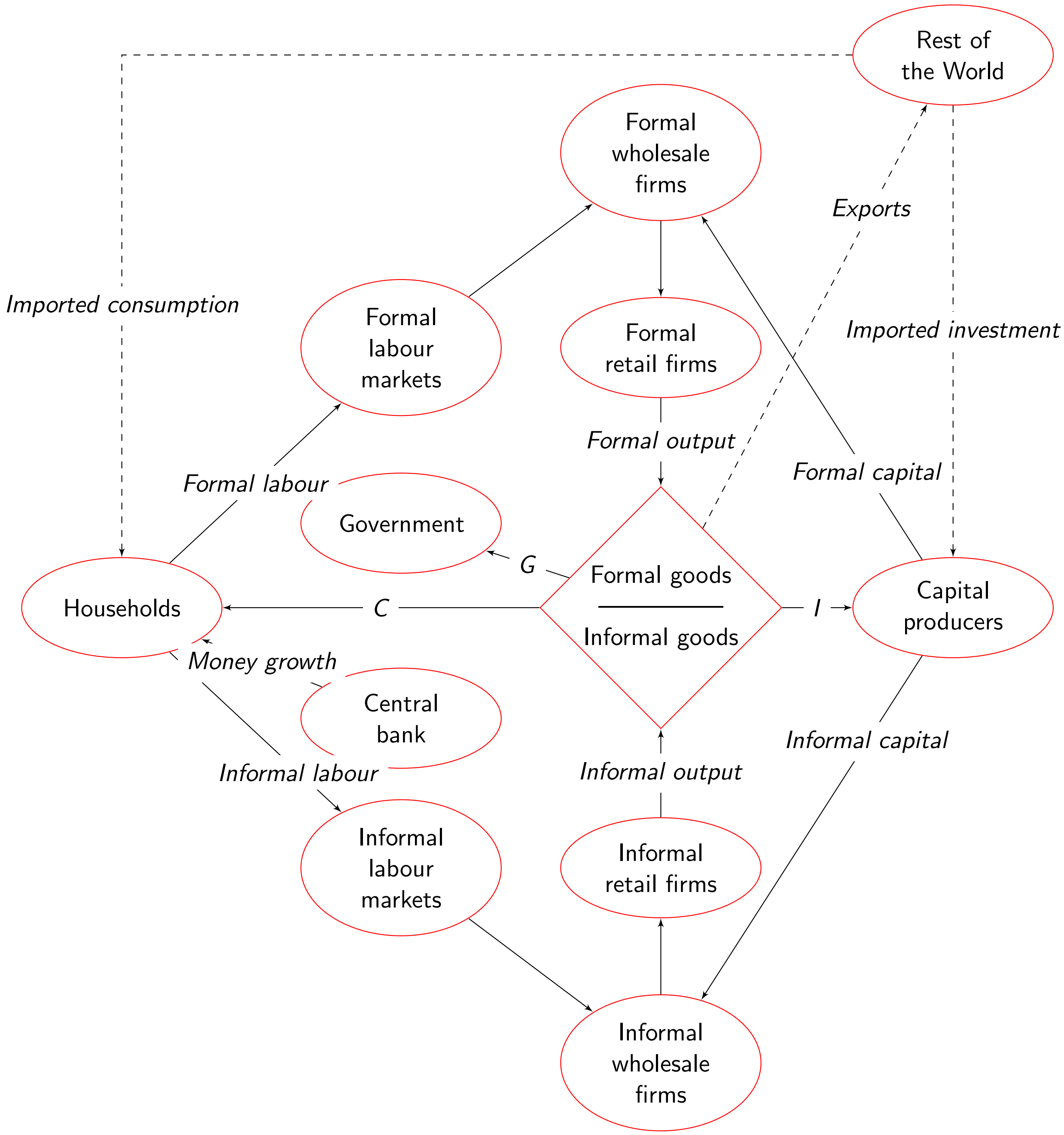
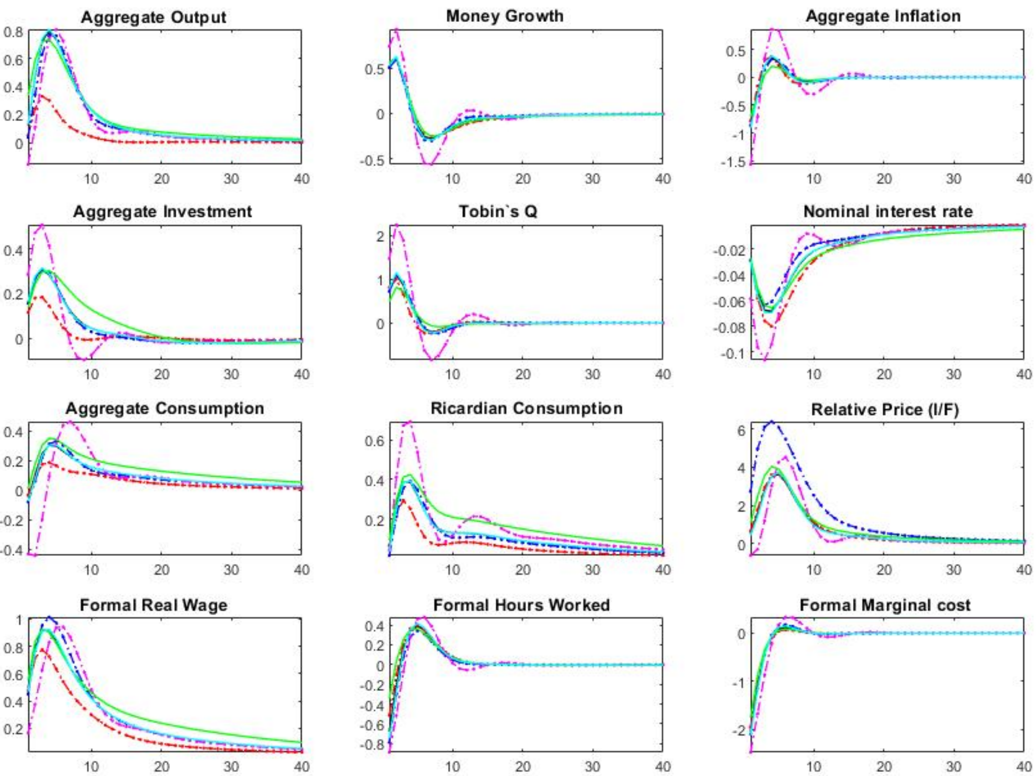
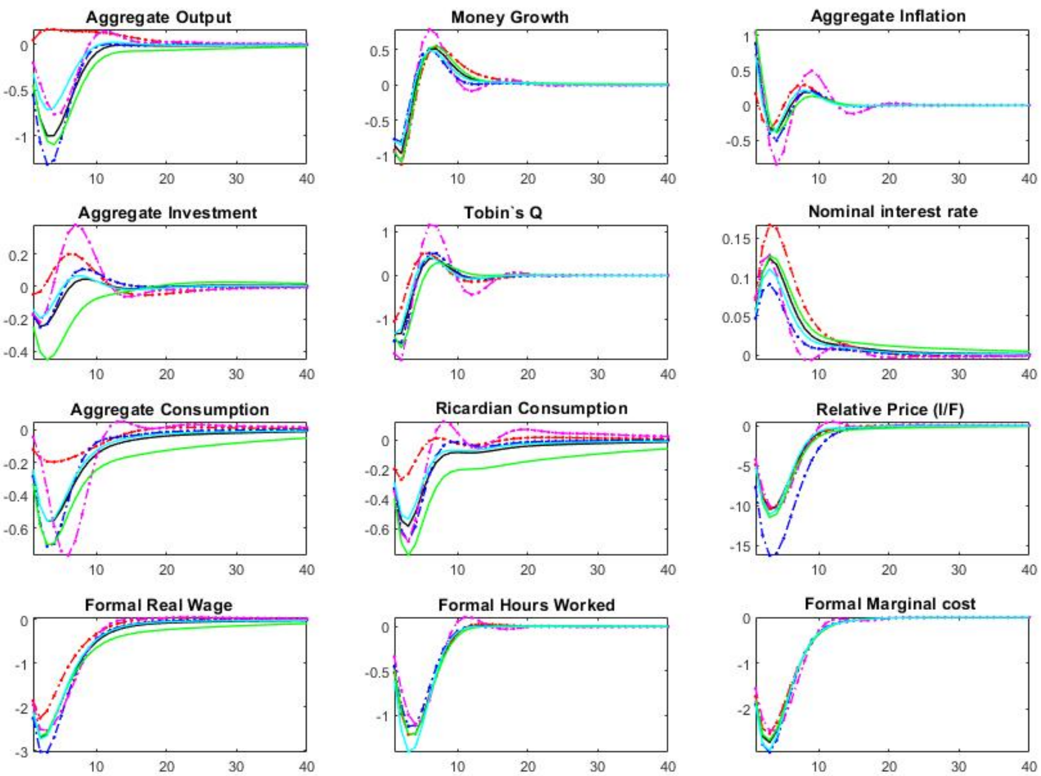


Figure 2: Estimated Impulse Responses to a Formal Productivity Shock



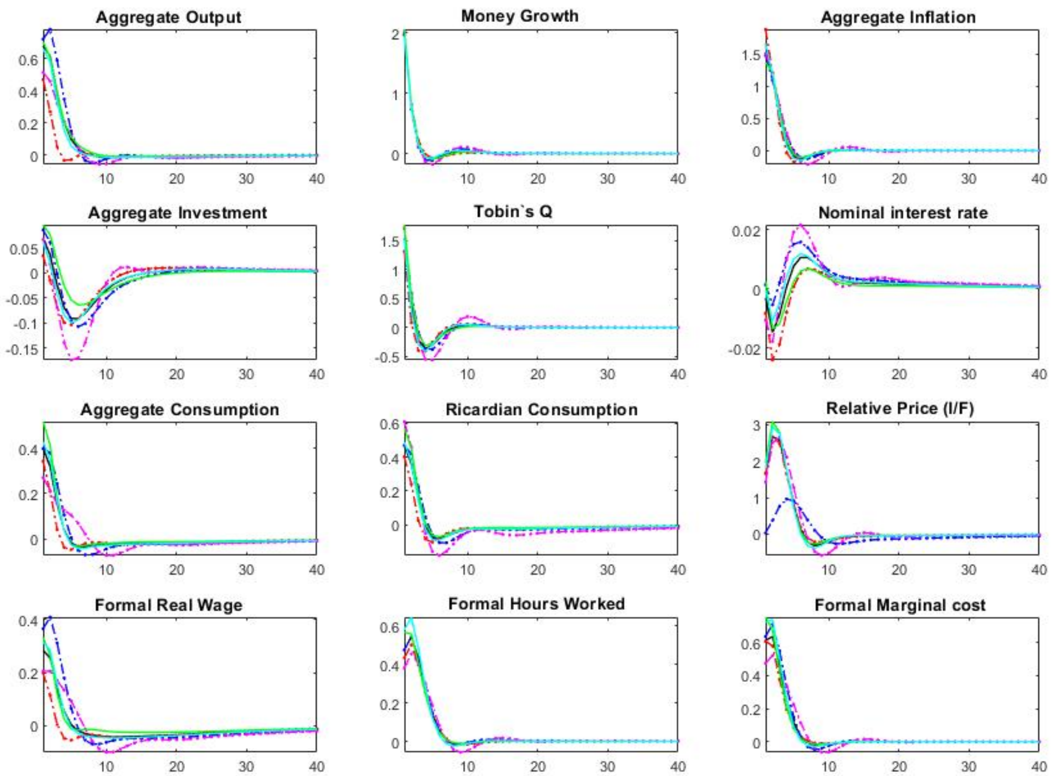
Notes: the black thick line represents our baseline model; the green line is the case of trade autarky; the blue line is model with the formal and informal sectors with similar levels of rigidities; the magenta line is the model with higher share of credit-constrained consumers, the red line is the model with a larger informal sector; the indigo line is the model with no LCP specification.

