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HYSTERESIS IN THE NEW KEYNESIAN THREE EQUATION MODEL

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Hysteresis in the New Keynesian three equation model

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Abstract

This paper introduces unemployment hysteresis into a tractable New Keynesian three equation model using an insider-outsider labour market. We demonstrate that strict inflation targeting can lead to a unit root in the unemployment rate, but dual mandate monetary policy can stabilise the economy around its efficient employment rate.

Keywords: unemployment, hysteresis, indeterminacy.

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1 Introduction

The unemployment hysteresis hypothesis states that temporary economic shocks can have permanent effects on unemployment, and as a result, unemployment rates can suffer from non-stationarity (Røed, 1997). Hysteresis enjoyed a resurgence of interest following the 2008 financial crisis, and is now a firm fixture of macroeconomic policy debates. To take one example, Janet Yellen, current Secretary of the US Treasury, has described the reasoning behind the Biden administration's American Rescue Plan as follows:

> We've decided to go big, because we think that the risks are of severe scarring if we allow there to be long-term unemployment . . . We are going to be careful to learn the lesson of the financial crisis, which is don't withdraw support too quickly. (Shalal & Lawder, 2021).

In other words, persistently high unemployment has serious welfare consequences; mitigating these is a valid objective of demand management policy.

In this paper, we embed hysteresis into the workhorse New Keynesian three equation model. Using this model, we demonstrate that strict inflation targeting implies indeterminacy, and that the type of indeterminacy implied by strict inflation targeting can result in a unit root in the unemployment rate. A corollary is that dual mandate Taylor rules can stabilise economies that suffer from hysteresis. This result supports recent calls for expansive government support to mitigate labour market scarring effects from the Covid-19 pandemic.

While the hysteresis hypothesis emerged in its modern form in the 1980s (Blanchard & Summers, 1986; Cross, 1988), relatively few papers study hysteresis in New Keynesian DSGE models. Some of these papers study optimal monetary policy (Kapadia, 2005; Galí, 2020), some study the welfare costs of conventional monetary policy (Tervala, 2021), while others study the effects of fiscal policy (Engler & Tervala, 2018). Our paper is closely related to Kienzler & Schmid (2014), who examine the ability of dual mandate Taylor rules to stabilise economies in which hysteresis mechanisms induce endogenous persistence in potential output. By assuming the existence of 'full' or 'pure' hysteresis, we build on Kienzler & Schmid (2014) by presenting analytical stability conditions and a reduced-form solution. In doing so, we provide analytical results on (in)determinacy in New Keynesian models with hysteresis.

2 A tractable model

The model relies on a simplified version of the labour market in Galí (2020). This is discussed in section 2.1, and is embedded in the New Keynesian three equation model in section 2.2.

2.1 The Galí (2020) labour market

The labour market is an insider-outsider market in which unions set wages. Specifically, a union resetting the wage for occupation j in period t in Galí (2020) is assumed to choose a wage $w_t^*(j)$ such that,

$$(1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t[n_{t+k|t}(j)] = n_t^*(j),$$

in which β is the household discount factor, θ_w is the fraction of unions that keep nominal wages unchanged within the period, n_t is the (log) employment rate, $n_t^*(j)$ is some target employment rate for occupation j, and $n_{t+k|t}(j)$ is the employment rate for occupation j whose wage has been reset for the last time in period t, i.e.,

$$n_{t+k|t}(j) - n_{t+k} = -\epsilon_w (w_t^*(j) - w_{t+k}),$$

in which n_{t+k} is (log) average employment in period $t+k$ and ϵ_w is the wage elasticity of labour demand. Calvo pricing implies that average wages evolve according to,

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^*,$$

and target employment is assumed to evolve according to,

$$n_t^*(j) = \gamma n_{t-1}(j) + (1-\gamma) n^*,$$

in which n^* is some long-run target, and γ determines the extent to which the number of labour market 'insiders' responds to actual employment. Taken together, these insider-outsider labour market conditions yield the New Keynesian wage Phillips curve,

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + (1-\gamma)\lambda_n (1-\beta\theta_w)(n_t - n^*) + \gamma \lambda_n \Delta n_t,$$

in which $\lambda_n = (1-\theta_w)(\theta_w \epsilon_w)^{-1}$.

2.2 A three equation model

To yield a tractable New Keynesian model of hysteresis, we place two assumptions on the Galí (2020) labour market:

1. Hysteresis is of the 'full' or 'pure' variety, i.e., $\gamma = 1$;
2. Firms can reset their prices freely in each period.

