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# OPTIMAL LOCKDOWN IN AN EPIDEMIOLOGICAL-MACROECONOMIC MODEL

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# Optimal Lockdown in an Epidemiological-Macroeconomic Model

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**Abstract**

This paper sets out a coherent framework for studying the economic effects of the Covid-19 pandemic, and policies aimed at controlling both the health and economic trade-offs that it poses. It does this by combining two key epidemiological and macroeconomic models: the SIR model and the RBC model. We argue that much of the present literature can be understood using this framework. The SIR-type epidemiology model in the paper has the novel feature of both no-disease and endemic steady states, two possible outcomes of Covid-19. The stability properties of these equilibria are examined and are shown to depend on the reproduction number and also, possibly, on the complex dynamics introduced by 'predator-prey' behaviour of the virus. In addition, we show how endogenous social interaction fits within the model. Lockdown – reducing the size of the susceptible population – is then introduced into the RBC model as a social planner's problem. By linking this epidemiolgy model with a simple RBC model, we provide an integrated framework for examining the economic effects of Covid-related policies and the economic cost of lockdown policies of particular scope and duration. In principle an empirical implementation of this framework can be used to deduce the price of a life implied by a particular lockdown policy. Looking forward, extensions of our framework offer the chance to study economic challenges in areas such as debt financing, human capital shocks, or vaccine production and roll-out, all of which are inevitably emerging.

**Keywords:** lockdown, epidemiology-RBC model, mortality-macroeconomic trade-off
**JEL Codes**: C63, D58, E24, E27, E32, E37

# Contents

# 1 Introduction

The Covid-19 pandemic has posed profound challenges for societies across the world and these have been reflected in the remarkable development of responses and analyses amongst physical and social scientists. Work by economists has grown rapidly since early awareness of the virus in the Spring of 2020 and has generated several new series such as Covid Economics (CEPR) and Covid-19 Economic Research (INET) as well as appearing elsewhere. At the most general, this work has focused on the policy trade-offs at the heart of the pandemic (controlling the virus at the expense of economic activity), and a central feature has been the integration of epidemiological models of virus proliferation with various models of the economy in order to specify the trade-offs and analyse possible policy responses.

More specifically, three broad strands can be discerned. One, exemplified by Miles *et al.* (2020), seeks to quantify the cost and benefits of controlling the virus, by balancing its costs in terms of QALYs against the output costs of seeking to limit these by curtailing economic activity. The analysis is numerical, rather than technical, but emphasises the importance, and feasibility, of an evidence-based approach to this naturally emotive context. Two other strands use more technical approaches to analyse the economic effects of the Covid pandemic and to assess the effects of alternative policy responses. The first concentrates on how virus transmission is linked to social and economic activity and how it might be tempered by policies focusing on these. As such, it explicitly models the epidemiology of virus transmission using the classic "Susceptible, Infected, Recovered or Removed" (SIR) model of Kermack and McKendrick (1927) and embeds this, to varying extents, within an economic model. Both Brodeur *et al.* (2020) and Lewis (2020) provide good summaries of this work but we give several examples since the current paper also makes use of this framework. In a sense, the important feature here is how policy can influence the interactions between individuals that promote the spread of the virus. Thus, Eichenbaum *et al.* (2020) endogenise transitions between the stages of the SIR model via the way that economic interactions in consumption and work (themselves resulting from optimising behaviour) affect the probabilities of encountering and contracting the virus. Alternatively, Farboodi *et al.* (2020) consider the effect of social distancing on these transition probabilities. Other work using the SIR model has focused more explicitly on its structure and how this influences the economic effects of the pandemic. For instance,

Acemoglu *et al.* (2020) partition the occupants of the various stages into different groups of the population so that different age-groups can be allowed for[1], while Piguillem and Shi (2020) consider symptomatic and asymptomatic carriers and Gollier (2020) introduces uncertainty about the underlying parameters characterising the virus.

A third strand of literature examines the macroeconomic consequences of the pandemic. Mihailov (2020) derives these economic effects from an explicit macro model (the Gali-Smets-Wouters model) but does not incorporate a specific model of virus transition. Instead, the virus is modelled as a labour supply shock. Chudik *et al.* (2020) also treat the virus as an exogenous shock to employment and trace its effects through that. Like Mihailov, McKibbin and Fernando (2020) examine a formal (DSGE) macro model, analysing the effects of seven scenarios across a number of countries in a Computable General Equilibrium Framework. Other authors aim to link the macroeconomic consequences of Covid-19 to the SIR model of virus transmission. Eichenbaum *et al.* (2020) build their macro-model onto their version of the SIR model (described above), paying particular attention to how policies impact on labour market and consumption activity and, thus, to the probabilities of virus transmission they endogenise through these routes. Bayraktar *et al.* (2020) offer a model where a social planner chooses policies (in particular, relating to lockdown) to minimise a macroeconomic loss function that includes a production function linking output to labour supply. In common with others, the authors investigate several interesting amendments to the SIR model, including 'old' and 'young' population segments and behaviour-dependent transmission, captured by an inter-group interaction parameter, and a 'penalty' for policies that allow excessive pressure to be placed on ICU facilities in hospitals. Their simulations suggests that optimal lockdown, coupled with other policies to slow transmission, can achieve herd immunity before the need for vaccines. Hindsight, of course, raises questions about these findings but does not undermine the interesting attempts to build lockdown, heterogeneous groups and several behavioural policies into a combined SIR/macro model.

The current paper contributes to this literature by presenting a framework for studying the economic effects of the Covid-19 pandemic that, in principle, can incorporate many of the developments and policy responses described above: in this way, the paper can also

[1]As well as age, other groups to have attracted significant attention during the pandemic, and to be amenable to Acemoglu *et al.* (2020)'s approach, would include care home residents.

be read as a survey of the main elements of work to date. We also show how several new developments of existing literature can be treated within the framework. We do this by first developing the SIR model in order to identify its steady state equilibria and then showing how the introduction of 'predator-prey' behaviour enables population dynamics to influence these equilibria. We then integrate this with a simple RBC model with fiscal policy and illustrate how this can be used to study optimal lockdown policies.

To be more precise, the epidemiological component of the paper extends the SIR model, drawing upon and adapting the recent work of Cui *et al.* (2020), which is a model of hepatitis C. The SIR model is extended to a SIHR model by adding a hospitalized (H) group. Sections 2 and 3 set out this component. We show that the epidemiology dynamics have both a 'no-disease' steady state and an endemic one in which the disease does not 'die away' in equilibrium, thus offering two possible outcomes of Covid-19. The stability properties of these equilibria are examined and are shown to depend on the reproduction number. Section 2.4 adapts the SIR model to allow for 'predator-prey' behaviour along the lines of the seminal Lotka-Volterra model, thereby allowing it to illustrate the complex dynamics that a virus can exhibit, perhaps along the lines of the more recent 'waves' associated with Covid-19.

Section 4 shows how the SIHR epidemiology model can incorporate endogenous social interaction, along the lines of Farboodi *et al.* (2020). The process for solving the decentralized laissez-faire market equilibrium where the level of social activity is chosen by households is compared with that for solving the social planner's problem, where the quantity of interaction is chosen to maximise social welfare. We note that treating this as a deterministic model with perfect foresight rational expectations admits a standard solution technique but we also discuss what would be required to solve a stochastic version, with information frictions relating to (say) perceptions of risk under uncertainty.

Section 5 then presents a simple model that, when integrated with the SIR model, allows us to indicate how an optimal Covid policy (in our case, lockdown) can be studied. Our RBC model is a richer macroeconomic setting compared with Eichenbaum *et al.* (2020); it includes capital as a factor of production, capital as well as labour taxes and a government non-balanced budget constraint that enables us to study the debt aftermath of lockdown. Medium-sized New Keynesian DSGE models, such as Smets and Wouters

(2007), have an RBC core in the limit as sticky prices and wages become flexible. Our model is then are well-suited to interesting extensions that can incorporate, alongside Keynesian features, financial frictions along the lines of Bernanke *et al.* (1999), Gertler and Karadi (2011), Gertler and Kiyotaki (2012) and Iacoviello (2015). It can also be extended to include an R&D (perhaps pharmaceutical) sector that results in endogenous technical change as in Comin and Gertler (2006) and Comin *et al.* (2016). In addition, the trade-off between lives saved and foregone economic activity suggests an implicit value of life, as indicated in Section 6, where lockdown is chosen as a social planner's problem. Our SIR/RBC model, based on well-established modelling, should therefore be seen as a helpful framework for thinking about many features of the economic effects of Covid-19, and policies to combat it.

