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Imperfect Exchange Rate Pass-through: Empirical Evidence and Monetary Policy Implications

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Abstract

We construct a small open economy (SOE) DSGE model interacting with the rest of the world (ROW). We depart from the standard SOE model along several dimensions. Firstly, we nest two different pricing paradigms: local currency pricing (LCP) alongside producer currency pricing (PCP). Second, the production function incorporates capital and intermediate inputs produced domestically and abroad. Finally, international asset markets are incomplete. Using US and Canadian data, we explore the empirical evidence for PCP vs LCP pricing paradigms through a Bayesian estimation likelihood race and a comparison with the second moments of the data. We then examine the implications of these two paradigms for the conduct of monetary policy using optimized Taylor-type inertial interest rate rules with a zero lower bound constraint. The main results are: first, in a likelihood race LCP easily beats PCP and fits reasonably the second moments of the data; second, whereas for the closed economy ROW the price-level rule closely mimics the optimized general inflation-output rule, for the SOE the corresponding result requires a nominal income rule.

Keywords: Imperfect Exchange Rate Pass-through; Producer and Local Currency Pricing; Small Open Economy; Zero Lower Bound.

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1 Introduction

The open-economy dimensions of monetary policy continue to raise an important set of questions: what are the size and sign of the spillovers, what are the channels via which domestic monetary policy transmits internationally and should policymakers respond to exchange rate fluctuations? The growing literature on this is far from having yet establish a consensus. This disagreement is due to differences in data sources, methodology and modelling choices. However, if one is to allow for price rigidity, as one does in a Mundell-Fleming setup, it is crucial to know in which currency prices are rigid. In this regard, a distinction among three different paradigms has emerged: prices may be set in the producer’s currency (Producer Currency Pricing, PCP henceforth), in the buyer’s currency (Local Currency Pricing, LCP), or in a local currency that is neither the buyer’s nor seller’s (Dominant Currency Pricing, DCP), often the US dollar.

Each of these pricing models will have different implications on both domestic and international levels. Consider a small open economy (SOE) trading with the rest of the world (ROW), which in our paper is modelled as Canada and the US, respectively. In the basic Mundell-Fleming model, which assumes PCP and the law of one price, a ROW monetary expansion (a cut to its nominal interest rate) will lead to a depreciation of the US dollar, a rise in the price of the SOE exported goods in ROW currency, thus resulting in a lower demand for them (a phenomenon known as expenditure switching). In other words, under the PCP assumption, ‘local’ (US) export prices will fluctuate in response to changes in the nominal exchange rates and the exchange rate pass-through into prices is 100% (i.e. perfect pass-through). Alternatively, under a LCP regime, export prices from the SOE are sticky in the destination country’s currency. In this case, shocks to the nominal exchange rate will not affect the prices in the ROW currency - in the short-run there will be a deviation from the law of one price and exchange rate pass-through into imported prices is 0%.

Recent studies, however, suggest that both of these pricing schemes seem to be at odds with the trends seen in international trade transactions. Firms set prices in very few currencies (with the US dollar being the most frequently used currency) and do not change prices often (Goldberg and Tille (2010), Gopinath (2015)). These observations have led to the recent emergence of a literature that considers a DCP paradigm. Under this regime, prices are anchored in a third currency (the US dollar, say), in which case changes in nominal exchange rate will only weakly impact the terms of trade, while the main factor in terms of prices and quantity of imported goods will be the SOE’s currency value vis-à-vis the US dollar.

Motivated by the recent studies on pricing paradigms, we attempt to contribute to the literature by shedding light on the impact of a large economy monetary policy on small open economies with which it trades. More specifically, in this paper, we step back from the n -country framework of Gopinath (2015) for $n \geq 3$ and reconsider a conventional 2-country framework. We then investigate the interaction between US monetary policy and that of its neighbour Canada under two different pricing regimes for this small open economy, PCP and LCP.

We use a structural DSGE model and historical data in order to estimate the impact of US domestic monetary policy on its trading partners. We model the US as a closed NK economy that is not affected by shocks from the other, small, countries. Canada is, in turn, modelled as small open economies (SOE) that is affected by shocks from the ROW. The model is estimated for both

pricing regimes and the results under these pricing models are compared.

Our contribution to the literature is threefold. First, we add to the empirical literature on producer versus local currency pricing by comparing the data fit for Canada-US in terms of a likelihood race and second-moment validation for these two pricing regimes. Second, using the estimated models we further the understanding of the transmission of monetary policy for small open economies by comparing the impulse responses of shocks to US and Canadian monetary rules. Third, we examine the implications of these two paradigms for the conduct of monetary policy using welfare-optimized Taylor-type nominal interest rate rules subject to a zero-lower-bound constraint with a particular focus on price level versus nominal income targeting.

1.1 Related Literature

The impact of domestic monetary policy on other countries has been a very active area of research in the last few decades. The main questions that it discusses revolve around the following. Does a monetary contraction in the U.S. lead to recessions or expansions in other countries? Does it improve or worsen financial conditions abroad? Does it lead to capital inflows or outflows? Are spillovers different across advanced and emerging economies, or across countries pegging their exchange rate to the dollar and those retaining monetary autonomy? The existing literature suggests that spillover effects can be sizeable, but that there is considerable heterogeneity across countries in the response of macroeconomic variables, asset prices, and financial flows, with no discernible link between effects and country characteristics (see Dedola *et al.* (2017)).

To address these questions many of the previous studies focused on one or two of these areas. Many studies have focused on investigating the main channels through which shocks are transmitted. Using a VAR model, Kim (2001) found that a decrease in the world interest rate to play a major role in the propagation of the shocks while trade balance to play a much less important role. Regarding the credit channel, Romer and Romer. (1993) and Ramey (1993) found that it plays an insignificant role, while Bernanke and Gertler (1995) in contrast find that there is a direct relationship between monetary policy shocks.

Another area of research under this topic is measuring the spill-over impact on other countries. This is done through the employment of modelling and empirical methods including DSGE and VAR models. The idea in this type of research is to quantify the size and sign of the impact of the international spill-overs coming from domestic policies. A useful general paper that sets the scene is that of Ammer *et al.* (2016) in which it attempts to compute the impact of US monetary policy internationally. Using back-of-the-envelope calculation, it shows that it is important to consider the various channels that come into play. The conclusion is that without considering all these different channels it is hard to be certain on the sign or size of this impact. In their simple example, they show that it is actually possible that an expansionary US monetary policy can lead to a positive international impact.

A number of structural VAR studies have contributed to this literature. An example of this type is Canova (2005) which investigates the impact of US monetary policy on Latin America countries. The paper uses monthly data with VAR and sign restrictions and found that the US monetary policy shocks have a significant impact on the eight Latin American countries that have

been studied. Georgiadis (2016), using quarterly data for 61 countries with a global VAR method, studies the main factors that influence the impact of the US monetary policy internationally. It showed that the impact of monetary policy globally is not the same across countries and that it is influenced by factors such as the exchange rate regime, the degree of openness, trade and financial integration, financial market development, and industry structure. The paper suggested that countries could eventually protect themselves or minimize the impact of US shocks through close trading ties with the US, by improving their domestic financial and labour markets and by adopting a more flexible exchange rate regime.

Another dimension to this literature is the differences in terms of effectiveness domestically as well as in terms of global effects between conventional and unconventional monetary policy. While the main belief is that unconventional monetary policy has a larger propagation effect, a recent paper by Curcuru *et al.* (2018) investigates this assumption. That paper used a method to disentangle the long term return on bond into expected short rate and term premium components to compare the impact of these different components. The results concludes that in contrast to the common belief, conventional monetary policy has a higher impact.

The focus of our paper is on the currency in which prices are sticky. As explained earlier, this currency pricing literature can be traced back to the seminal framework of Mundell-Fleming (Mundell, 1963, Fleming, 1962), studying the interaction between macroeconomic variables within an open economy. In this model, and in the literature that followed, the maintained assumption is that prices are pre-set in the producer currency PCP. A few decades later, other studies came to suggest that the first framework fails to explain the trends that are observed in the data. This gave rise to a new paradigm, local currency pricing (LCP), in which prices are rigid in the destination currency (see Betts and Devereux, 2000 and Devereux and Engel., 2003).

A more recent and influential paper in this literature is Gopinath *et al.* (2020), which draws on Gita Gopinath's and co-authors' work over the past ten years on international trade transactions and firms' choice of currency.¹ This study suggests that most firms set their prices in a few currencies and that they do not change them frequently. Also, it shows that among these few currencies the US dollar remains the most often used currency, possibly for two reasons: first, strategic complementarities in prices and second, intermediate goods prices for imported intermediate inputs. From these empirical facts, Gopinath *et al.* (2020) build a theoretical model to test the impact of this new paradigm on the transmission of shocks. Comparing the predictions of the three pricing paradigms (i.e. PCP, LCP and DCP), Gopinath *et al.* (2020) show that under DCP terms of trade is less sensitive to the changes in the bilateral nominal exchange rate, but more affected by the changes in relation to the US dollar. They also report that a constant increase in value of the dollar may weaken the global trade, as prices of goods will increase globally.

The optimal policy literature is reviewed in Corsetti *et al.* (2010) and Gali (2015). Here a distinction between optimal policy and simple rules should be emphasized. Whereas Corsetti *et al.* (2010) and Gali (2015) focus on the former, our paper conducts policy analysis in terms of the latter. Section 5 elaborates this point. Overall in our estimated model, our results for

¹This include Gopinath and Rigobon. (2008); Gopinath and Neiman (2014), Gopinath (2015); and Boz *et al.* (2017)

policy conduct suggest that the optimal level of inflation target set by the central bank is much lower than the typical target inflation of 2%, and that the inflation target may be too blunt an instrument to efficiently reduce the severe costs of zero-bound episodes. In addition, we find that for the closed economy ROW the optimized a price-level rule closely mimics the optimized general inflation-output rule, while for the SOE the corresponding result requires a nominal income rule.

1.2 Roadmap

The rest of the paper’s structure is organized as follows: Section 2 sets out the model, which is estimated in Section 3 by Bayesian methods. Section 4 brings together empirical results. Section 5 computes optimized simple rules, which impose a zero-lower bound constraint on the nominal interest rate in the form of a delegation game between the government and an instrument-independent central bank. Section 6 of the paper concludes.

2 Model Description

The model economy is a two-block dynamic general equilibrium model. We first set out a general model of two economies of different population sizes and then consider as a limiting case a small open economy interacting with the ROW, but with no policy strategic interdependence.²

In each bloc, domestically produced and imported goods are consumed with prices denominated in the country’s currency with notation summarized in Table 1.

< Table 1 here >

2.1 Households

Households in the H bloc hold both domestic and foreign bonds, but those in the F bloc only hold domestic bonds. In order to accommodate financial frictions in a simple way, households are divided in those who participate in the financial sector and can lend or borrow to each other. These are *Ricardian consumers*. The remaining rule-of-thumb consumers are credit-constrained and must consume out of wage income net of tax.³

In a stochastic environment, household j of both types maximizes

$$V_0(j) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U_t(j) \right] \tag{1}$$

where, following Smets and Wouters (2007), the single-period utility is of the general form proposed by King *et al.* (1988):

$$U_t(j) = \frac{[C_t(j) - \chi C_{t-1}(j)]^{1-\sigma_c}}{1 - \sigma_c} \exp \left[(\sigma_c - 1) \frac{H_t(j)^{1+\psi}}{1 + \psi} \right] \tag{2}$$

²Full details of the first order condition, equilibrium and deterministic steady state about which the perturbation solution is obtained is provided in a Supplementary Appendix.

³See Table 18 for evidence of financial exclusion in advanced as well as in emerging economies.

where $C_t(j)$ is real consumption, $H_t(j)$ is hours supplied, β is the discount factor, χ controls habit formation, σ_c is the inverse of the elasticity of intertemporal substitution (for constant labour), and ψ is the inverse of the Frisch labour supply elasticity. Note that, unlike in the original SW model, we use internal instead of external habit formation.

There are $(1 - \lambda)$ non-credit constrained Ricardian (R) consumers. The household solves

$$\max_{C_t^R, L_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, H_{t+s}^R) \right] \quad (3)$$

subject to a nominal budget constraint given by

$$P_t^B B_{H,t} + P_t^{B^*} S_t B_{F,t}^* = B_{H,t-1} + S_t B_{F,t-1}^* + P_t W_t (1 - \tau_t^w) H_t^R - P_t C_t^R + \Gamma_t \quad (4)$$

with nominal profits given by Γ_t and a proportional labour tax given by τ_t^w . $B_{H,t}$ and $B_{F,t}^*$ are domestic and foreign bonds respectively, bought at nominal prices P_t^B and $P_t^{B^*}$ and denominated in the respective currencies. P_t is the CPI index that includes an imported component (see (88) below) and S_t is the nominal exchange rate.

The first order conditions for the Ricardian households then give a UIP condition modified to allow for risk:

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t = R_t^* \phi \left(\frac{S_t B_{F,t}^*}{P_{H,t} Y_t} \right) \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \Pi_{t,t+1}^S \right] \quad (5)$$

The remaining λ consumers are credit constrained (C) and have no income from monopolistic retail firms. They must consume out of wage income and their consumption is given by

$$C_t^C = W_t (1 - \tau_t^w) H_t^C \quad (6)$$

Liquidity constrained consumers now choose C_t^C and $L_t^C = 1 - H_t^C$, to maximize an analogous welfare function to (73) subject to (6), with analogous equilibrium conditions resulting.⁴ Total labour supply by Ricardian and non-Ricardian households to the formal and informal sectors is then $\lambda H_t^C + (1 - \lambda) H_t^R$.

Consumption Demand for Domestic and Imported Goods

For given aggregate consumption $C_t = C_t^R$, C_t^C for both Ricardian and credit-constrained consumers, household demand for consumption goods from domestic retailers (C_H) and foreign retailers (C_F , i.e. imports) is chosen to maximise the Dixit-Stigitz quantity aggregator

$$C_t = \left[\frac{1}{w_C^{\mu_C}} C_{H,t}^{\mu_C} + (1 - w_C)^{\frac{1}{\mu_C}} C_{F,t}^{\mu_C} \right]^{\frac{\mu_C}{\mu_C - 1}} \quad (7)$$

⁴Households's conditions in the ROW are derived under the same assumptions, see the Supplementary Appendix for details.

The corresponding Dixit-Stigitz price index is given by

$$P_t = [w_C(P_{H,t})^{1-\mu_C} + (1-w_C)(P_{F,t})^{1-\mu_C}]^{\frac{1}{1-\mu_C}} \quad (8)$$

Analogous aggregates apply to the ROW.

Now define CPI, domestic and imported inflation rates over the time interval $[t-1, t]$ by $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$ respectively. Then from (8) we have

$$\Pi_t = \left[w_C \left(\Pi_{H,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-w_C) \left(\Pi_{F,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \quad (9)$$

Parameter μ_C is the elasticity of substitution between home and foreign goods, while parameter w_C is related to the degree of home-bias in preferences and plays a critical role in this paper. In turn, $1-w_C$ is interpreted as an index of openness to international trade in final goods: when $w_C = 1$, the share of foreign goods in the composite consumption index approaches zero. The degree of openness $1-w_C$ is identical across economies and $w_C = 1$ denotes an economy in autarky, i.e. a closed economy. In contrast, if $w_C = 0$, there is no home-bias in consumption. Note also that there is international trade in intermediate goods which enter into production (see Section 2.3 below).

Maximizing total consumption (88) subject to a given aggregate expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ yields

$$C_{H,t} = w_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \quad \text{and} \quad C_{F,t} = (1-w_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \quad (10)$$

In our general model set-up we assume that a fixed proportion of retail firms set prices in home currency (i.e. PCP), and the remaining proportion are local pricers (i.e. LCP) - see Section 2.4.2. For now, however, we assume PCP. Define the real exchange rate as the relative aggregate consumption price $RER_t \equiv \frac{P_t^* S_t}{P_t}$, where S_t is the nominal exchange rate. With PCP, because the home country is small, the law of one price (LOP), i.e. perfect exchange rate pass-through for imports, implies that $P_t^* = P_{F,t}^*$, $S_t P_t^* = P_{F,t}$, so $RER_t = \frac{P_{F,t}}{P_t}$ and *terms of trade* for the home country are defined as $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, i.e. the price of imported goods relative to domestic ones, and

$$\frac{\mathcal{T}_t \text{tot}_t}{\mathcal{T}_{t-1} \text{tot}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} \quad (11)$$

where we introduce a terms of trade shock, tot_t .

2.2 Capital Producers

Capital producers⁵ purchase investment goods from home and foreign retail firms at real price $\frac{P_t^I}{P_t}$ selling at real price Q_t to maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k} (1 - \mathcal{S}(I_{t+k}/I_{t+k-1})) I_{t+k} - \frac{P_t^I}{P_t} I_{t+k} \right]$$

where total capital accumulates according to

$$K_t = (1 - \delta)K_{t-1} + (1 - \mathcal{S}(X_t))I_t I S_t \quad (12)$$

$I S_t$ is an investment shock.

Dixit-Stiglitz aggregators over home and imported investment are:

$$I_t = \left[w_I^{\frac{1}{\mu_I}} I_{H,t}^{\frac{\mu_I-1}{\mu_I}} + (1 - w_I)^{\frac{1}{\mu_I}} I_{F,t}^{\frac{\mu_I-1}{\mu_I}} \right]^{\frac{\mu_I}{\mu_I-1}} \quad (13)$$

$$P_{I,t} = [w_I (P_{H,t})^{1-\mu_I} + (1 - w_I) (P_{F,t})^{1-\mu_I}]^{\frac{1}{1-\mu_I}} \quad (14)$$

and analogous demand for home and imported investment goods to (10) are

$$I_{H,t} = w_I \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_H} I_t \quad \text{and} \quad I_{F,t} = (1 - w_I) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t \quad (15)$$

For the FOCs, we define the gross real return on capital R_t^K as

$$R_t^K = \frac{r_t^K (1 - \tau_t^k) + (1 - \delta)Q_t}{Q_{t-1}} \quad (16)$$

such that the right-hand-side is the gross return to holding a unit of capital from $t-1$ to t , while the left-hand-side is the gross return from holding bonds and the opportunity cost of capital. τ_t^k is a tax on corporate profits assumed exogenous in the model. We further define investment adjustment costs and the rate of change of investment as

$$\mathcal{S}(X_t) \equiv \phi_X (X_t - X)^2 \quad (17)$$

$$X_t \equiv \frac{I_t}{I_{t-1}}; \quad S', S'' \geq 0; \quad S(1) = S'(1) = 0 \quad (18)$$

where ϕ_X is the elasticity of investment adjustment costs.

⁵Investment and capital accumulation decisions can be included in those of the household by allowing it to own the capital and rent to firms without changing the equilibrium. Separating out the decisions, as in our paper, is a useful modelling device for incorporating a banking sector as in Gertler and Kiyotaki (2010). This will be an avenue for future work.

2.3 Traded Intermediate Goods

We now introduce capital and trade in investment goods, and allow for intermediate inputs in production. Both are important channels for the effect of exchange rate changes on the supply side, but the intermediate goods channel is more direct. Moreover, a large proportion of trade is in intermediate goods.

Modelling trade in intermediate inputs, M_t , is analogous to investment goods with

$$M_t \equiv \left[w_M^{\frac{1}{\mu_M}} M_{I,t}^{\frac{\mu_M-1}{\mu_M}} + (1 - w_M)^{\frac{1}{\mu_M}} M_{F,t}^{\frac{\mu_M-1}{\mu_M}} \right]^{\frac{\mu_M}{\mu_M-1}} \quad (19)$$

$$P_t^M \equiv \left[w_M P_{H,t}^{1-\mu_M} + (1 - w_M) P_{F,t}^{1-\mu_M} \right]^{\frac{1}{1-\mu_M}} \quad (20)$$

Maximizing total intermediate input (109) subject to a given aggregate expenditure $P_t M_t = P_{H,t} M_{H,t} + P_{F,t} M_{F,t}$ yields

$$M_{H,t} = w_M \left(\frac{P_{H,t}}{P_t^M} \right)^{-\mu_M} M_t \quad (21)$$

$$M_{F,t} = (1 - w_M) \left(\frac{P_{F,t}}{P_t^M} \right)^{-\mu_M} M_t \quad (22)$$

2.4 Firms

There are wholesale and retail sectors. The former act in perfect competition producing a homogeneous intermediate good, the latter in monopolistic competition producing differentiated final goods.