Assumption 1 yields the simple New Keynesian wage Phillips curve,

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w + \lambda_n \Delta n_t. \tag{1}$$

If technology is such that $y_t = a_t + n_t$, assumption 2 implies that the real wage $w_t = a_t - \mu$ in each period, where μ is the mark-up of prices over wages. In this case,

$$\pi_t = \pi_t^w - \Delta a_t,$$

which in combination with the wage Phillips curve (1) yields the price Phillips curve,

$$\pi_t + \Delta a_t = \beta \mathbb{E}_t[\pi_{t+1} + \Delta a_{t+1}] + \lambda_n \Delta n_t.$$

Finally, if we assume that log productivity follows a random walk, i.e., $\Delta a_t = \epsilon_t$, where ϵ_t is white noise, then we have,

$$\begin{aligned} \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \lambda_n \Delta n_t - \epsilon_t, \\ &= \beta \mathbb{E}_t \pi_{t+1} + \lambda_n \Delta y_t - (1+\lambda_n)\epsilon_t. \end{aligned} \tag{2}$$

The Phillips curve can now be combined with an Euler equation,

$$y_t = \mathbb{E}_t y_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1}) + g_t, \tag{3}$$

in which g_t is a white noise demand shock, and a dual-mandate Taylor rule,

$$\begin{aligned} i_t &= \phi_\pi \pi_t + \phi_y (n_t - n^T), \\ &= \phi_\pi \pi_t + \phi_y (y_t - y_t^T), \end{aligned} \tag{4}$$

in which $y_t^T = a_t + n^T$ and n^T is the efficient level of employment, and ϕ_π and ϕ_y are the Taylor rule elasticities.

3 Indeterminacy and unit roots

Substitution of (4) into (2) and (3) yields the state space model,

$$\mathbb{E}_t z_{t+1} = B z_t + C\xi_t, \tag{5}$$

in which,

$$z_t = \begin{bmatrix} y_t - y_t^T \\ \pi_t \\ q_t \end{bmatrix},$$

$$B = \frac{1}{\beta}\begin{bmatrix} \beta(1+\phi_y)+\lambda_n & \beta\phi_\pi - 1 & -\lambda_n \\ -\lambda_n & 1 & \lambda_n \\ \beta & 0 & 0 \end{bmatrix},$$

$$\xi_t = \begin{bmatrix} g_t \\ \epsilon_t \end{bmatrix},$$

and $q_t = y_{t-1}$.

Now, consider the case of strict inflation targeting, i.e., $\phi_y = 0$. In this case the characteristic polynomial of the Jacobian derivative matrix B is given by,

$$-\mu^3 + \left(\frac{\beta+\lambda_n+1}{\beta}\right)\mu^2 - \left(\frac{\lambda_n+\lambda_n\phi_\pi+1}{\beta}\right)\mu + \frac{\lambda_n\phi_\pi}{\beta},$$

which can be factorised to yield,

$$-(\mu-1)\left(\mu^2 - \left(\frac{\lambda_n+1}{\beta}\right)\mu + \frac{\lambda_n\phi_\pi}{\beta}\right). \tag{6}$$

Following Blanchard & Kahn (1980), the model in (5) requires two eigenvalues outside the unit circle and one eigenvalue inside the unit circle for determinacy and stability, as there are two forward-looking variables (y_t and π_t) and one backwards-looking variable (q_t). But, as is apparent from (6), the model in (5) always has one real root exactly equal to unity when $\phi_y = 0$, regardless of the remaining parameterisation. This yields our first proposition:

Proposition 1: Strict inflation targeting is a sufficient condition for indeterminacy.

A corollary is that a dual mandate Taylor rule can stabilise the model in (5). In fact, as illustrated in figure 1, the model is saddle-path stable under plausible calibrations of a dual mandate Taylor rule.

In the absence of a dual mandate Taylor rule, a minimum state variable solution is,

$$y_t = y_{t-1} + \left(\frac{1}{\lambda_n\theta_\pi}\right) g_t + \left(\frac{1+\lambda_n}{\lambda_n}\right)\epsilon_t, \tag{7}$$

$$\pi_t = \left(\frac{1}{\phi_\pi}\right) g_t, \tag{8}$$

$$n_t = n_{t-1} + \left(\frac{1}{\lambda_n\theta_\pi}\right) g_t + \left(\frac{1}{\lambda_n}\right)\epsilon_t, \tag{9}$$