In Section 7, we provide conclusions and directions for future research. Our framework has the benefit of accommodating, and opening up, a number of policy and modelling opportunities which, we argue, can lead to empirically-backed contributions on the design and effects of policy, both now and in the future. Not least amongst these is the opportunity, offered by the incorporation of a fully specified macro model, to consider how the inevitable government expenditures of the past year can be financed, whether the pandemic is eliminated or remains endemic (possibly through variants), and against a background of shocks to human capital that may emerge from Covid-19's impact on education.[2]

## 2 Covid-19 Model

We adapt the seminal SIR model to model important features of the Covid-19 virus epidemic, drawing upon the recent work of Cui *et al.* (2020) which is a model of hepatitis C. We now allow for **four categories** of living people: susceptible (S), newly infected with the virus (I), seriously infected and hospitalized (H) and recovered (R). Then in discrete time in period $t$, the living population $N_t = S_t + I_t + H_t + R_t$. In addition there is a mortality stock of people ($D_t$) who have died from the virus. $N_t$, $S_t$, $I_t$, $H_t$ and $R_t$ are

[2]On completing this paper our attention was drawn to a special issue in the Journal of Mathematical Economics with an editorial introduction by Boucekkine *et al.* (2021). The articles address multiple topics where economic theory may help understand the evolution of a pandemic, the policies that may best deal with it, and its socio-economic implications. A survey of this impressive work is beyond the scope of our article, except to say that both complementary and contrasting approaches to the use of an integrated epidemiology-macro model for designing lock-down policy are to be found in this issue.

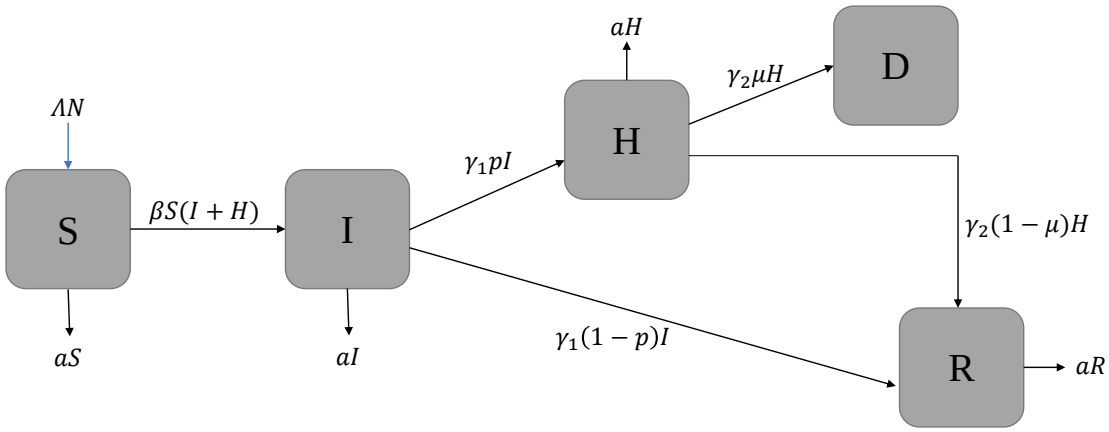


Figure 1: Covid-19 Model

end-of-period $t$ stocks of people.

Flows take place from the S-group to the I-group; then from the I-group to the H-group with probability $p$ and to the R-group with probability $(1-p)$ both at a rate $\gamma_1$; then from the H-group to the D-group with a probability $\mu$ and the R-group with probability $1-\mu$ at rate $\gamma_2$. The infection rate $\beta$ is defined by the flow of new infections $= \beta S_t(I_t + H_t)$. For all groups there is a natural mortality rate $a$. For the S-group there is a birth rate $\Lambda$ = the natural mortality rate $a$. Population then declines at the virus mortality rate $\mu$.

## 2.1 Dynamics with Full Immunity

The epidemiology model dynamics are given by:

$$S_t = S_{t-1} + \Lambda N_{t-1} - \beta S_{t-1}(I_{t-1} + H_{t-1}) - aS_{t-1} - \log IS_t \quad (1)$$

$$I_t = I_{t-1} + \beta S_{t-1}(I_{t-1} + H_{t-1}) - (a + \gamma_1)I_{t-1} + \log IS_t \quad (2)$$

$$H_t = H_{t-1} + p\gamma_1 I_{t-1} - (a + \mu)H_{t-1} - (1 - \mu)\gamma_2 H_{t-1} \quad (3)$$

$$
\begin{aligned}
R_t &= R_{t-1} + (1-p)\gamma_1 I_{t-1} + (1-\mu)\gamma_2 H_{t-1} - aR_{t-1} && (4)\\
D_t &= D_{t-1} + \gamma_2 \mu H_{t-1} && (5)\\
N_t &= S_t + I_t + H_t + R_t && (6)\\
IS_t &= IS_{t-1}^{\rho} \exp(\epsilon_{I,t}) && (7)
\end{aligned}
$$

An infection shock at time $t$, $\log IS_t$, transfers people from the S-group to the I-group.

### 2.2 No-Disease and Endemic Steady State Equilibria

Consider a zero-growth steady state with denoted by $S$, $I$, $H$, $R$ and $D$. There are two of these steady-state equilibria, one associated with zero disease and one with permanent disease. We consider these in turn.

The zero-growth steady state of (1)–(6) takes the form:

$$
\begin{aligned}
\Lambda N - \beta S(I+H) - aS &= 0\\
\beta S(I+H) - (a+\gamma_1)I &= 0\\
(1-p)\gamma_1 I + (1-\mu)\gamma_2 H - aR &= 0\\
D &= \gamma_2 \mu H
\end{aligned}
$$

Taking $N$ as given this has a solution

$$
\begin{aligned}
I &= \alpha_I N && (8)\\
H &= \alpha_H N && (9)\\
R &= \alpha_R N && (10)
\end{aligned}
$$

where

$$
\begin{aligned}
\beta S(\alpha_I + \alpha_H) &= (a+\gamma_1)\alpha_I && (11)\\
p\gamma_1 \alpha_I &= (a+\mu+(1-\mu)\gamma_2)\alpha_H && (12)\\
\alpha_R &= \frac{(1-p)\gamma_1\alpha_I + (1-\mu)\gamma_2\alpha_H}{a} && (13)
\end{aligned}
$$

Now introduce the *reproduction number* $\mathcal{R}_0$ that plays a central role in epidemiology

modelling. For now we define $\mathcal{R}_0$ in the steady state by

$$\mathcal{R}_0 \equiv \frac{\Lambda N}{aS} \tag{14}$$

Then from (8)–(14) a little algebra gives

$$\mathcal{R}_0 = \underbrace{\frac{\beta\Lambda N}{a+\gamma_1}}_{\text{primary infections}} + \underbrace{\left(\frac{\beta\Lambda N}{a+\gamma_1}\right)\left(\frac{p\gamma_1}{a+\mu+(1-\mu)\gamma_2}\right)}_{\text{secondary infections}} \tag{15}$$

This shows how the reproduction number $\mathcal{R}_0$ captures two flows: the first term is the flow from the S-group into the I-group; the second term is the flow from the I-group into the H-group.

Up to now the analysis is general and applies to two possible steady-state equilibria. The first is a *no-disease steady-state* with $I = H = R = 0$. Then from (8) we have $S = \frac{\Lambda N}{a}$. The second equilibrium is *endemic* with $I$, $H$, $R$ and $D$ all greater than zero. This has a unique solution for $\mathcal{R}_0 > 1$ of the form:

$$I = \alpha_I(\mathcal{R}_0 - 1) \tag{16}$$

$$H = \alpha_H(\mathcal{R}_0 - 1) \tag{17}$$

$$R = \alpha_R(\mathcal{R}_0 - 1) \tag{18}$$

$$\Lambda N = \beta S(\alpha_I + \alpha_H)(\mathcal{R}_0 - 1) + aS \tag{19}$$

$$p\gamma_1\alpha_I = (a + \mu + (1-\mu)\gamma_2)\alpha_H \tag{20}$$

$$\alpha_R = \frac{(1-p)\gamma_1\alpha_I + (1-\mu)\gamma_2\alpha_H}{a} \tag{21}$$

$$S = \frac{\Lambda N}{a\mathcal{R}_0} \tag{22}$$

Then from (16), (17) and (22) we have that

$$(\mathcal{R}_0 - 1) = \frac{\beta}{a}(\alpha_I + \alpha_H)(\mathcal{R}_0 - 1) \tag{23}$$

Hence for $\mathcal{R}_0 > 1$ we have

$$1 = \frac{\beta}{a}(\alpha_I + \alpha_H) \tag{24}$$

Equations (20) and (24) give us two equations in $\alpha_I$ and $\alpha_H$ leading to

$$\alpha_I = \frac{(a+\mu+(1-\mu)\gamma_2)a}{\beta(a+\mu+(1-\mu)\gamma_2+p\gamma_1)} \tag{25}$$

which completes the solution.