Figure 3: Estimated Impulse Responses to a Formal Price Markup Shock



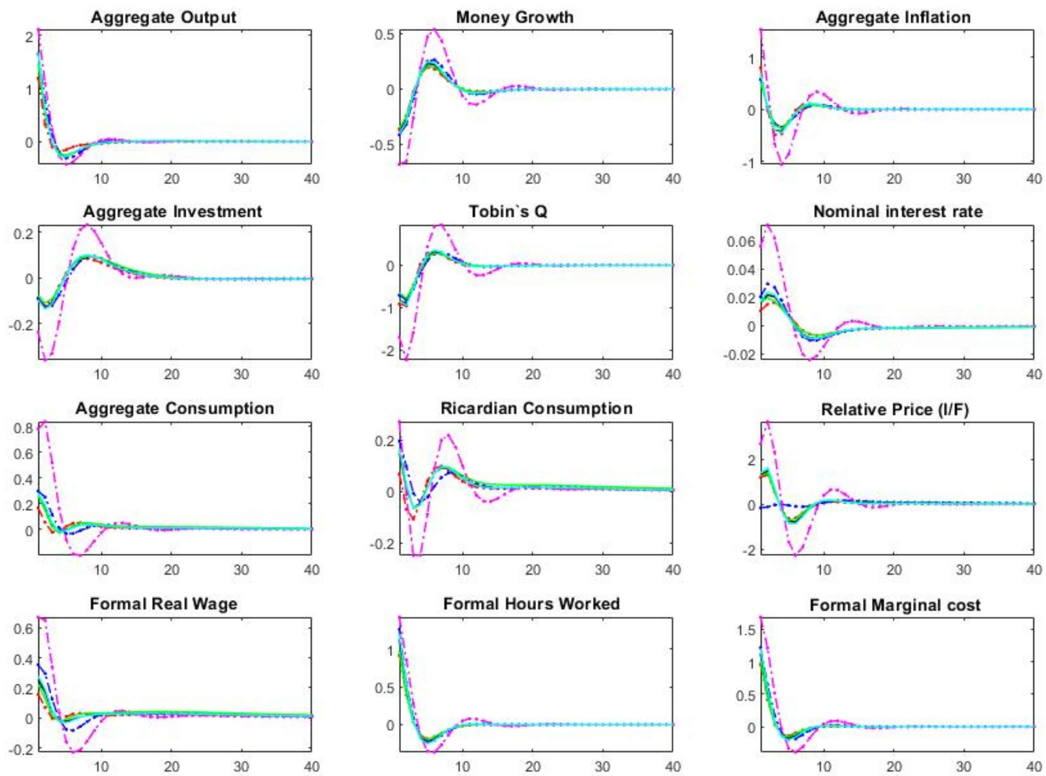
See notes to Figure 2

Figure 4: Estimated Impulse Response to a Monetary Policy Shock



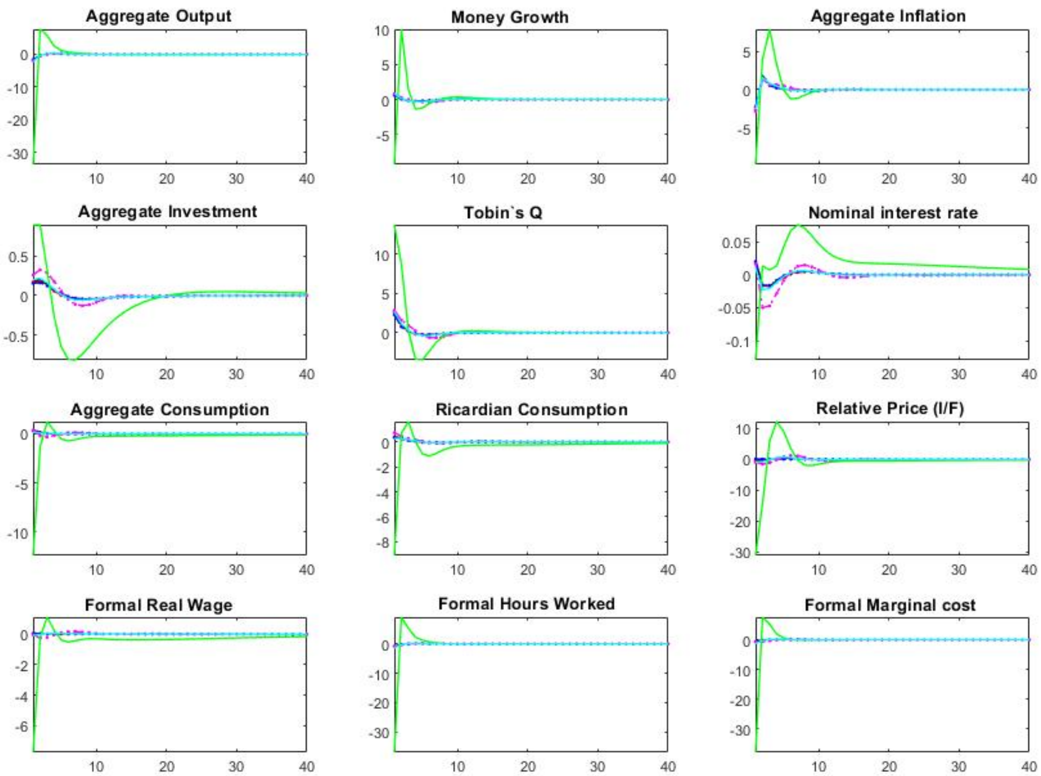
See notes to Figure 2

Figure 5: Estimated Impulse Responses to a Government Spending Shock



See notes to Figure 2

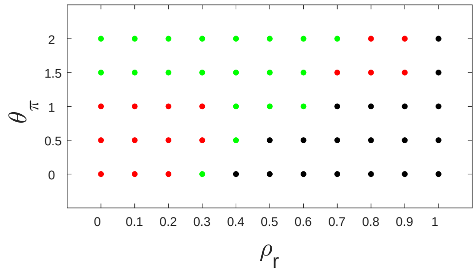
Figure 6: Estimated Impulse Responses to an Oil Price Shock



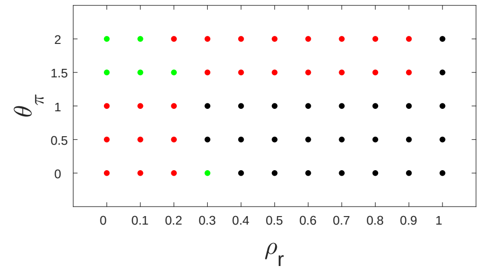
See notes to Figure 2

Figure 7: Interest Rate Rule: Instability and Indeterminacy ($\lambda = 0.5$)

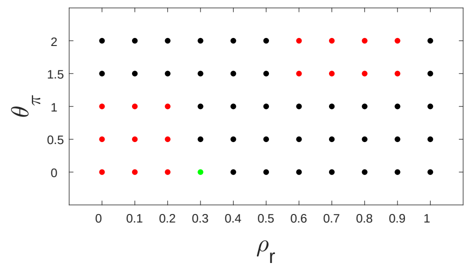
(a) Simple $\mathbb{E}_t \log\left(\frac{\Pi_{1,t}}{\Pi_1}\right)$



(b) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+1}}{\Pi_1}\right)$



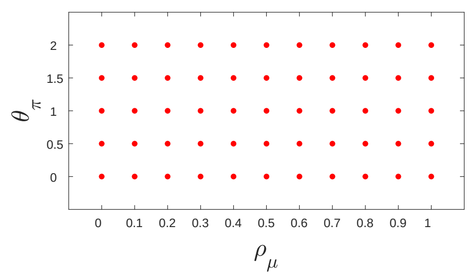
(c) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+4}}{\Pi_1}\right)$



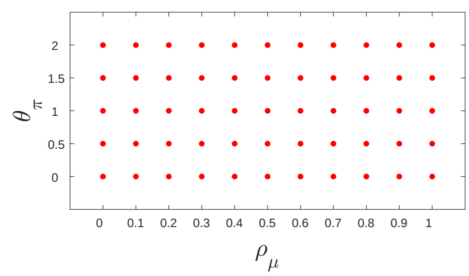
Notes: Determinacy (green); Indeterminacy (black); Stability (red)

Figure 8: Monetary Growth Rule: Determinacy and Stability ($\lambda = 0.5$)

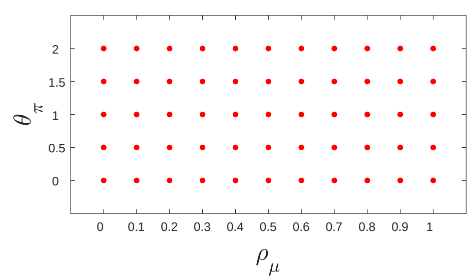
(a) Simple $\mathbb{E}_t \log\left(\frac{\Pi_{1,t}}{\Pi_1}\right)$



(b) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+1}}{\Pi_1}\right)$



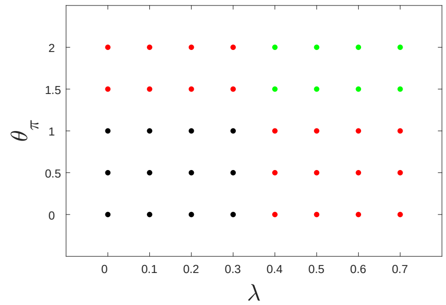
(c) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+4}}{\Pi_1}\right)$



