Discussion Papers in Economics

LIMITED ASSET MARKET PARTICIPATION AND MONETARY POLICY IN A SMALL OPEN ECONOMY

By

Paul Levine
(University of Surrey and CIMS),

Stephen McKnight
(El Colegio de Mexico),

Alexander Mihailov
(University of Reading)
&
Jonathan Swarbrick
(University of St Andrews).

DP 09/21

School of Economics
University of Surrey
Guildford
Surrey GU2 7XH, UK
Telephone +44 (0)1483 689380
Facsimile +44 (0)1483 689548
Web https://www.surrey.ac.uk/school-economics
ISSN: 1749-5075

Limited Asset Market Participation and Monetary Policy in a Small Open Economy

Paul Levine, Stephen McKnight,
Alexander Mihailov and Jonathan Swarbrick*

October, 2021

Abstract

Limited asset market participation (LAMP) and trade openness are crucial features that characterize all real-world economies. We study equilibrium determinacy and optimal monetary policy in a model of a small open economy with LAMP. With low enough participation in asset markets, the conventional wisdom concerning the stabilizing benefits of policy inertia can be overturned irrespective of the constraint of a zero lower bound on the nominal interest rate. In contrast to recent studies, in LAMP economies trade openness can play an important stabilizing role. We also show that the central bank must balance the opposing influence of openness and LAMP on the aggressiveness of optimal policy, and that the equivalence between efficient and equitable optimal allocation found in closed economies breaks down in open economies. We derive targeting rules and demonstrate the superiority of commitment over discretion in implementable optimal interest rate rules.

JEL: E31; E44; E52; E58; E63; F41

Keywords: limited asset market participation; small open economy; inverted aggregate demand logic; equilibrium determinacy; policy inertia; optimal monetary policy

---

*Levine: School of Economics, University of Surrey, Guildford, GU2 7XH, UK (p.levine@surrey.ac.uk); McKnight: Centro de Estudios Económicos, El Colegio de México, Camino al Ajusco 20, Col. Pedregal de Santa Teresa, Mexico City, 10740, Mexico (mcknight@colmex.mx); Mihailov: Department of Economics, University of Reading, Whiteknights, Reading, RG6 6AA, UK (a.mihailov@reading.ac.uk); Swarbrick: School of Economics and Finance, University of St Andrews, St Andrews, KY16 9AZ, UK (jms48@st-andrews.ac.uk). We are grateful to Tom Holden, Ralph Luetticke and seminar participants at the University of Surrey, the Bank of Canada, the conference on "Macroeconomics and Reality: Where Are We Now?" at the University of Reading, the annual conferences of the Royal Economic Society (2021), the Canadian Economic Association (2021), the GdRE on Money, Banking and Finance at the Banque de France (2021) and the European Economic Association (2021). All remaining errors and misinterpretations are ours.

1 Introduction

Limited asset market participation (LAMP) is a well documented feature of both developing and developed economies. While its implications for monetary policy have recently been studied, the focus has been limited to closed economies. This paper seeks to address this gap. Our results suggest that trade openness and LAMP have important consequences for the design of monetary policy. First, we challenge the conventional wisdom on the benefits of inertia in monetary policy rules in preventing indeterminacy and self-fulfilling expectations. Second, we show that optimal monetary policy faces a difficult trade-off whereby openness requires an aggressive response to inflation, but LAMP a cautious response.

LAMP is commonly introduced into two-agent New Keynesian (TANK) models by allowing for a share of 'rule-of-thumb consumers', a concept coined by Mankiw (2000) and further popularized by Galí et al. (2004).[1] Often referred to as 'hand-to-mouth consumers' (e.g, by Kaplan et al., 2014), these households differ from Ricardian consumers in that they hold no assets and consume all current income. The empirical evidence supports the inclusion of a large share of hand-to-mouth (H2M) behaviour. For example, Aguiar et al. (2020) estimate that 40% of US households are H2M based on the Panel Study of Income Dynamics. This share is likely to be significantly higher in middle and low income countries.[2]

This paper makes two main contributions to the literature. First, we examine the determinacy properties of a small open New Keynesian (NK) economy with LAMP, focusing on the role of monetary policy inertia and trade openness for indeterminacy of Taylor-type feedback rules both with and without the zero lower bound on the nominal interest rate. Then in the second half of the paper, we replace the feedback rule with a microfounded welfare criterion and examine the implications of LAMP and trade openness for optimal targeting and implementable interest-rate rules under both discretion and commitment.