### 2.3 Stability Properties about the Two Steady States

We generalize the following propositions of Cui *et al.* (2020) obtained for $\gamma_2 = 0$ case (Hepatitis C) to the $\gamma_2 > 0$ case (Covid19):

**Proposition 1.**

*The model is locally stable in the vicinity of the no-disease steady state iff the reproduction number* $\mathcal{R}_0 < 1$.

*The model is always locally stable in the vicinity of the positive-disease steady state.*

**Proof.** A straightforward generalization of those in Cui *et al.* (2020) confirmed by numerical simulations. □

### 2.4 Epidemic Dynamics as Predator-Prey Models

Consider a small change to the SIR model, and the S-group dynamics (1) in particular, to give

$$S_t = S_{t-1} + \Lambda S_{t-1} - \beta S_{t-1} I_{t-1} - a S_{t-1} \tag{26}$$

Here flows into the S-group are $\Lambda S_{t-1}$ rather than $\Lambda N_{t-1}$. Now, away from the steady state, $N_t - N_{t-1} = \Lambda S_{t-1} - a N_{t-1} \neq 0$: the total population can grow or decline away from a **unique** zero growth steady state given by

$$\begin{aligned} S &= \frac{\gamma_1 + a}{\beta} \\ I &= \frac{\Lambda - a}{\beta} \\ R &= \frac{\gamma_1 I}{a} \end{aligned}$$

Since we must have that $I, R \geq 0$, it follows that $\Lambda \geq a$ and as $\Lambda \to a$ we then arrive at a no-disease steady state, $I = R = 0$, $N = S$.

In continuous time we can write the SIR model as

$$\begin{aligned} \dot{S} &= S(\Lambda - a - bI) && (27) \\ \dot{I} &= I(cS - d) && (28) \\ \dot{R} &= \gamma_1 I - aR && (29) \\ N &= S + I + R && (30) \end{aligned}$$

where $b = c = \beta$ and $d = \gamma_1 + a$. Equations (27) and (28) form the seminal Lotka-Volterra (LV) predator-prey model in population dynamics extensively studied in Murray (2002). They describe the dynamic interaction of two species, the prey (population S in our SIR model) and predators (population I). As such, $\Lambda - a$ is the growth rate of the S-group in the absence of a virus, $bI$ is the removal rate to the I-group (mortality in the predator-prey interpretation), $c = \beta S$ is the growth rate of the "predator", the virus-infected population in our case and $d = \gamma_1 + a$ is its removal rate either to the R-group or directly to mortality.

To make further progress, following Murray (2002), Chapter 3, subsection 3.1, we first de-dimensionalise the system by defining new variables

$$\begin{aligned} u(\tau) &\equiv \frac{cS(t)}{d} \frac{\beta S(t)}{\gamma_1 + a} \\ v(\tau) &\equiv \frac{bI(t)}{a} = \frac{\beta I(t)}{\Lambda - a} \\ \tau &\equiv et \\ e &\equiv \frac{\gamma_1 + a}{\Lambda - a} \end{aligned}$$

The system becomes

$$\begin{aligned} \frac{du}{d\tau} &= u(1 - v) && (31) \\ \frac{dv}{d\tau} &= ev(u - 1) && (32) \end{aligned}$$

In the space of $(u, v)$ (31) and (32) give

$$\frac{du}{dv} = e\frac{v(u-1)}{u(1-v)} \tag{33}$$

which can be written

$$\int \left(\frac{1}{v} - 1\right) dv = e \int \left(1 - \frac{1}{u}\right) du \tag{34}$$

Integrating we have

$$H + \log v - v = e(-\log u + u) \Rightarrow eu + v - \log u^e v = H(u, v) = \text{constant} \tag{35}$$

where $H$ is a constant.

We can now show that $H(u, v)$ has a minimum at $u = v = 1$.

**Proof**:

First order (necessary) conditions are:

$$\begin{aligned} \frac{\partial H}{\partial u} &= e\left(1 - \frac{1}{u}\right) = 0 \Rightarrow u = 1 \\ \frac{\partial H}{\partial v} &= 1 - \frac{1}{v} = 0 \Rightarrow v = 1 \end{aligned}$$

The second order (sufficient) condition is that the Hessian

$$\begin{bmatrix} \frac{\partial^2 H}{\partial u^2} & \frac{\partial^2 H}{\partial u \partial v} \\ \frac{\partial^2 H}{\partial v \partial u} & \frac{\partial^2 H}{\partial v^2} \end{bmatrix} = \begin{bmatrix} \frac{e}{u^2} & 0 \\ 0 & \frac{1}{v^2} \end{bmatrix} = \begin{bmatrix} e & 0 \\ 0 & 1 \end{bmatrix} \tag{36}$$

at the turning point is positive definite, which is clearly the case. Hence $H \geq 1 + e$ and in periodic solutions initial conditions $u(0)$ and $v(0)$ determine the constant in H.[3]

The steady state relevant for the SIR model is $u = v = 1$. Then Murray (2002), page 82, shows (for a continuous time model) that the eigenvalues are purely imaginary, so the solution is periodic: 'waves' in the language of the Covid pandemic. Figure 2 illustrates this feature in a discretized version of the model.

Sections 3.3 and 3.4 in Murray (2002) extend the L-V model to interesting cases of bifurcation, limit cycles and, for three or more species, chaos. Our 4-group Covid model can

[3]See Murray (2002), page 81, for the phase plane trajectories.

possibly be seen as a 4-species L-V model with the possibility of these forms of complexity.

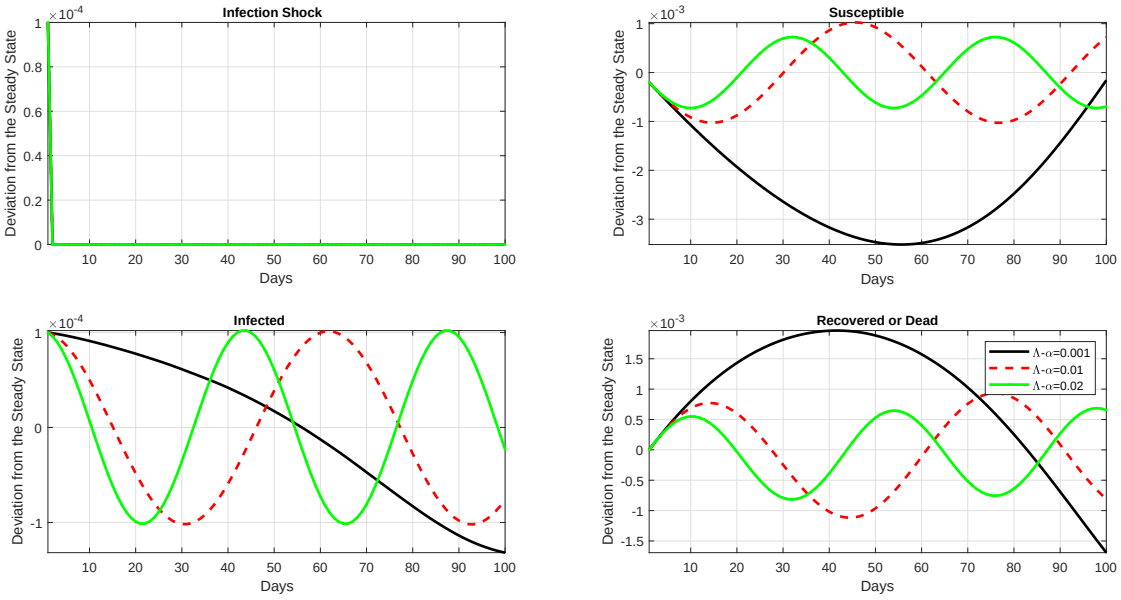


Figure 2: **SIR-Lotka-Volterra Model: No-Disease Steady State**. $R_0 = 1$; $a = 0.01/365$; $p = 0.2$; $\gamma_1 = 0.05$; $\gamma_2 = 0.025$; $\mu = 0.5/365$; $\rho = 0.75$; $\Lambda > a$;

# 3 Quantitative Analysis of Covid-19 Model

This section first sets out a calibration strategy that can be adopted given emerging available data and then proceeds to examine the impulse responses following an infection shock at the no-disease and endemic steady states examined in Section 2.2.