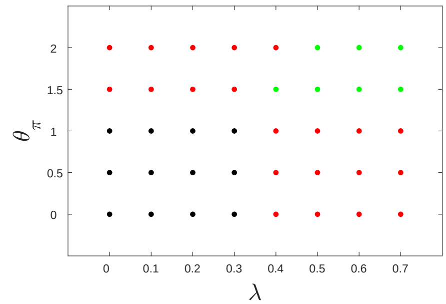
Notes: Determinacy (green); Indeterminacy (black); Stability (red)

Figure 9: LAMP and Interest Rate Rule: Instability and Indeterminacy ($\rho_r = 0$)

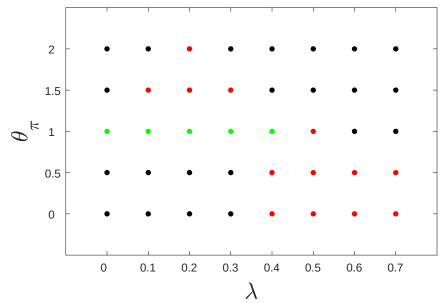
(a) Simple $\mathbb{E}_t \log\left(\frac{\Pi_{1,t}}{\Pi_1}\right)$



(b) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+1}}{\Pi_1}\right)$



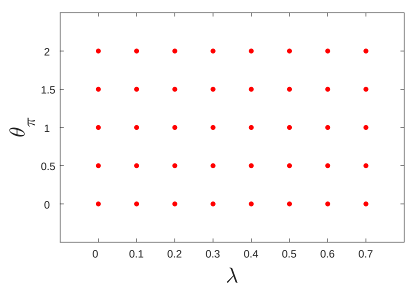
(c) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+4}}{\Pi_1}\right)$



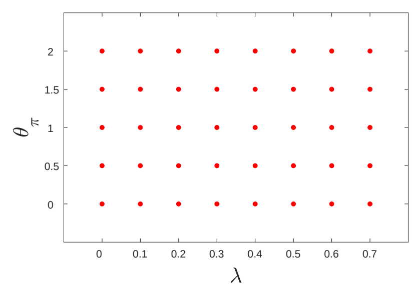
Notes: Determinacy (green); Indeterminacy (black); Stability (red)

Figure 10: LAMP and Monetary Growth Rule: Determinacy and Stability ($\rho_\mu = 0$)

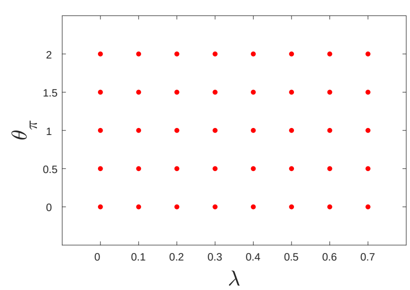
(a) Simple $\mathbb{E}_t \log\left(\frac{\Pi_{1,t}}{\Pi_1}\right)$



(b) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+1}}{\Pi_1}\right)$



(c) Forward looking $\mathbb{E}_t \log\left(\frac{\Pi_{1,t+4}}{\Pi_1}\right)$



Notes: Determinacy (green); Indeterminacy (black); Stability (red)

C Supplement to "Monetary Growth Rules in an Emerging Open Economy"

A The 2-bloc SOE-ROW model: solution and steady-state details

This section provides further details on the model, steady state and equilibrium conditions. The SOE model builds upon the framework of Gali and Monacelli (2005), in which the world economy is modelled as a continuum of SOEs on the unit interval. The latter feature is introduced by assuming that some households are excluded from financial markets which can neither borrow nor save, and hence they do not smooth consumption over time. These households consume their current labour income each period. These consumers are labelled non-Ricardian, as they break the Ricardian Equivalence, but in the main text we use interchangeably 'credit-constrained', 'liquidity-constrained', 'LAMP' or 'rule-of-thumb' (RoT) agents. Based on Smets and Wouters (2007) the model incorporates imperfect labour market and monopolistic trade union. A role for monetary policy is introduced by assuming that prices are slow to adjust.

There is a continuum of households, a single perfectly competitive intermediate good producer and a continuum of monopolistically competitive final producers setting prices and trade union setting wages on a Calvo-type staggered basis. We further develop the model by allowing for the existence of a formal and less-capital intensive informal sector, producing different goods with different technologies sold at different prices with the following features distinguishing it from the formal sector:

- Not taxed
- Non-traded
- Only produces consumption goods
- Only producer currency pricing
- Different labour shares (α_i) in wholesale sectors
- Different degree of price stickiness ($\xi_{p,i}$) and elasticity of demand ($\zeta_{p,i}$)
- Different degree of wage stickiness ($\xi_{w,i}$) and elasticity of demand ($\zeta_{w,i}$)
- Different technology shock ($A_{i,t}$).
- Different markup shock ($MS_{i,t}$).
- Different wage markup shock ($MSw_{i,t}$).

The monetary authority sets its policy instrument, the money growth rate, with respect to government budget constraint. The demand for domestic goods, goods sold by the formal and informal sectors, imports, formal and informal labour, the corresponding CPI prices and wage indexes are characterized by CES Dixit-Stiglitz aggregators. For modelling convenience, we introduce capital producers who, rather than households, accumulate the capital stock and rent it to firms. We introduce exogenous distortionary tax rates on wage and capital income to pay for government spending which is given by a Taylor-type rule. We also allow the government to run a fiscal deficit, use the government spending rule as a stabilization instrument and to borrow only from domestic investors. We also introduce an oil exporting sector and follow Gabriel *et al.* (2012) and Khera (2016) for our model structure. As we set up the model in a complete financial autarky economy, we have no foreign bonds in the households' budget constraint, so that the UIP condition does not hold. Also, the government does not issue any bonds in foreign currency and thus has no foreign liabilities.

A.1 Households

Recall that, given the utility function in equation (1) in the paper, the (Ricardian) household solves

$$\max_{C_t^R, L_t^R, m_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} PS_t \, \beta^s \, U(C_{t+s}^R, H_{t+s}^R, m_{t+s}^R) \right] \tag{52}$$

subject to a nominal budget constraint given by

$$P_t{}^B B_{H,t} + P_t C_t^R + P_t m_t^R = B_{H,t-1} + W_{1,t}^{nh} \left(1 - \tau_t^w\right) H_{1,t}^R + W_{2,t}^{nh} H_{2,t}^R + P_{t-1} m_{t-1}^R + \Gamma_t \tag{53}$$

(see the paper for the notation details).

Maximizing (52) subject to 53, we have

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t+1}} \right] = \frac{1}{R_t} \tag{54}$$

where $\Lambda_{t,t+1} \equiv \beta \frac{U_{C,t+1}^R}{U_{C,t}^R}$ is the real stochastic discount rate over the interval $[t, t+1]$, being convenient to write the Euler consumption equation as $\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t+1}} \right] R_t = 1$, which equates the expected discounted return on a riskless bond over the period $[t, t+1]$,

$$\frac{U_{H,1,t}^R}{U_{C,t}^R} = W_{1,t}^h \left(1 - \tau_t^w\right) \tag{55}$$

$$\frac{U_{H,2,t}^R}{U_{C,t}^R} = W_{2,t}^h \tag{56}$$

$$U_{m,t}^R = U_{C,t}^R - \beta \mathbb{E}_t \left[\frac{U_{C,t+1}^R}{\Pi_{t+1}} \right] = U_{C,t}^R \left[1 - \frac{1}{R_t} \right] \tag{57}$$

where equation (55 and 56) are formal and informal labour supply and the marginal utilities given by

$$U_{m,t}^R = \Psi m_t^{R^{-\psi}} \tag{58}$$

$$U_{C,t}^R = (1-\varrho)(C_t^R - \chi C_{t-1}^R)^{(1-\varrho)(1-\sigma_R)-1} (1 - H_t^R)^{\varrho(1-\sigma_R)} \tag{59}$$

$$U_{H,1,t}^R = \varrho(C_t^R - \chi C_{t-1}^R)^{(1-\varrho)(1-\sigma_R)} (1 - H_{1,t}^R)^{\varrho(1-\sigma_R)-1} \tag{60}$$

$$U_{H,2,t}^R = \varrho(C_t^R - \chi C_{t-1}^R)^{(1-\varrho)(1-\sigma_R)} (1 - H_{2,t}^R)^{\varrho(1-\sigma_R)-1} \tag{61}$$

complete the equilibria, with $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ as the home CPI inflation rate.
Similar steps can be derived for the remaining λ non-Ricardian agents, who choose C_t^{RoT} and $L_t^{RoT} = 1 - H_t^{RoT}$ to maximize an analogous welfare function to (52) subject to their respective budget constraint (equation (4) in the paper).

The first-order conditions are now the same for both types:

$$\frac{U_{H,1,t}^{RoT}}{U_{C,t}^{RoT}} = W_{1,t}^h \left(1 - \tau_t^w\right) \tag{62}$$

$$\frac{U_{H,2,t}^{RoT}}{U_{C,t}^{RoT}} = W_{2,t}^h \tag{63}$$

Thus, hours worked by liquidity constrained consumers are constant,

$$U_{m,t}^{RoT} = U_{C,t}^{RoT} - \beta \mathbb{E}_t \left[\frac{U_{C,t+1}^{RoT}}{\Pi_{t+1}} \right] = U_{C,t}^{RoT} \left[1 - \frac{1}{R_t} \right] \tag{64}$$

where the marginal utilities given by

$$U_{m,t}^{RoT} = \Psi m_t^{RoT^{-\psi}} \tag{65}$$
$$U_{C,t}^{RoT} = (1-\varrho)(C_t^{RoT} - \chi C_{t-1}^{RoT})^{(1-\varrho)(1-\sigma_{RoT})-1}(1-H_t^{RoT})^{\varrho(1-\sigma_{RoT})} \tag{66}$$
$$U_{H,1,t}^{RoT} = \varrho(C_t^{RoT} - \chi C_{t-1}^{RoT})^{(1-\varrho)(1-\sigma_{RoT})}(1-H_{1,t}^{RoT})^{\varrho(1-\sigma_{RoT})-1} \tag{67}$$
$$U_{H,2,t}^{RoT} = \varrho(C_t^{RoT} - \chi C_{t-1}^{RoT})^{(1-\varrho)(1-\sigma_{RoT})}(1-H_{2,t}^{RoT})^{\varrho(1-\sigma_{RoT})-1} \tag{68}$$

which completes the equilibria.