1.1 Policy Inertia and Trade Openness

As shown by Bilbiie (2008), LAMP can either reinforce or overturn the contractionary aggregate demand effect of a real interest rate increase in a closed economy. This can lead to an 'inverted aggregate demand logic' (IADL) that requires an 'inverted Taylor principle'

[1] It has been shown that for many purposes TANK models provide an appropriate theoretical shortcut to fully heterogeneous-agent New Keynesian models. See, e.g., Debortoli and Galí (2017) and Bilbiie (2020).

[2] For example, Boerma (2014) reports that half of all adults in upper middle income countries do not even have a bank account. This number rises to over 80% for low income countries.

for determinacy. The presence of IADL depends on whether the profit channel has a larger role than labor income. When asset market participation is low, the profit channel via the interest rate dominates the wage effect, leading to an expansionary effect of increasing interest rates. Boerma (2014) and Buffie and Zanna (2017) examine determinacy in the open economy with LAMP, but limit attention only to simple inflation targeting rules. We add to this literature by focusing on the role of monetary policy inertia, a well-documented feature of central bank behavior, including price-level targeting rules (so-called Wicksellian rules).

We find that interest-rate inertia has contrasting effects on the determinacy properties of standard and IADL economies. In the standard case, policy inertia reduces the possibility of indeterminacy, whereas it increases the likelihood of indeterminacy under IADL. This highlights an important caveat concerning the potential benefits of adopting Wicksellian rules. In the absence of LAMP, several studies find that price-level targeting improves the determinacy and stability properties compared to inflation targeting.[3] More recently, these benefits have been described in terms of "make-up" strategies for central banks (see Powell, 2020; Svensson, 2020).[4] Indeed, while determinacy is always possible under a price-level targeting rule in the standard case, under IADL, there are many degrees of LAMP for which determinacy is not possible. These findings are shown to be robust to a variety of popular specifications for the interest-rate rule, including the choice of inflation target and a policy response to output, and generalize to a medium-scale NK model with capital, incomplete asset markets, and positive trend inflation.

Trade openness is also found to have contrasting effects on determinacy in the standard and IADL economies, although this depends on both the degree of LAMP and interest-rate smoothing. While trade openness reduces the determinate policy space in the standard case, under IADL, openness increases the policy space when the degree of LAMP is high. Indeed, we find that under price-level targeting, closed IADL economies are more prone to indeterminacy than open IADL economies.

[3] See, e.g., Carlstrom and Fuerst (2002), Woodford (2003), Vestin (2006), Gaspar et al. (2007), Dib et al. (2013), Giannoni (2014), Bernanke (2017), and McKnight (2018). As Holden (2016) shows, these benefits extend to a zero lower bound setting.

[4] Under such strategies policymakers seek to redress past deviations of inflation from its target. Assuming a make-up rule enjoys credibility, undershooting (overshooting) the target will raise (lower) inflation expectations, lower (raise) the real interest rate and help to stabilize the economy. Inertial Taylor rules have by design the make-up feature as they commit to a response of the nominal interest rate to a weighted average of past inflation with the weights increasing with the degree of interest-rate smoothing.

In an extension of Bilbiie (2008) to the open economy, Boerma (2014) and Buffie and Zanna (2017) show that the Taylor principle is more likely to hold because the terms of trade channel of monetary policy can exert sufficient contractionary pressure after a rise in the real interest rate. It follows that monetary authorities could mistakenly adopt a 'passive' policy stance if they do not take into account the impact of both trade openness and LAMP on the monetary policy transmission mechanism. In contrast, we find the benefits of openness in restoring the Taylor principle are undermined by policy inertia. The scope for active policy is limited due to a lower bound on the inflation response coefficient, which becomes very large with even a small amount of inertia. While a policy response to output can help for some degrees of LAMP, this requires the central bank placing a large weight on output stabilization.

Following the analysis of the linear model, we examine the determinacy conditions in the presence of a zero lower bound (ZLB). We show that policy inertia can increase indeterminacy in standard economies under a ZLB, and the determinate region generated in IADL economies under the Taylor principle is found to be extremely unstable under a ZLB. While price-level targeting can be effective in preventing indeterminacy stemming from the ZLB, in general policy inertia is not effective in inducing determinacy unless the policy rule responds to a lagged 'shadow rate'.