2.4.1 Wholesale sector

The production technology is given by

$$Y_t^W = F(A_t, H_t^d, K_{t-1}, M_t) = (A_t H_t^d)^{\alpha_H} M_t^{\alpha_M} (K_{t-1})^{1-\alpha_H-\alpha_M} \quad (23)$$

where A_t is labour augmenting productivity. Wholesale firms sell at nominal price P_t^W to retailers, so profit maximisation implies

$$F_{H,t} = \alpha_H \frac{Y_t^W}{H_t^d} \frac{P_t^W}{P_t} \frac{P_{H,t}}{P_t} = W_t \quad (24)$$

$$F_{K,t} = (1 - \alpha_H - \alpha_M) \frac{Y_t^W}{u_t K_{t-1}} \frac{P_t^W}{P_t} \frac{P_{H,t}}{P_t} = r_t^K \quad (25)$$

$$F_{M,t} = \frac{\alpha_M P_t^W Y_t^W}{M_t} = \frac{P_t^M}{P_t^W} \quad (26)$$

where P_t is price index of final consumption goods.

2.4.2 Retail Sector and Incomplete Exchange Rate Pass-through For Exports

Each home retailer $m \in (0, 1)$ purchases output from the intermediate good sector at price $P_{H,t}^W$ and converts into a differentiated good sold at price $P_{H,t}(m)$ to households, capital good producers and governments, who use the technology

$$C_{H,t} = \left(\int_0^1 C_{H,t}(m)^{(\zeta-1)/\zeta} dm \right)^{\zeta/(\zeta-1)} \quad (27)$$

to combine into baskets, where ζ is the elasticity of substitution (similarly for $I_{H,t}$ and G_t).

Maximising (27) subject to $P_t C_t = \int_0^1 P_t(m) C_t(m) dm$ implies a set of demand equations for each intermediate good m with price $P_t(m)$ of the form

$$C_{H,t}(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta_p} C_{H,t} \quad (28)$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta_p} dm \right]^{\frac{1}{1-\zeta_p}}$. P_t is the aggregate price index of home produced goods. There are equivalent demand schedules for investment goods, government consumption and for foreign demand. Summing the demand schedules from each buyer implies a total demand for home produced good m given by

$$Y_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta_p} Y_t \quad (29)$$

Every period, each firm faces a fixed probability $1 - \xi_p$ that they will be able to update their prices.⁶

We must now distinguish the price setting in domestic and foreign markets. We assume that the F bloc is large and firms set producer currency prices (see Section 2.5), while in the H bloc the prices of goods sold domestically, $C_{H,t} + I_{H,t} + M_{H,t} + G_t$, are set in domestic currency, but those exported, $C_{H,t}^* + I_{H,t}^* + M_{H,t}^*$, are invoiced in either the home or foreign currency. These are the Producer and Local Currency Pricing cases.

Consider first the PCP case. Denoting the optimal price at time t for good m as $P_t^O(m)$, the firms allowed to re-optimize prices maximise expected discounted profits by solving

$$\max_{P_t^O(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) [P_t^O(m) - P_{t+k}^W MS_{t+k}] \quad (30)$$

with MS_t as a markup shock in each sector and real marginal cost is given by $MC_t \equiv \frac{P_t^W}{P_t}$.

Turning to the LCP case, denoting the optimal price at time t for exported good m as $P_{H,t}^{*O}(m)$ in F currency, the firms allowed to re-optimize prices maximise real (consumption price) expected

⁶We also allow for prices to be indexed to last period's aggregate inflation, with a price indexation parameter $\gamma_i \in [0, 1]$.

discounted profits by solving

$$\max_{P_t^O(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (C_{t+k}^*(m) + I_{t+k}^*(m) + M_{t+k}^*(m)) [S_{t+k} P_{H,t}^{*O}(m) - P_{t+k}^W] \quad (31)$$

subject to the demand schedule which now becomes

$$C_{t+k}^*(m) + I_{t+k}^*(m) + M_{t+k}^*(m) = \left(\frac{P_{H,t}^{*O}(m)}{P_{H,t+k}^*} \right)^{-\zeta} (C_{t+k}^* + I_{t+k}^* + M_{t+k}^*) \quad (32)$$

Now note that the real (own exported good price) marginal cost for each retailer is given by

$$MC_{H,t}^* \equiv \frac{P_t^W}{S_t P_{H,t}^*} = \frac{MC_t \frac{P_{H,t}}{P_t}}{\frac{S_t P_{H,t}^*}{P_t}} \quad (33)$$

Foreign exporters from the large ROW bloc are PCPers, so we have

$$P_{F,t} = S_t P_{F,t}^* \quad (34)$$

As before, define the terms of trade for the home bloc (import/export prices in one currency) as $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$. Define the terms of trade for the foreign bloc as $\mathcal{T}_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*}$. With PCPers, only the law of one price holds and $\mathcal{T}_t^* = \frac{S_t P_{H,t}^*}{S_t P_{F,t}^*} = \frac{P_{H,t}}{P_{F,t}} = \frac{1}{\mathcal{T}_t}$, but with LCPers this no longer is the case. Now we have that

$$\mathcal{T}_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{P_{H,t}^*/P_{H,t}}{P_{F,t}^*/P_{H,t}} = \frac{1}{\mathcal{T}_t} \frac{P_{H,t}^* S_t}{P_{H,t}} = \frac{1}{\mathcal{T}_t} \frac{S_t P_{H,t}^*}{P_t} \quad (35)$$

Hence, we can write $\frac{S_t P_{H,t}^*}{P_t} = \frac{S_t P_{H,t}^*}{P_{H,t} P_t / P_{H,t}} = \frac{\mathcal{T}_t \mathcal{T}_t^*}{P_t / P_{H,t}}$ or alternatively from 149 we have $\mathcal{T}_t^* \mathcal{T}_t = \frac{MC_t}{MC_{H,t}^*}$, which completes the set-up. Table 2 summarizes the notation used.

< Table 2 here >

Using this, aggregate final output is divided between exports $EX_t = C_{H,t}^* + I_{H,t}^*$ and domestic consumption $Y_t - EX_t = C_{H,t} + I_{H,t} + G_t$. Then, allowing for dispersion we have

$$Y_t = \left(\frac{\frac{EX_t}{Y_t}}{\Delta_{H,t}^*} + \frac{\left(1 - \frac{EX_t}{Y_t}\right)}{\Delta_{H,t}} \right) Y_t^W \quad (36)$$

2.5 Bloc Size Effects and the SOE

In our representative agent model, all variables such as Y_t and Y_t^* are *per capita* quantities and can differ, for example, because labour productivity in the steady state is $A \neq A^*$. The implication up to now is that population sizes are the same in both blocs. We now let the F bloc have a population n times that of the H bloc. In the limit, as $n \rightarrow \infty$ we get to the SOE-ROW model .

$$\begin{aligned}
Y_t &= C_{H,t} + n C_{H,t}^* + I_{H,t} + n I_{H,t}^* + M_{H,t} + n M_{H,t}^* + G_t \\
&= C_{H,t} + I_{H,t} + M_{H,t} + G_t + EX_t
\end{aligned} \tag{37}$$

$$\begin{aligned}
n Y_t^* &= n C_{F,t}^* + C_{F,t} + n I_{F,t}^* + I_{F,t} + n M_{F,t}^* + M_{F,t} + n G_t^* \\
&= n (C_{F,t}^* + I_{F,t}^* + M_{F,t}^* + G_t^* + EX_t^*)
\end{aligned} \tag{38}$$

where *per capita* exports by the Home and Foreign Country are respectively given by

$$\begin{aligned}
EX_t &\equiv n C_{H,t}^* + n I_{H,t}^* + n M_{H,t}^* \\
&= n(1 - w_C^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* + n(1 - w_I^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu_I^*} I_t^* + n(1 - w_M^*) \left(\frac{P_{H,t}^*}{P_t^{M^*}} \right)^{-\mu_M^*} M_t^*
\end{aligned} \tag{39}$$

$$\begin{aligned}
EX_t^* &\equiv C_{F,t}/n + I_{F,t}/n + M_{F,t}/n \\
&= (1 - w_C)/n \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t + (1 - w_I)/n \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_I} I_t + (1 - w_M)/n \left(\frac{P_{F,t}}{P_t^M} \right)^{-\mu_M} M_t
\end{aligned} \tag{40}$$

Nominal trade balance in the Home and Foreign blocs are now respectively

$$\begin{aligned}
P_t TB_t &= P_{H,t} Y_t - P_t C_t - P_t^I I_t - P_t^M M_t - P_{H,t} G_t \\
&= P_{H,t} Y_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} - P_{H,t} I_{H,t} \\
&\quad - P_{F,t} I_{F,t} - P_{H,t} M_{H,t} - P_{F,t} M_{F,t} - P_{H,t} G_t
\end{aligned} \tag{41}$$

$$\begin{aligned}
P_t^* TB_t^* &= P_{F,t}^* Y_t^* - P_t^* C_t^* - P_t^{I^*} I_t^* - P_t^{M^*} M_t^* - P_{F,t}^* G_t^* \\
&= P_{F,t}^* Y_t^* - P_{F,t}^* C_{F,t}^* - P_{H,t}^* C_{H,t}^* - P_{F,t}^* I_{F,t}^* - P_{H,t}^* I_{H,t}^* \\
&\quad - P_{F,t}^* M_{F,t}^* - P_{H,t}^* M_{H,t}^* - P_{F,t}^* G_t^*
\end{aligned} \tag{42}$$

Then, combining (37)–(38) and (41)–(42) we have

$$\begin{aligned}
P_t TB_t &= P_{H,t} n (C_{H,t}^* + I_{H,t}^* + M_{H,t}^*) - P_{F,t} (C_{F,t} + I_{F,t} + M_{F,t}) \\
&= P_{H,t} EX_t - P_{F,t} n EX_t^*
\end{aligned} \tag{43}$$

$$\begin{aligned}
P_t^* TB_t^* &= P_{F,t}^* (C_{F,t} + I_{F,t} + M_{F,t})/n - P_{H,t}^* (C_{H,t}^* + I_{H,t}^* + M_{H,t}^*) \\
&= P_{F,t}^* EX_t^* - P_{H,t}^* EX_t/n
\end{aligned} \tag{44}$$

denominated in units of H and F currency respectively for the two blocs.

For any $1 \leq n < \infty$ we can set up the model with output and trade equilibria given by (37) –

(42) with TB_t given by

$$\begin{aligned}
TB_t &= \frac{P_{H,t}}{P_t} EX_t - \frac{P_{F,t}}{P_t} n EX_t^* \\
&= \frac{P_{H,t}}{P_t} n (1 - w_C^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* + \frac{P_{H,t}}{P_t} n (1 - w_I^*) \left(\frac{P_{H,t}^*}{P_t^{I^*}} \right)^{-\mu_I^*} I_t^* \\
&+ \frac{P_{H,t}}{P_t} n (1 - w_M^*) \left(\frac{P_{H,t}^*}{P_t^{M^*}} \right)^{-\mu_M^*} M_t^* \\
&- \frac{P_{F,t}}{P_t} (1 - w_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t - \frac{P_{F,t}}{P_t} (1 - w_I) \left(\frac{P_{F,t}}{P_t^I} \right)^{-\mu_I} I_t \\
&- \frac{P_{F,t}}{P_t} (1 - w_M) \left(\frac{P_{F,t}}{P_t^M} \right)^{-\mu_M} M_t
\end{aligned} \tag{45}$$

Then, using trade data, we can *calibrate* w_C , w_I , w_M , w_C^* , w_I^* and w_M^* in the steady state as follows: in the steady state, without loss of generality, by a choice of output units we can put all prices equal to unity. We can thus write (45) in terms of observable non-dimensional quantities as

$$\begin{aligned}
\frac{TB}{Y} &\equiv \text{tb} \\
&= \left(n(1 - w_C^*) \frac{C^*}{Y^*} + n(1 - w_I^*) \frac{I^*}{Y^*} + n(1 - w_M^*) \frac{M^*}{Y^*} \right) \frac{Y^*}{Y} \\
&- (1 - w_C) \frac{C}{Y} - (1 - w_I) \frac{I}{Y} - (1 - w_M) \frac{M}{Y} \\
&\equiv \text{excs} + \text{exis} + \text{exims} - \text{imcs} - \text{imis} - \text{imims}
\end{aligned} \tag{46}$$

The first term on the rhs of (46) is the share consumption goods exports, *excs* say. Given trade data for *excs* and letting n be the relative population size, we can then calibrate w_C^* to hit *excs*. Similarly, we can calibrate w_I^* and w_M^* to the fourth, fifth and sixth terms on the rhs of (46) are the shares of consumption, investment and intermediate goods imports. Let these be *imcs*, *imis* and *imims*, respectively, and they can be used to calibrate w_C , w_I and w_M in a similar fashion. Letting $\text{tb} = \frac{TB}{Y}$ we must have from (46) that

$$\text{tb} = \text{excs} + \text{exis} + \text{exims} - \text{imcs} - \text{imis} - \text{imims} \tag{47}$$

Thus, our trade data and trade balance data must be chosen to satisfy (47).

2.6 Closed ROW-SOE Special Case

Now consider the ROW-SOE case as $n \rightarrow \infty$ and $w_C^* \rightarrow 1$, $w_I^* \rightarrow 1$ and $w_M^* \rightarrow 1$. Then, for the F-bloc:

$$EX_t^* = \frac{C_{F,t}}{n} + \frac{I_{F,t}}{n} + \frac{M_{F,t}}{n} \rightarrow 0 \tag{48}$$

$$TB_t^* = P_{F,t}^* EX_t^* - P_{H,t}^* EX_t/n \rightarrow 0 \text{ as } n \rightarrow \infty \tag{49}$$

Hence, the ROW becomes a *closed economy* bloc, but $TB_t \neq 0$. Then, in (46) in the home country $n(1 - w_C^*)$ is replaced with $\frac{exc_s}{C^*/Y}$ and $n(1 - w_I^*)$ replaced with $\frac{exis}{M^*/Y}$. Therefore, exports (40) is equal to:

$$EX_t = \frac{targexc_s}{C^*/Y} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{\mu_C^*} C_t^* + \frac{targexis}{I^*/Y} \left(\frac{P_{H,t}^*}{P_{I,t}^*} \right)^{\mu_I^*} I_t^* + \frac{targexim}{M^*/Y} \left(\frac{P_{H,t}^*}{P_{M,t}^*} \right)^{\mu_M^*} M_t^* \quad (50)$$

2.7 The Commodity Sector and Trade Balance

We introduce a commodity sector (e.g. oil) which is an exogenous process, subject to a shock $\epsilon_{P_O^*,t}$

$$\log \frac{P_{O,t}^*}{P_t^*} - \log \frac{P_O^*}{P^*} = \rho_{P_O^*} \left(\log \frac{P_{O,t-1}^*}{P_{t-1}^*} - \log \frac{P_O^*}{P^*} \right) + \epsilon_{P_O^*,t} \quad (51)$$

The commodity is entirely exported and the only channel through which oil production and price affects the model is via the trade balance and the government budget constraint.⁷ A tax rate τ_t^o applies to this sector.

The nominal trade balance

$$P_t TB_t = S_t P_{O,t}^* Y_t^O + P_{H,t} Y_t - P_t C_t - P_{I,t} I_t - P_{M,t} M_t - P_{H,t} G_t \quad (52)$$

is the difference between output, commodity revenue, private and public consumption, and investment.

2.8 Financial Intermediation

Efficient financial intermediation within the Home country implies the zero arbitrage condition:

$$\mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^K \right] = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t = 1 \quad (53)$$

which we take as the equilibrium equation for Q_t .

2.9 Central Bank, Foreign Assets and Monetary Policy

The nominal interest rate R_t is a policy variable, typically given in the literature by a standard Taylor-type rule⁸ that includes an exchange rate depreciation term:

$$\begin{aligned} \log \left(\frac{R_t}{R} \right) &= \rho_r \log \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_r) \left[\theta_\pi \log \left(\frac{\Pi_{t-1,t}}{\Pi} \right) + \theta_s \log \left(\frac{\Pi_{S,t-1,t}}{\Pi_S} \right) \right. \\ &\quad \left. + \theta_y \log \left(\frac{Y_t}{Y} \right) + \theta_{dy} \log \left(\frac{Y_t}{Y_{t-1}} \right) \right] + \epsilon_{M,t} \end{aligned} \quad (54)$$

⁷Given the empirical case we study below, we focus on an oil sector, but this setup can easily be applied to capture commodity dependence on energy, minerals or agricultural exports.

⁸In a closed-economy NK model with credit-constrained consumers, Bilbiie (2008) shows that an inversion of the Taylor principle occurs with a sufficient high proportion of such households.

Foreign bond holdings evolves according to home country nominal terms

$$P_t^{B^*} S_t B_{F,t}^* = S_t B_{F,t-1}^* + P_t T B_t \quad (55)$$

Now define $B_{F,t} \equiv \frac{S_t B_{F,t}^*}{P_t}$ to be the stock of foreign bonds in home country consumption units. Then,

$$P_t^{B^*} B_{F,t} = \frac{\Pi_{t-1,t}^S}{\Pi_{t-1,t}} B_{F,t-1} + T B_t \quad (56)$$

Finally, a government nominal balanced budget constraint gives

$$P_{H,t} G_t = P_t W_t H_t \tau_t^w + (1 - \alpha) Y_t^W P_{H,t} M C_t \tau_t^k + R E R_t P_t^{*O} Y^O \tau^O \quad (57)$$

recalling that $M C_t \equiv \frac{P_t^W}{P_{H,t}}$.

$$\tau_t^w = \frac{P_{H,t} G_t - (1 - \alpha) Y_t^W M C_t \tau_t^k - R E R_t P_t^{*O} Y^O \tau^O}{W_t H_t} \quad (58)$$

and τ_t^o is a tax on oil revenue.

We assume that tax rates $(\tau_t^k, \tau_t^k, \tau_t^o)$ are held fixed at the steady state value of τ^K . The fiscal stabilization instrument G_t follows a Taylor-type rule

$$\log G_t - \log G = \rho_G (\log G_{t-1} - \log G) + \epsilon_{G,t} \quad (59)$$

where $\epsilon_{G,t}$ is a fiscal policy shock process.

Monetary Policy in ROW Block

The ROW nominal interest rate is given by the following Taylor-type rule

$$\log \left(\frac{R_t^*}{R^*} \right) = \rho_r \log \left(\frac{R_{t-1}^*}{R^*} \right) + (1 - \rho_r^*) \left[\theta_{\pi^*} \log \left(\frac{\Pi_{t-1,t}^*}{\Pi^*} \right) + \theta_{y^*} \log \left(\frac{Y_t^*}{Y^*} \right) + \theta_{dy^*} \log \left(\frac{Y_{*t}^*}{Y_{*t-1}^*} \right) \right] + \epsilon_{M^*,t} \quad (60)$$

This completes the specification of the two-bloc open-economy model.

3 Estimation

3.1 Data

To estimate the model, we use quarterly information on seven key variables for Canada: GDP, consumption, investment, consumer price index (CPI), nominal exchange rate, nominal interest rate (including Wu-Xia-type shadow rates computed in MacDonald and Popiel, 2017), oil prices and five US variables: output, consumption, investment, consumer price index (CPI) and the Wu

and Xia (2016) nominal shadow Federal Funds rates for the sample period from 1993Q2 to 2018Q4. Quarterly crude oil prices were obtained from FRED Economic Data and is deflated with US CPI, all other quarterly data are from IMF’s International Financial Statistics. Real variables are in log-differences and seasonally adjusted. In addition, we calibrate share of the trade targets using data from World Integrated Trade Solution (WITS)⁹.

3.2 Calibrated Parameters

We use a combination of estimated and calibrated parameters, with Tables 3 and 17 summarizing the calibration of parameters and the steady state values of selected endogenous variables, matching, as accurately as possible, the empirical evidence and available (quarterly) data on key statistics of these economies.

< Table 3 here >

As in much of the literature, the depreciation rate of capital δ is set at 10 per cent per annum, implying a quarterly value of 0.025. The home discount rate is set at $\beta = 0.99$, while we assume the value of 2.00 for Ricardian risk aversion. The substitution elasticity between imported and home goods (μ_C) is calibrated at 1.50 and following Medina *et al.* (2005), Chang *et al.* (2015) and Adler *et al.* (2016), the export elasticities μ_C^* and μ_I^* are set to 1.50. In terms of the elasticity of substitution among different retail varieties, we adopt a mean of 7 for ζ .