Households in ROW bloc

The same structure is used for foreign part and we have the following dynamics:

$$\begin{aligned}
U_t^* &= PS_t^* \frac{(C_t^{*(1-\varrho^*)}(1-H_t^*)^{\varrho})^{1-\sigma_c^*} - 1}{1-\sigma_c^*} \\
U_{C^*,t}^* &= PS_t^*(1-\varrho^*)C_t^{*(1-\varrho^*)(1-\sigma_c^*)-1}(1-H_t^*)^{\varrho^*(1-\sigma_c^*)} \\
U_{H^*,t}^* &= -PS_t^*\varrho^* C_t^{*(1-\varrho^*)(1-\sigma_c^*)}(1-H_t^*)^{\varrho^*(1-\sigma_{c^*}^*)-1} \\
R_t^{**} &= \frac{R_{t-1}^*}{\Pi_t^*} \\
\text{(Fischer Equation)} \quad R_t^{**} & \quad \text{is now the } \textit{ex-post} \text{ real interest rate in interval } [t-1,t]
\end{aligned}$$

$$\begin{aligned}
U^*{}_{C^*,t} &= \beta^* \mathbb{E}_t \left[R_{t+1}^* U^*{}_{C^*,t+1} \right] \\
\text{where } \Lambda_{t-1,t}^* &\equiv \beta^* \frac{U_{C^*,t}^*}{U_{C^*,t-1}^*} \\
\frac{U_{H^*,t}^*}{U_{C^*,t}^*} &= -W_t^*
\end{aligned}$$

Aggregate Consumption, Labour and Money Balance

Total consumption, hours and money balances are then

$$C_t = \lambda C_t^{RoT} + (1-\lambda)C_t^R \tag{69}$$
$$m_t M_t = \lambda m_t^{RoT} + (1-\lambda)m_t^R \tag{70}$$
$$H_{1,t} = \lambda H_{1,t}^{RoT} + (1-\lambda)H_{1,t}^R \tag{71}$$
$$H_{2,t} = \lambda H_{2,t}^{RoT} + (1-\lambda)H_{2,t}^R \tag{72}$$
$$H_t = H_{1,t} + H_{2,t} \tag{73}$$
$$H_t^R = H_{1,t}^R + H_{2,t}^R \tag{74}$$
$$H_t^{RoT} = H_{1,t}^{RoT} + H_{2,t}^{RoT} \tag{75}$$

where M_t is the money demand shock.

Consumption and Investment Demand

In the main paper, we focus on aggregate demand. Here we provide some additional details on intermediate steps. Demand for goods sold by the formal and informal sectors are chosen to maximise

$$C_{H,t} = \left[\mathrm{w}_S^{\frac{1}{\mu_S}} C_{1,t}^{\frac{\mu_S-1}{\mu_S}} + (1-\mathrm{w}_S)^{\frac{1}{\mu_S}} C_{2,t}^{\frac{\mu_S-1}{\mu_S}} \right]^{\frac{\mu_S}{\mu_S-1}} \tag{76}$$

where w_S and $1-\mathrm{w}_S$ are sector shares and μ_S is the elasticity of substitution between formal and informal goods. The corresponding price index and domestic CPI inflation is given by

$$P_{H,t} = \left[\mathrm{w}_S (P_{1,t})^{1-\mu_S} + (1-\mathrm{w}_S)(P_{2,t})^{1-\mu_S} \right]^{\frac{1}{1-\mu_S}} \tag{77}$$

$$\Pi_{H,t+1} = \left[\mathrm{w}_S \left(\Pi_{1,t+1} \frac{P_{1,t}}{P_{H,t}} \right)^{1-\mu_S} + (1-\mathrm{w}_S) \left(\Pi_{2,t+1} \frac{P_{2,t}}{P_{H,t}} \right)^{1-\mu_S} \right]^{\frac{1}{1-\mu_S}} \tag{78}$$

where $\Pi_{H,t,t+1}$, $\Pi_{1,t+1}$ and $\Pi_{2,t+1}$ are home, formal and informal CPI inflation, respectively. $C_{1,t}$ and $C_{2,t}$ are baskets of differentiated consumption goods with price index $P_{1,t}$ and $P_{2,t}$ - these are defined in further detail below.
Maximising total consumption (76) subject to $P_{H,t}C_{H,t} = P_{1,t}C_{1,t} + P_{2,t}C_{2,t}$ yields

$$C_{1,t} = \mathrm{w}_S \left(\frac{P_{1,t}}{P_{H,t}} \right)^{-\mu_S} C_{H,t} \tag{79}$$

$$C_{2,t} = (1-\mathrm{w}_S) \left(\frac{P_{2,t}}{P_{H,t}} \right)^{-\mu_S} C_{H,t} \tag{80}$$

In the small open economy, we take foreign aggregate consumption and investment, denoted by C_t^* and I_t^*, as exogenous processes. Recall the definition of the real exchange rate as the relative aggregate consumption price $RER_t \equiv \frac{P_t^* S_t}{P_t}$, where S_t is the nominal exchange rate. Then, the foreign counterpart of the above defining demand for the export of the home goods are

$$C_{H,t}^* = (1-\mathrm{w}_C^*) \left(\frac{P_{1,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* \tag{81}$$

where $1-\mathrm{w}_C^*$ determines the share of domestic goods in the foreign consumption bundle. $\mu_C^* > 1$ is the substitution elasticity between exports and foreign domestic goods. $P_{1,t}^*$ and P_t^* denote the price of home consumption and aggregate consumption goods in foreign currency. Because the home country is small, the LOP implies that $P_t^* = P_{F,t}^*$, $S_t P_t^* = P_{F,t}$, so $RER_t = \frac{P_{F,t}}{P_t}$. We can then write

$$C_{H,t}^* = (1-\mathrm{w}_C^*) \left(\frac{P_{1,t}^*}{P_{F,t}^*} \frac{P_{F,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^*$$

$$C_{H,t}^* = (1-\mathrm{w}_C^*) (\mathcal{T}_t^*)^{-\mu_C^*} C_t^* \tag{82}$$

and $\mathcal{T}_t^* \equiv \frac{P_{1,t}^*}{P_{F,t}^*}$ for the foreign bloc, so that we have

$$\frac{\mathcal{T}_t^*}{\mathcal{T}_{t-1}^*} = \frac{\Pi_{1,t}^*}{\Pi_{F,t}^*} \tag{83}$$

which is the foreign bloc counterpart to (9) in the paper.

Analogous conditions to (8) in the paper and (82) hold for domestic demand, import and export demand for investment goods, respectively. We denote the aggregate price index for investment goods $P_{I,t}$ with weights w_I such that

$$I_{H,t} = \mathrm{w}_I \left(\frac{P_{H,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{84}$$

$$I_{F,t} = (1-\mathrm{w}_I) \left(\frac{P_{F,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{85}$$

$$I_{H,t}^* = (1-\mathrm{w}_I^*) \left(\mathcal{T}_t^*\right)^{-\mu_I^*} I_t^* \tag{86}$$

where

$$\frac{P_{H,t}}{P_t} = \frac{1}{\left(\mathrm{w}_C + (1-\mathrm{w}_C)\,\frac{P_{1,t}}{P_{H,t}}\,\mathcal{T}_t^{1-\mu_C}\right)^{1/(1-\mu_C)}} \tag{87}$$

$$\frac{P_{F,t}}{P_t} = \frac{1}{\left(\mathrm{w}_C\,\frac{P_{1,t}}{P_{H,t}}\,\mathcal{T}_t^{\mu_C-1} + (1-\mathrm{w}_C)\right)^{1/(1-\mu_C)}} \tag{88}$$

$$\frac{P_{H,t}}{P_t^I} = \frac{1}{\left(\mathrm{w}_I + (1-\mathrm{w}_I)\mathcal{T}_t^{1-\mu_I}\right)^{1/(1-\mu_I)}} \tag{89}$$

$$\frac{P_{F,t}}{P_t^I} = \frac{1}{\left(\mathrm{w}_I\mathcal{T}_t^{\mu_I-1} + (1-\mathrm{w}_I)\right)^{1/(1-\mu_I)}} \tag{90}$$

$$\frac{P_{1,t}}{P_{H,t}} = \frac{1}{\left(\mathrm{w}_S + (1-\mathrm{w}_S)\,\frac{P_{2,t}}{P_{1,t}}^{(1-\mu_S)}\right)^{1/(1-\mu_S)}} \tag{91}$$

$$\frac{P_{2,t}}{P_{H,t}} = \frac{1}{\left((1-\mathrm{w}_S) + \mathrm{w}_S\,\frac{P_{2,t}}{P_{1,t}}^{(\mu_S-1)}\right)^{1/(1-\mu_S)}} \tag{92}$$

$$\frac{P_{2,t}}{P_{1,t}} = \frac{P_{2,t-1}}{P_{1,t-1}}\frac{\Pi_{2,t}}{\Pi_{1,t}} \tag{93}$$

A.2 Labor market and Wage setting

Recall that the trade union chooses the optimal wage $W_{i,t}^{nO}(j)$ to maximize real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_{w,i}^k \frac{\Lambda_{t,t+k}}{P_{i,t+k}} H_{i,t+k}(j) \left[W_{i,t}^{nO}(j) \left(\frac{P_{i,t+k-1}}{P_{i,t-1}}\right)^{\gamma_i^w} - W_{i,t+k}^{nh} MS_{W_{i,t}}\right], i=1,2 \tag{94}$$

Similarly to (11) in the paper, we can derive labour demand with indexing $H_{i,t+k}^d(j)$ as

$$H_{i,t+k}(j) = \left(\frac{W_{i,t}^{nO}(j)}{W_{i,t+k}^n} \left(\frac{P_{i,t+k-1}}{P_{i,t-1}}\right)^{\gamma_i^w}\right)^{-\zeta_{w,i}} H_{i,t+k}^d, i=1,2 \tag{95}$$

which leads to the following first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_{w,i}^k \frac{\Lambda_{t,t+k}}{P_{i,t+k}} H_{i,t+k}(j) \Big[W_{i,t}^{nO}(j) \left(\frac{P_{i,t+k-1}}{P_{i,t-1}} \right)^{\gamma_i^w} - W_{i,t+k}^{nh} \, MS_{W_{i,t}} \Big] = 0 \, , i = 1,2 \tag{96}$$

and hence this leads to the optimal real wage

$$\frac{W_{i,t}^{nO}}{P_{i,t}} \equiv W_{i,t}^O = \frac{1}{(1 - 1/\zeta_{w,i})} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_{w,i}^k \Lambda_{t,t+k} \left(\Pi_{i,t+k}^W \right)^{\zeta_{w,i}} H_{i,t+k}^d \, \frac{W_{i,t+k}^{nh}}{P_{i,t+k}} \, MS_{W_{i,t}}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_{w,i}^k \Lambda_{i,t+k} \left(\Pi_{i,t+k}^W \right)^{\zeta_{w,i}} \left(\Pi_{i,t+k} \right)^{-1} H_{i,t+k}^d} \, , i = 1,2 \tag{97}$$