## 3.1 Calibration of the SIHR Model

The parameter values chosen in Table 1 are tentative, drawing upon the existing literature; they enable us to explore some interesting dynamics without providing a definitive quantitative assessment.

| Calibrated Parameters | Value | Data |
|---|---|---|
| Average Death Rate $a$ | 0.01 (annual) | Available |
| Infection Rate $\beta$ | Calibrated to $R_0$ | Available |
| Proportion with mild attack $1-p$ | 0.8 | Available |
| Recovery rate for mild attach group $\gamma_1$ | 0.05 (daily) | Available |
| Recovery rate for serious attack group $\gamma_2$ | 0.025 (daily) | Available |
| Serious Covid Death Rate $\mu$ | 0.5/365 (daily) | Available |

Table 1: Calibration of Epidemiology Model

### 3.2 Impulse Response Functions Following an Infection Shock

Figures 3 and 4 show responses to a persistent infection for different values of the reproduction $\mathcal{R}_0$ parameter. Starting from a no-disease steady state the elimination of the virus and a return to the same steady state requires that $\mathcal{R}_0 < 1$. This is illustrated in Figure 3 which shows the 'flattening of the curve' and the substantial mortality gains from lowering the reproduction number to well below unity. By contrast, if the goal is to return to an endemic steady state, a reproduction number above unity is possible, though again there are substantial gains from keeping the reproduction number close to unity.

Now consider a lockdown policy that at time $t$ reduces the size of the susceptible population by a proportion $\lambda_t$ and follows a process:

$$\lambda_t = \lambda_{t-1}^{\rho_\lambda} \exp(-\epsilon_{\lambda,t}) \tag{37}$$

$\lambda = 1$ in the steady state and $\epsilon_{\lambda,0} = 0.0, 0.1, 0.25, 0.5$, a one-off shock as in the Figure, $\epsilon_{\lambda,t} = 0$ for $t > 0$. $\epsilon_{\lambda,0}$ then measures the **scope of lockdown** and $\rho_\lambda$ is the **speed of lockxit**.

In Figure 5, $\rho_\lambda = 0.95$ and $\lambda_t$ returns to 1 after 100 days following the same infection shock as Figure 3. Lockdown then flattens the curve and acts in the same way as reducing the $\mathcal{R}_0$ number.

## 4 Endogenous Social Interaction

Up to now the degree of social interaction has been exogenous. We now introduce into our SIHR epidemiology model a decentralized laissez-faire market equilibrium. Following

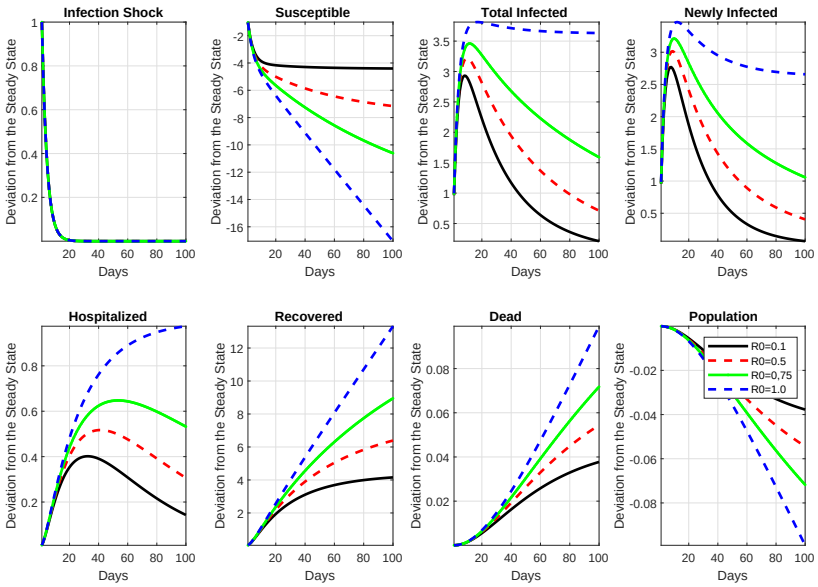


Figure 3: **No-Disease Steady State**. $a = 0.01/365$; $p = 0.2$; $\gamma_1 = 0.05$; $\gamma_2 = 0.025$; $\mu = 0.5/365$; $\rho = 0.75$; $\Lambda = a$;

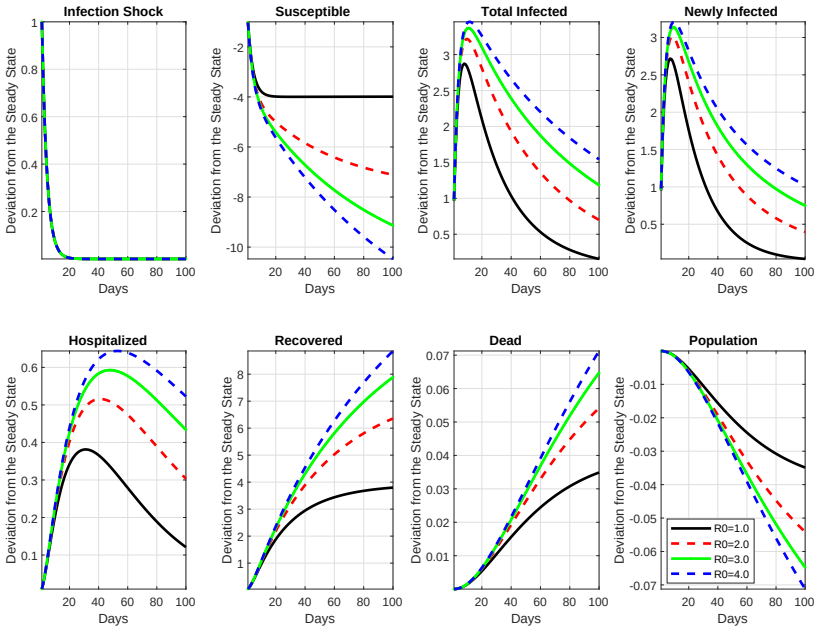


Figure 4: **Endemic Steady State**. $a = 0.01/365$; $p = 0.2$; $\gamma_1 = 0.05$; $\gamma_2 = 0.025$; $\mu = 0.5/365$; $\rho = 0.75$; $\Lambda = a$;

Farboodi *et al.* (2020), social interaction is endogenously chosen by households in a rational expectations equilibrium where they know the model and can form beliefs of their

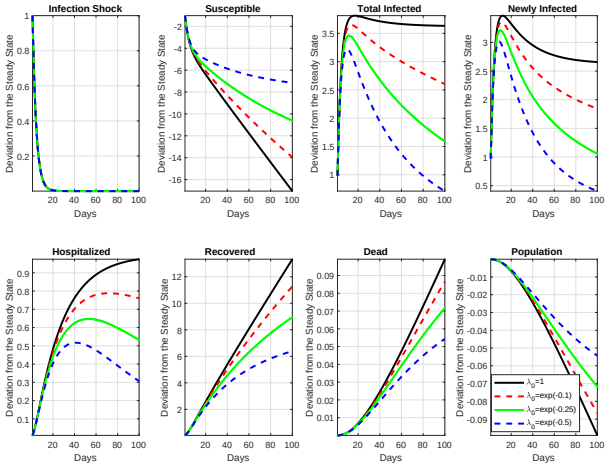


Figure 5: No Disease Steady State: Dynamics following Lockdown. $\mathcal{R}_0 = 1$; $a = 0.01/365$; $p = 0.2$; $\gamma_1 = 0.05$; $\gamma_2 = 0.025$; $\mu = 0.5/365$; $\rho = 0.75$; $\Lambda = a$;

probabilities of being in each of the SIHR groups. We first assume a deterministic world with no uncertainty and study a perfect foresight equilibrium.