1.2 Optimal Monetary Policy

Our second contribution is to extend the optimal monetary policy analysis of Bilbiie (2008) to the open economy dimension. In addition to the three market imperfections standard in the literature,[5] a behavioral constraint is also present in our model. This constraint has three aspects; namely, the inability of LAMP consumers to invest in either (i) domestic or (ii) foreign shares, and (iii) to pool risk in complete international markets.

We consider three welfare-relevant output gap concepts and three corresponding equilibrium baseline allocations around which to approximate a social welfare function. These are: the flexible-price decentralized equilibrium allocation; the efficient social planner allocation; the equitable social planner allocation. We show that the equivalence between the efficient and equitable equilibrium allocation in the closed LAMP economy of Bilbiie (2008) breaks down in open economies. The efficient allocation is not equitable in the

[5] Market power, relative price distortion and terms-of-trade manipulation incentive - see, e.g., Galí and Monacelli (2005).

open economy; it should use only employment subsidies to firms, as in Galí and Monacelli (2005), which do not restore equality across households. Furthermore, with trade openness the equitable allocation is not efficient; it adds the wage subsidy to the constrained households, but this is similar to an unemployment benefit, which introduces a labor market distortion and implies a trade-off between efficiency and equality.

We derive optimal monetary policy under an equitable allocation using government transfers.[6] Solving for the optimal targeting rules, a trade-off emerges regarding the degree of aggressiveness of the response of monetary policy to inflation, under both discretion and commitment. While higher trade openness requires a stronger response, a higher LAMP mitigates it. The intuition is as follows. The more open the economy, the less the output gap depends on domestic inflation, so the central bank needs to be more aggressive. In contrast, the higher the degree of LAMP, the larger is H2M behavior, and the more the output gap depends on domestic inflation, reducing the need for aggressiveness.

Finally, we derive implementable optimal interest-rate rules. Our results demonstrate that commitment is superior to discretion for two main reasons. First, commitment enhances welfare by avoiding the inflation bias in the steady state typical of discretion. Second, the targeting rule under commitment can be implemented as a saddle-path stable, robust interest rate rule, whereas the targeting rule under discretion cannot.

1.3 Road-Map

The rest of the paper is structured as follows. Section 2 sets out the SOE model with LAMP. For the determinacy analysis in Section 3, we take a linear approximation around a zero inflation, equitable steady state. We also consider a richer medium-scale model that allows for capital and investment spending, incomplete asset markets, and positive trend inflation to test the robustness of our analytical results. Section 4 derives a welfare-theoretic social loss function for the SOE LAMP model focusing on the equitable allocation. We first analyze optimal monetary policy in the form of targeting rules under both discretion and commitment, before deriving the corresponding implementable interest-rate rules. Finally, Section 5 concludes. Detailed derivations and proofs are provided in an online appendix.

[6] In doing so, our normative analysis relates to empirical studies which find that consumption inequality tracks closely income inequality (Aguiar and Bils, 2015).

2 A Small Open Economy Model with LAMP

This section presents our theoretical setup. It nests both the influential SOE framework of Galí and Monacelli (2005) and the closed-economy LAMP model of Bilbiie (2008). The economy is comprised of perfectly competitive wholesale firms that produce a final good and monopolistically competitive retailers that sell intermediate tradable goods under Calvo (1983) price setting. There are two types of households in the economy. In addition to standard Ricardian households, we include an exogenous fraction of constrained households that do not have access to asset markets.

2.1 Households

Households are divided into two types. A fraction of households, $\lambda \in (0,1)$, participate in domestic and international financial markets; these are referred to as Ricardian households and are denoted by superscript R. The remaining households $1-\lambda$, referred to as constrained households and denoted by superscript C, have no assets and must consume out of wage income without borrowing or risk-pooling options.