Denoting the numerator and denominator $JJ_{i,t}^w$ and $J_{i,t}^w$, we write in recursive form

$$JJ_{W_{i,t}} = \frac{\zeta_{w,i}}{\zeta_{w,i} - 1} \, H_{i,t}^d \, W_{i,t}^h \, MS_{W_{i,t}} + \xi_{w,i} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\Pi_{i,t+1}^W \right)^{\zeta_{w,i}} JJ_{W_{i,t+1}} \right] \, , i = 1,2 \tag{98}$$

$$J_{Wi,t} = H_{i,t}^d + \xi_{w,i} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\Pi_{i,t+1}^W \right)^{\zeta_{w,i}} \left(\Pi_{i,t+1} \right)^{-1} J_{W_{i,t+1}} \right] \, , i = 1,2 \tag{99}$$

where $\Pi_{i,t}^W = \frac{W_{i,t}}{W_{i,t-1}} \Pi_{i,t}$ and $\Pi_t^W = \frac{W_t}{W_{t-1}} \Pi_t$ as before. Using the aggregate wage index $W_{i,t}$ and the fact that all resetting packers in sector i will choose the same wage, by the Law of Large Numbers we can find the evolution of the wage index as given by

$$W_{i,t}{}^{1-\zeta_{w,i}} = \xi_{w,i} W_{i,t-1}{}^{1-\zeta_{w,i}} + (1 - \xi_{w,i}) W_{i,t}^O{}^{1-\zeta_{w,i}} \, , i = 1,2 \tag{100}$$

which can be written in the form required

$$1 = \xi_{w,i} \left(\Pi_{i,t}^W \right)^{\zeta_{w,i} - 1} + (1 - \xi_{w,i}) \left(\frac{W_{i,t}^O}{W_{i,t}} \right)^{1-\zeta_{w,i}} \, , i = 1,2 \tag{101}$$

where $W_{i,t}^O = \frac{JJ_{W_{i,t}}}{J_{W_{i,t}}}$. Whilst the distribution of wages is not required to track the evolution of the aggregate wage index, (104) below implies a loss of labour due to dispersion in wages. Using the demand schedules, we can write the wage dispersion that gives the average loss in labour as

$$\Delta_{H_i,t} = \frac{1}{M} \sum_{j=1}^{M} \left(\frac{W_{i,t}\,(m)}{W_{i,t}} \right)^{-\zeta_{w,i}} \, , i = 1,2 \tag{102}$$

for non-optimizing firms $j = 1, ..., J$. It is not possible to track all $W_{j,t}$, but it is known that a proportion $1 - \xi_{w,i}$ of packers will optimize prices in period t, and from the Law of Large Numbers, that the distribution of non-optimized prices will be the same as in the overall distribution. Therefore, wage dispersion can be written as a law of motion

$$\Delta_{H_i,t} = \xi_{w,i} \Pi_{i,t}^W{}^{\zeta_{w,i}} \Delta_{H_i,t-1} + (1 - \xi_{w,i}) \left(\frac{JJ_{W_{i,t}}}{J_{W_{i,t}} \, W_{i,t}} \right)^{-\zeta_{w,i}} \, , i = 1,2 \tag{103}$$

Using this, final labour is given as a proportion of the intermediate labour

$$H_{i,t}^d = \frac{H_{i,t}}{\Delta_{H_i,t}} \, , i = 1,2 \tag{104}$$

where

$$H_t = H_{1,t} + H_{2,t} \tag{105}$$

A.3 Firms

Wholesale Sector in ROW bloc

As in the SOE bloc, we have as following

$$Y_t^{*W} = F(A_t^*, H_t^*, K_{t-1}^*) = (A_t^* H_t^*)^{\alpha^*} {K_{t-1}^*}^{1-\alpha^*}$$

$$F_{H,t}^* = \frac{\alpha_t^* Y_t^{*W}}{H_t^*} MC^* = W_t^*$$

$$F_{K,t}^* = \frac{(1-\alpha^*)_t Y_t^{*W}}{K_{t-1}^*} MC^* = r_t^{*K}$$

A.4 Capital Producers

Capital producers maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k}(1 - \mathcal{S}\left(I_{t+k}/I_{t+k-1}\right)) I_{t+k} - I_{t+k} \right]$$

subject to the law of motions of capital and investment - (17) and (18) in the paper. This results in the first-order condition

$$IS_t\, Q_t(1 - \mathcal{S}(X_t) - X_t \mathcal{S}'(X_t)) + E_t \left[\Lambda_{t,t+1}\, Q_{t+1}\, IS_{t+1} \mathcal{S}'(X_{t+1}) \frac{I_{t+1}^2}{I_t^2} \right] = 1 \tag{106}$$

Therefore, we have

$$\mathbb{E}_t \left[R_{t+1}^K \Lambda_{t,t+1} \right] = 1 \tag{107}$$

where capital demand equates the expected discounted return on capital over the period $[t, t+1]$ and must satisfy

$$R_t^K = \frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}} \tag{108}$$

Capital Producers in ROW bloc

Similarly, we have the following capital dynamics equations,

$$\begin{aligned}
X_t^* &\equiv \frac{I_t^*}{I_{t-1}^*} \\
S^*(X_t^*) &= \phi_X^* (X_t^* - 1)^2 \\
S'^*(X_t^*) &= 2\phi_X^* (X_t^* - 1) \\
K_t^* &= (1-\delta^*) K_{t-1}^* + (1 - S^*(X_t^*)) IS_t^*\, I_t^* \\
IS_t^* Q_t^* (1 - S^*(X_t^*) - X_t S'^*(X_t^*)) &+ \mathbb{E}_t \left[\Lambda_{t,t+1}^*\, IS_{t+1}^* Q_{t+1}^* S'^*(X_{t+1}^*) {X^*}_{t+1}^2 \right] = 1
\end{aligned}$$

$$
\begin{aligned}
R_t{}^{*K} &= \frac{\left[r_t^{*K} + (1-\delta^*)Q_t^*\right]}{Q_{t-1}^*} \\
1 &= \mathbb{E}_t\left[R_{t+1}^{*}{}^{K}\Lambda_{t,t+1}^*\right]
\end{aligned}
$$

A.5 Retail Sector and Incomplete Exchange Rate Pass-through For Exports

Price setting in export markets by domestic LCP exporters follows in a very similar fashion to domestic pricing. The optimal price in units of domestic currency is $\hat{P}_{1,t}^{*\,\ell}S_t$, costs are as for domestically marketed goods, so firms maximise expected discounted profits by solving

$$
\max_{P_{H,t}^{*O\,\ell}(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_{p,1}^k \frac{\Lambda_{t,t+k}}{P_{1,t+k}^{*\,\ell}} Y_{1,t+k}^{*\,\ell}(m)\left[P_{1,t}^{*O\,\ell}(m)S_{t+k} - P_{1,t+k}^W MS_{1,t+k}^{*\,\ell}\right] \tag{109}
$$

with real marginal cost

$$
MC_{1,t}^{*\,\ell} \equiv \frac{MC_{1,t}\,P_{1,t}}{S_t\,P_{1,t}^{*\,\ell}} \tag{110}
$$

Substituting this in the demand schedule (27) in the paper, taking first-order conditions with respect the new price and rearranging leads to

$$
P_{i,t}^O = \frac{\zeta_{p,i}}{\zeta_{p,i}-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,i}^k\frac{\Lambda_{t,t+k}}{P_{t+k}}\left(P_{i,t+k}\right)^{\zeta_{p,i}}Y_{i,t+k}P_{i,t+k}^W\,MS_{i,t+k}}{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,i}^k\frac{\Lambda_{t,t+k}}{P_{t+k}}\left(P_{i,t+k}\right)^{\zeta_{p,i}}Y_{i,t+k}}\,, i=1,2 \tag{111}
$$

$$
P_{1,t}^{O\,*\,\ell} = \frac{\zeta_{p,1}}{\zeta_{p,1}-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,1}^k\frac{\Lambda_{t,t+k}}{P_{t+k}^{*\,\ell}}\left(P_{1,t+k}^{*\,\ell}\right)^{\zeta_{p,1}}Y_{1,t+k}^{*\,\ell}P_{1,t+k}^W MS_{1,t+k}^{*\,\ell}}{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,1}^k\frac{\Lambda_{t,t+k}}{P_{t+k}^{*\,\ell}}\left(P_{1,t+k}^{*\,\ell}\right)^{\zeta_{p,1}}Y_{1,t+k}^{*\,\ell}S_{t+k}} \tag{112}
$$

where the m index is dropped, as all firms face the same marginal cost, so the right-hand side of the equation is independent of firm size or price history. $MS_{i,t}$ is a mark-up shock in each sector and real marginal cost is given by $MC_{i,t} \equiv \frac{P_{i,t}^W}{P_{i,t}}$.

We also index prices to last period's aggregate inflation, with a price indexation parameter γ_i. Then, the price trajectory with no re-optimization is given by $P_{i,t}^O(j)$, $P_{i,t}^O(j)\left(\frac{P_{i,t}}{P_{i,t-1}}\right)^{\gamma_i}$, $P_{i,t}^O(j)\left(\frac{P_{i,t+1}}{P_{i,t-1}}\right)^{\gamma_i}$, etc. With indexing by an amount $\gamma_i \in [0,1]$, the optimal price-setting first-order condition for a firm j setting a new optimized price $P_{i,t}^O(j)$ is now given by

$$
\frac{P_{i,t}^O}{P_{i,t}} = \frac{\zeta_{p,i}}{\zeta_{p,i}-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,i}^k\Lambda_{t,t+k}\left(\tilde{\Pi}_{i,t+k}\right)^{\zeta_{p,i}}Y_{i,t+k}MC_{i,t+k}\,MS_{i,t}}{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,i}^k\Lambda_{t,t+k}\left(\tilde{\Pi}_{i,t+k}\right)^{\zeta_{p,i}}\left(\Pi_{i,t+k}\right)^{-1}Y_{i,t+k}}\,, i=1,2 \tag{113}
$$

$$
\frac{P_{1,t}^{O\,*\,\ell}}{P_{1,t}^{*\,\ell}} = \frac{\zeta_{p,1}}{\zeta_{p,1}-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,1}^k\Lambda_{t,t+k}\left(\tilde{\Pi}_{1,t+k}^{*\,\ell}\right)^{\zeta_{p,i}}Y_{1,t+k}^{*\,\ell}MC_{1,t+k}^{*\,\ell}S_{t+k}MS_{1,t+k}^{*\,\ell}}{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_{p,1}^k\Lambda_{t,t+k}\left(\tilde{\Pi}_{1,t+k}^{*\,\ell}\right)^{\zeta_{p,1}}\left(\Pi_{1,t,t+k}^{*\,\ell}\right)^{-1}Y_{1,t+k}^{*\,\ell}S_{t+k}} \tag{114}
$$