Using IMF and World Bank data on broad aggregates, we calibrate the government share of production (g_y) at 21%. On the other hand, the trade weights, foreign productivity and foreign discount factor are calibrated to hit the trade targets explained in section 2. Trade shares (consumer, investment and intermediate goods in exports) are calibrated using World Bank and World Integrated Trade Solution (WITS) data, suitably normalised to account for the fact that GDP does not include intermediate inputs by construction. Regarding the oil targets, we calibrate share of oil production in GDP by taking the value of average oil production divided by GDP, resulting in a value of 0.0568 for Canada. For the oil taxation rate, we use the value of 15%. Turning to the calibration of the proportion of constrained consumers, we use a prior mean of 0.2 for US and Canada.¹⁰

3.3 Bayesian Estimation

We estimate the model by Bayesian methods, which entails retrieving the posterior distribution of the model’s parameters, say Θ , conditional on the data. Using the Bayes’ theorem, the posterior distribution is obtained as

$$p(\Theta|Y^T) = \frac{L(Y^T|\Theta)p(\Theta)}{\int L(Y^T|\Theta)p(\Theta)d\Theta}$$

where $p(\Theta)$ denotes the prior density of the parameter vector Θ , $L(Y^T|\Theta)$ is the likelihood of the sample Y^T with T observations (evaluated with the Kalman filter) and $\int L(Y^T|\Theta)p(\Theta)d\Theta$ is the

⁹<https://wits.worldbank.org>

¹⁰See Supplementary Appendix for a more detailed review of the literature.

marginal likelihood. Since there is no closed form analytical expression for the posterior, this must be simulated.¹¹

In our two-bloc setup, the ROW bloc does not depend on interactions with the SOE, so it can be estimated separately, which is carried out using US data.¹² The price of oil ($P_{O,t}^*$) is an exogenous process in the model, so we have estimated the standard deviation of this shock separately by fitting an AR(1) process and then use the estimated coefficient and standard deviation in the SOE model.

3.4 Prior distributions

In order to implement Bayesian estimation, prior distributions must be defined for the parameters and the structural shocks. This choice is usually guided by inherent theoretical restrictions and evidence from previous studies. We use normal distributions as priors for unbounded parameters when more informative priors seem to be necessary, while beta distributions are used for all parameters bounded between 0 and 1, i.e., fractions or probabilities. We use inverse gamma distributions as priors when non-negativity constraints are necessary. All priors are assumed to be the same across specifications.

Tables 4 and 5 list the prior distribution along with the prior mean and standard deviation of all the estimated parameters. We assume a normal distribution centred at 2 and a standard deviation of 0.25 for the risk aversion parameter of non-Ricardian households (σ_C), in line with the literature on (Ricardian) risk aversion. Moreover, following Gabriel *et al.* (2010), the prior for the proportion λ of credit-constrained consumers is a beta distribution with mean of 0.5 and standard deviation of 0.1. To ensure that the consumption habit persistence, χ , is bounded between 0 and 1, we assign a beta distribution (mean of 0.5, standard deviation of 0.1). The prior for price indexation parameters are beta distributions with mean of 0.5 and standard deviation of 0.1, while we assume a normal distribution of mean 4 and standard deviation of 1.5 for investment adjustment costs (ϕ_I).

Regarding the monetary policy rule, we assign a normal prior with a mean of 2.00 and standard deviation of 0.25 to θ_{Π} , while for the feedback parameter on GDP (θ_y), GDP growth (θ_{dy}) and depreciation rate (θ_{ds}), we assume normal distribution of mean 0.10 and standard deviation of 0.05. In turn, a beta distribution with mean of 0.75 and standard deviation of 0.1 is assigned to ρ_M .

As for the shock processes, we use a beta distribution for the persistence of all shocks with a mean of 0.5 and a standard deviation of 0.10. Given the uncertainty regarding the sources of business cycle fluctuations, we adopt uninformative gamma distributions for the standard deviations of all shocks, with a prior mean of 0.10 along with a standard deviation of 0.20. For input shares, we follow Gopinath *et al.* (2020) in using 0.67 and 0.33 for the means of intermediate and labour shares, respectively.

¹¹In a first step, the posterior mode and corresponding Hessian are obtained and then the posterior density is approximated by using the Monte-Carlo Markov Chain Metropolis-Hastings (MCMC-MH) algorithm, with two parallel chains with 250,000 draws, sufficient to ensure convergence according to the the indicators recommended by Brooks and Gelman (1998).

¹²Full results are available in the Supplementary Appendix.

4 Empirical Results

4.1 Posterior Estimates, Model Comparisons and Fit

Tables 4 and 5 display the posterior means of the Bayesian estimation along with 90% high-posterior density intervals. Parameter estimates are generally plausible, with the posterior estimates more sharply peaked than the prior distribution for most parameters, implying that the data is reasonably informative, away from the priors.¹³ The volatility of shocks is high, which is consistent with the literature on open economies. The standard deviations of investment, terms of trade and markup shocks are large relative to other shocks, with the government spending shock having a lesser role in driving fluctuations in Canada. Note the higher estimated volatility of the terms of trade shock in the PCP model compared to the LCP version, suggesting that without the appropriate modelling of the imperfect exchange rate pass-through setting, mis-specifications in the rest of the models are magnified, as important channels in the economy are otherwise shut down.

< Tables 4 and 5 here >

Turning to Calvo price stickiness, there is a significant difference between the two price regimes, with PCP estimated to be more flexible than LCP, a result that highlights the importance of the pricing mechanisms for policy considerations. Regarding the policy parameters, we observe a high degree of policy inertia ($\rho_M = 0.93$) and that the central bank responds strongly to inflationary pressures, with θ_Π larger than 2, but only modestly to output fluctuations, with a slightly stronger feedback on exchange rate movements.

To further evaluate the absolute performance of our model against data, we compare the model's implied characteristics, evaluated at the posterior means, with those of the actual data. Table 6 displays a second moments comparison between the data and the two models. These are able to reproduce the volatility of the main variables under study, with a slight tendency for overshooting it. Table 6 also reports the auto-correlation up to order one and cross-correlations of the six observable variables vis-a-vis output. Both models perform rather poorly at capturing the auto-correlation and contemporaneous correlations observed in the data, which is a well-known result for estimated SOE models, with mixed results regarding LCP vs PCP for second moments fit.

< Table 6 here >

Furthermore, Tables 4 and 5 also provide a formal Bayesian likelihood race comparison, suggesting that setting the imperfect exchange rate pass-through via LCP is very relevant and improves model fit remarkably.

4.2 Variance Decomposition Analysis

What are the driving forces of the observed business cycle fluctuations? What are the impacts of the structural shocks on the main macroeconomic time series? To address these questions, we

¹³See Supplementary Appendix; the procedures of Ratto and Iskrev (2011) provide more formal checks and indicate that all parameters are reasonably well identified.

investigate the contribution of each of the structural shocks to the variance of endogenous variables in the model, i.e. the underlying sources of business cycle dynamics.

The most significant observation in both models is the considerable role of foreign shocks, specifically markup and investment shocks, in explaining the dynamics of the model - slightly magnified in the LCP setting. The second noticeable result is the dominance of the terms of trade shock, followed by the price markup, then investment shocks, with a relatively smaller contribution from other domestic shocks.

The variation in total output is mainly explained by both domestic and foreign price markup shock, while for consumption the terms of trade shock has the dominant role. In the PCP setting, exchange rate movements, as well as exports and imports, are mainly driven by terms of trade shocks, while the latter are more prominent in explaining changes in inflation and imported intermediate inputs in the case of LCP.

< Table 7 here >

< Table 8 here >

4.3 Impulse Response to a Monetary Policy Shock

We now contrast the responses to a monetary policy shock under the different pricing regimes under study for the SOE. As the IRFs reveal, there are different implications for the degree of exchange rate pass-through, terms-of-trade and the volume of exports and imports under the different currency pricing regimes. Figure 1 plots the impulse response to a 0.25 basis point exogenous cut in domestic interest rates. Following the monetary shock, domestic interest rate declines but less than one-to-one in the PCP case, as the exchange rate depreciates by 0.8%, raising inflationary pressures on the economy. This in turn dampens the fall in nominal interest rates via the monetary rule. However, results for the LCP case indicate that in this pricing regime the monetary rule reacts more aggressively to inflation, which leads to a larger decline in the domestic interest rate. This is due to the fact that exchange rate movements have a smaller impact on domestic prices when export prices are sticky in foreign currency and, therefore, the increase in inflation under PCP exceeds that of LCP.

On the other hand, the home currency price of imported goods increases with the exchange rate depreciation. As we can observe, terms of trade rise much more with LCP rather than PCP, indicating a much higher exchange rate pass-through under LCP. Moreover, under LCP we observe a decline in exported goods, not only because of pricing exports in foreign currency, which mutes export demand, but also due to the rise in the price of imported intermediate goods, thus making exports more expensive. This contrasts with the PCP case, which generates an increase in exports (due to the currency depreciation) in spite of the increase in the price of intermediate inputs. The decline in imports of consumption and investment goods is lower for PCP than LCP, again confirming the higher degree of exchange rate pass-through in a LCP setting. However, we can observe that imports of intermediate inputs increase in both cases due to the use of intermediate inputs in production. Because LCP misses out the export expansion, the increase of imported intermediate inputs is smaller than in the case of PCP. Overall, the dominant share of intermediate

inputs in the variation of aggregate imports leads to the increase of total imports in the event of a domestic monetary policy easing.

Regarding output, the expansionary impact is dampened under LCP relative to PCP - with LCP there is an expenditure switching effect from imports towards domestic output, while missing the expansionary impact on exports under PCP. As export prices depend on imported input prices, which are sticky in the local currency, it reinforces the lack of an export expansion effect. Comparing the inflation-output IRFs in Figure 1, the trade-off worsens under LCP, as output does not expand much, but inflation increases the least with PCP: for LCP, inflation rises by 0.55% on impact and output by 0.50%, a ratio of 1.1, while with PCP this ratio is reduced to $0.7/0.8=0.87$.

Finally, we note that the strengthening of the local currency is associated with a decline in overall trade, in contrast to the case of PCP. In the case of LCP, trade declines by 0.1% as the increase of import prices reduces the export quantity. However, in the case of PCP trade expands by 0.5% because of the induced demand for imported inputs arising from the export expansion.

< Figure 1 here >

4.4 Spillover effects of the foreign monetary policy on the SOE

Figure 2 plots the impulse responses to a 0.25 basis point exogenous cut in foreign interest rates. Like in Figure 1, in each plot we contrast the response under the LCP and PCP regimes. Indeed, we observe that the spillover effects are quite significant under LCP, highlighting the importance of currency fluctuations for a small open economy. The lessening of the foreign interest rate and the appreciation of local currency reduces import prices, leading to an increase in imports and a fall in domestic demand and aggregate output. In response to the ROW expansionary policy, the SOE central bank aggressively cuts the domestic interest rate.

< Figure 2 here >

Moreover, we note that output in LCP falls less than under PCP and this is due to the export reduction in the PCP setting. As before, exchange rate pass-through into import prices and real exchange rate is higher in LCP case. However, the exchange rate pass-through into domestic prices is higher under PCP due to domestic export pricing. Imported inputs decrease with PCP, but grow with LCP because of export expansion. Also, imports are more responsive under LCP than PCP. Overall, local currency depreciation in the LCP case leads to an increase in global trade.

5 Optimized Simple Rules with a ZLB

As observed in 1.1, policy analysis can be conducted in terms of optimal policy or simple rules. An acknowledged problem with the former is that such rules make unrealistic observability assumptions regarding macroeconomic variables such as the output gap, the natural real rate of interest and even shock processes. Simple rules, by contrast, make the policy instrument a function of observable variables only. These are then both easy to implement and monitor by the public. Since simple rule

are forms of commitment (and time-inconsistent), the latter feature is important for credibility; or, in other words, establishing a reputation for such commitment.

In this paper, we adopt a mandate framework for implementing simple nominal interest rate rules that consists of four components: (i) a welfare objective delegated to the central bank in the form of the household inter-temporal utility; (ii) a form of Taylor-type instrument rule that responds to specified observable macroeconomic variables, including for the SOE the depreciation rate, to capture ‘exchange rate targeting’; (iii) a specified low probability of hitting the zero-bound constraint on the nominal interest rate and (iv) a long-run (steady-state) inflation rate. With these four features the mandate, makes the central bank goal-dependent, but instrument-independent in the sense that it remains free to choose the strength of its response to the targets in the rule.

There are two central banks given mandates of this form; that in the rest ROW and that in the SOE in question. We consider these in turn.

5.1 The Optimized Rule in the ROW

Recall the nominal interest rate rule for the ROW in ‘implementable form’, which has been estimated as

$$\log\left(\frac{R_{n,t}^*}{R_n^*}\right) = \rho_r^* \log\left(\frac{R_{n,t-1}^*}{R_n^*}\right) + (1 - \rho_r^*) \left(\theta_\pi^* \log\left(\frac{\Pi_t^*}{\Pi^*}\right) + \theta_y^* \log\left(\frac{Y_t^*}{Y^*}\right) + \theta_{dy}^* \log\left(\frac{Y_t^*}{Y_{t-1}^*}\right) \right) + \epsilon_{M,t}^* \quad (61)$$

For optimal policy purposes, we remove the policy shock $\epsilon_{M,t}^*$ and re-parameterize this rule as

$$\log\left(\frac{R_{n,t}^*}{R_n^*}\right) = \rho_r^* \log\left(\frac{R_{n,t-1}^*}{R_n^*}\right) + \alpha_\pi^* \log\left(\frac{\Pi_t^*}{\Pi^*}\right) + \alpha_y^* \log\left(\frac{Y_t^*}{Y^*}\right) + \alpha_{dy}^* \log\left(\frac{Y_t^*}{Y_{t-1}^*}\right) \quad (62)$$

which allows for the possibility of an integral rule with $\rho_r^* = 1$.

Two forms of rules found in the literature are special cases of (62). First, put $\alpha_{dy}^* = \alpha_y^* = 0$ to get:

$$\log\left(\frac{R_{n,t}^*}{R_n^*}\right) = \log\left(\frac{R_{n,t-1}^*}{R_n^*}\right) + \alpha_\pi^* \log\left(\frac{\Pi_t^*}{\Pi^*}\right) \quad (63)$$

which integrating gives

$$\log\left(\frac{R_{n,t}^*}{R_n^*}\right) = \alpha_\pi^* \log\left(\frac{P_t^*}{\bar{P}_t^*}\right) \quad (64)$$

which is a *price level rule* with the trend price-level given by $\frac{\bar{P}_t^*}{\bar{P}_{t-1}^*} = \Pi$.

The benefits of price-level targeting versus inflation targeting have been studied in the literature for some time (see, *inter alia*, Svensson, 1999, Schmitt-Grohe and Uribe, 2000, Vestin, 2006, Gaspar *et al.*, 2010, Giannoni, 2014, Deak *et al.*, 2019). These papers examine the good determinacy/stability and robustness properties of price-level targeting. Holden (2016) shows these benefits extend to a ZLB setting. Our paper shows that these results for a closed economy carry over to

an open-economy setting. The intuition for the benefits of price-level targeting is as follows: faced with an unexpected temporary rise in inflation, price-level stabilization commits the policymaker to bring inflation below the target in subsequent periods. In contrast, with inflation targeting, the drift in the price level is accepted.

Alternatively, put $\alpha_{dy}^* = \alpha_\pi^*$ and $\alpha_y^* = 0$. Integrating as before, we then arrive at

$$\log\left(\frac{R_{n,t}^*}{R_n^*}\right) = \alpha_\pi^* \left(\log\left(\frac{P_t^*}{\bar{P}_t^*}\right) + \log\left(\frac{Y_t^*}{Y^*}\right) \right) = \alpha_\pi^* \log\left(\frac{P_t^* Y_t^*}{\bar{P}_t^* Y^*}\right) \quad (65)$$

which is an *nominal income rule* with a zero trend growth rate.

Let $\rho \equiv [\rho_r^*, \alpha_\pi^*, \alpha_y^*, \alpha_{dy}^*]$ be the policy choice of feedback parameters that defines the form of the rule. The equilibrium is solved by backward induction in the following three-stage *delegation game*.

1. **Stage 1:** The policymaker chooses a per-period probability of hitting the ZLB and designs the optimal loss function in the mandate.
2. **Stage 2:** The optimal steady state inflation rate consistent with stage 1 is chosen.
3. **Stage 3:** The CB receives the mandate in the form of a welfare criterion and rule of the form (62). Welfare is then optimized with respect to ρ resulting in an optimized rule.

This delegation game is then solved by backwards induction as follows:

Stage 3: The CB Mandate

Given a steady state inflation rate target, Π^* , the ROW Central Bank (CB) receives a mandate to implement the rule (62) and to maximize with respect to ρ a modified welfare criterion in recursive form:

$$\begin{aligned} (\Omega_t^*)^{mod} &\equiv \mathbb{E}_t \left[(1 - \beta^*) \sum_{\tau=0}^{\infty} (\beta^*)^\tau \left(U_{t+\tau}^* - 100w_r^* (R_{n,t+\tau}^* - R_n^*)^2 \right) \right] \\ &= (1 - \beta^*) \left(U_t^* - 100w_r^* (R_{n,t}^* - R_n^*)^2 \right) + \beta \mathbb{E}_t \left[(\Omega_{t+1}^*)^{mod} \right] \end{aligned} \quad (66)$$

This results in a probability of hitting the ZLB:¹⁴

$$p^* = p(\Pi^*, \rho^*(\Pi^*, w_r^*)) \quad (67)$$

where $\rho^*(\Pi^*, w_r^*)$ is now the *optimized form* of the rule given the steady state target Π^* and the weight on the interest rate volatility, w_r^* .

Stage 2: Choice of Π^*

Given a target low probability \bar{p}^* and given w_r^* , Π^* is now chosen so satisfy

$$p^*(R_{n,t}^* \leq 1) \equiv p^*(\Pi^*, \rho^*(\Pi^*, w_r^*)) \leq \bar{p}^* \quad (68)$$

¹⁴We assume that the nominal interest rate, $R_{n,t} \sim N(R_n, VAR(R_n))$, has a normal distribution with mean at its steady state value and variance as its theoretical variance, then we can pin down $p(R_{n,t} \leq 1)$ with this normal PDF.

This then achieves the ZLB constraint

$$R_{n,t}^* \geq 1 \text{ with high probability } 1 - \bar{p}^* \quad (69)$$

Stage 1: Design of the Mandate

The policymaker first chooses a per period probability \bar{p}^* of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the *actual* household inter-temporal welfare again in recursive form

$$\Omega_t^* = \mathbb{E}_t \left[(1 - \beta^*) \sum_{\tau=0}^{\infty} \beta^\tau U_{t+\tau}^* \right] = (1 - \beta^*) U_t^* + \beta^* \mathbb{E}_t [\Omega_{t+1}^*] \quad (70)$$

with respect to w_r^* .

This three-stage delegation game defines an equilibrium in choice variables w_r^* , ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (69).

< Table 9 here >

Table 9 shows the results for the ROW using the general rule (62). The probability per-period of hitting the nominal interest rate ZLB is reported, as are the consumption equivalent variations (CEV) which are calculated as follows:

$$CEV(w_r^*, \Pi^*, \rho_1^*) = \frac{\Omega^*(w_r^*, \Pi^*, \rho_1^*) - \Omega^*(0, 1, \rho_2^*)}{CE_{ss}} \quad (71)$$

CE_{ss} is the consumption equivalent at the steady state, which represents the utility gain when consumption increases by 1%; that is $CE_t^* = U_t^*(1.01C_t^*, H_t^*) - U_t^*(C_t^*, H_t^*) / (1 - \beta) \times 100$.¹⁵ Hence, the CEV is the welfare gain (loss) for different settings of the rule $\rho^* = \rho_1^*, \rho_2^*$ with different values on the weight of nominal interest rate's variation in our penalty function and on trend inflation to household welfare. In the table, ρ_2^* is the optimized rule with the weight and net inflation set at zero ($w_r^* = 0, \Pi^* = 1$).

The bottom three rows of the Table compares the welfare outcomes for the optimized rule of this form with the estimated rule as part of the Bayesian estimation of the model. The CEV loss for the rule with the estimated trend is 2.91% but this is largely the consequence of an estimated trend net inflation of 0.47% per quarter, or almost 2% annually. When this contribution is removed the loss becomes 0.624% per quarter, which falls further to 0.618% when the estimated monetary shock $\epsilon_{M,t}$ is removed. These are then the pure business cycle costs in the model under the estimated rule compared with the optimized rule in the middle row of the Table. Total business cycle costs of shocks that can be found by comparing the stochastic and deterministic welfare outcomes are higher and much higher than those reported in the seminal study by Lucas (1987) and updated in Lucas (2003) which are less than 0.01%. The reason for this is his choice of utility function which, unlike our NK-type models, excludes hours (and therefore leisure) in these original studies.

¹⁵For the steady state in the estimated we have $CE_{ss} = 45.04$.