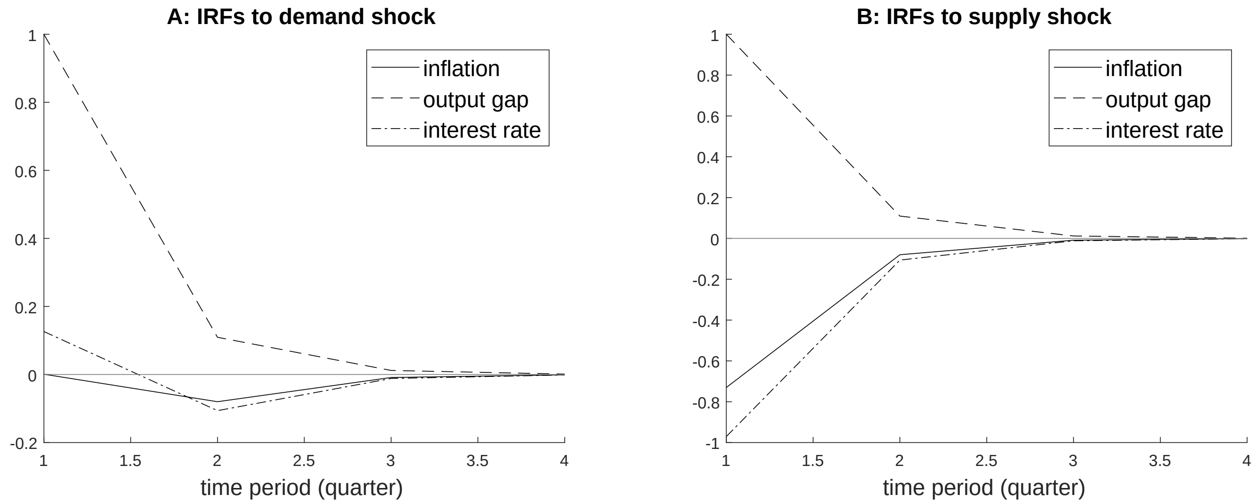

Figure 1: Impulse response functions to demand and supply shocks: $\beta = 0.99$, $\lambda_n = 0.08$, $\phi_y = 0.125$, $\phi_\pi = 1.5$. We follow Galí (2020) in our calibration of λ_n, which assumes that $\theta_w = 0.75$ and $\epsilon_w = 4.3$.

which can be verified by substituting $\mathbb{E}_t y_{t+1} = y_t$ and $\mathbb{E}_t \pi_{t+1} = 0$ into (2) and (3) and rearranging. In this particular solution, output and employment follow random walks and inflation is stationary, yielding our second proposition:

Proposition 2: The type of indeterminacy implied by strict inflation targeting can result in a unit root in the (un)employment rate.

This proposition links the New Keynesian model in this paper to Post Keynesian hysteresis models with backwards-looking expectations, in which strict inflation targeting is sufficient for a unit root in the unemployment rate and output targeting can stabilise economies (e.g., Michl & Oliver, 2019).

4 Concluding remarks

As discussed in Kienzler & Schmid (2014), an inflation-targeting central bank aims to increase the real interest rate in response to an inflationary shock. In a non-hysteretic economy, the resulting decline in output causes a decline in the output gap, and thus inflation falls. In such an economy, inflation will stabilise around the central bank's target when the output gap is equal to zero, which occurs when output is equal to its supply-side potential.

In an economy suffering from full hysteresis, inflation targeting can still stabilise inflation by increasing the real interest rate until the output gap falls, and inflation will still stabilise around the central bank's target when the output gap is equal to zero. However, because potential output is equal to lagged output in such an economy, the output gap can be forced to zero at any level of output. Unless, therefore, demand management policy targets a specific level of output or (un)employment, these variables can suffer from non-stationarity.

Arriving at a tractable model of unemployment hysteresis has required a number of assumptions. Restricting log productivity to follow a random walk and household utility to have a unitary intertemporal elasticity of substitution are relatively weak assumptions. Assumptions 1 and 2 are somewhat more restrictive, and relaxing them requires numerical computation. Future work will estimate a medium-sized New Keynesian model that explores more general hysteresis wage setting processes. In particular we will employ a version of Smets & Wouters (2007) which replaces the standard Calvo-wage setting with a labour market in which the optimized nominal wage is set by trade-unions such that in expectation a weighted average of employment equals an exogenous target over the period for which the wage remains effective. Research questions to be explored are: first, does the hysteresis model estimated by Bayesian methods using Euro-zone provide a better fit for the data when the latter includes unemployment (or even when it doesn't). Second, how does hysteresis change the nature of the optimized simple inertial rule? Is there a case for wage inflation and/or real wage targeting in such rules? Third, does hysteresis lead to more significant consumption equivalent gains compared with the rather small ones that emerge from such exercises in the literature?

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