Let $\Theta_{st}$, $\Theta_{it}$ and $\Theta_{ht}$ be the levels of social interaction chosen by the S, I and H groups respectively. Then the epidemiology model aggregate dynamics (1)–(2) become

$$S_t = S_{t-1} + \Lambda - \beta\Theta_{st}S_{t-1}(\Theta_{it}I_{t-1} + \Theta_{ht}H_{t-1}) - aS_{t-1} \tag{38}$$

$$I_t = I_{t-1} + \beta\Theta_{st}S_{t-1}(\Theta_{it}I_{t-1} + \Theta_{ht}H_{t-1}) - (a + \gamma_1)I_{t-1} \tag{39}$$

$$H_t = H_{t-1} + p\gamma_1 I_{t-1} - (a + \mu)H_{t-1} - (1 - \mu)\gamma_2 H_{t-1} \tag{40}$$

$$R_t = R_{t-1} + (1 - p)\gamma_1 I_{t-1} + (1 - \mu)\gamma_2 H_{t-1} - aR_{t-1} \tag{41}$$

$$D_t = D_{t-1} + \gamma_2\mu H_{t-1} \tag{42}$$

Since we have now normalized the total population at unity and assumed zero population growth we can interpret the proportions $S_t$, $I_t$, $H_t$, $R_t$ as probabilities of susceptibility, infection, hospitalization and recovery. Following Farboodi *et al.* (2020) we now assume a common level of social interaction $\Theta_{st} = \Theta_{it} = \Theta_{ht} = \Theta_t$ across S, I and H groups. The household discounts the future at a rate $\beta_h$ and a cure is found for the disease at a rate $\beta_c$. The cost of hospitalization and death are given by the functions $\kappa_H(H_t)$ and $\kappa_D(D_t)$ respectively.

It is assumed that households in the S and H groups choose levels of social activity and

enjoy utility $U(\Theta)$ where $U$ is a single-peaked utility function with a maximium attained at $\Theta^* > 0$. A normalization $\Theta^* = 1$ and $U(\Theta^*) = 0$ means $U$ is a measure of the utility loss from social distancing.

Denote by lower case the beliefs facing the individual household. With rational beliefs, the household solves

$$\max_{\Theta_t} \Sigma_{t=0}^{\infty} \bar{\beta}^t ((s_t + i_t)U(\Theta_t) - \kappa_H(h_t)h_t - \kappa_D(d_t)d_t) \tag{43}$$

where $\bar{\beta} \equiv \beta_h \beta_c$ subject to the constraints of the model

$$\begin{aligned}
s_t &= s_{t-1} + \Lambda - \beta\Theta_t^2 s_{t-1}(I_{t-1} + H_{t-1}) - a s_{t-1} \\
i_t &= i_{t-1} + \beta\Theta_t^2 s_{t-1}(I_{t-1} + H_{t-1}) - (a + \gamma_1) i_{t-1} \\
h_t &= h_{t-1} + p\gamma_1 i_{t-1} - (a + \mu) h_{t-1} - (1 - \mu)\gamma_2 h_{t-1} \\
r_t &= r_{t-1} + (1 - p)\gamma_1 i_{t-1} + (1 - \mu)\gamma_2 h_{t-1} - a r_{t-1} \\
d_t &= d_{t-1} + \gamma_2 \mu h_{t-1}
\end{aligned}$$

taking the aggregate states $I_{t-1}$ and $H_{t-1}$ as given. This leads to the *laissez-faire equilibrium.*

By contrast, the social planner solves

$$\max_{\Theta_t} \Sigma_{t=0}^{\infty} \bar{\beta}^t ((S_t + I_t)U(\Theta_t) - \kappa_H(H_t)H_t - \kappa_D(D_t)D_t)$$

subject to the *aggregate model* (38)–(42) with $\Theta_{st} = \Theta_{it} = \Theta_{ht} = \Theta_t$. The difference between the inter-temporal utilities arising from these two problems is a measure of the potential benefits from lock-down.

So far the model is deterministic with a perfect foresight rational expectations equilibrium. In a stochastic model we can allow exogenous parameters $\beta$, $p$, $\gamma_1$, $\gamma_2$, and $\mu$ to be log-normal AR1 time-varying processes as well. Now *informational frictions* are potentially important and we have a model where perceptions of risk under uncertainty are important.[4] In the epidemiology model there are five shocks for the household to respond to in each period. If they have accurate data on infections $I_t$, those hospitalized $H_t$,

[4]See Angeletos and Lian (2016) for a survey of recent developments in this literature.

recovered $R_t$ and deceased $D_t$ they can in principle estimate a finite VAR approximation of the current values of four of the now time-varying parameters $\beta_t$, $p_t$, $\gamma_{1,t}$, $\gamma_{2,t}$, and $\mu_t$. This is the ABC and D of VARs in Fernandez-Villaverde *et al.* (2007) and the shocks can be backed-out from data observed by agents. But there are five shocks and, moreover, it is reasonable to assume that they are measured with error. It follows from Levine *et al.* (2019) that the standard perfect information assumption in solving for the rational expectations equilibrium, adopted in our solution, is no longer valid, and the imperfect information solution of Pearlman *et al.* (1986) must be assumed.[5] This is an important area for future research on epidemiology-macroeconomic models.

# 5 The Economic Model

We use a simple RBC with fiscal policy model to assess the economic and welfare cost of reducing the size of the susceptible working population by enforced 'leisure'.

## 5.1 Households

In the absence of lockdown households freely choose between work and leisure and therefore how much labour they supply. They also own the capital stock which is rented to firms at a rental rate $r_t^K$ and choose an optimal investment path. Let the total time available for work (say 16 hours per day) be normalized at unity and consist of leisure time $L_t$ and $M_t = 1 - L_t$, the proportion of this time spent at work. The single-period utility is

$$U = U(C_t, L_t) \tag{44}$$

and we assume that[6]

$$U_C > 0,\ U_L > 0\ U_{CC} \leq 0,\ U_{LL} \leq 0$$

In a stochastic environment, the value function of the representative household at time $t$ is given by

$$\Omega_t = \Omega_t(B_{t-1}) = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta_h^s U(C_{t+s}, L_{t+s}) \right] \tag{45}$$

[5] See for example Collard *et al.* (2009) and Levine *et al.* (2012) for applications of this imperfect information solution which is now available as a Dynare option, as explained in Levine *et al.* (2020).

[6] Our notation is $U_C \equiv \frac{\partial U}{\partial C}$, $U_{CC} \equiv \frac{\partial^2 U}{\partial C^2}$ etc.

The household's problem at time $t$ is to choose paths for consumption $\{C_t\}$, leisure, $\{L_t\}$, labour supply $\{M_t = 1 - L_t\}$, capital stock $\{K_t\}$, investment $\{\text{Inv}_t\}$ and bond holdings to maximize $\Omega_t$ in (45), given its *nominal* budget constraint in period $t$

$$P_t^B B_t = B_{t-1} + P_t(1-\tau_k) r_t^K K_{t-1} + P_t(1-\tau_w) W_t M_t - P_t C_t - P_t \text{Inv}_t - P_t T_t \qquad (46)$$

where $B_t$ is the number of 1-period bonds held by the household at the end of period $t$ with face value unity, $P_t^B = \frac{1}{R_{n,t}}$ is the price of bonds where $R_{n,t}$ is the nominal interest rate, $B_{t-1}$ is the value of these bonds purchased at time $t-1$ at maturity in period $t$, $r_t^K$ is the rental rate on capital received from firms, $W_t$ is the real wage rate $\text{Inv}_t$ is real investment, $\tau_k$ and $\tau_w$ are capital and labour tax rates and $T_t$ are real lump-sum taxes; and given that capital stock accumulates according to

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + (1 - AC(X_t))\text{Inv}_t\,; & (47)\\ X_t &\equiv \frac{\text{Inv}_t}{\text{Inv}_{t-1}}\,;\ AC',\, AC'' \geq 0\,;\ AC(1) = AC'(1) = 0 & (48) \end{aligned}$$

In (48), $AC(X_t)$ are investment adjustment costs, $\text{Inv}_t$ units of output converts to $(1 - AC(X_t))\text{Inv}_t$ of new capital sold at a real price $Q_t$ (Tobin's Q). All variables are expressed in real terms relative to the price of output.