So the optimized simple rule with a zero inflation trend will improve household utility by a CEV of almost 3%. But it comes at the cost of a very high probability of a ZLB of 0.374 per quarter or, in other words, spending over a third of the time with ZLB episodes. The upper three rows of the Table compute the revised policy rules and welfare costs of reducing this probability. This involves reducing the very aggressive optimized rule in the middle row and shifting the inflation distribution to the right. Thus, if the aim is to reduce the probability (0.05, 0.025, 0.01), then the rule becomes increasingly less aggressive in response to the mandate with higher weights w_r^* at a CEV welfare cost of 0.278%, 0.361%, 0.480%, respectively. These then are means and the welfare costs by imposing the ZLB constraint using our penalty function approach.

Now turn to the price-level rule. We have seen from Table 9 that the OSR with ZLB is very close to a price-level rule. So what are the welfare costs of using an optimized form of such a rule? Table 10 provides the answer. First, the welfare costs of the optimized price-level rule compared with the general one is quite low; namely around 0.07% CEV, but seen a lower probability of hitting the ZLB. The costs of imposing the same low probabilities of ZLB episodes are only slightly higher to those before. So, overall, the price-level targeting rule brings low welfare costs, which should be set against the greater simplicity the rule brings.

< Table 10 here >

5.2 The Optimized Rule in the SOE

The well-known traditional recommendation to monetary policymakers in open economies is that an optimal monetary policy in an open economy requires exchange rate flexibility. But the argument relies on the notion that exchange-rate movements have a large immediate impact on aggregate demand, by allowing instantaneous adjustment of relative prices. Empirical studies, however, indicate that in the short run there is very little response of consumer prices to changes in nominal exchange rates. The short-run adjustment role of nominal exchange rates is eliminated to the extent that consumer prices are unresponsive to exchange-rate changes: the so-called “expenditure-switching effect” may be negligible.

The distinction between price-setting specifications is critical for our analysis of optimal monetary policy and exchange-rate flexibility. Based on Betts and Devereux (2000), under PCP, optimal monetary policy relies on nominal exchange rate adjustment. The exchange rate must be employed as part of optimal monetary policy in order to achieve a change in relative prices. In fact, with exchange-rate flexibility, optimal monetary policy can replicate the equilibrium of the economy with fully flexible prices. Flexible exchange rates are a perfect substitute for flexible goods prices. Therefore, expanding the above intuition, when prices are set in local currency, there is added benefit to exchange-rate flexibility. Under the optimal monetary policy, followed by policymakers in each country, exchange rates are much more flexible. Intuitively, when the policymaker chooses an optimal monetary rule under LCP, monetary policy is employed to alter the relative price of home to foreign goods, because movements in exchange rates have a greater effect on prices faced by consumers. Like the policy rule for the PCP economy, monetary authorities under LCP replicate the flexible price equilibrium, but in a more pronounced manner.

We now consider the following SOE counterpart of the ROW rule (62)

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \alpha_\pi \log\left(\frac{\Pi_{t-1,t}}{\Pi}\right) + \alpha_s \log\left(\frac{\Pi_{S,t-1,t}}{\Pi_S}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) \quad (72)$$

(72) includes an interest rate response to a nominal exchange depreciation captured by the magnitude of α_s . We impose a constant nominal exchange rate in the steady state, $\Pi_S = 1$, which requires the steady state inflation rate in the SOE to be the same as the ROW, i.e. $\Pi = \Pi^*$. This removes Stage 2 of the delegation game for the ROW, leaving the policymaker with only one means of imposing the ZLB constraint, namely the choice of the weight on the variance of the nominal interest rate w_r . The rest of the delegation game is as before, with unstarred variables replacing starred ones.

< Table 11 here >

< Table 12 here >

Tables 11 and 12 now repeat the exercise in Table 9 for the two price-setting regimes PCP and LCP. A number of results stand out. First, for the optimal rule without ZLB considerations (the middle rows), the response of the nominal interest rate to exchange rate changes is stronger for the LCP regime than for PCP and the policy rule is generally far more aggressive in terms of responses to inflation and output. This is in accordance with our intuition above and the findings of Betts and Devereux (2000). Second, as a consequence, the probability of hitting the ZLB is far higher under LCP. It then follows from the first three rows that the welfare costs of imposing the ZLB relative to the optimal policy with no ZLB constraint are far higher and now in the region of 2-3% CEV. Third, from the bottom three rows, the welfare costs associated with non-optimized estimated rule are also very high. Finally, and this is a key result, whereas for the ROW the optimized rule under a ZLB constraint was close to a price-level rule, for the SOE, under both PCP and LCP pricing regimes, it is close to a *nominal-income regime*.

Finally, in Tables 13 and 14 we constrain the rules to be optimized price-level and nominal interest rate rules, respectively. Both rules achieve welfare outcomes very close to the optimized hybrid rules, confirming the earlier discussion.

< Table 13 here >

< Table 14 here >

6 Conclusions

In this study, we present an analysis of the empirical relevance of imperfect exchange rate pass-through and the consequences for domestic and foreign monetary policy. We build a comprehensive two-country New Keynesian DSGE model with limited asset market participation which interacts with the rest of the world (ROW). Two currency pricing regimes for the exports of the SOE are

considered - PCP and LCP. We also estimate our proposed models for different economies, the ROW model on US data and SOE model on Canadian data using Bayesian estimation techniques. This provides not only a better understanding about the consequences of shocks that generate fluctuations in the exchange rate on different small open economies, but also an interesting international comparison of spillover effects of monetary policy as well. Our findings can be summarized as following:

First, the striking result is that in a likelihood race LCP beats PCP with large marginal likelihood differences. Our model comparison analysis suggests that setting the imperfect exchange rate pass-through in the model seems to be significantly relevant and does improve model fit remarkably.

Second, from the IRFs, we find distinct and important implications for exchange rate pass-through, terms-of-trade and volumes of exports and imports under the different currency pricing regimes and across countries. Notably, we find that the expansionary impact of a cut in the domestic interest rate (or the contractionary impact of a cut in the foreign interest rate) is muted under LCP relative to PCP. For the former, under LCP, there is an expenditure switching effect from imports towards domestic output, missing out the expansionary impact on exports seen under PCP. Hence, we highlight important propagation channels active in the SOE, essential for any policy-related study.

Finally, our analysis of welfare-optimized interest rate rules show there is no role for exchange rate targeting in either PCP or LCP regimes. In both cases, optimized rules are 100% inertial with the muted response to exchange rate changes under LCP (noted above) resulting in less aggressive interest rate responses to inflation and output fluctuations. While for the closed economy ROW the welfare-optimized price-level rule closely mimics the optimized general inflation-output rule, for the SOE the corresponding result requires a nominal income rule. Nevertheless, in both cases the LCP rule is again less aggressive than its PCP counterpart. These results then clearly demonstrate important implications of both openness and the currency pricing regime for monetary policy conducted using Taylor-type simple nominal interest rate rules.

Our paper has worked largely within the framework of a SOE or 2-country set-up exemplified by Gali and Monacelli (2005a). Future work will extend our research along the following dimensions: firstly, we will extend the study to $n \geq 3$ countries as in Gopinath *et al.* (2020) thus allowing for the possibility of the dominant currency pricing regime. Secondly, we will then allow for strategic complementarity in pricing using a Kimball aggregator as in Kimball (1995) and Dotsey and King (2005), giving rise to variable, as opposed to constant, mark-ups. Thirdly, we will introduce wage stickiness (highlighted by Gali and Monacelli, 2016 for the open economy) and capacity utilization, thus making our model an open-economy counterpart of Smets and Wouters (2007). Fourthly, our current model overestimates the variances observed in the data. This suggests that using “endogenous priors” as in Del Negro and Schorfheide (2008) instead of imposed ones will improve our model fit. Finally, we will extend the policy analysis to other targeting rules including the nominal wage (as in Levine *et al.*, 2008).

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Appendices

A Tables

Table 1: Home and ROW Notation

	Domestic Production	Imported Good	Aggregate
Home Country Quantity	$C_{H,t}$	$C_{F,t}$	C_t
Home Country Price	$P_{H,t}$	$P_{F,t}$	P_t
Foreign Country Quantity	$C_{F,t}^*$	$C_{H,t}^*$	C_t^*
Foreign Country Price	$P_{F,t}^*$	$P_{H,t}^*$	P_t^*

Table 2: The Law of One Price under PCP and LCP

Origin of Good	Domestic Market	Export Market (PCP)	Export Market (LCP)
Home	$P_{H,t}$	$P_H^* = \frac{P_H}{S_t}$	$P_H^* \neq \frac{P_H}{S_t}$
Foreign	P_F^*	$P_F^* = \frac{P_F}{S_t}$	$P_F^* = \frac{P_F}{S_t}$

Table 3: Calibrated Parameters

Calibrated/Imposed parameter	Symbol	Value
Depreciation rate	δ	0.025
Risk aversion	σ	2.00
Labour share	α	0.70
Risk premium elasticity	ϕ_B	0.001
Price Substitution elasticity (Others)	ζ	7.00
Substitution elasticity (Home/Foreign goods)	$\mu_C = \mu_I$	1.50
Foreign Substitution elasticity (Export/Foreign goods)	$\mu_{C^*} = \mu_{I^*}$	1.50
Capital taxation rate	τ_r	0.00
Standard deviation of shocks	σ_i	1.00
Government spending	g_y	0.21
Oil taxation rate	τ_o	0.15
Home Bloc domestic share of Consumption	w_C	calibrated so excs=0.0782
Home Bloc domestic share of Investment	w_I	calibrated so exis=0.0580
Home Bloc domestic share of Intermediate goods	w_M	calibrated so exis=0.0668
Home Bloc Imported share of Consumption	$1 - w_C$	calibrated so excs=0.0780
Home Bloc Imported share of Investment	$1 - w_I$	calibrated so exis=0.0941
Home Bloc Imported share of Intermediate goods	$1 - w_M$	calibrated so exis=0.0636
Oil output	Y^O	calibrated so exco=0.0568
Home Discount factor	β	calibrated so tb=.019
Home productivity	A	calibrated so $Y^*/Y=1.22$

Table 4: Estimated shocks and parameter values in Canada case of PCP

Estimated Parameter Values	Prior		Posterior		
	Symbole	Dist. (Mean,Std Dev)	Mean	90% HPD Interval	
Log data density is -595.78					
Canada-PCP					
Technology shock	ϵ_A	IG	0.10, 2.00	1.0620	0.7761 , 1.3496
Monetary policy shock	ϵ_M	IG	0.10, 2.00	0.5144	0.4018 , 0.6211
Markup shock	ϵ_{MS}	IG	0.10, 2.00	4.1018	2.4834 , 5.5026
Government shock	ϵ_G	IG	0.10, 2.00	0.0732	0.0229 , 0.1355
Investment shock	ϵ_{IS}	IG	0.10, 2.00	3.3000	2.2317 , 4.4189
Terms of trade shock	ϵ_{tot}	IG	0.10, 2.00	5.1686	4.5531 , 5.8788
Technology shock persistence	ρ_A	β	0.50,0.20	0.8735	0.7859 , 0.9466
Markup shock persistence	ρ_G	β	0.50,0.20	0.8729	0.8149 , 0.9289
Government shock persistence	ρ_{IS}	β	0.50,0.20	0.5132	0.1833 , 0.8362
Investment shock persistence	ρ_{MS}	β	0.50,0.20	0.6924	0.5406 , 0.8470
Terms of trade shock persistence	ρ_{tot}	β	0.5,0.10	0.9916	0.9867 , 0.9956
Monetary Policy shock persistence	ρ_M	β	0.70,0.10	0.9294	0.9124 , 0.9449
Feedback from inflation	θ_π	N	2.00,0.25	2.1540	1.8053 , 2.4833
Feedback from output	θ_y	N	0.10,0.05	-0.0043	-0.0408 , 0.0342
Feedback from output growth	θ_{dy}	N	0.10,0.05	0.0964	0.0215 , 0.1742
Feedback from exchange rate depreciation	θ_{ds}	N	0.10,0.05	0.1275	0.0639 , 0.1898
Calvo price stickiness	ξ	β	0.50,0.10	0.5387	0.4629 , 0.6100
Consumption habit formation	χ	β	0.50,0.10	0.7757	0.6963 , 0.8489
Price index	γ	β	0.50,0.10	0.4938	0.3444 , 0.6373
Elasticity of Investment adjustment cost	ϕ_I	N	2.00,0.75	3.2026	2.2988 , 4.1065
Labour Share	α	β	0.40,0.05	0.4299	0.3746 , 0.4843
Intermediate goods Share	α_M	β	0.20,0.05	0.1565	0.1005 , 0.2056
Share of non-Ricardian consumers	λ	N	0.20,0.05	0.2593	0.2200 , 0.2956
Ricardian frisch elasticity	ψ_R	N	2.00,0.75	2.0752	1.3545 , 2.7436
Non-Ricardian frisch elasticity	ψ_c	N	2.00,0.75	2.0825	0.9081 , 3.2584
Ricardian risk aversion	σ_R	N	1.50,0.40	1.2967	0.9637 , 1.6128
Non-Ricardian risk aversion	σ_C	N	1.50,0.40	1.1749	0.6704 , 1.6452

Table 5: Estimated shocks and parameter values in Canada case of LCP

Estimated Parameter Values	Prior		Posterior		
	Symbole	Dist. (Mean,Std Dev)	Mean	90% HPD Interval	
Log data density is -582.60					
Canada-LCP					
Technology shock	ϵ_A	IG	0.10, 2.00	1.0854	0.7905 , 1.3962
Monetary policy shock	ϵ_M	IG	0.10, 2.00	0.4936	0.4003 , 0.5844
Markup shock	ϵ_{MS}	IG	0.10, 2.00	5.9811	2.6021 , 9.2563
Government shock	ϵ_G	IG	0.10, 2.00	0.0761	0.0220 , 0.1433
Investment shock	ϵ_{IS}	IG	0.10, 2.00	2.6756	1.6146 , 3.7153
Terms of trade shock	ϵ_{tot}	IG	0.10, 2.00	4.2965	3.6776 , 4.9287
Technology shock persistence	ρ_A	β	0.50,0.20	0.9099	0.8416 , 0.9739
Markup shock persistence	ρ_G	β	0.50,0.20	0.7749	0.6206 , 0.9171
Government shock persistence	ρ_{IS}	β	0.50,0.20	0.4900	0.1613 , 0.8253
Investment shock persistence	ρ_{MS}	β	0.50,0.20	0.7704	0.6048 , 0.9305
Terms of trade shock persistence	ρ_{tot}	β	0.5,0.10	0.9945	0.9908 , 0.9981
Monetary Policy shock persistence	ρ_M	β	0.70,0.10	0.9316	0.9141 , 0.9484
Feedback from inflation	θ_π	N	2.00,0.25	2.0628	1.6969 , 2.4010
Feedback from output	θ_y	N	0.10,0.05	-0.0074	-0.0583 , 0.0351
Feedback from output growth	θ_{dy}	N	0.10,0.05	0.1045	0.0239 , 0.1799
Feedback from exchange rate depreciation	θ_{ds}	N	0.10,0.05	0.1242	0.0603 , 0.1850
Calvo price stickiness	ξ	β	0.50,0.10	0.5976	0.5305 , 0.6691
Consumption habit formation	χ	β	0.50,0.10	0.8056	0.7182 , 0.8994
Price index	γ	β	0.50,0.10	0.4685	0.3209 , 0.6086
Elasticity of Investment adjustment cost	ϕ_I	N	2.00,0.75	2.9098	1.9923 , 3.8759
Labour Share	α	β	0.40,0.05	0.4404	0.3903 , 0.4846
Intermediate goods Share	α_M	β	0.20,0.05	0.1589	0.0927 , 0.2257
Share of non-Ricardian consumers	λ	N	0.20,0.05	0.2543	0.2158 , 0.2987
Ricardian frisch elasticity	ψ_R	N	2.00,0.75	2.1811	1.3649 , 3.1417
Non-Ricardian frisch elasticity	ψ_c	N	2.00,0.75	2.0521	0.8801 , 3.2697
Ricardian risk aversion	σ_R	N	1.50,0.40	1.4091	1.0017 , 1.8006
Non-Ricardian risk aversion	σ_C	N	1.50,0.40	1.1764	0.6991 , 1.6509

Table 6: Selected Second Moments Comparison

	Output	Inflation	Interest rate	Consumption	Investment	Exchange Rate
Standard Deviation						
Data	0.6076	0.7804	0.4927	0.4689	3.2787	3.3701
LCP Model	0.7495	1.1432	0.4337	0.6832	3.4650	3.7350
PCP Model	0.7847	1.1405	0.4373	0.6715	3.5328	4.0730
Cross-correlation with Output						
Data	1.00	0.3712	0.1568	0.3438	0.5524	-0.0634
LCP Model	1.00	-0.1149	-0.0095	0.5511	0.3514	0.1090
PCP Model	1.00	-0.0073	0.0356	0.6266	0.4053	0.1885
Autocorrelations (Order=1)						
Data	0.4981	0.3381	0.9449	0.1969	0.4760	0.2259
LCP Model	0.3210	0.3216	0.9470	0.5409	0.7213	0.0629
PCP Model	0.3808	0.2402	0.9452	0.5884	0.7104	0.0659

Table 7: Variance Decomposition of Estimated Model (%) - PCP case

	ϵ_A	ϵ_G	ϵ_{MS}	ϵ_m	ϵ_{tot}	ϵ_{IS}	$\epsilon_{P_O^*}$	ϵ_{A^*}	ϵ_{G^*}	ϵ_{MS^*}	ϵ_{m^*}	ϵ_{IS^*}
Output	3.32	0.00	39.61	0.12	7.82	4.61	0.07	3.77	0.30	17.73	0.04	22.62
Consumption	1.32	0.00	10.42	0.01	50.41	1.62	0.35	2.75	0.23	16.47	0.10	16.34
Inflation	5.67	0.01	0.72	36.58	5.42	7.60	0.01	29.63	3.26	1.06	1.01	9.04
Nominal Interest Rate	2.74	0.00	2.44	0.15	5.73	11.19	0.04	16.72	1.61	15.92	0.29	43.18
Depreciation	0.22	0.00	1.14	3.94	85.29	0.16	0.01	2.01	0.25	3.18	0.34	3.45
Real Exchange Rate	1.24	0.00	10.10	0.02	19.51	2.93	0.06	2.37	0.18	25.24	0.09	38.26
Export	1.52	0.00	12.34	0.03	23.86	3.59	0.07	5.65	0.46	18.51	0.01	33.96
Imported Consumption Goods	0.32	0.00	1.21	0.01	44.25	0.42	0.22	2.62	0.20	24.10	0.10	26.55
Imported Investment Goods	0.39	0.00	9.54	0.00	9.90	8.32	0.08	5.01	0.41	17.38	0.06	48.91
Imported Intermediate Inputs	0.84	0.00	39.64	1.44	15.68	1.12	0.07	4.06	0.39	19.36	0.09	17.32
Terms of Trade	1.24	0.00	10.10	0.02	19.51	2.93	0.06	2.37	0.18	25.24	0.09	38.26

Table 8: Variance Decomposition of Estimated Model (%) - LCP case

	ϵ_A	ϵ_G	ϵ_{MS}	ϵ_m	ϵ_{tot}	ϵ_{IS}	$\epsilon_{P_O^*}$	ϵ_{A^*}	ϵ_{G^*}	ϵ_{MS^*}	ϵ_{m^*}	ϵ_{IS^*}
Output	7.21	0.00	25.51	0.07	5.17	7.23	0.08	4.71	0.38	22.46	0.04	27.14
Consumption	3.29	0.00	7.22	0.03	42.47	2.13	0.43	3.53	0.30	21.06	0.13	19.41
Inflation	6.09	0.01	6.43	25.49	21.60	8.69	0.00	20.46	2.12	1.24	0.32	7.55
Nominal Interest Rate	2.86	0.00	2.11	0.85	4.69	13.72	0.04	12.12	1.12	17.85	0.12	44.53
Depreciation	0.38	0.00	0.88	5.38	81.39	0.28	0.02	2.88	0.35	3.86	0.44	4.14
Real Exchange Rate	2.23	0.00	4.91	0.17	12.46	4.48	0.06	2.89	0.23	29.82	0.13	42.62
Export	2.86	0.00	6.18	0.08	19.94	5.68	0.08	4.75	0.36	22.71	0.02	37.33
Imported Consumption Goods	0.61	0.00	0.74	0.07	33.58	0.76	0.25	3.18	0.25	29.72	0.14	30.71
Imported Investment Goods	0.73	0.00	3.70	0.02	4.85	11.74	0.08	5.75	0.47	19.44	0.07	53.13
Imported Intermediate Inputs	1.66	0.00	33.52	0.61	24.23	1.28	0.06	3.68	0.33	18.85	0.06	15.71
Terms of Trade	2.23	0.00	4.91	0.17	12.46	4.48	0.06	2.89	0.23	29.82	0.13	42.62