where

$$
\tilde{\Pi}_{i,t+1} = \frac{\Pi_{i,t+1}}{\Pi_{i,t}^{\gamma_i}}\,, i=1,2 \tag{115}
$$

$$\tilde{\Pi}^{*\,\ell}_{1,t+1} = \frac{\Pi^{*\,\ell}_{1,t+1}}{\Pi^{*\,\ell\,\gamma_\ell}_{1,t}} \tag{116}$$

where $\Pi_{i,t+1} \equiv \frac{P_{i,t+1}}{P_{i,t}}$ and $\Pi_{t+1,t} \equiv \frac{P_{t+1}}{P_t}$ as before and $\Pi^{*\,\ell}_{1,t+1} = \frac{P^{*\,\ell}_{H,t+1}}{P^{*\,\ell}_{1,t}}$ is the part of formal sector inflation set in terms of foreign currency. Denoting the numerator and denominator $JJ_{i,t}$ and $J_{i,t}$ we write in recursive form

$$JJ_{i,t} = \frac{\zeta_{p,i}}{\zeta_{p,i}-1} Y_{i,t} MC_{i,t}\, MS_{i,t} + \xi_{p,i}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\tilde{\Pi}_{i,t+1}\right)^{\zeta_{p,i}} JJ_{i,t+1}\right], i = 1,2 \tag{117}$$

$$J_{i,t} = Y_{i,t} + \xi_{p,i}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\tilde{\Pi}_{i,t+1}\right)^{\zeta_{p,i}} \left(\Pi_{i,t+1}\right)^{-1} J_{i,t+1}\right], i = 1,2 \tag{118}$$

$$JJ^{*\,\ell}_{1,t} = \frac{\zeta_{p,1}}{\zeta_{p,1}-1} Y^{*\,\ell}_{1,t}\, MC^{*\,\ell}_{1,t}\, MS^{*\,\ell}_{1,t} + \xi_{p,1}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\tilde{\Pi}^{*\,\ell}_{1,t+1}\right)^{\zeta_{p,1}} JJ^{*\,\ell}_{1,t+1}\Pi^S_{t+k}\right] \tag{119}$$

$$J^{*\,\ell}_{1,t} = Y^{*\,\ell}_{1,t} + \xi_{p,1}\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\tilde{\Pi}^{*\,\ell}_{1,t+1}\right)^{\zeta_{p,1}} \left(\Pi^{*\,\ell}_{1,t+1}\right)^{-1} J^{*\,\ell}_{1,t+1}\Pi^S_{t+k}\right], \tag{120}$$

Using the aggregate producer price index $P_{H,t}$ and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

$$P_{i,t}{}^{1-\zeta_{p,i}} = \xi_{p,i} P_{i,t-1}{}^{1-\zeta_{p,i}} + (1-\xi_{p,i}) P^{O\;1-\zeta_{p,i}}_{i,t}, i = 1,2 \tag{121}$$

$$P^{*\,\ell\,1-\zeta_{p,1}}_{1,t} = \xi_{p,1} P^{*\,\ell}_{1,t-1}{}^{1-\zeta_{p,1}} + (1-\xi_{p,1}) P^{O\,*\,\ell\,1-\zeta_{p,1}}_{1,t} \tag{122}$$

which can be written in the form required

$$1 = \xi_{p,i}\left(\tilde{\Pi}_{i,t}\right)^{\zeta_{p,i}-1} + (1-\xi_{p,i})\left(\frac{P^O_{i,t}}{P_{i,t}}\right)^{1-\zeta_{p,i}}, i = 1,2 \tag{123}$$

$$1 = \xi_{p,1}\left(\tilde{\Pi}^{*\,\ell}_{1,t}\right)^{\zeta_{p,1}-1} + (1-\xi_{p,1})\left(\frac{P^{O\,*\,\ell}_{1,t}}{P^{*\,\ell}_{1,t}}\right)^{1-\zeta_{p,1}} \tag{124}$$

Whilst the distribution of prices is not required to track the evolution of the aggregate price index, (129) and (130) below implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

$$\Delta_{i,t} = \frac{1}{M}\sum_{j=1}^{M}\left(\frac{P_{i,t}(m)}{P_{i,t}}\right)^{-\zeta_{p,i}}, i = 1,2 \tag{125}$$

$$\Delta^{*\,\ell}_{1,t} = \frac{1}{M}\sum_{j=1}^{M}\left(\frac{P^{*\,\ell}_{1,t}(m)}{P^{*\,\ell}_{1,t}}\right)^{-\zeta_{p,1}} \tag{126}$$

for non-optimizing firms $j = 1, ..., J$. It is not possible to track all $P_{j,t}$, but it is known that a proportion $1-\xi_{p,i}$ of firms will optimize prices in period t and from, the Law of Large Numbers, that the distribution of non-optimized prices will be the same in as the overall distribution. Using the above, the law of motion for price dispersion can then be derived as

$$\Delta_{i,t} = \xi_{p,i}\left(\tilde{\Pi}_{i,t}\right)^{\zeta_{p,i}} \Delta_{i,t-1} + (1-\xi_{p,i})\left(\frac{JJ_{i,t}}{J_{i,t}}\right)^{-\zeta_{p,i}}, i = 1,2 \tag{127}$$

$$\Delta_{1,t}^{*\,\ell} = \xi_{p,1} \left(\tilde{\Pi}_{1,t}^{*\,\ell}\right)^{\zeta_{p,1}} \Delta_{1,t-1}^{*\,\ell} + (1-\xi_{p,1}) \left(\frac{JJ_{1,t}^{*\,\ell}}{J_{1,t}^{*\,\ell}}\right)^{-\zeta_{p,1}} \tag{128}$$

so that output in each sector is given as a proportion of the intermediate output

$$Y_{2,t} = \frac{Y_{2,t}^W}{\Delta_{2,t}} \tag{129}$$

$$Y_{1,t} = (1-(1-\theta_1)(1-\theta)) \frac{Y_{1,t}^W}{\Delta_{1,t}} \tag{130}$$

$$Y_{1,t}^{*\,\ell} = (1-\theta_1)(1-\theta) \frac{Y_{1,t}^W}{\Delta_{1,t}^{*\,\ell}} \tag{131}$$

$$Y_1 T = \frac{P_{1,t}Y_{1,t} + S_t P_{1,t}^{*\,\ell} Y_{1,t}^{*\,\ell}}{(1-(1-\theta_1)(1-\theta)) P_{1,t} + (1-\theta_1)(1-\theta) S_t P_{1,t}^{*\,\ell}} \tag{132}$$

where θ_1 is the export share of production.

Price Dynamics of ROW bloc

Price dynamics in the ROW bloc follows in a similar fashion:

$$\begin{aligned}
\frac{P_t^{0*}}{P_t^*} &= \frac{J_t^*}{JJ_t^*} \\
J_t^* - \xi^* \mathbb{E}_t[\Lambda_{t,t+1}^* \tilde{\Pi^*}_{t+1}^{\zeta^*} J_{t+1}] &= \frac{1}{1-\frac{1}{\zeta^*}} Y_t^* MC_t^* MS_t^* \\
JJ_t^* - \xi \mathbb{E}_t[\Lambda_{t,t+1}^* \tilde{\Pi^*}_{t+1}^{\zeta^*-1} JJ_{t+1}] &= Y_t^* \\
\tilde{\Pi^*}_t &\equiv \frac{\Pi_t^*}{{\Pi_{t-1}^*}^{\gamma^*}} \\
\xi^* \tilde{\Pi^*}_t^{\zeta^*-1} + (1-\xi^*) \left(\frac{J_t^*}{JJ_t^*}\right)^{1-\zeta^*} &= 1 \\
\Delta_t^* - \xi^* \tilde{\Pi^*}_t^{\zeta^*} \Delta_{t-1}^* &= (1-\xi^*) \left(\frac{J_t^*}{JJ_t^*}\right)^{-\zeta^*}
\end{aligned}$$

A.6 Market Clearing

The resource constraint implies

$$Y_{1,t} + \frac{S_t P_{1,t}^{*\,\ell}}{P_{1,t}} Y_{1,t}^{*\,\ell} = C_{1,t} + I_{1,t} + \left(\theta + (1-\theta) \frac{S_t P_{1,t}^{*\,\ell}}{P_{1,t}}\right) EX_t + G_t \tag{133}$$

$$Y_{2,t} = C_{2,t} \tag{134}$$

$$Y_t = \frac{P_{1,t}}{P_{H,t}} Y_{1,t} + \frac{S_t P_{1,t}^{*\,\ell}}{P_{H,t}} Y_{1,t}^{*\,\ell} + \frac{P_{2,t}}{P_{H,t}} Y_{2,t} \tag{135}$$

$$Y_t^* = C_t^* + G_t^* + I_t^* \tag{136}$$

A.7 Monetary Policy in ROW bloc

The ROW nominal interest rate is given by the following Taylor-type rule

$$\log\left(\frac{R_t^*}{R*}\right) = \rho_r \log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\rho_r^*)\left[\theta_{\pi^*}\log\left(\frac{\Pi_t^*}{\Pi^*}\right) + \theta_{y*}\log\left(\frac{Y_t^*}{Y}\right)\right] + \epsilon_{M^*,t} \tag{137}$$

A.8 Deterministic Non-Zero Net-Inflation and Zero-Growth Steady State of the SOE model

The steady is solved allowing for a non-zero steady state inflation ($\Pi > 1$) and it is obtained by solving the following for formal labour supply H_1, consumption preference parameter ϱ, money preference parameters Ψ, aggregate money demand m, non-Ricardian money demand m^{RoT}, non-Ricardian formal labour supply H_1^{RoT}, non-Ricardian informal labour supply H_2^{RoT}, and informal/formal price ratio $\frac{P_2}{P_1}$ using

$$\begin{aligned}
&\frac{U_{H,1}^R}{U_C^R} = W_1^h\left(1-\tau_t^w\right)\\
&\frac{U_{H,1}^{RoT}}{U_C^{RoT}} = W_1^h\left(1-\tau_t^w\right)\\
&\frac{U_{H,2}^R}{U_C^R} = W_2^h\\
&\frac{U_{H,2}^{RoT}}{U_C^{RoT}} = W_2^h\\
&C_2 = Y_2\\
&\Psi m^{RoT^{-\psi}} = U_C^{RoT}\left[1-\frac{1}{R}\right]\\
&m = \lambda m^{RoT} + (1-\lambda)m^R
\end{aligned}$$