We can write the household budget constraint in terms of the *nominal value* of bond holdings, $B_t^n \equiv P_t^B B_t$ and the *real value* $B_t^r \equiv \frac{P_t^B B_t}{P_t}$ as follows:

$$\begin{aligned} B_t^n &= \frac{1}{P_{t-1}^B} P_{t-1}^B B_{t-1} + P_t r_t^K K_{t-1} + P_t W_t M_t - P_t C_t - P_t \text{Inv}_t - P_t T_t \\ &= r_{n,t-1} B_{t-1}^n + P_t r_t^K K_{t-1} + P_t W_t M_t - P_t C_t - P_t \text{Inv}_t - P_t T_t \\ B_t^r &\equiv \frac{B_t^n}{P_t} = \frac{P_{t-1}}{P_t} r_{n,t-1} \frac{B_{t-1}^n}{P_{t-1}} + r_t^K K_{t-1} + W_t M_t - C_t - \text{Inv}_t - T_t \\ &= \frac{r_{n,t-1}}{\Pi_t} B_{t-1}^r + r_t^K K_{t-1} + W_t M_t - C_t - \text{Inv}_t - T_t \\ &= R_{t-1}^B B_{t-1}^r + r_t^K K_{t-1} + W_t M_t - C_t - \text{Inv}_t - T_t \end{aligned}$$

where $R_{t-1}^B = \frac{r_{n,t-1}}{\Pi_t}$ is the ex post real interest rate on bonds set in period $t-1$ taking into account inflation $\Pi \equiv \frac{P_t}{P_{t-1}}$ in the interval $[t-1, t]$. The standard first-order conditions for

this optimization problem are

$$\begin{aligned}
\text{Euler Consumption} \quad &: \quad U_{C,t} = \beta_h R_t^B \mathbb{E}_t \left[U_{C,t+1}\right] && (49)\\
\text{Labour Supply} \quad &: \quad \frac{U_{N,t}}{U_{C,t}} = -\frac{U_{L,t}}{U_{C,t}} = -W_t(1-\tau_w) && (50)\\
\text{Leisure and Hours} \quad &: \quad L_t \equiv 1 - M_t && (51)\\
\text{Investment FOC} \quad &: \quad Q_t(1 - AC(X_t) - X_t AC'(X_t)) \\
&+ \quad E_t\left[\Lambda_{t,t+1}\, Q_{t+1} AC'(X_{t+1}) X_{t+1}^2\right] = 1 && (52)\\
\text{Capital Supply} \quad &: \quad \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}^K\right] = 1 && (53)
\end{aligned}$$

where $\Lambda_{t,t+1} \equiv \beta_h \frac{U_{C,t+1}}{U_{C,t}}$ is the *real stochastic discount factor* over the interval $[t, t+1]$, $X_t = \text{Inv}_t/\text{Inv}_{t-1}$ is the rate of change of investment and $R_t^K$ is the gross return on capital net of tax is given by

$$R_t^K = \frac{\left[r_t^K(1-\tau_k) + (1-\delta)Q_t\right]}{Q_{t-1}}$$

The Euler consumption equation, (49), where $U_{C,t} \equiv \frac{\partial U_t}{\partial C_t}$ is the marginal utility of consumption and $\mathbb{E}_t[\cdot]$ denotes rational expectations based on agents observing all current macroeconomic variables (i.e., 'perfect information'), describes the optimal consumption-savings decisions of the household. It equates the marginal utility from consuming one unit of income in period $t$ with the discounted marginal utility from consuming the gross income acquired, $R_t^B$, by saving the income. For later use it is convenient to write the Euler consumption equation as

$$1 = R_t^B \mathbb{E}_t \left[\Lambda_{t,t+1}\right] \tag{54}$$

Equation (50) equates the real wage with the marginal rate of substitution between consumption and leisure. Note that (49) and (54) imply that bonds are real and there is therefore no inflation risk.

Equation (52) is the first-order condition for investment where $\text{Inv}_t$ units of output converts to $(1 - AC(X_t))\text{Inv}_t$ of new capital sold at a real price $Q_t$. (53) and (54) equate the expected discounted return on a riskless bond with that of capital over the period $[t, t+1]$.

### 5.2 Firms

Output and the firm's behaviour is summarized by:

$$\begin{aligned} \text{Output} &: Y_t = F(A_t, M_t, K_{t-1}) && (55)\\ \text{Labour Demand} &: F_{M,t} = W_t && (56)\\ \text{Capital Demand} &: F_{K,t} = r_t^K && (57) \end{aligned}$$

where (55) is a production function. Note here $K_t$ is *end-of-period* $t$ capital stock. Equation (56), where $F_{M,t} \equiv \frac{\partial F_t}{\partial M_t}$, equates the marginal product of labour with the real wage. (57), where $F_{K,t} \equiv \frac{\partial F_t}{\partial K_t}$, equates the marginal product of capital with the rental rate $r_t^K$.

### 5.3 Output Equilibrium and the Government Budget Constraint

The model is completed with an output equilibrium and a government budget constraint. The former is given by

$$Y_t \;=\; C_t + G_t + \text{Inv}_t$$

Corresponding to (46) we have a Government Budget Constraint

$$P_t^B B_t = B_{t-1} + P_t G_t - P_t(\tau_w W_t M_t + \tau_k r_t^K \; K_{t-1} + T_t) \tag{58}$$

where $B_t$ are bonds issues by the government and we have excluded money creation by the Central Bank. Corresponding to (49) we then have

$$B_t^r = R_{t-1}^B B_{t-1}^r + G_t - (\tau_w W_t M_t + \tau_k r_t^K K_{t-1} + T_t) \tag{59}$$

Combining (49) and (59) we arrive at the economy's consolidated budget (resource) constraint:

$$r_t^K K_{t-1} + W_t M_t - C_t - \text{Inv}_t - T_t = G_t - T_t \tag{60}$$

Equating real output and real income we have $Y_t = r_t^K K_{t-1} + W_t M_t$. Hence (60) implies

$$Y_t = C_t + \text{Inv}_t + G_t \tag{61}$$

Thus the resource constraint is the same as the output equilibrium. The reason for this is that the household and government budget constraints determine the supply and demand for government bonds respectively. Thus the consolidated budget constraint is the financial market equilibrium. Our model imposes labour market equilibrium, the equality of the supply of hours by households and demand by firms. We have three markets – financial, labour and output. By Walras' Law, equilibrium in the first two of these markets implies equilibrium in the third.

### 5.4 Functional Forms

We now specify functional forms for production and utility and AR(1) processes for exogenous variables $A_t$ and $G_t$. For production we assume a Cobb-Douglas function. The utility function is non-separable and consistent with a balanced growth path when the inter-temporal elasticity of substitution, $1/\sigma$ is not unitary. These functional forms, the associated marginal utilities and marginal products, and exogenous processes are given by

$$F(A_t, M_t, K_{t-1}) = (A_t M_t)^{\alpha} K_{t-1}^{1-\alpha} \tag{62}$$

$$F_M(A_t, M_t, K_{t-1}) = \frac{\alpha Y_t}{M_t} \tag{63}$$

$$F_K(A_t, M_t, K_{t-1}) = \frac{(1-\alpha) Y_t}{K_{t-1}} \tag{64}$$

$$\log A_t - \log \bar{A}_t = \rho_A (\log A_{t-1} - \log \bar{A}_{t-1}) + \epsilon_{A,t} \tag{65}$$

$$\log G_t - \log \bar{G}_t = \rho_G (\log G_{t-1} - \log \bar{G}_{t-1}) + \epsilon_{G,t} \tag{66}$$

$$\begin{aligned} U_t &= \frac{(C_t^{(1-\varrho)}(1-M_t)^{\varrho})^{1-\sigma} - 1}{1-\sigma} \\ &\to (1-\varrho)\log C_t + \varrho \log(1-M_t) \text{ as } \sigma \to 1 \end{aligned} \tag{67}$$

$$U_{C,t} = (1-\varrho) C_t^{(1-\varrho)(1-\sigma)-1} (1-M_t)^{\varrho(1-\sigma)} \tag{68}$$

$$U_{N,t} = -\varrho C_t^{(1-\varrho)(1-\sigma)} (1-M_t)^{\varrho(1-\sigma)-1} \tag{69}$$

$$AC(X_t) = \phi_X (X_t - 1)^2 \tag{70}$$

The equations (54)–(70) describe an equilibrium in $U_t$, $C_t$, $W_t$, $Y_t$, $L_t$, $H_t$, $K_t$, $\text{Inv}_t$, $R_t^B$, $T_t$, given parameter values and exogenous processes, $A_t$ and $G_t$, where for the latter we assume AR1 processes about possibly trending steady states $\bar{A}_t, \bar{G}_t$ driven by zero mean

iid shocks $\epsilon_{A,t}$ and $\epsilon_{G,t}$. The zero-growth deterministic steady state is set out in Appendix A.