Table 9: ROW: Inflation-Output Targeting Rule

Optimized simple rule with ZLB Mandate										
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	sd($\epsilon_{M,t}$)
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.4269	0.0012	0.0037	1.0007	-3020.85	-0.480	0.010	85	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.6804	0.0011	0.0085	1.0005	-3015.51	-0.361	0.025	45	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	1.2337	0.0003	0.0259	1.0004	-3011.78	-0.278	0.050	23	0.0000
The Optimized simple rule (without ZLB)										
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r	sd($\epsilon_{M,t}$)
OSR without ZLB ($\Pi = 1.0$)	0.9863	77.6646	0.0090	8.0913	1.0000	-2999.23	0	0.3736	0	0.0000
Estimated model										
Regimes	ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	sd($\epsilon_{M,t}$)
Estimated rule ($\Pi^* = 1.0047$)	0.8357	1.8420	0.0735	0.1482	1.0047	-3130.17	-2.91	0.0903	-	0.0025
Estimated rule ($\Pi^* = 1$)	0.8357	1.8420	0.0735	0.1482	1.0000	-3027.33	-0.624	0.1894	-	0.0025
Estimated rule ($\Pi^* = 1$)	0.8357	1.8420	0.0735	0.1482	1.0000	-3027.07	-0.618	0.1849	-	0.00

Table 10: ROW: Price Level Rule

Optimized simple rule with ZLB Mandate										
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.4641	0.0000	0.000	1.0008	-3021.0064	-0.483	0.010	85	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.7195	0.0000	0.000	1.0005	-3015.5697	-0.363	0.025	45	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	1.0715	0.0000	0.000	1.0001	-3011.8160	-0.279	0.050	25	0.0000
Optimized simple price-level rule without ZLB										
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π	Ω^*	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB ($\Pi = 1.0$)	1.0000	41.0235	0.0000	0.000	1.0000	-3002.3330	-0.069	0.3248	0	0.0000

Notes: The CEV is measured relative to the optimized simple rule (without ZLB) in Table 9

Table 11: SOE: Hybrid Rule (PCP)

Optimized simple rule with ZLB Mandate											
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.2697	0.0000	0.2867	0.0000	1.0007	-215.01	-0.477	0.010	4.0	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	0.9999	0.3856	0.0001	0.4440	0.0000	1.0007	-214.70	-0.313	0.025	2.0	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.5985	0.0000	0.7773	0.0000	1.0007	-214.41	-0.160	0.050	1.2	0.0000
Optimized simple rule without ZLB											
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB ($\Pi = 1.0$)	0.9396	1.6877	0.0004	3.1688	0.0005	1.0000	-214.11	0	0.1619	0	0.0000
Estimated model											
Regimes	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	$\frac{\alpha_{ds}}{1-\rho_r}$	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
Estimated rule ($\Pi = 1.0047$)	0.9294	2.1540	-0.0043	0.0964	0.1275	1.0047	-217.8887	-2.002	0.0020	-	0.0013
Estimated rule ($\Pi = 1$)	0.9294	2.1540	-0.0043	0.0964	0.1275	1.0000	-217.3805	-1.733	0.0234	-	0.0013
Estimated rule ($\Pi = 1$)	0.9294	2.1540	-0.0043	0.0964	0.1275	1.0000	-217.3776	-1.731	0.0234	-	0.0000

Table 12: SOE Hybrid Rule (LCP)

Optimized simple rule with ZLB Mandate											
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω (%)	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0000	0.1393	0.0077	0.1729	0.0000	1.0007	-391.8956	-2.420	0.010	7.5	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.025$)	1.0000	0.1699	0.0103	0.2204	0.0000	1.0007	-391.3414	-2.192	0.025	6	0.0000
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0000	0.2409	0.0167	0.3401	0.0000	1.0007	-390.3439	-1.781	0.050	4	0.0000
Optimized simple rule without ZLB											
Regimes	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
OSR without ZLB ($\Pi = 1.0$)	0.8092	5.1067	0.1142	16.8983	0.0340	1.0000	-386.0228	0	0.3239	0	0.0000
Estimated model											
Regimes	ρ_r	$\frac{\alpha_\pi}{1-\rho_r}$	$\frac{\alpha_y}{1-\rho_r}$	$\frac{\alpha_{dy}}{1-\rho_r}$	$\frac{\alpha_{ds}}{1-\rho_r}$	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
Estimated rule ($\Pi = 1.0047$)	0.9316	2.0628	-0.0074	0.1045	0.1242	1.0047	-396.9998	-4.523	0.0050	-	0.0012
Estimated rule ($\Pi = 1$)	0.9316	2.0628	-0.0074	0.1045	0.1242	1.0000	-396.2183	-4.201	0.0321	-	0.0012
Estimated rule ($\Pi = 1$)	0.9316	2.0628	-0.0074	0.1045	0.1242	1.0000	-396.2155	-4.200	0.0320	-	0.0000

Table 13: SOE: Price level Rule

Regimes -LCP	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Act welfare	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.00039$)	1.0000	0.2528	0.0000	0.0000	0.0000	1.0007	-395.6978	-3.986	0.00039	0	0.0000
OSR without ZLB ($\Pi = 1.0$)	1.0000	0.2301	0.0000	0.0000	0.0000	1.0000	-395.5960	-3.944	0.00047	0	0.0000
Regimes - PCP	ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Act welfare	CEV (%)	p_zlb	w_r	MPS
OSR with ZLB ($\bar{p}_{zlb} = 0.0036$)	1.0000	0.5142	0.0000	0.0000	0.0000	1.0007	-217.2380	-0.0017	0.00036	0	0.0000
OSR without ZLB ($\Pi = 1.0$)	1.0000	0.5220	0.0000	0.0000	0.0000	1.0000	-217.1775	-0.0016	0.0058	0	0.0000

Table 14: SOE: Nominal Income Rule

LCP model										
ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
1.0000	0.2031	0.0000	0.2031	0.0000	1.0007	-392.1280	-2.516	0.010	7	0.0000
1.0000	0.2691	0.0000	0.2691	0.0000	1.0007	-391.4788	-2.248	0.025	5.5	0.0000
1.0000	0.3963	0.0000	0.3963	0.0000	1.0007	-390.9098	-2.014	0.050	4	0.0000
PCP model										
ρ_r	α_π	α_y	α_{dy}	α_{ds}	$\Pi = \Pi^*$	Ω	CEV (%)	p_zlb	w_r	MPS
1.0000	0.3209	0.0000	0.3209	0.0000	1.0007	-214.9135	-0.426	0.010	3.5	0.0000
1.0000	0.5347	0.0000	0.5347	0.0000	1.0007	-214.6941	-0.309	0.025	2	0.0000
1.0000	1.3906	0.0000	1.3906	0.0000	1.0007	-214.4421	-0.176	0.050	0.5	0.0000

Figure 1: Impulse Response to a Domestic Negative Monetary Policy Shock

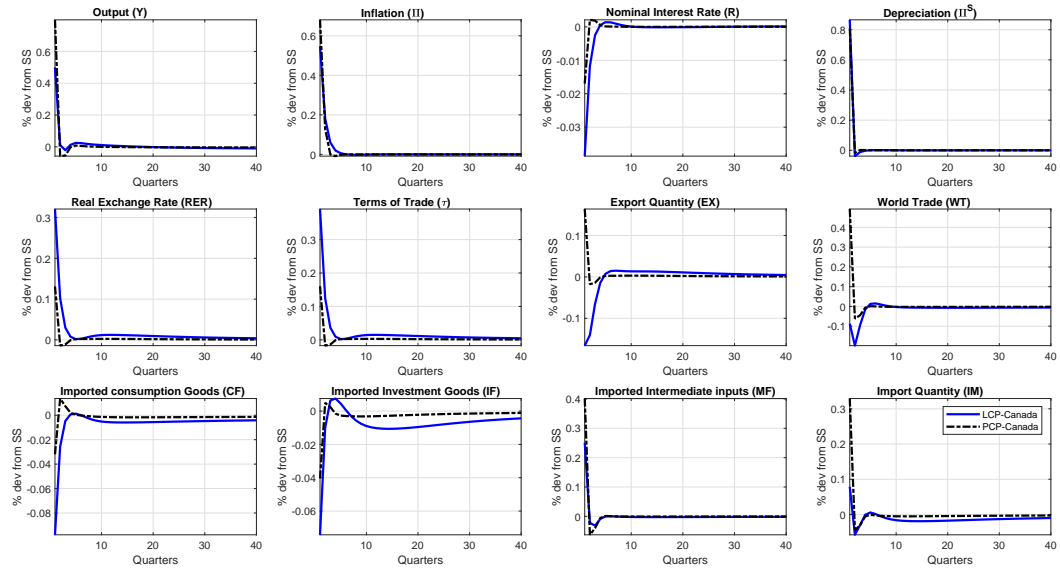
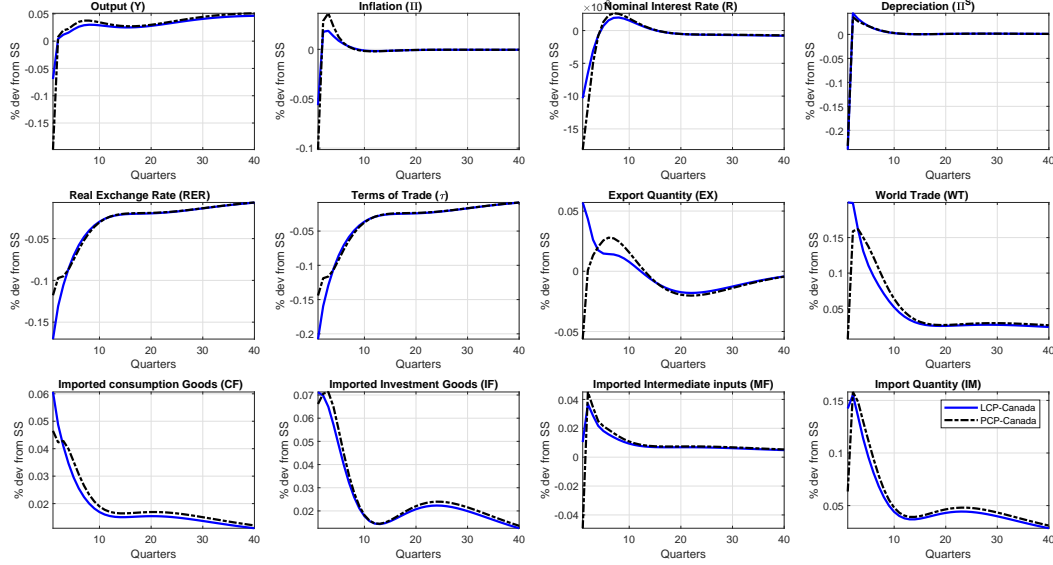


Figure 2: Impulse Response to a ROW Negative Monetary Policy Shock



Supplement to "Imperfect Exchange Rate Pass-through: Empirical Evidence and Monetary Policy Implications" (not for publication)

B The 2-bloc SOE-ROW model: solution and steady-state details

This section provides further details on the model, steady state and equilibrium conditions. The SOE model departs from the standard framework of Gali and Monacelli (2005b) in three dimensions. First we model two different pricing paradigms, local currency pricing (LCP) alongside the producer currency pricing (PCP). Second is the use of intermediate input and capital in production function and finally the international asset markets is incomplete. The world economy is modelled as a continuum of SOEs on the unit interval. The latter feature is introduced by assuming that some households are excluded from financial markets which can neither borrow nor save, and hence they do not smooth consumption over time. These households consume their current labour income each period. These consumers are labelled non-Ricardian, as they break the Ricardian Equivalence, but in the main text we use interchangeably 'credit-constrained', 'liquidity-constrained' or 'rule-of-thumb' agents.

There is a continuum of households, a single perfectly competitive intermediate good producer and a continuum of monopolistically competitive final producers setting prices on a Calvo-type staggered basis.

A Households

Recall that, given the utility function in equation (1) in the paper, the (Ricardian) household solves

$$\max_{C_t^R, L_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, H_{t+s}^R) \right] \quad (73)$$

subject to a nominal budget constraint given by

$$P_t^B B_{H,t} + P_t^{B^*} S_t B_{F,t}^* = B_{H,t-1} + S_t B_{F,t-1}^* + P_t W_t (1 - \tau_t^w) H_t^R - P_t C_t^R + \Gamma_t \quad (74)$$

(see the paper for the notation details).

Maximizing (73) subject to the budget constraint we have

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] \quad (75)$$

$$P_t^{B^*} = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \frac{S_{t+1}}{S_t} \right] \quad (76)$$

$$U_t^R = \frac{\beta^t}{1 - \sigma_c^R} [C_t^R - \chi C_{t-1}^R]^{-\sigma_c^R + 1} \exp \left[(\sigma_c^R - 1) \frac{H_t^{R^{1+\psi^R}}}{1 + \psi^R} \right] \quad (77)$$

$$\lambda_t = \frac{(1 - \sigma_c^R) U_t^R}{C_t^R - \chi C_{t-1}^R} - \beta \chi \frac{(1 - \sigma_c^R) U_{t+1}^R}{C_{t+1}^R - \chi C_t^R} \quad (78)$$

$$W_{h,t} (1 - \tau_t^w) = \frac{[C_t^R - \chi C_{t-1}^R] H_t^{R\psi^R}}{1 - \beta \chi \frac{U_{t+1}^R}{U_t^R} \frac{C_t^R - \chi C_{t-1}^R}{C_{t+1}^R - \chi C_t^R}} \quad (79)$$

where $\Lambda_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$. Nominal return on home bonds is by definition $R_t = \frac{1}{P_t^B}$, where R_t is set by the central bank. We assume foreign bonds are subject to a risk premium that depends on the exposure to foreign debt, $R_t^* = \frac{1}{P_t^{B^*} \phi \left(\frac{S_t B_{F,t}^*}{P_{H,t} Y_t} \right)}$. Additionally, we assume $\phi(0) = 0$ and $\phi' < 0$. A functional form with these properties is

$$\phi(x) = \exp(-\phi_B x); \quad \phi_B > 0 \quad (80)$$

Write equation (75) as

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t \quad (81)$$

and equation (76) as

$$1 = R_t^* \phi \left(\frac{S_t B_{F,t}^*}{P_{H,t} Y_t} \right) \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \Pi_{t,t+1}^S \right] \quad (82)$$

where $\Pi_{t,t+1}^S \equiv \frac{S_{t+1}}{S_t}$ is the rate of change of the nominal exchange rate over the interval $[t, t+1]$ (i.e., the depreciation rate). Equations (81) and (82) together give a **UIP condition** modified to allow for risk:

$$\mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \right] R_t = R_t^* \phi \left(\frac{S_t B_{F,t}^*}{P_{H,t} Y_t} \right) \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\Pi_{t,t+1}} \Pi_{t,t+1}^S \right] \quad (83)$$

To understand the UIP condition better it is useful to linearize (83) about a zero net inflation a net depreciation steady state ($\Pi = \Pi^S = 1$) to give

$$\mathbb{E}_t \pi_{t+1}^S \equiv \mathbb{E}_t [s_{t+1} - s_t] = r_t - r_t^* - \phi_t \quad (84)$$

Hence solving forward in time

$$s_t = r_t^* + \phi_t - r_t + \mathbb{E}_t s_{t+1} = \sum_{\tau=t}^{\infty} [r_\tau^* + \phi_\tau - r_\tau] \quad (85)$$

Hence an expected drop *fall* in $r_t^* + \phi_t - r_t$ in the future arising from a *domestic monetary relaxation* (a fall in r_t) or the opposite in the foreign bloc (a rise in r_t^*) results in a *rise* in s_t (a depreciation).

Similar steps can be derived for the remaining λ non-Ricardian agents, who choose C_t^{RoT} and $L_t^{RoT} = 1 - H_t^{RoT}$ to maximize an analogous welfare function to (73) subject to their respective budget constraint (equation (4) in the paper).

The first-order conditions are now the same for both types:

$$U_t^C = \frac{\beta^t}{1 - \sigma_c^C} [C_t^C - \chi C_{t-1}^C]^{-\sigma_c^C + 1} \exp \left[(\sigma_c^C - 1) \frac{H_t^C^{1+\psi^C}}{1 + \psi^C} \right] \quad (86)$$

$$W_{h,t}(1 - \tau_t^w) = \frac{[C_t^C - \chi C_{t-1}^C] H_t^C \psi^C}{1 - \beta \chi \frac{U_{t+1}^C}{U_t^C} \frac{C_t^C - \chi C_{t-1}^C}{C_{t+1}^C - \chi C_t^C}} \quad (87)$$

Households in ROW bloc

The same structure is used for foreign part and we have the following dynamics:

$$\begin{aligned} U_t^{*R} &= \frac{[C_t^{*R} - \chi^* C_{t-1}^{*R}]^{1-\sigma_R^*}}{1 - \sigma_R^*} \exp \left[(\sigma_R^* - 1) \frac{H_t^{*R^{1+\psi_R^*}}}{1 + \psi_R^*} \right] \\ W_t^* (1 - \tau_t^{*w}) &= \frac{[C_t^{*C} - \chi^* C_{t-1}^{*C}] H_t^{*C} \psi^*}{1 - \beta^* \chi^* \frac{U_{t+1}^{*C}}{U_t^{*C}} \frac{C_t^{*C} - \chi^* C_{t-1}^{*C}}{C_{t+1}^{*C} - \chi^* C_t^{*C}}} \\ W_t^* (1 - \tau_t^{*w}) &= \frac{[C_t^{*R} - \chi^* C_{t-1}^{*R}] H_t^{*R} \psi^*}{1 - \beta^* \chi^* \frac{U_{t+1}^{*R}}{U_t^{*R}} \frac{C_t^{*R} - \chi^* C_{t-1}^{*R}}{C_{t+1}^{*R} - \chi^* C_t^{*R}}} \\ \lambda_t^* &= \frac{(1 - \sigma_R^*) U_t^{*R}}{C_t^{*R} - \chi^* C_{t-1}^{*R}} - \beta^* \chi^* \frac{(1 - \sigma_R^*) U_{t+1}^{*R}}{C_{t+1}^{*R} - \chi^* C_t^{*R}} \end{aligned}$$

$$\begin{aligned}
\Lambda_{t,t+1}^* &\equiv \beta^* \frac{\lambda_{t+1}^*}{\lambda_t^*} \\
1 &= \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^*}{\Pi_{t,t+1}^*} \right] R_t^* \\
U_t^* &= \lambda^* U_t^{*C} + (1 - \lambda^*) U_t^{*R} \\
C_t^* &= \lambda^* C_t^{*C} + (1 - \lambda^*) C_t^{*R} \\
H_t^* &= \lambda^* H_t^{*C} + (1 - \lambda^*) H_t^{*R}
\end{aligned}$$

Aggregate Utility, Consumption and Labour

Total utility, consumption and hours are then

$$U_t = \lambda U_t^C + (1 - \lambda) U_t^R \quad (88)$$

$$C_t = \lambda C_t^C + (1 - \lambda) C_t^R \quad (89)$$

$$H_t = \lambda H_t^C + (1 - \lambda) H_t^R \quad (90)$$

Consumption, Investment and Intermediate Goods Demand

In the main paper, we focus on aggregate demand. Here we provide some additional details on intermediate steps. For the Home country, aggregate Dixit-Stiglitz consumption and price indices are given by

$$C_t = \equiv \left[w_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C-1}{\mu_C}} + (1 - w_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C-1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C-1}} \quad (91)$$

$$P_t = \equiv \left[w_C P_{H,t}^{1-\mu_C} + (1 - w_C) P_{F,t}^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \quad (92)$$

Maximising total consumption (91) subject to a given aggregate expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ yields

$$C_{H,t} = w_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \quad (93)$$

$$C_{F,t} = (1 - w_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \quad (94)$$

For the Foreign country, aggregate Dixit-Stiglitz consumption and price indices are given by

$$C_t^* = C^{DS}(w_C^*, \mu_C^*, C_{F,t}^*, C_{H,t}^*) \quad (95)$$

$$P_t^* = P^{DS}(w_C^*, \mu_C^*, C_{F,t}^*, C_{H,t}^*) \quad (96)$$

$$P_t^* C_t^* = P_{F,t}^* C_{F,t}^* + P_{H,t}^* C_{H,t}^* \quad (97)$$

Then, foreign consumption functions corresponding to (93) and (94) are:

$$C_{F,t}^* = w_C^* \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* \quad (98)$$

$$C_{H,t}^* = (1 - w_C^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* \quad (99)$$