In doing so, we calibrate ϱ and Ψ to target average hours $H = H_1 + H_2 = 1/3$ and $\frac{m}{Y_1^T} = 1$, respectively. Then, in a non-zero net-inflation steady state, with appropriate choice of units and in recursive form, we have:

$$\begin{aligned}
&\mathcal{T} = \mathcal{T}^* = 1\\
&\Pi = \Pi_1 = \Pi_2 = \Pi_H = \Pi_F = \Pi^* = \Pi_1^{*\ell} = \Pi_1^{*P}\\
&\Pi_S = \frac{\Pi_F}{\Pi^*}\\
&\tilde{\Pi}_i = (\Pi_i)^{1-\gamma_i}, i = 1,2\\
&\tilde{\Pi}_i^W = \frac{\Pi_i^W}{(\Pi_i)^{\gamma_i^W}}, i = 1,2\\
&\tilde{\Pi}_1^{*\ell} = \left(\Pi_1^{*\ell}\right)^{1-\gamma^{*\ell}}\\
&\Lambda = \beta\\
&R^* = \frac{\Pi^*}{\beta^*}\\
&R = \frac{\Pi}{\beta}\\
&PB = \frac{1}{R}
\end{aligned}$$

$$Q = 1$$
$$H = \bar{H}$$
$$\mu = \Pi_1$$
$$\frac{P_H}{P} = \frac{1}{\left(\mathrm{w}_C + (1-\mathrm{w}_C)\left(\frac{P_1}{P_H}\mathcal{T}\right)^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}}}$$
$$\frac{P_F}{P} = \frac{1}{\left(\mathrm{w}_C\left(\frac{P_1}{P_H}\mathcal{T}\right)^{\mu_C-1} + (1-\mathrm{w}_C)\right)^{\frac{1}{1-\mu_C}}}$$
$$\frac{P_H}{P_I} = \frac{1}{\left(\mathrm{w}_I + (1-\mathrm{w}_I)(\mathcal{T})^{1-\mu_I}\right)^{\frac{1}{1-\mu_I}}}$$
$$\frac{P_F}{P_I} = \frac{1}{\left(\mathrm{w}_I(\mathcal{T})^{\mu_I-1} + (1-\mathrm{w}_I)\right)^{\frac{1}{1-\mu_I}}}$$
$$\frac{P_1}{P} = \frac{\frac{P_1}{P_H}}{\frac{P_H}{P_I}}$$
$$\frac{P_1}{P_H} = \frac{P_1}{P} = \frac{1}{\left(\mathrm{w}_S + (1-\mathrm{w}_S)\left(\frac{P_2}{P_1}\right)^{1-\mu_S}\right)^{\frac{1}{1-\mu_S}}}$$
$$\frac{P_2}{P_H} = \frac{P_2}{P} = \frac{1}{\left(\mathrm{w}_S\left(\frac{P_2}{P_1}\right)^{\mu_S-1} + (1-\mathrm{w}_S)\right)^{\frac{1}{1-\mu_S}}}$$
$$JJ_i^w = \frac{\frac{\zeta_{i,w}}{\zeta_{i,w}-1} H_i^d W_i^h}{1-\xi_{i,w}\beta\left(\tilde{\Pi}_i^w\right)^{\zeta_{i,w}}}; i = 1, 2$$
$$J_i^w = \frac{H_i^d}{1-\xi_i\beta\left(\tilde{\Pi}_i^w\right)^{\zeta_i-1}}; i = 1, 2$$
$$\frac{JJ_i^w}{J_i^w} = W_i\left(\frac{1-\xi_{i,w}\left(\tilde{\Pi}_i^w\right)^{\zeta_{i,w}-1}}{1-\xi_{i,w}}\right)^{\frac{1}{1-\zeta_{i,w}}}; i = 1, 2$$
$$W_i^W = \frac{JJ_i^w}{J_i^w}\frac{\zeta_{i,w}}{\zeta_{i,w}-1}\frac{1-\xi_{i,w}\beta\left(\tilde{\Pi}_i^w\right)^{\zeta_{i,w}}}{1-\xi_{i,w}\beta\left(\tilde{\Pi}_i^w\right)^{\zeta_{i,w}-1}}, i = 1, 2$$
$$\Delta_i^w = \frac{(1-\xi_{i,w})\left(\frac{JJ_i^w}{W_i J_i^w}\right)^{-\zeta_{i,w}}}{1-\xi_{i,w}\left(\tilde{\Pi}_i^w\right)^{\zeta_{i,w}}}, i = 1, 2$$
$$H_i^d = \frac{H_i}{\Delta_i^w}, i = 1, 2$$
$$JJ_i = \frac{\frac{\zeta_i}{\zeta_i-1} Y_i MC_i}{1-\xi_i\beta\left(\tilde{\Pi}_i\right)^{\zeta_i}}, i = 1, 2$$

$$J_i = \frac{\frac{P_i}{P} Y_i}{1 - \xi_i \beta \left(\tilde{\Pi}_i\right)^{\zeta_i} (\Pi)^{-1}}$$

$$\frac{JJ_i}{J_i} = \left(\frac{1 - \xi_i \left(\tilde{\Pi}_i\right)^{\zeta_i - 1}}{1 - \xi_i} \right)^{\frac{1}{1-\zeta_i}}, i = 1, 2$$

$$MC_i = \frac{JJ_i}{J_i} \frac{\zeta_i - 1}{\zeta_i} \frac{1 - \xi_i \beta \left(\tilde{\Pi}_i\right)^{\zeta_i}}{1 - \xi_i \beta \left(\tilde{\Pi}\right)^{\zeta_i} (\Pi_i)^{-1}}, i = 1, 2$$

$$\Delta_i = \frac{(1 - \xi_i) \left(\frac{JJ_i}{J_i}\right)^{-\zeta_i}}{1 - \xi_i \left(\tilde{\Pi}_i\right)^{\zeta_i}}, i = 1, 2$$

$$JJ_1^{*\ell} = \frac{\frac{\zeta_1}{\zeta_1 - 1} Y_1^{*\ell} MC_1^{*\ell}}{1 - \xi_1 \beta \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_1}}$$

$$J_1^{*\ell} = \frac{\frac{P_1^{*\ell}}{P^*} Y_1^{*\ell}}{1 - \xi_1 \beta \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_1} (\Pi^*)^{-1}}$$

$$\frac{JJ_1^{*\ell}}{J_1^{*\ell}} = \left(\frac{1 - \xi_1 \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_1 - 1}}{1 - \xi_1} \right)^{\frac{1}{1-\zeta_1}}$$

$$MC_1^{*\ell} = \frac{JJ_1^{*\ell}}{J_1^{*\ell}} \frac{\zeta_1 - 1}{\zeta_1} \frac{1 - \xi_1 \beta \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_1}}{1 - \xi_1 \beta \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_1} (\Pi_1^*)^{-1}}$$

$$\Delta_1^{*\ell} = \frac{(1 - \xi_1) \left(\frac{JJ_1^{*\ell}}{J_1^{*\ell}}\right)^{-\zeta_i}}{1 - \xi_1 \left(\tilde{\Pi}_1^{*\ell}\right)^{\zeta_i}}$$

$$\frac{P_1^{*\ell}}{P^*} = \frac{P_1 \left(\mathcal{TT}^* - \theta\right)}{P(1 - \theta) RER_t}$$

$$\frac{H_1^d}{K_1} = \left(\frac{\frac{1}{\beta} - (1 - \delta)}{\frac{P_1}{P} MC_1 \left(1 - \alpha_1\right) \left(1 - \tau^k\right)} \right)^{1/\alpha_1};$$

$$\frac{H_2^d}{K_2} = \left(\frac{\frac{1}{\beta} - (1 - \delta)}{\frac{P_2}{P} MC_2 \left(1 - \alpha_1\right)} \right)^{1/\alpha_2};$$

$$W_i = \alpha_i \, MC_i \left(\frac{H_i^d}{K_i} \right)^{\alpha_i - 1}; i = 1, 2$$

$$Y_i^W = H_i^d \left(\frac{H_i^d}{K_i} \right)^{\alpha_i - 1}; i = 1, 2$$

$$Y_1 = \theta \frac{Y_1^W}{\Delta_1}$$

$$Y_2 = \frac{Y_2^W}{\Delta_2}$$

$$Y_1^{*\ell} = \frac{(1-\theta)Y_1^W}{\Delta_1^{*\ell}}$$

$$Y = \frac{P_1}{P_H}Y_1 + \frac{P_2}{P_H}Y_2 + \frac{SP_1^{*\ell}}{P_H}Y_1^{*\ell}$$

$$Y_1^T = \frac{Y_1 + \frac{SP_1^{*\ell}}{P_1}Y_1^{*\ell}}{\theta + (1-\theta)\frac{SP_1^{*\ell}}{P_1}}$$

$$Y^O = kY_1^T$$

$$K_i = \frac{Y_i^W}{\left(\frac{H_i^d}{K_i}\right)^{\alpha_i}};, i = 1, 2$$

$$K = K_1 + K_2$$

$$I = \delta(K_1 + K_2)$$

$$I_H = \mathrm{w}_I \left(\frac{P_H}{P}\right)^{-\mu_I} I$$

$$I_F = (1-\mathrm{w}_I)\left(\frac{P_H}{P}\right)^{-\mu_I} I$$

$$\bar{G} = g_y Y_1^T$$

$$G = \bar{G}$$

$$B_G = b_g Y_1^T$$

$$TB = 0$$

$$D = \left(\frac{1}{R} - \frac{1}{\Pi}\right) B_G$$

$$\tau^w = \frac{\frac{P_1}{P}G - P^{*O}Y^O RER_t - (1-\alpha_1)Y_1^W \frac{P_1}{P} MC_1 \tau^k - \left(m - \frac{m}{\Pi}\right) - D}{W_1^h H_1}$$

$$C = Y + Y^O - I - \frac{P_1}{P}G - TB$$

$$C_H = \mathrm{w}_C \left(\frac{P_H}{P}\right)^{-\mu_C} C$$

$$C_F = (1-\mathrm{w}_C)\left(\frac{P_H}{P}\right)^{-\mu_C} C$$

$$C_1 = \mathrm{w}_S \left(\frac{P_1}{P_H}\right)^{-\mu_S} C_H l$$

$$C_2 = (1-\mathrm{w}_S)\left(\frac{P_2}{P_H}\right)^{-\mu_S} C_H$$

$$C^{RoT} = H_1^{RoT} W_1^h (1-\tau^w) + H_2^{RoT} W_2^h + m^{RoT} - \frac{m^{RoT}}{\Pi}$$