### 5.5 Calibration of the RBC Model

The deterministic steady state of the RBC model can be used to calibrate a number of parameters. The idea is to assume an observed baseline steady state equilibrium. We then use this observed equilibrium to solve for model parameters consistent with this observation. In general terms, our baseline steady state can be described in terms of a vector $\underline{X} = f(\underline{\theta})$ of outcomes where $\underline{\theta}$ is a vector of parameters. The calibration strategy is to choose a subset $\underline{X}_1$ of $n$ observed outcomes to calibrate a subset $\underline{\theta}_1$ of $n$ parameters. Partition $\underline{X} = [\underline{X}_1, \underline{X}_2]$ and $\underline{\theta} = [\underline{\theta}_1, \underline{\theta}_2]$. Then $\underline{\theta}_1$ is found by solving

$$[\underline{X}_1, \underline{X}_2] = f([\underline{\theta}_1, \underline{\theta}_2]) \tag{71}$$

for $\underline{X}_2$ and $\underline{\theta}_1$, given $\underline{X}_1$ and $\underline{\theta}_2$. If such a solution exists for economically meaningful parameter values for $\underline{\theta}_1$ then a successful calibration has been achieved.

To apply this we use data for factor shares in the production sector, hours as a proportion of the available leisure time ($H$), the real interest rate ($R$) and expenditure shares $c_y \equiv \frac{C}{Y}$, $i_y \equiv \frac{I}{Y}$ and $g_y \equiv \frac{G}{Y}$. First we calibrate $\alpha$ to be the observed wage share in the wholesale sector.

We can choose units of output and capital stock so that $A = 1$. Then using $K/Y = \frac{1-\alpha}{R^B-1+\delta}$ from the RBC steady state we can now write

$$i_y \equiv \frac{I}{Y} \quad = \quad \frac{\delta K}{Y} = \frac{\delta K}{Y} = \frac{\delta(1-\alpha)}{R^B - 1 + \delta}$$

from which $\delta$ can be calibrated.

From the steady state equation $\frac{\varrho C}{(1-\varrho)(1-H)} = W$ we have seen that

$$H = \frac{\alpha(1-\varrho)}{\varrho C/Y + \alpha(1-\varrho)}$$

from which the calibrated $\varrho$ is obtained as:

$$\varrho = \frac{(1-H)\alpha}{(1-H)\alpha + c_y H} \tag{72}$$

Finally from an observation of $R$ we can calibrate $\beta_h$ from

$$R^B = \frac{1}{\beta_h}$$

| Observed Equilibrium | Value |
|---|---|
| $H$ | 0.35 |
| wage share $= \alpha$ | 0.7 |
| $c_y$ | 0.6 |
| $i_y$ | 0.2 |
| $g_y$ | 0.2 |
| $R^B$ | 1.01 |
| $\tau_w = \tau_k$ | 0.25 |
| **Calibrated Parameters** | **Value** |
| $\varrho$ | 0.6842 |
| $\delta$ | 0.0202 |
| $\beta_h$ | 0.990 |

Table 2: Calibration of RBC Model

Some remaining parameters need to be set: $\phi_X$ is needed for investment adjustment costs, and the persistence parameters, $\rho_A$ and $\rho_G$ are needed for the AR 1 shock processes, as are the standard deviations of the shocks. These can be estimated by Bayesian methods; but for the purposes of the simulations here we choose values $\sigma_{es} = \phi_X = 2.0$ roughly reflecting the empirical literature. For all exogenous processes in our models, persistence parameters are set at 0.75, and shocks have a standard deviation of 1%. This completes the calibration, and typical US observations and calibrated parameters are illustrated in Table 2.

# 6 Quantitative Analysis of Lock-Down Policy

We now use this RBC with fiscal policy model to assess the economic and welfare cost of reducing the size of the susceptible working population by enforced "leisure". In the absence of lockdown households freely choose how much labour they supply.

## 6.1 Modelling Lockdown

Consider the utility of a representative household, $U(C_t, 1 - M_t^s)$, where $M_t^s$ is the proportion of hours available chosen for work. As we have seen, $M_t^s$ satisfies the condition $\frac{U_{N^s,t}}{U_{C,t}} = -W_t(1 - \tau_w)$ where $W_t$ is the wage and $\tau_w$ is the income tax rate. Without lockdown, the labour market clears so $M_t^d = M_t^s$. Under lockdown firms can only employ $M_t = \lambda_t M_t^d$ where the lockdown variable has been described in Section 3.2.

## 6.2 The Fiscal Aftermath of Lockdown

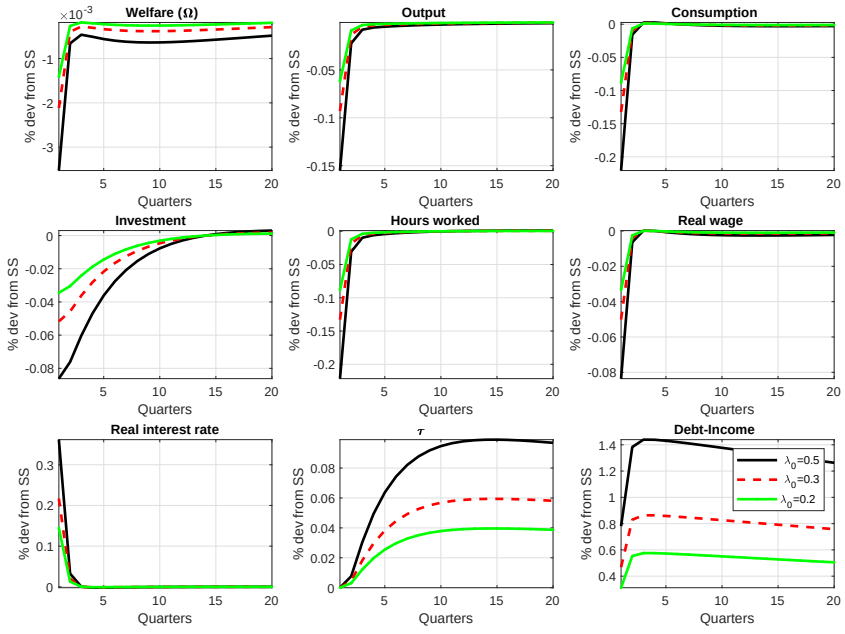


Figure 6: **Macroeconomic Impulse Responses Following Lockdown**. $\rho_\lambda = 0.1$; $\lambda_0 = 0.2, 0.3, 0.5$.

Figure 6 compares the fiscal consequences of different degrees of lockdown as captured by the proportions of the workforce ($\lambda_0$) instructed to leave employment. Workers affected

are compensated with lost wages paid by the government. Distortionary taxes adjust to stabilize and then reduce the debt-income ratio. A drawn-out recession occurs with output, consumption, investment and hours sharply falling before gradually returning to the steady state.

### 6.3 Welfare Costs of Lockdown

Utility is measured at **actual consumption** but **keeping hours at the non-lockdown value**. Then no 'leisure' with $M_t < M_t^s$ brings no utility gains and

$$U_t = \frac{(C_t^{(1-\varrho)}(1-M_t^s)^{\varrho})^{1-\sigma}-1}{1-\sigma}$$

In a non-stochastic environment, the value function of the representative household at time $t$ is given by

$$\Omega_t = (1-\beta_h)\left[\sum_{\tau=0}^{\infty}\beta_h^{\tau}U(C_{t+\tau}, 1-M_{t+\tau}^s)\right]$$

This can be computed from

$$\Omega_t = (1-\beta_h)U_t + \beta_h\Omega_{t+1}$$

Thus $\Omega = U$ in the steady state.