For the Home country, aggregate Dixit-Stiglitz investment and price indices are given by

$$I_t = \equiv \left[w_I^{\frac{1}{\mu_I}} I_{I,t}^{\frac{\mu_I-1}{\mu_I}} + (1 - w_I)^{\frac{1}{\mu_I}} I_{F,t}^{\frac{\mu_I-1}{\mu_I}} \right]^{\frac{\mu_I}{\mu_I-1}} \quad (100)$$

$$P_t^I = \equiv \left[w_I P_{H,t}^{1-\mu_I} + (1 - w_I) P_{F,t}^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}} \quad (101)$$

Maximising total investment (100) subject to a given aggregate expenditure $P_t^I I_t = P_{H,t} I_{H,t} + P_{F,t} I_{F,t}$ yields

$$I_{H,t} = w_I \left(\frac{P_{H,t}}{P_t^I} \right)^{-\mu_I} I_t \quad (102)$$

$$I_{F,t} = (1 - w_I) \left(\frac{P_{F,t}}{P_t^I} \right)^{-\mu_I} I_t \quad (103)$$

For the Foreign country, aggregate Dixit-Stiglitz investment and price indices are given by

$$I_t^* = I^{DS}(w_I^*, \mu_I^*, I_{F,t}^*, I_{H,t}^*) \quad (104)$$

$$P_t^{I^*} = P^{DS}(w_I^*, \mu_I^*, I_{F,t}^*, I_{H,t}^*) \quad (105)$$

$$P_t^{I^*} I_t^* = P_{F,t}^* I_{F,t}^* + P_{H,t}^* I_{H,t}^* \quad (106)$$

Then, foreign investment functions corresponding to (102) and (103) are:

$$I_{F,t}^* = w_I^* \left(\frac{P_{F,t}^*}{P_t^{I^*}} \right)^{-\mu_I^*} I_t^* \quad (107)$$

$$I_{H,t}^* = (1 - w_I^*) \left(\frac{P_{H,t}^*}{P_t^{I^*}} \right)^{-\mu_I^*} I_t^* \quad (108)$$

For the Home country, aggregate Dixit-Stiglitz intermediate goods and price indices are given by

$$M_t = \equiv \left[w_M^{\frac{1}{\mu_M}} M_{M,t}^{\frac{\mu_M-1}{\mu_M}} + (1 - w_M)^{\frac{1}{\mu_M}} M_{F,t}^{\frac{\mu_M-1}{\mu_M}} \right]^{\frac{\mu_M}{\mu_M-1}} \quad (109)$$

$$P_t^M = \equiv \left[w_M P_{H,t}^{1-\mu_M} + (1 - w_M) P_{F,t}^{1-\mu_M} \right]^{\frac{1}{1-\mu_M}} \quad (110)$$

Maximising total intermediate goods (109) subject to a given aggregate expenditure $P_t^M M_t =$

$P_{H,t}M_{H,t} + P_{F,t}M_{F,t}$ yields

$$M_{H,t} = w_M \left(\frac{P_{H,t}}{P_t^M} \right)^{-\mu_M} M_t \quad (111)$$

$$M_{F,t} = (1 - w_M) \left(\frac{P_{F,t}}{P_t^M} \right)^{-\mu_M} M_t \quad (112)$$

For the Foreign country, aggregate Dixit-Stiglitz intermediate goods and price indices are given by

$$M_t^* = M^{DS}(w_M^*, \mu_M^*, M_{F,t}^*, M_{H,t}^*) \quad (113)$$

$$P_t^{M^*} = P^{DS}(w_M^*, \mu_M^*, M_{F,t}^*, M_{H,t}^*) \quad (114)$$

$$P_t^{M^*} M_t^* = P_{F,t}^* M_{F,t}^* + P_{H,t}^* M_{H,t}^* \quad (115)$$

Then, foreign investment functions corresponding to (111) and (112) are:

$$M_{F,t}^* = w_M^* \left(\frac{P_{F,t}^*}{P_t^{M^*}} \right)^{-\mu_M^*} M_t^* \quad (116)$$

$$M_{H,t}^* = (1 - w_M^*) \left(\frac{P_{H,t}^*}{P_t^{M^*}} \right)^{-\mu_M^*} M_t^* \quad (117)$$

The Law of One Price, Terms of Trade and Inflation

Let S_t be the nominal exchange rate defined as the cost of one unit of Foreign currency in the Home bloc. With producer currency pricing (one of our assumptions in the price-setting model below) the *law of one price* holds and hence

$$S_t P_{H,t}^* = P_{H,t} \quad (118)$$

$$S_t P_{F,t}^* = P_{F,t} \quad (119)$$

The *terms of trade* for the home country are defined as $\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, the price of the imported good relative to the domestic one, and $\mathcal{T}_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*}$ for the Foreign bloc. Hence from the law of one price

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P_{F,t}^*}{S_t P_{H,t}^*} = \frac{P_{F,t}^*}{P_{H,t}^*} = \frac{1}{\mathcal{T}_t^*} \quad (120)$$

Now define CPI, domestic and imported inflation rates over the interval $[t-1, t]$ for the Home bloc by

$$\Pi_{t-1,t} \equiv \frac{P_t}{P_{t-1}} \quad (121)$$

$$\Pi_{H,t-1,t} \equiv \frac{P_{H,t}}{P_{H,t-1}} \quad (122)$$

$$\Pi_{F,t-1,t} \equiv \frac{P_{F,t}}{P_{F,t-1}} \quad (123)$$

The foreign counterparts of CPI, domestic and imported inflation rates over the interval $[t-1, t]$ are defined by

$$\Pi_{t-1,t}^* \equiv \frac{P_t^*}{P_{t-1}^*} \quad (124)$$

$$\Pi_{F,t-1,t}^* \equiv \frac{P_{F,t}^*}{P_{F,t-1}^*} \quad (125)$$

$$\Pi_{H,t-1,t}^* \equiv \frac{P_{H,t}^*}{P_{H,t-1}^*} \quad (126)$$

Then from (92) and (96) we have

$$\Pi_{t-1,t} = \left[w_C \left(\Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-w_C) \left(\Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \quad (127)$$

$$\Pi_{t-1,t}^* = \left[w_C^* \left(\Pi_{F,t-1,t}^* \frac{P_{F,t-1}^*}{P_{t-1}^*} \right)^{1-\mu_C^*} + (1-w_C^*) \left(\Pi_{H,t-1,t}^* \frac{P_{H,t-1}^*}{P_{t-1}^*} \right)^{1-\mu_C^*} \right]^{\frac{1}{1-\mu_C^*}} \quad (128)$$

where for the Home bloc:

$$\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{F,t-1,t}}{\Pi_{H,t-1,t}} \quad (129)$$

$$\frac{P_t}{P_{H,t}} = \left(w_C + (1-w_C) \mathcal{T}_t^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \quad (130)$$

$$\frac{P_t}{P_{F,t}} = \left(w_C \mathcal{T}_t^{\mu_C-1} + (1-w_C) \right)^{\frac{1}{1-\mu_C}} \quad (131)$$

$$\frac{P_t^I}{P_{H,t}} = \left(w_I + (1-w_I) \mathcal{T}_t^{1-\mu_I} \right)^{\frac{1}{1-\mu_I}} \quad (132)$$

$$\frac{P_t^I}{P_{F,t}} = \left(w_I \mathcal{T}_t^{\mu_I-1} + (1-w_I) \right)^{\frac{1}{1-\mu_I}} \quad (133)$$

$$\frac{P_t^M}{P_{H,t}} = \left(w_M + (1-w_M) \mathcal{T}_t^{1-\mu_M} \right)^{\frac{1}{1-\mu_M}} \quad (134)$$

$$\frac{P_t^M}{P_{F,t}} = \left(w_M \mathcal{T}_t^{\mu_M-1} + (1-w_M) \right)^{\frac{1}{1-\mu_M}} \quad (135)$$

and for the Foreign bloc:

$$\frac{\mathcal{T}_t^*}{\mathcal{T}_{t-1}^*} = \frac{\Pi_{H,t-1,t}^*}{\Pi_{F,t-1,t}^*} \quad (136)$$

$$\frac{P_t^*}{P_{F,t}^*} = \left(w_C^* + (1 - w_C^*)(\mathcal{T}_t^*)^{1-\mu_C^*} \right)^{\frac{1}{1-\mu_C^*}} \quad (137)$$

$$\frac{P_t^*}{P_{H,t}^*} = \left(w_C^*(\mathcal{T}_t^*)^{\mu_C^*-1} + (1 - w_C^*) \right)^{\frac{1}{1-\mu_C^*}} \quad (138)$$

$$\frac{P_t^{I^*}}{P_{F,t}^{I^*}} = \left(w_I^* + (1 - w_I^*)(\mathcal{T}_t^*)^{1-\mu_I^*} \right)^{\frac{1}{1-\mu_I^*}} \quad (139)$$

$$\frac{P_t^{I^*}}{P_{H,t}^{I^*}} = \left(w_I^*(\mathcal{T}_t^*)^{\mu_I^*-1} + (1 - w_I^*) \right)^{\frac{1}{1-\mu_I^*}} \quad (140)$$

$$\frac{P_t^{M^*}}{P_{F,t}^{M^*}} = \left(w_M^* + (1 - w_M^*)(\mathcal{T}_t^*)^{1-\mu_M^*} \right)^{\frac{1}{1-\mu_M^*}} \quad (141)$$

$$\frac{P_t^{M^*}}{P_{H,t}^{M^*}} = \left(w_M^*(\mathcal{T}_t^*)^{\mu_M^*-1} + (1 - w_M^*) \right)^{\frac{1}{1-\mu_M^*}} \quad (142)$$

The real exchange rate is defined as $RER_t = \frac{S_t P_t^*}{P_t}$. Then from (131) and (137) and the law of one price we have

$$RER_t = \frac{S_t P_t^*}{P_t} = \frac{S_t P_t^* / P_{F,t}^*}{P_t / P_{F,t}^*} = \frac{P_t^* / P_{F,t}^*}{P_t / S_t P_{F,t}^*} = \frac{P_t^* / P_{F,t}^*}{P_t / P_{F,t}^*} \quad (143)$$

$$= \frac{(w_C^* + (1 - w_C^*)(\mathcal{T}_t^*)^{1-\mu_C^*})^{\frac{1}{1-\mu_C^*}}}{\left(w_C \mathcal{T}_t^{\mu_C-1} + (1 - w_C) \right)^{\frac{1}{1-\mu_C}}} \quad (144)$$

Thus in the *symmetric bloc case* where $w_C = w_C^*$ and $\mu_C = \mu_C^*$, since $\mathcal{T}_t = 1/\mathcal{T}_t^*$ the law of one price ($RER_t = 1$) holds for the aggregate price indices. Otherwise it does not.

This completes the equilibrium for Home variables $\{\mathcal{T}_t, \frac{P_t}{P_{H,t}}, \frac{P_t}{P_{F,t}}, \Pi_{t-1,t}, \Pi_{F,t-1,t}, C_{H,t}, C_{F,t}, I_{H,t}, I_{F,t}, M_{H,t}, M_{F,t}\}$ given C_t, I_t and domestic inflation $\Pi_{H,t-1,t}$, and for the corresponding Foreign variables $\{\mathcal{T}_t^*, \frac{P_t^*}{P_{F,t}^*}, \frac{P_t^*}{P_{H,t}^*}, \Pi_{t-1,t}^*, \Pi_{H,t-1,t}^*, C_{F,t}^*, C_{H,t}^*, I_{F,t}^*, I_{H,t}^*, M_{F,t}^*, M_{H,t}^*\}$ given C_t^*, I_t^*, M_t^* and foreign domestic inflation $\Pi_{F,t-1,t}^*$.

B Capital Producers

Capital producers maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k} (1 - \mathcal{S}(I_{t+k}/I_{t+k-1})) I_{t+k} - \frac{P_t^I}{P_t} I_{t+k} \right]$$

subject to the law of motions of capital and investment - (17) and (18) in the paper. This results in the first-order condition

$$Q_t (1 - \mathcal{S}(X_t) - X_t \mathcal{S}'(X_t)) + E_t \left[\Lambda_{t,t+1} Q_{t+1} \mathcal{S}'(X_{t+1}) \frac{I_{t+1}^2}{I_t^2} \right] = \frac{P_t^I}{P_t} \quad (145)$$

Therefore, we have

$$\mathbb{E}_t [R_{t+1}^K \Lambda_{t,t+1}] = 1 \quad (146)$$

where capital demand equates the expected discounted return on capital over the period $[t, t+1]$ and must satisfy

$$R_t^K = \frac{r_t^K (1 - \tau_t^k) + (1 - \delta)Q_t}{Q_{t-1}} \quad (147)$$

Capital Producers in ROW bloc

Similarly, we have the following capital dynamics equations,

$$\begin{aligned} X_t^* &\equiv \frac{I_t^*}{I_{t-1}^*} \\ S^*(X_t^*) &= \phi_X^*(X_t^* - 1)^2 \\ S'^*(X_t^*) &= 2\phi_X^*(X_t^* - 1) \\ K_t^* &= (1 - \delta^*)K_{t-1}^* + (1 - S^*(X_t^*))IS_t^* I_t^* \\ IS_t^* Q_t^* (1 - S^*(X_t^*) - X_t S'^*(X_t^*)) + \mathbb{E}_t \left[\Lambda_{t,t+1}^* IS_{t+1}^* Q_{t+1}^* S'^*(X_{t+1}^*) X_{t+1}^{*2} \right] &= \frac{P_t^{*I}}{P_t^*} \\ R_t^{*K} &= \frac{[r_t^{*K} (1 - \tau_t^{*k}) + (1 - \delta^*)Q_t^*]}{Q_{t-1}^*} \\ 1 &= \mathbb{E}_t \left[R_{t+1}^{*K} \Lambda_{t,t+1}^* \right] \end{aligned}$$

C Firms

Wholesale Sector in ROW bloc

As in the SOE bloc, we have as following

$$\begin{aligned} Y_t^{*W} &= F(A_t^*, M_t^*, H_t^*, K_{t-1}^*) = (A_t^* H_t^{*d})^{\alpha_H^*} M_t^{*\alpha_M^*} (K_{t-1}^*)^{1 - \alpha_H^* - \alpha_M^*} \\ F_{K,t}^* &= (1 - \alpha^*) \frac{Y_t^{*W}}{K_{t-1}^*} MC_t^* \frac{P_{F,t}^*}{P_t^*} = r_t^{*K} \\ F_{H,t}^* &= \alpha^* \frac{Y_t^{*W}}{H_t^*} MC_t^* \frac{P_{F,t}^*}{P_t^*} = W_t^* \\ F_{M,t}^* &= \frac{\alpha_M^* P_t^{*W} Y_t^{*W}}{M_t^*} = \frac{P_t^{*M}}{P_t^{*W}} \end{aligned}$$

C.1 Retail Sector and Incomplete Exchange Rate Pass-through For Exports

Price setting in export markets by domestic LCP exporters follows in a very similar fashion to domestic pricing. The optimal price in units of domestic currency is $\hat{P}_{1,t}^{*\ell} S_t$, costs are as for domes-

tically marketed goods, so firms maximise expected discounted profits by solving

$$\max_{P_t^O(m)} \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (C_{t+k}^*(m) + I_{t+k}^*(m) + M_{t+k}^*(m)) [S_{t+k} P_{H,t}^{*O}(m) - P_{t+k}^W] \quad (148)$$

with real marginal cost

$$MC_{H,t}^* \equiv \frac{P_t^W}{S_t P_{H,t}^*} = \frac{MC_t \frac{P_{H,t}}{P_t}}{\frac{S_t P_{H,t}^*}{P_t}} \quad (149)$$

Substituting in this demand schedule, taking first-order conditions with respect the new price and rearranging leads to

$$P_{H,t}^{*O} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} \left(P_{H,t+k}^* \right)^\zeta (C_{t+k}^* + I_{t+k}^* + M_{t+k}^*) P_{t+k}^W}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} \left(P_{H,t+k}^* \right)^\zeta (C_{t+k}^* + I_{t+k}^* + M_{t+k}^*)} \quad (150)$$

where the m index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history.

We can now write the fraction (150)

$$\frac{P_{H,t}^{*O}}{P_{H,t}^*} = \frac{\zeta}{\zeta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^* \right)^\zeta \left(\frac{P_{H,t+k}^* S_{t+k}}{P_{t+k}} \right) (C_{t+k}^* + I_{t+k}^* + M_{t+k}^*) MC_{H,t+k}^*}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^* \right)^{\zeta-1} \left(\frac{S_{t+k} P_{H,t+k}^*}{P_{t+k}} \right) (C_{H,t+k}^* + I_{H,t+k}^* + M_{H,t+k}^*)} \quad (151)$$

Denoting the numerator and denominator by $J J_{H,t}^*$ and $J_{H,t}^*$ respectively, and introducing a mark-up shock MS_t to MC_t , we write in recursive form

$$\begin{aligned} \frac{P_{H,t}^{*O}}{P_{H,t}^*} &= \frac{J J_{H,t}^*}{J_{H,t}^*} \\ J J_{H,t}^* - \xi \beta \mathbb{E}_t [\Pi_{H,t,t+1}^* \zeta J J_{H,t+1}^*] &= \frac{1}{1 - \frac{1}{\zeta}} \frac{S_t P_{H,t}^*}{P_t} (C_{H,t+k}^* + I_{H,t+k}^* + M_{H,t+k}^*) U_{C,t} MC_{H,t}^* MS_t \\ J_{H,t}^* - \xi \beta \mathbb{E}_t [\Pi_{H,t,t+1}^* \zeta^{-1} J_{H,t+1}^*] &= \frac{S_t P_{H,t}^*}{P_t} (C_{H,t+k}^* + I_{H,t+k}^* + M_{H,t+k}^*) U_{C,t} \end{aligned}$$

Using the aggregate producer price index $P_{H,t}$ and the fact that all resetting firms will choose the same price, by the Law of Large Numbers we can find the evolution of the price index as given by

$$P_{H,t}^{*1-\zeta} = \xi P_{H,t-1}^{*1-\zeta} + (1 - \xi) (P_{H,t}^{*O})^{1-\zeta} \quad (152)$$

which can be written in the form required

$$1 = \xi (\Pi_{H,t-1,t}^*)^{\zeta-1} + (1 - \xi) \left(\frac{P_{H,t}^{*O}}{P_{H,t}} \right)^{1-\zeta} \quad (153)$$

Whilst the distribution of prices is not required to track the evolution of the aggregate price index, (158) below implies a loss of output due to dispersion in prices. Using the demand schedules, we can write the price dispersion that gives the average loss in output as

$$\Delta_{H,t} = \frac{1}{M} \sum_{m=1}^M \left(\frac{P_{H,t}(m)}{P_{H,t}} \right)^{-\zeta} \quad (154)$$

$$\Delta_{H,t}^* = \frac{1}{M} \sum_{m=1}^M \left(\frac{P_{H,t}^*(m)}{P_{H,t}^*} \right)^{-\zeta} \quad (155)$$

for firms $m = 1, \dots, M$. It is not possible to track all $P_t(m)$ but as it is known that a proportion $1 - \xi$ of firms will optimise prices in period t , and from the Law of Large Numbers, that the distribution of non-optimised prices will be the same in as the overall distribution. Therefore, price dispersion can be written as a law of motion

$$\Delta_{H,t} = \xi (\Pi_{H,t-1,t})^{\zeta} \Delta_{H,t-1} + (1 - \xi) \left(\frac{JJ_{H,t}}{J_{H,t}} \right)^{-\zeta}. \quad (156)$$

$$\Delta_{H,t}^* = \xi (\Pi_{H,t-1,t}^*)^{\zeta} \Delta_{H,t-1}^* + (1 - \xi) \left(\frac{JJ_{H,t}^*}{J_{H,t}^*} \right)^{-\zeta}. \quad (157)$$

Using this, aggregate final output is divided between exports $EX_t = C_{H,t}^* + I_{H,t}^* + M_{H,t}^*$ and domestic consumption $Y_t - EX_t = C_{H,t} + I_{H,t} + M_{H,t} + G_t$. Then allowing for dispersion we have

$$Y_t = \left(\frac{EX_t}{Y_t} + \frac{\left(1 - \frac{EX_t}{Y_t}\right)}{\Delta_{H,t}} \right) Y_t^W \quad (158)$$