$$C^R = \frac{1}{1-\lambda}C - \frac{\lambda}{1-\lambda}C^{RoT}$$

$$H_1^R = \frac{1}{1-\lambda}H_1 - \frac{\lambda}{1-\lambda}H_1^{RoT}$$

$$H_2^R = \frac{1}{1-\lambda}H_2 - \frac{\lambda}{1-\lambda}H_2^{RoT}$$

$$H^R = H_1^R + H_2^R$$

$$H^{RoT} = H_1^{RoT} + H_2^{RoT}$$

$$
\begin{aligned}
&U_C^R = (1-\varrho)(C^R - \chi C^R)^{(1-\varrho)(1-\sigma_R)-1}(1-H^R)^{\varrho(1-\sigma_R)} \\
&U_C^{RoT} = (1-\varrho)(C^{RoT} - \chi C^{RoT})^{(1-\varrho)(1-\sigma_C)-1}(1-H^{RoT})^{\varrho(1-\sigma_{RoT})} \\
&U_{H,1}^R = \varrho(C^R - \chi C^R)^{(1-\varrho)(1-\sigma_R)}(1-H_1^R)^{\varrho(1-\sigma_R)-1} \\
&U_{H,2}^R = \varrho(C^R - \chi C^R)^{(1-\varrho)(1-\sigma_R)}(1-H_2^R)^{\varrho(1-\sigma_R)-1} \\
&U_{H,1}^{RoT} = \varrho(C^{RoT} - \chi C^{RoT})^{(1-\varrho)(1-\sigma_{RoT})}(1-H_1^{RoT})^{\varrho(1-\sigma_{RoT})-1} \\
&U_{H,2}^{RoT} = \varrho(C^{RoT} - \chi C^{RoT})^{(1-\varrho)(1-\sigma_{RoT})}(1-H_2^{RoT})^{\varrho(1-\sigma_{RoT})-1} \\
&m^{RoT} = \left(\frac{1}{\Psi}U_C^{RoT}\left[1-\frac{1}{R}\right]\right)^{\frac{1}{-\psi}} \\
&m^R = \left(\frac{1}{\Psi}U_C^{R}\left[1-\frac{1}{R}\right]\right)^{\frac{1}{-\psi}} \\
&f = Y - \frac{P_1}{P_H}MC_1Y_1^W - \frac{P_2}{P_H}MC_2Y_2^W \\
&EX = \frac{\left(Y_1 + \frac{SP_1^{*\ell}}{P_1}Y_1^{*\ell} - C_1 - \frac{P_H}{P_I}I_H - G\right)}{\left(\theta + (1-\theta)\left(\frac{S_tP_{1,t}^{*\ell}}{P_1}\right)\right)} \\
&r^K = (1-\tau_K)\,MC_1\frac{P_1}{P}\,(1-\alpha_1)\,\frac{Y_1^W}{K_1} \\
&R^K = r^K + (1-\delta) \\
&S(X) = 0
\end{aligned}
$$

B Bayesian Estimation Summary Of The ROW Closed Economy Model

This section presents results for the Bayesian estimation of the rest-of-the-world bloc using the same dataset as in Smets and Wouters (2007), i.e. quarterly data on the log difference of real GDP, log difference of real consumption, the log difference of the GDP deflator and the federal funds rate. All series are seasonally adjusted, taken from the FRED Database.

Some structural parameters are kept fixed, as is standard in the literature (see Table S1).

Calibrated parameter	Symbol	Value
Discount factor	β^*	0.99
Depreciation rate	δ^*	0.025
Growth rate	g^*	0.004
Substitution elasticity of goods	ζ^*	7
Government expenditure-output ratio	g_y^*	0.2
Hours worked	h^*	1/3
Preference parameter	$\varrho*$	calibrated to hit h

Table S1: Calibrated parameters in the ROW bloc

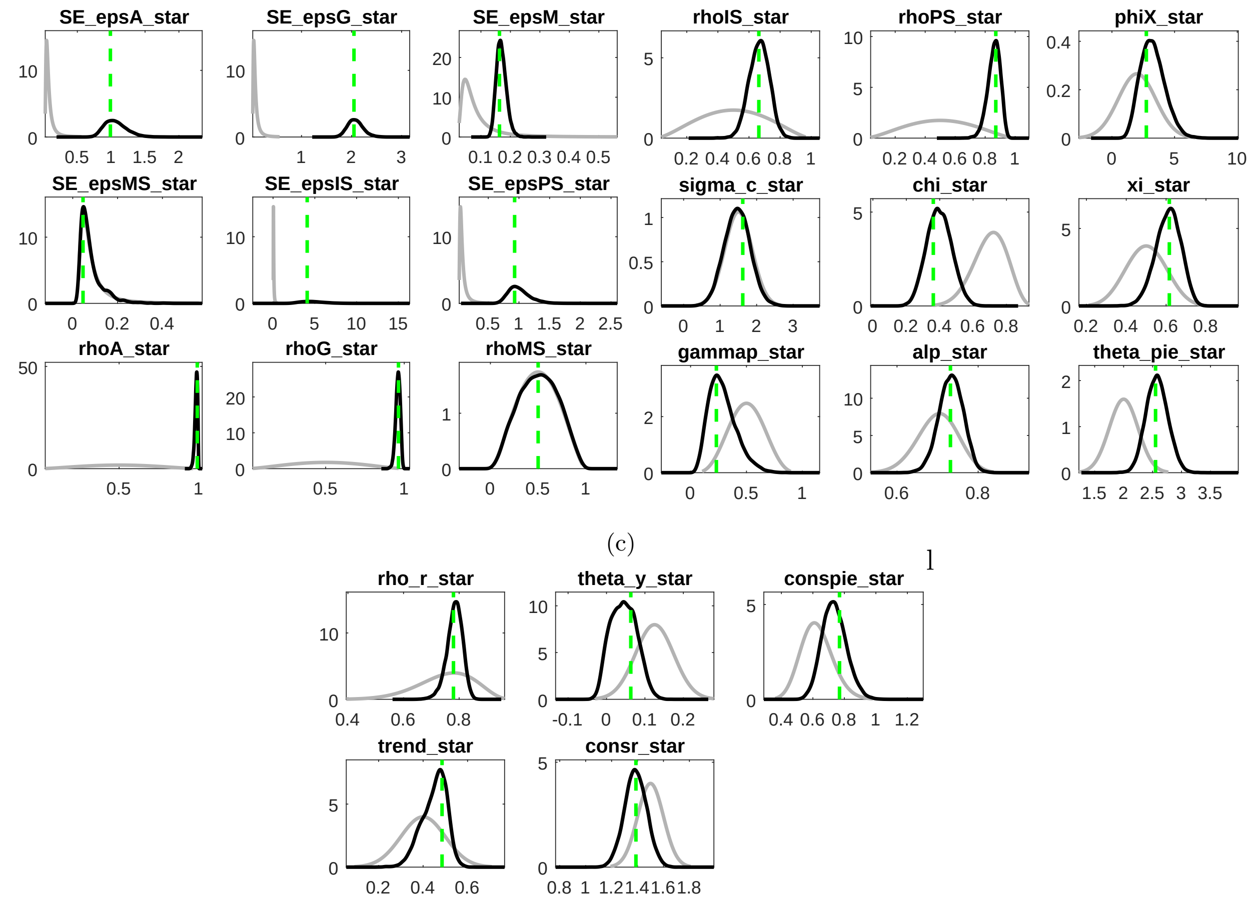
Table 8 summarizes the prior distribution, estimated posterior means and 90% confident intervals, with the marginal data density of the model computed using the Geweke (1999) modified harmonic-mean estimator. Figure 11 depicts the corresponding prior and posterior distributions.

Estimated Parameter Values		Prior		Posterior	
	Parameter	Dist.	(Mean, Std Dev)	Mean	90% HPD Interval
ROW					
Technology shock	ϵ_{A*}	IG	$0.10, 2.00$	1.0647	0.8064 , 1.3224
Markup shock	ϵ_{MS*}	IG	$0.10, 2.00$	2.2182	0.0806 , 0.1529
Investment shock	ϵ_{IS*}	IG	$0.10, 2.00$	4.7214	2.4719 , 6.8916
Government shock	ϵ_{G*}	IG	$0.10, 2.00$	2.0739	1.8241 , 2.3175
Monetary policy shock	ϵ_{M*}	IG	$0.10, 2.00$	0.1693	0.1417 , 0.1958
Preference shock	ϵ_{PS*}	IG	$0.10, 2.00$	1.0003	0.7321 , 1.2816
Technology shock persistence	ρ_{A*}	β	0.50,0.10	0.9835	0.9706 , 0.9974
Markup shock persistence	ρ_{MS*}	β	0.50,0.10	0.5037	0.1685 , 0.8254
Investment shock persistence	ρ_{IS*}	β	0.50,0.10	0.6645	0.5566 , 0.7644
Government shock persistence	ρ_{G*}	β	0.50,0.10	0.9595	0.7409 , 0.8316
Monetary Policy shock persistence	ρ_{M*}	β	0.75,0.10	0.7834	0.7409 , 0.8316
Preference shock persistence	ρ_{M*}	β	0.75,0.10	0.8577	0.7901 , 0.9243
Consumption habit formation	$\chi*$	β	0.70,0.05	0.3966	0.2711 , 0.5139
Labour Share	$\alpha*$	β	0.70,0.05	0.7327	0.6859 , 0.7826
Calvo price stickiness	$\xi*$	β	0.50,0.05	0.6093	0.5065 , 0.7104
Elasticity of demand	$\zeta*$	N	6.00,2.50	3.9817	2.4804 , 5.7917
Price index	$\gamma*$	β	0.50,0.10	0.2862	0.1009 0.4816
Elasticity of Investment adjustment cost	ϕ_I*	N	2.00,1.50	1.4651	1.7187 , 4.8394
Non-Ricardian risk aversion	σ_c*	N	2.00,0.25	1.5757	0.8871 , 2.0337
Feedback from inflation	$\theta_{\pi*}$	N	2.00,0.25	2.5745	2.2693 , 2.8822
Feedback from output	θ_{y*}	N	0.125,0.05	0.0444	-0.0091 , 0.0974
Constant π	$conspie*$	G	0.625,0.10	0.7351	0.6084 , 0.8584
Trend growth	$trend*$	N	0.4, 0.10	0.4493	0.3546 , 0.5304
Constant R_n^*	$consr*$	N	1.5, 0.10	1.3840	1.2439 , 1.5243

Table 8: Estimated parameter values and standard deviations of shocks

Figure 11: Prior and Posterior Distributions for the ROW estimation

(a) (b)



C Bayesian estimation of the SOE bloc

This section presents additional results mentioned in the main text concerning the SOE bloc estimation, namely the prior and posterior distributions for the SOE parameters in Figure 12.

Figure 12: Prior and Posterior Distributions for the SOE model estimation