The consumption equivalent (CEQ) measure is given by $CEQ \equiv (\Omega_0^{\text{no lockdown}} - \Omega_0^{\text{lockdown}})/CE$, where $CE$ is the the steady state of

$$CE_t \equiv ((1.01C_t^{(1-\varrho)}(1-M_t^s)^{\varrho})^{(1-\sigma)}-1)/(1-\sigma) - U_t$$

That is, CE is the per period *utility gain from a permanent* 1% *increase in consumption* with fixed hours worked. Since $\Omega = U$, it is the permanent inter-temporal welfare increase as well in CEQ units.

Table 3 shows the economic costs of lockdown in consumption equivalent units. Now recall from (37) that $\lambda_0$ is a measure of the scope of lockdown and $\rho_\lambda$ is a measure of its persistence. A lockdown with the largest scope and slowest exit brings a cost that is equivalent to a 0.21% permanent reduction in consumption. Our epidemiology model

| $\lambda_0$ | $\rho_\lambda$ | Days | $\Omega_0$ | CEQ (%) |
|---|---|---|---|---|
| 0.2 | 0.95 | 100 | $5 \times 10^{-4}$ | 0.0175 |
| 0.3 | 0.95 | 100 | $7 \times 10^{-4}$ | 0.0246 |
| 0.5 | 0.95 | 100 | $1.3 \times 10^{-3}$ | 0.0456 |
| 0.2 | 0.99 | 400 | $5 \times 2.4^{-3}$ | 0.0842 |
| 0.3 | 0.99 | 400 | $5 \times 3.5^{-3}$ | 0.1228 |
| 0.5 | 0.99 | 400 | $5 \times 6.0^{-3}$ | 0.2105 |

Table 3: Consumption Equivalent Costs with $CE = 0.0286$ in Steady State.

enables us in principle to link this to lives saved and the cost implied by this policy,[7] but both the economic and empirical models need stronger empirical foundations to come to conclusions.

# 7 Conclusions

Both the SIR model of virus proliferation, and the RBC model of macroeconomic activity, are 'workhorse' models in their fields. They have been developed in many directions and, in the current Covid-19 pandemic, have been brought closer by the need to understand the interactions between the virus's health and social effects, and the economic cost of the measures that have necessarily sought to limit its spread by curtailing social and economic activity. An appreciable body of work has developed in this direction, and an important objective of the current paper has been to provide a framework for understanding and developing this work by integrating the SIR and RBC models. Such a framework allows a coherent presentation of the existing literature, as well as access to developments in epidemiology and macroeconomics that can enhance current (and future) work in this area.

A natural example of where such a framework can be helpful is in studying the lockdown-mortality gain and the value of life implied by how this is approached. Presenting this as a social planning problem, nested within a full macroeconomic model, permits investigation of the basis and effects of such policies. In turn, this opens a further set of topics: for instance, incorporating models of social and voter preferences can explicitly capture tastes for equality, or public choice influences on such decisions. Of course,

[7] See Miles *et al.* (2020) and Bayraktar *et al.* (2020).

decisions about how to finance the significant deficits arising from Covid-19 also need careful economic analysis, and if these are taken against a background of Covid variants (and possible future waves – as seems likely), integrated macroeconomic and epidemiological models can offer important insights here. The framework can also be applied to a much richer economic model with New Keynesian features, while possible regional (or national) variations in lockdown policy can be studied using trade and factor mobility adaptations. Incorporating an R&D sector (alluded to in the introduction) would allow incorporation of vaccine production, while questions vaccine distribution and roll-out could be studied by including features from trade and aid literature. The epidemiology model can also be developed; for example, to allow for only temporary immunity and the possibility of bifurcation and periodic solutions such as the 'waves' currently observed with Covid-19, as in Murray (2002). Section 2.4 illustrates possible research in this direction. Another path for research is the need to develop models of endogenous social interaction that take into account perceptions of risk reflected in inferences of key parameters such as the risk of infection and the mortality rate if infected. Section 4 illustrates this, along the lines of Farboodi *et al.* (2020), and suggests possible routes for solving such models.

A striking feature of Covid-19 is its huge differential health impact on individuals across different ages and social classes; an issue that will also be central to global vaccination initiatives. We believe that a multi-faceted approach across many disciplines is necessary to address the questions posed for countries, governments and individuals over the past year and into the future, and the current paper argues that there is a need to bridge the gap between the two disciplines of economics and epidemiology as part of this approach (see Yates (2020)), drawing in particular on mathematical models of epidemiology (see, for example, Funk *et al.* (2010) and Metcalf *et al.* (2015)) and the macroeconomy. This can enhance the recognition, and empirically-based study, of the policy choices and trade-offs involved. It seems highly likely, for example, that an optimal lockdown policy that takes full account of the immediate and long-term intergenerational aspects in an integrated macro-epidemiology framework would look quite different from the blanket Covid-19 lockdowns pursued in most countries at various points in the last year; an issue that will be relevant as countries seek to open up externally as well as internally in the future.

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# A The Zero-Growth Steady State of the RBC Model

We assume a zero-growth steady state with $\bar{A}_t = \bar{A}_{t-1} = A$ say and $\bar{G}_t = \bar{G}_{t-1} = G$. $K_t = K_{t-1} = K$, etc. Then the full steady state of the standard RBC model is given by:

$$
\begin{aligned}
Q &= 1 \\
X &= 1 \\
AC &= 0 \\
R^B &= \frac{1}{\beta_h} \\
R^K &= R^B = r_K(1-\tau_k) + 1 - \delta \\
r^K &= \frac{(1-\alpha)Y}{K} \\
Y &= (AM)^\alpha K^{1-\alpha} \\
\frac{\varrho C}{(1-\varrho)(1-M)} &= W(1-\tau_w) \\
\frac{\alpha Y}{M} &= W \\
\frac{K}{Y} &= \frac{1-\alpha}{R^B - 1 + \delta} \\
\text{Inv} &= \delta K \\
Y &= C + \text{Inv} + G \\
G &= T \\
U &= \frac{(C^{(1-\varrho)}(1-M)^\varrho)^{1-\sigma} - 1}{1-\sigma} \\
&\to (1-\varrho)\log C_t + \varrho \log(1-M) \text{ as } \sigma \to 1 \\
U_C &= (1-\varrho)C^{(1-\varrho)(1-\sigma)-1}((1-M)^{\varrho(1-\sigma)}) \\
U_M &= -\varrho C^{(1-\varrho)(1-\sigma)}(1-M)^{\varrho(1-\sigma)-1}
\end{aligned}
$$

Given $A$ and $G$, the steady state above gives 8 equations in 8 stationary variables $R^B$, $C$, $Y$, $W$, $M$, Inv, $K$ , $T$. This describes the zero-growth steady-state equilibrium.

In *recursive form* this steady state can be written

$$
\begin{aligned}
R^B &= \frac{1}{\beta_h} \\
R^K &= R^B
\end{aligned}
$$

$$
\begin{aligned}
r^K &= R^K(1-\tau_k) - 1 + \delta \\
\frac{K}{Y} &= \frac{1-\alpha}{r^K} = \frac{(1-\alpha)}{R^B - 1 + \delta} \\
\frac{I}{Y} &= \frac{\delta K}{Y} = \frac{(1-\alpha)\delta}{R^B - 1 + \delta} \\
\frac{C}{Y} &= 1 - \frac{I}{Y} - \frac{G}{Y} = 1 - \frac{I}{Y} - g_y \\
\frac{M\varrho}{(1-M)(1-\varrho)} &= \frac{WM}{C} = \frac{WM/Y}{C/Y} = \frac{\alpha}{C/Y} \\
&\Rightarrow \quad M = \frac{\alpha(1-\varrho)}{\varrho C/Y + \alpha(1-\varrho)} \\
Y &= (AM)^{\alpha} K^{1-\alpha} = (AM)^{\alpha} \left(\frac{K}{Y}\right)^{1-\alpha} (Y)^{1-\alpha} \Rightarrow \; Y = AM(K/Y)^{\frac{1-\alpha}{\alpha}} \\
G &= g_y Y \\
W &= \alpha \frac{Y}{M} \\
I &= \frac{I}{Y} Y \\
C &= \frac{C}{Y} Y \\
K &= \frac{K}{Y} Y
\end{aligned}
$$