Price Dynamics of ROW bloc

Price dynamics in the ROW bloc follows in a similar fashion:

$$\begin{aligned} 1 &= \xi^* (\Pi_{F,t-1,t}^*)^{\zeta^*-1} + (1 - \xi^*) \left(\frac{JJ_t^*}{J_t^*} \right)^{1-\zeta^*} \\ \Delta_t^* &= \xi^* (\Pi_{F,t-1,t}^*)^{\zeta^*} \Delta_{t-1}^* + (1 - \xi^*) \left(\frac{JJ_t^*}{J_t^*} \right)^{-\zeta^*} \\ JJ_t^* &= \frac{\zeta^*}{\zeta^* - 1} \frac{P_{F,t}^*}{P_t^*} Y_t^* MS_t^* MC_t^* + \xi^* \mathbb{E}_t \left[\Lambda_{t,t+1}^* (\Pi_{F,t,t+1}^*)^{\zeta^*} JJ_{t+1}^* \right] \\ J_t^* &= \frac{P_{F,t}^*}{P_t^*} Y_t^* + \xi^* \mathbb{E}_t \left[\Lambda_{t,t+1}^* \frac{\left(\Pi_{F,t,t+1}^* \right)^{\zeta^*}}{\Pi_{t,t+1}^*} J_{t+1}^* \right] \end{aligned}$$

$$\tilde{\Pi}^*_t \equiv \frac{\Pi_t^*}{\Pi_{t-1}^* \gamma^*}$$

D Deterministic Non-Zero Net-Inflation and Zero-Growth Steady State of the SOE model

The steady is solved allowing for a non-zero steady state inflation ($\Pi > 1$) and it is obtained by solving the following for aggregate and Ricardian labour supply both in SOE and ROW model.

$$W(1 - \tau^w) = \frac{[1 - \chi] C^C H^{C\psi}}{1 - \beta\chi}$$

$$W(1 - \tau^w) = \frac{[1 - \chi] C^R H^{R\psi}}{1 - \beta\chi}$$

$$W^*(1 - \tau^{*w}) = \frac{[1 - \chi^*] C^{*C} H^{*C\psi^*}}{1 - \beta^*\chi^*}$$

$$W^*(1 - \tau^{*w}) = \frac{[1 - \chi^*] C^{*R} H^{*R\psi^*}}{1 - \beta^*\chi^*}$$

Then, in a non-zero net-inflation steady state, with appropriate choice of units and in recursive form, we have:

$$A = 1 \tag{159}$$

$$MS = 1 \tag{160}$$

$$\mathcal{S}(X) = 0 \tag{161}$$

$$\mathcal{S}'(X) = 0 \tag{162}$$

$$\Lambda = \beta \tag{163}$$

$$H = \bar{H} \tag{164}$$

$$H^C = \frac{1 - \varrho}{1 - \chi\varrho} \tag{165}$$

$$H^R = \frac{H - \lambda H^C}{1 - \lambda} \tag{166}$$

$$\frac{P}{P_H} = 1 \tag{167}$$

$$\frac{P}{P_F} = 1 \tag{168}$$

$$\frac{P^I}{P_H} = 1 \tag{169}$$

$$\frac{P^I}{P_F} = 1 \tag{170}$$

$$\frac{P^I}{P} = 1 \quad (171)$$

$$\frac{P^M}{P_H} = 1 \quad (172)$$

$$\frac{P^M}{P_F} = 1 \quad (173)$$

$$\frac{P^M}{P} = 1 \quad (174)$$

$$Q = 1 \quad (175)$$

$$\Pi_H = \bar{\Pi} \quad (176)$$

$$\Pi_F = \bar{\Pi} \quad (177)$$

$$\Pi = \bar{\Pi} \quad (178)$$

$$R^K = \frac{R}{\bar{\Pi}} \quad (179)$$

$$\frac{JJ}{J} = \left(\frac{1 - \xi (\bar{\Pi})^{\zeta-1}}{1 - \xi} \right)^{\frac{1}{1-\zeta}} \quad (180)$$

$$MC = \frac{JJ \zeta - 1}{J} \frac{1 - \xi \beta (\bar{\Pi}^H)^\zeta}{1 - \xi \beta (\bar{\Pi})^{\zeta-1}} \quad (181)$$

$$\Delta = \frac{(1 - \xi) \left(\frac{JJ}{J} \right)^{-\zeta}}{1 - \xi (\bar{\Pi}^H)^\zeta} \quad (182)$$

$$KY^W = \frac{MC \frac{P^H}{P} (1 - \alpha_H - \alpha_M)}{\frac{RK Q - (1-\delta) Q}{(1-\tau^K)}} \quad (183)$$

$$Y^W = KY^W \frac{1 - \alpha_H - \alpha_M}{\alpha_H} H \quad (184)$$

$$M = \alpha_M MC Y^W \quad (185)$$

$$W = \alpha_H \frac{Y^W}{H} MC \quad (186)$$

$$K = Y^W KY^W \quad (187)$$

$$Y = \frac{Y^W}{\Delta} \quad (188)$$

$$I = \delta K \quad (189)$$

$$G = g_y Y \quad (190)$$

$$\bar{G} = g_y Y \quad (191)$$

$$r^K = (1 - \alpha_H - \alpha_M) \frac{Y}{K} MC (1 - \tau^k) \quad (192)$$

$$\tau^w = \frac{G - (1 - \alpha_H - \alpha_M) Y MC \tau^k - Y^O RER P^{*O} \tau^O}{WH} \quad (193)$$

$$C^C = H^C (1 - \tau_w) W \quad (194)$$

$$JJ = \frac{\frac{\zeta}{\zeta-1} YMC}{1 - \xi\beta (\Pi_H)^\zeta} \quad (195)$$

$$J = \frac{\frac{P_H}{P} Y}{1 - \xi\beta (\Pi)^\zeta} \quad (196)$$

$$U^R = \frac{[C^R - \chi C^R]^{1-\sigma_R}}{1 - \sigma_R} \exp \left[(\sigma_R - 1) \frac{H^{R1+\psi_R}}{1 + \psi_R} \right] \quad (197)$$

$$U^C = \frac{[C^C - \chi C^C]^{1-\sigma_C}}{1 - \sigma_C} \exp \left[(\sigma_C - 1) \frac{H^{C1+\psi_C}}{1 + \psi_C} \right] \quad (198)$$

$$\lambda = \frac{(1 - \sigma_R)U^R}{C^R - \chi C^R} - \beta\chi \frac{(1 - \sigma_R)U^R}{C^R - \chi C^R} \quad (199)$$

$$U = \lambda U_t^C + (1 - \lambda)U_t^R \quad (200)$$

$$V = \frac{U}{1 - \beta} \quad (201)$$

$$\frac{EX}{Y} = \text{targexcs} + \text{targexis} + \text{targexim} \quad (202)$$

$$Y_t = \left(\frac{(\text{targexcs} + \text{targexis} + \text{targexim})}{\Delta_{H,t}^*} + \frac{(1 - (\text{targexcs} + \text{targexis} + \text{targexim}))}{\Delta_t} \right) Y_t^W \quad (203)$$

$$JJ = \frac{\frac{\zeta}{\zeta-1} \frac{P_H}{P} (Y - EX)MC}{1 - \xi\beta (\Pi_H)^\zeta} \quad (204)$$

$$J = \frac{\frac{P_H}{P} (Y - EX)}{1 - \xi\beta (\Pi_H)^\zeta} \quad (205)$$

$$JJ_H^* = \frac{\frac{\zeta}{\zeta-1} \frac{S P_H^*}{P} EXMC_H^*}{1 - \xi\beta (\Pi_H^*)^\zeta} \quad (206)$$

$$J_H^* = \frac{\frac{S P_H^*}{P} EX}{1 - \xi\beta (\Pi_H^*)^\zeta} \quad (207)$$

$$MC_H^* = \frac{JJ_H^* \zeta - 1}{J_H^* \zeta} \frac{1 - \xi\beta (\Pi_H^*)^\zeta}{1 - \xi\beta (\Pi_H^*)^\zeta} \quad (208)$$

$$\Delta_H^* = \frac{(1 - \xi) \left(\frac{JJ_H^*}{J_H^*} \right)^{-\zeta}}{1 - \xi (\Pi_H^*)^\zeta} \quad (209)$$

$$\frac{S P_H^*}{P} = \frac{MC \frac{P_H}{P}}{MC_H^*} \quad (210)$$

$$A^* = 1 \quad (211)$$

$$MS^* = 1 \quad (212)$$

$$S^*(X) = 0 \quad (213)$$

$$S^{*I}(X) = 0 \quad (214)$$

$$\Lambda^* = \beta^* \quad (215)$$

$$H^* = \bar{H}^* \quad (216)$$

$$H^{*C} = \frac{1 - \varrho^*}{1 - \chi^* \varrho^*} \quad (217)$$

$$H^{*R} = \frac{H^* - \lambda^* H^{*C}}{1 - \lambda^*} \quad (218)$$

$$\mathcal{T}^* = \frac{1}{\bar{\mathcal{T}}^*} \quad (219)$$

$$\frac{P^*}{P_F^*} = 1 \quad (220)$$

$$\frac{P^*}{P_H^*} = 1 \quad (221)$$

$$\frac{P^{*I}}{P_F^*} = 1 \quad (222)$$

$$\frac{P^{*I}}{P_H^*} = 1 \quad (223)$$

$$\frac{P^{*I}}{P^*} = 1 \quad (224)$$

$$Q^* = 1 \quad (225)$$

$$\frac{P^{*M}}{P_F^*} = 1 \quad (226)$$

$$\frac{P^{*M}}{P_H^*} = 1 \quad (227)$$

$$\frac{P^{*M}}{P^*} = 1 \quad (228)$$

$$RER = 1 \quad (229)$$

$$\Pi_H^* = \bar{\Pi} \quad (230)$$

$$\Pi_F^* = \bar{\Pi} \quad (231)$$

$$\Pi^* = \bar{\Pi} \quad (232)$$

$$R^* = \frac{\Pi^*}{\beta^*} \quad (233)$$

$$R^{*K} = \frac{R^*}{\Pi^*} \quad (234)$$

$$\Pi_S = 1 \quad (235)$$

$$\phi = \frac{R}{R^* \Pi_S} = \frac{\Pi}{\Pi^* \Pi_S} \frac{\beta^*}{\beta} = \frac{\beta^*}{\beta} \quad (\text{using } R^* = \frac{\Pi^*}{\beta^*}) \quad (236)$$

$$P^{B^*} = \frac{1}{R^* \phi} \quad (237)$$

$$B_F = -\frac{Y \log(\phi)}{\phi_B} \geq 0 \text{ iff } \beta \geq \beta^* \quad (238)$$

$$TB = \left(\frac{1}{\phi R^*} - \frac{\Pi^S}{\Pi} \right) B_F \quad (239)$$

$$C = Y^O RER P^{*O} + Y - I - M - G - TB \quad (240)$$

$$C^R = \frac{1}{1-\lambda} (C - \lambda) C^C \quad (241)$$

$$U_C^R = (1-\varrho) (C^R - \chi C^R)^{(1-\varrho)(1-\sigma)-1} (1 - H^R)^{\varrho(1-\sigma)} \quad (242)$$

$$\frac{JJ^*}{J^*} = \left(\frac{1 - \xi^* (\Pi^{*F})^{\zeta^*-1}}{1 - \xi^*} \right)^{\frac{1}{1-\zeta^*}} \quad (243)$$

$$MC^* = \frac{JJ^* \zeta^* - 1}{J^* \zeta^*} \frac{1 - \xi^* \beta^* (\Pi^{*F})^{\zeta^*}}{1 - \xi^* \beta^* (\Pi^{*F})^{\zeta^*-1}} \quad (244)$$

$$\Delta^* = \frac{(1 - \xi^*) \left(\frac{JJ^*}{J^*} \right)^{-\zeta^*}}{1 - \xi (\Pi^{*F})^{\zeta^*}} \quad (245)$$

$$KY^{*W} = \frac{MC^* (1 - \alpha_H^* - \alpha_M^*)}{\frac{RK^* Q^* - (1-\delta^*) Q^*}{(1-\tau^{K^*})}} \quad (246)$$

$$Y^{*W} = KY^{*W} \frac{1 - \alpha_H^* - \alpha_M^*}{\alpha_H^*} H^* \quad (247)$$

$$M^* = \alpha_M^* MC^* Y^{*W} \quad (248)$$

$$W^* = \alpha_H^* \frac{Y^{*W}}{H^*} MC^* \quad (249)$$

$$K^* = Y^{*W} KY^{*W} \quad (250)$$

$$Y^* = \frac{Y^{*W}}{\Delta^*} \quad (251)$$

$$I^* = \delta^* K^* \quad (252)$$

$$G^* = g_y^* Y^* \quad (253)$$

$$r^{*K} = (1 - \alpha_H^* - \alpha_M^*) \frac{Y^*}{K^*} MC^* (1 - \tau^{*k}) \quad (254)$$

$$\tau^{*w} = \frac{G^* - (1 - \alpha_H^*) Y^* MC^* \tau^{*k}}{W^* H^*} \quad (255)$$

$$C^{*C} = H^{*C} (1 - \tau_w^*) W^* \quad (256)$$

$$JJ^* = \frac{\frac{\zeta^*}{\zeta^*-1} Y^* MC^*}{1 - \xi^* \beta^* (\Pi^{*F})^{\zeta^*}} \quad (257)$$

$$J^* = \frac{\frac{P_E^*}{P^*} Y^*}{1 - \xi^* \beta^* (\Pi^{*F})^{\zeta^*-1}} \quad (258)$$

$$EX^* = 0; TB^* = 0; C^* = Y^* - I^* - M^* - G^* - TB^* \quad (259)$$

$$C^{*R} = \frac{1}{1-\lambda^*} (C^* - \lambda^*) C^{*C} \quad (260)$$

$$I_F = (1 - w_I) I \quad (261)$$

$$C_F = (1 - w_C)C \quad (262)$$

$$M_F = (1 - w_M)M \quad (263)$$

$$I_H = (w_I)I \quad (264)$$

$$C_H = (w_C)C \quad (265)$$

$$M_H = (w_M)M \quad I_F^* = w_I^* I^* \quad (266)$$

$$C_F^* = w_C^* C^* \quad (267)$$

$$M_F^* = w_M^* M^* \quad (268)$$

$$I_H^* = (1 - w_I^*) I^* \quad (269)$$

$$C_H^* = (1 - w_C^*) C^* \quad (270)$$

$$M_H^* = (1 - w_M^*) M^* \quad (271)$$

$$U^{*R} = \frac{[C^{*R} - \chi^* C^{*R}]^{1-\sigma_R^*}}{1 - \sigma_R^*} \exp \left[(\sigma_R^* - 1) \frac{H^{*R^{1+\psi_R^*}}}{1 + \psi_R^*} \right] \quad (272)$$

$$U^{*C} = \frac{[C^{*C} - \chi^* C^{*C}]^{1-\sigma_C^*}}{1 - \sigma_C^*} \exp \left[(\sigma_C^* - 1) \frac{H^{*C^{1+\psi_C^*}}}{1 + \psi_C^*} \right] \quad (273)$$

$$Lam^* = \frac{(1 - \sigma_R^*) U^{*R}}{C^{*R} - \chi^* C^{*R}} - \beta^* \chi^* \frac{(1 - \sigma_R^*) U^{*R}}{C^{*R} - \chi^* C^R} \quad (274)$$

$$U^* = \lambda^* U_t^{*C} + (1 - \lambda^*) U_t^{*R} \quad (275)$$

$$V^* = \frac{U^*}{1 - \beta^*} \quad (276)$$

$$(277)$$

$$imcs = (1 - w_C) \frac{C}{Y} \quad (278)$$

$$imis = (1 - w_I) \frac{I}{Y} \quad (279)$$

$$imim = (1 - w_M) \frac{M}{Y} \quad (280)$$

$$exco = \frac{Y^O RER P^{*O}}{Y} \quad (281)$$

$$tb \equiv \frac{TB}{Y} = exco + excs + exis + exim - imcs - imis - imim \quad (282)$$

In this calibrated version of the model, the external steady state also solves using fsolve the following for w_C , w_I , w_M , β , Y^O and A to target $imcs = targimcs$, $imis = targimis$, $imim = targimim$, $exco = targexco$, $tb \equiv \frac{TB}{Y} = targtb$ and $\frac{Y^*}{Y} = targY^*byY$

$$imcs = targimcs \quad (283)$$

$$imis = targimis \quad (284)$$

$$imim = targimim \quad (285)$$

$$exco = targexco \quad (286)$$

$$tb = targtb \quad (287)$$

$$\frac{Y^*}{Y} = targY^*byY \quad (288)$$

Note that targets have been imposed in export equations which make the steady state above recursive. Note also that we introduce a new variable *CheckTB* into the code verifies that $CheckTB = 0$.

E Data Preparation

E.1 US and Canada Preparations and the ‘filter’ used to stationarize the non-stationary data

The data set are taken from the FRED Database available through the Federal Reserve Bank of St.Louis. The sample period is 1993-2018 in first difference at quarterly frequency. Namely, these observable variables are the log difference of real GDP, log difference of real consumption, the log difference of the GDP deflator and the federal funds rate. All series are seasonally adjusted. Since the variables in the model state space are measured as deviations from a constant steady state, we take the first difference of the real GDP in order to obtain DSP. For real variables we take the log of the original data. Inflation and nominal interest rates are used as they are in perproportion terms. A full description of the data used in the subsequent Bayesian Estimation of the ROW closed economy model are summarized below:

- NGDP: Gross domestic product; BEA table 1.1.5, line 1
- RGDP: Real Gross domestic product; BEA table 1.1.6, line 1
- PCE: Personal Consumption expenditure; BEA Table 1.1.5, line 2
- PFI: Private Fixed Investment; BEA Table 5.3.5; line 1
- CNP16OV: Civilian non institutional population, FRED database, BLS
- *CNP16OV_index*: $CNP16OV(2005:2)=1$
- FFR: Federal Fund Rate, Fred Database, FED St. Louis
- *GDP_Deflator*: $NGDP/RGDP*100$
- BAA: Moody’s seasoned Baa corporate bond yields, FED Board.

The five observables used in the estimation are then constructed as follows:

- $Y_t^{obs} = D \left(LN \left(\frac{RGDP_t}{index_t} \right) \right)$
- $C_t^{obs} = D \left(LN \left(\frac{PCE_t/index_t}{GDP_Deflator_t} \right) \right)$
- $I_t^{obs} = D \left(LN \left(\frac{PFI_t/index_t}{GDP_Deflator_t} \right) \right)$
- $\Pi_t^{obs} = D (LN (GDP_Deflator_t))$

- $R_{n,t}^{obs} = FFR_t/4/100$

The corresponding measurement equations for the 5 observables are:

$$\begin{bmatrix} Y_t^{obs} \\ C_t^{obs} \\ I_t^{obs} \\ \Pi_t^{obs} \\ R_{n,t}^{obs} \end{bmatrix} = \begin{bmatrix} \log\left(\frac{Y_t}{Y_{t-1}}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-2}}\right) \\ \log\left(\frac{C_t}{C_{t-1}}\right) - \log\left(\frac{C_{t-1}}{C_{t-2}}\right) \\ \log\left(\frac{I_t}{I_{t-1}}\right) - \log\left(\frac{I_{t-1}}{I_{t-2}}\right) \\ \log\left(\frac{\Pi_t}{\Pi}\right) \\ \log\left(\frac{R_{n,t}}{R_n}\right) \end{bmatrix} \quad (289)$$

E.2 Canada Data Preparation and the ‘filter’ used to stationarize the non-stationary data

We use data on output growth, consumption growth, investment growth, inflation, interest rates, exchange rates growth and terms of trade growth. Output, consumption and investment growth are the log difference of real GDP, real INV and real CONS, inflation is the log difference in CPI in proportion terms, nominal interest rates are measure by federal reserve (divided by 4 to be consistent in quarterly terms), depreciation rates are computed as the log difference of the exchange rate. Thus, we have applied a first-differences filter to the raw data, with the exception of inflation and interest rates. All data is seasonally adjusted where relevant. In addition, we demean the data, as the sample averages are not consistent with the steady-state relationships. The data sources and transformation used in the above descriptive analysis and in the subsequent Bayesian Estimation of the open economy model are described below:

- *GDP* : Real Gross Domestic Product - Constant prices, Seasonally Adjusted. *Source: Federal Reserve Bank of St. Louis.*
- *PCE* : Personal Consumption Expenditures - Constant prices, Seasonally Adjusted. *Source: Federal Reserve Bank of St. Louis.*
- *GFKF* : Gross Fixed Capital Formation - Constant prices, Seasonally Adjusted. *Source: Federal Reserve Bank of St. Louis.*
- Policy Rate (PR) :Wu-Xia Shadow Federal Funds Rate.textitSource: Federal Reserve Bank of Atlanta.
- ER : Reference Exchange Rate - Not Seasonally Adjusted. *Source: Federal Reserve Bank of St. Louis.*
- *CPI* : Consumer Price Index - 2015=100, Raw data not seasonally adjusted, seasonal adjustment done using the Eviews’ X-12 filter.*Source: Federal Reserve Bank of St. Louis.*
- OP: Real Oil Price the original data is nominal which is made real by the CPI of US- Raw data not seasonally adjusted, seasonal adjustment done using the Eviews’ X-12 filter. *Source: Federal Reserve Bank of St. Louis*

The observables used in the estimation are then constructed as follows:

- $Y_t^{obs} = D(LN(GDP_t^C))$
- $C_t^{obs} = D(LN(PCE_t^C))$
- $I_t^{obs} = D(LN(GFKF_t^C))$
- $er_t^{obs} = D(LN(ER_t))$
- $\Pi_t^{obs} = D(LN(CPI_t^C))$
- $R_{n,t}^{obs} = FFR_t/4/100$
- $OP_t^{obs} = D(LN(OP_t))$

$$\begin{bmatrix} Y_t^{obs} \\ C_t^{obs} \\ I_t^{obs} \\ OP_t^{obs} \\ er_t^{obs} \\ \Pi_t^{obs} \\ R_{n,t}^{obs} \end{bmatrix} = \begin{bmatrix} \log\left(\frac{Y_t}{Y_{t-1}}\right) - \log\left(\frac{Y_{t-1}}{Y_{t-2}}\right) \\ \log\left(\frac{C_t}{C_{t-1}}\right) - \log\left(\frac{C_{t-1}}{C_{t-2}}\right) \\ \log\left(\frac{I_t}{I_{t-1}}\right) - \log\left(\frac{I_{t-1}}{I_{t-2}}\right) \\ \log\left(\frac{OP_t}{OP_{t-1}}\right) - \log\left(\frac{OP_{t-1}}{OP_{t-2}}\right) \\ \log\left(\frac{\Pi_t^S}{\Pi^S}\right) \\ \log\left(\frac{\Pi_t}{\Pi}\right) \\ \log\left(\frac{R_{n,t}}{R_n}\right) \end{bmatrix} \quad (290)$$

C Bayesian Estimation Summary Of The ROW Closed Economy Model

This section presents results for the Bayesian estimation of the rest-of-the-world bloc using quarterly data on the log difference of real GDP, log difference of real consumption, the log difference of the GDP deflator and the federal funds rate. All series are seasonally adjusted, taken from the FRED Database.

Some structural parameters are kept fixed, as is standard in the literature (see Table 15).

Calibrated parameter	Symbol	Value
Discount factor	β^*	0.99
Depreciation rate	δ^*	0.025
Government expenditure-output ratio	g_y^*	0.15
Capital taxation rate	τ_r^*	0.00

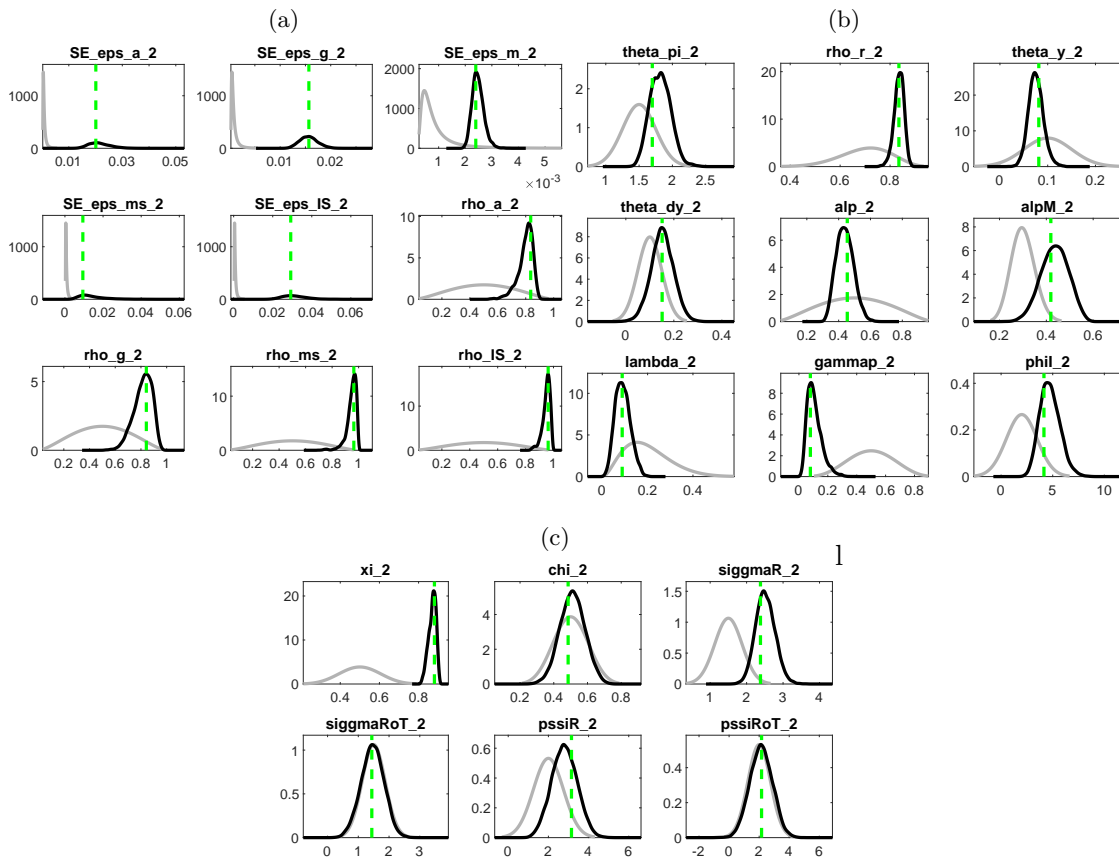
Table 15: Calibrated parameters in the ROW bloc

Table 16 summarizes the prior distribution, estimated posterior means and 90% confident intervals, with the marginal data density of the model computed using the Geweke (1999) modified harmonic-mean estimator. Figure 3 depicts the corresponding prior and posterior distributions.

Estimated Parameter Values	Prior		Posterior		
	Symbole	Dist. (Mean,Std Dev)	Mean	90% HPD Interval	
ROW					
Technology shock	ϵ_{A^*}	IG	0.001, 0.02	0.0214	0.0150 , 0.0276
Government shock	ϵ_{G^*}	IG	0.001, 0.02	0.0156	0.0128 , 0.0185
Monetary policy shock	ϵ_{M^*}	IG	0.001, 0.02	0.0025	0.0021 , 0.0028
Markup shock	ϵ_{MS^*}	IG	0.001, 0.02	0.0140	0.0045 , 0.0238
Investment shock	ϵ_{IS^*}	IG	0.001, 0.02	0.0311	0.0215 , 0.0406
Technology shock persistence	ρ_{A^*}	β	0.50,0.20	0.7979	0.7152 , 0.8777
Markup shock persistence	ρ_{G^*}	β	0.50,0.20	0.8142	0.7003 , 0.9234
Investment shock persistence	ρ_{MS^*}	β	0.50,0.20	0.9466	0.8996 , 0.9958
Government shock persistence	ρ_{IS^*}	β	0.50,0.20	0.9486	0.9098 , 0.9924
Monetary Policy shock persistence	ρ_{M^*}	β	0.70,0.10	0.8357	0.8049 , 0.8686
Feedback from inflation	θ_{π^*}	N	1.50,0.25	1.8174	1.5559 , 2.0585
Feedback from output	θ_{y^*}	N	0.10,0.05	0.0752	0.0493 , 0.0994
Feedback from output growth	θ_{dy^*}	N	0.10,0.05	0.1512	0.0756 , 0.2295
Share of non-Ricardian consumers	λ^*	β	0.20,0.10	0.0900	0.0385 , 0.1454
Consumption habit formation	χ^*	β	0.50,0.10	0.5094	0.3859 , 0.6275
Labour Share	α^*	β	0.50,0.20	0.4391	0.3509 , 0.5234
Intermediate goods Share	α_M^*	β	0.30,0.05	0.4349	0.3373 , 0.5282
Calvo price stickiness	ξ^*	β	0.50,0.10	0.8695	0.8379 , 0.8995
Price index	γ^*	β	0.50,0.15	0.1070	0.0305 , 0.1806
Elasticity of Investment adjustment cost	ϕ_I^*	N	2.00,1.50	4.7200	3.1459 , 6.2543
Ricardian risk aversion	σ_R^*	N	1.50,.375	2.5116	2.0671 , 2.9302
Non-Ricardian risk aversion	σ_C^*	N	1.50,.375	1.4574	0.8361 , 2.0457
Ricardian frisch elasticity	ψ_R^*	N	2.00,0.75	2.7899	1.7332 , 3.7896
Non-Ricardian frisch elasticity	ψ_c^*	N	2.00,0.75	2.1194	0.9064 , 3.3487

Table 16: Estimated shocks and parameter values in the ROW

Figure 3: Prior and Posterior Distributions for the ROW model estimation



D Bayesian estimation of the SOE model

This section presents additional results mentioned in the main text concerning the SOE bloc estimation, namely the prior and posterior distributions for the SOE parameters in Figures 4 and 5.

Figure 4: Prior and Posterior Distributions for the SOE model estimation- PCP case

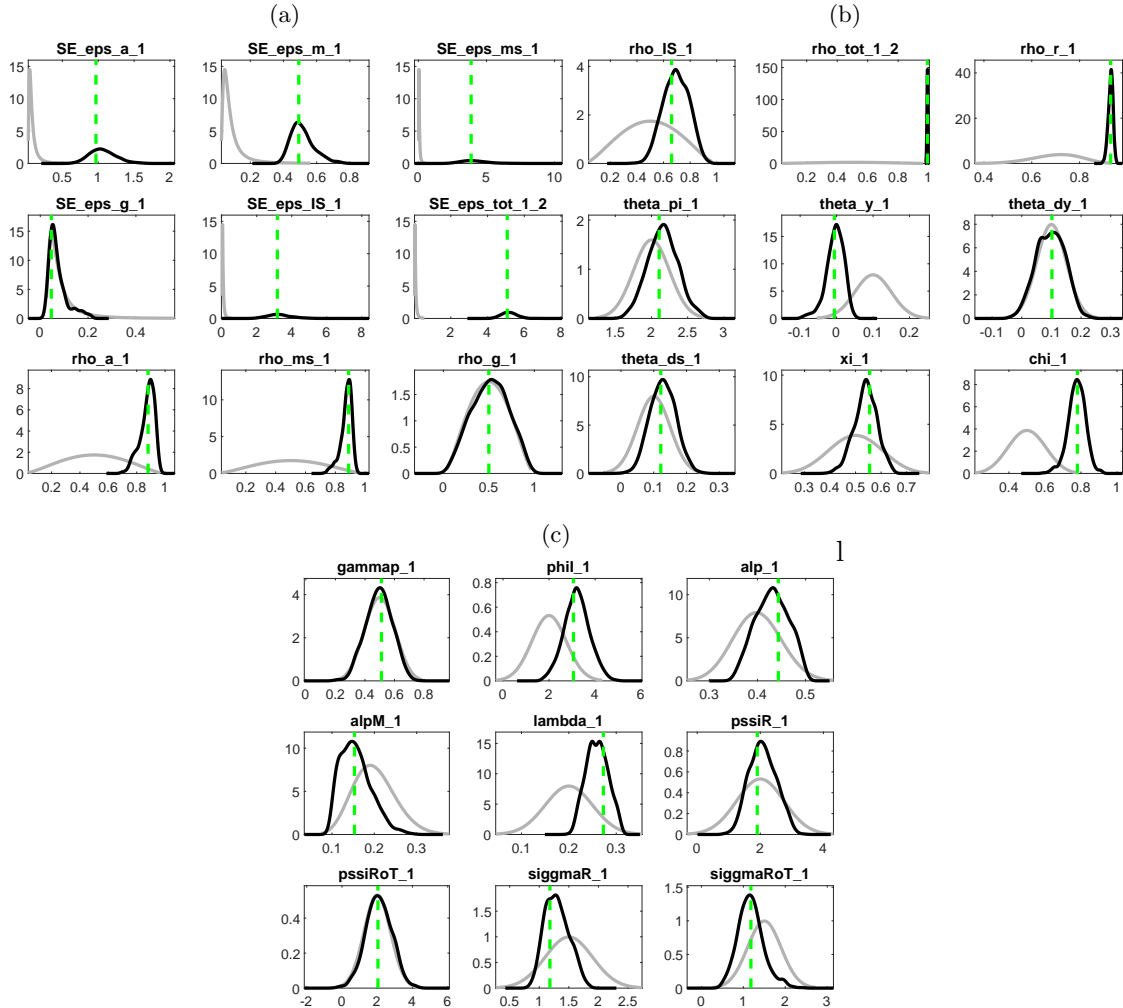
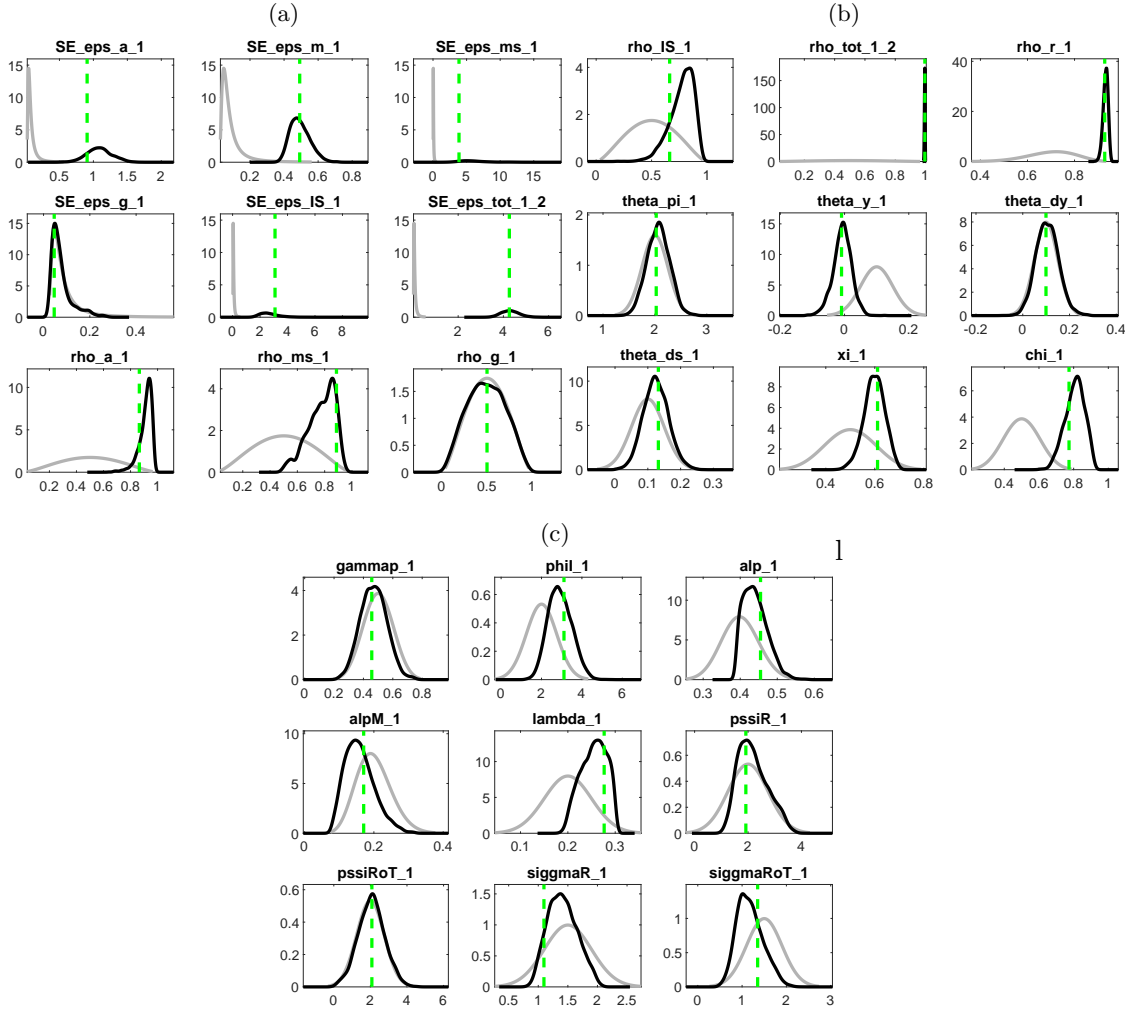


Figure 5: Prior and Posterior Distributions for the SOE model estimation- LCP case



E Calibration of the proportion of rule-of-thumb households

According to the Federal Deposit Insurance Corporation – FDIC (2014), 7.7% of the US households in 2013 did not have a bank account and an additional 20% were underbanked, meaning that they had a bank account, but also used alternative financial services (AFS) outside of the banking system (Mylonidis et al, 2019).

Wang and Guan (2016) measure the level of financial inclusion across 127 countries using the index of financial inclusion (IFI) and the World Bank Global Findex database, with a ranking for Canada and United states of 0.7232 (2) and 0.6870 (4), respectively. Gali et al. (2007) and De Graeve et al. (2010) employ 0.5 and 0.6 for the share of rule-of-thumb consumers, respectively. Campbell and Mankiw (1989 and 1990) use 35 percent, while Fuhrer (2000) employs the estimate in the range of 26–29 percent depending on the econometric method used.

Furlanetto and Seneca (2009) calibrate the share of rule-of-thumb consumers to be between 29 percent and 35 percent, while Kaszab (2016) set it to 0.3, which is more plausible empirically than

the 0.5 used by Gali, Lopez-Salido and Valles (2007).

In other literature, Campbell and Mankiw (1991) extend their earlier work to evaluate other developing nations - United Kingdom, Canada, France, Japan and Sweden. Their estimates indicate differences across countries concerning the effect of current income and consumption. To surmise, consumption is affected least by current income in Canada (0.236), but more in the US (0.363), Sweden (0.357), UK (0.372) and most of all in the France (0.974).

Many of the previous papers follow Campbell and Mankiw (1989), which assume that 50% of US HHs are liquidity constrained. Table 17 summarises this information:

Table 17: Calibrated share of credit-constrained consumers

Paper	Country	value
Hall and Mishkin (1982)	US	0.2
Campbell and Mankiw (1989)	US	0.5
Juppelli (1990)	US	0.19
Garcia, Lusardi and Ng (1997)	US	0.16
Gali et al. (2007)	US	0.5
Grant (2007)	US	0.31
Kumhof and Laxton (2007)	US	0.33
Benito and Mumtaz (2006)	US	0.2-0.4
Faruqui and Torchani (2012)	Canada	0.23
Gervais and Gosselin (2014)	Canada	0.14
Alichi, Shibata and Tanyeri (2019)	SOE	0.15

In sum, the introduction of non-Ricardian households into a small open economy model is motivated by the work Boerma (2014) done on cross-country data on household participation in financial markets, and by cross-country variation in the degree of openness¹⁶. The data¹⁷ is summarized in Table (18), which shows that the level of financial inclusion, as measured by the percentage of adults with a bank account at a formal financial institution, varies significantly across countries¹⁸. In low income countries, only 19% of the population has access to basic financial products. In high income countries this figure amounts to 89%. High income countries also import a greater share of their consumption bundle. Based on the literature, as our sample period is over 1993-2018, we are suggesting the prior of 0.2 for US and Canada.

¹⁶The degree of openness is approximated by domestic imports over domestic spending.

¹⁷Source: World Bank Development Indicators, World Bank Global Financial Inclusion Database, and author's own calculations

¹⁸This 'narrow' definition of asset market participation precludes the use of LAMP as a free parameter to capture the impact of financial frictions, uncertainty, and sub-optimal decision-making on the aggregate marginal propensity to consume.

Table 18: Cross-Country Data on Financial Inclusion, and Openness

Country Classification	No. of Countries	Financial Exclusion	Openness
Low Income	25/36	0.81	0.40
Lower Middle Income	31/48	0.72	0.45
Upper Middle Income	33/55	0.50	0.46
High Income	40/78	0.11	0.55

Notes: Financial inclusion is measured by the percentage of adults with a bank account at a formal financial institution. This ‘narrow’ definition of asset market participation precludes the use of non-Ricardian Consumers as a free parameter to capture the impact of financial frictions, uncertainty, and suboptimal decision-making on the aggregate marginal propensity to consume; the degree of openness is approximated by domestic imports over domestic spending.

Source: World Bank Development Indicators, World Bank Global Financial Inclusion Database, and author’s own calculations.