For both household types, $i = \{C, R\}$, single-period utility is assumed to be:

$$\begin{aligned} U_t^i = U(C_t^i, N_t^i) &= \frac{\left(C_t^i\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}; \quad \text{for } \sigma \neq 1 \\ &= \log\left(C_t^i\right) - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}; \quad \text{as } \sigma \to 1 \end{aligned} \tag{2.1}$$

where C_t^i is real consumption by household type i, σ is the coefficient of relative risk aversion (CRRA), N_t^i is labor supply of type i, and φ is the inverse of the Frisch elasticity.[7]

2.1.1 Ricardian Households

Ricardian households solve an intertemporal consumption problem:

$$\max_{C_t^R, N_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, N_{t+s}^R) \right] \tag{2.2}$$

[7] If $\sigma \to 1$ the functional form is consistent with a balanced growth path concept of the steady state.

subject to a sequence of nominal budget constraints given by:

$$P_t^B B_{H,t} + P_t^{B*} \mathcal{E}_t B_{F,t}^* = B_{H,t-1} + \mathcal{E}_t B_{F,t-1}^* + P_t W_t N_t^R - P_t C_t^R + \Gamma_t. \tag{2.3}$$

$B_{H,t}$ and $B_{F,t}^*$ are domestic and foreign bonds, denominated in the respective currencies, bought at the nominal price $P_t^B = 1/R_t$ and $P_t^{B*} = 1/R_t^*$, where R_t and R_t^* denote the domestic and foreign nominal interest rate, respectively. P_t is the consumer price index (CPI) and $\mathcal{E}_t$ is the nominal exchange rate, measured as the domestic price of a unit of foreign currency. Finally, W_t and Γ_t denote the real wage rate and nominal profits, respectively.

Maximizing (2.2) subject to the budget constraint we obtain:

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \right], \tag{2.4}$$

$$P_t^{B*} = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right], \tag{2.5}$$

$$\frac{U_{N,t}^R}{U_{C,t}^R} = -\left(C_t^R\right)^\sigma \left(N_t^R\right)^\varphi = -W_t, \tag{2.6}$$

where $\Pi_{t,t+1} \equiv \frac{P_{t+1}}{P_t}$ denotes the CPI inflation rate and $\Lambda_{t,t+1}^R \equiv \beta \frac{U_{C,t+1}^R}{U_{C,t}^R}$ is the stochastic discount factor for Ricardian consumers.

2.1.2 Consumption Demand

Households demand consumption goods from domestic H and foreign F retailers (imports):

$$C_t = \left[\mathrm{w}_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C - 1}{\mu_C}} + (1 - \mathrm{w}_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}. \tag{2.7}$$

The corresponding price index and CPI inflation rate are given by:

$$P_t = \left[\mathrm{w}_C (P_{H,t})^{1-\mu_C} + (1 - \mathrm{w}_C)(P_{F,t})^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}, \tag{2.8}$$

$$\Pi_{t-1,t} = \left[\mathrm{w}_C \left(\Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1 - \mathrm{w}_C) \left(\Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}, \tag{2.9}$$

where $\Pi_{H,t-1,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_{F,t-1,t} \equiv \frac{P_{F,t}}{P_{F,t-1}}$. The weight w_C in the consumption basket attached to domestic consumption demand is a measure of home bias (where $\mathrm{w}_C = 1$ is the autarky case). Maximizing total consumption (2.7) subject to a given aggregate expenditure $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ yields:

$$C_{H,t} = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t, \tag{2.10}$$

$$C_{F,t} = (1 - \mathrm{w}_C) \left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t. \tag{2.11}$$

Foreign aggregate consumption C_t^* is given by an exogenous process. The real exchange rate is defined as the relative aggregate consumption price $Q_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}$. Then the foreign counterpart of the import demand schedule (2.11), which determines the export demand of the home good, is

$$C_{H,t}^* = (1 - \mathrm{w}_C^*) \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\mu_C^*} C_t^* = (1 - \mathrm{w}_C^*) \left(\frac{P_{H,t}}{P_tQ_t}\right)^{-\mu_C^*} C_t^*. \tag{2.12}$$

$P_{H,t}^*$ and P_t^* denote the respective prices of home-produced (i.e., imported) consumption goods and of aggregate consumption goods in the rest of the world (RoW) in foreign currency, and we have used the law of one price for differentiated goods, $\mathcal{E}_tP_{H,t}^* = P_{H,t}$. We impose perfect exchange rate pass-through for imports and because the home country is small, the law of one price implies that $P_t^* = P_{F,t}^*$, $\mathcal{E}_tP_t^* = P_{F,t}$, so $Q_t = \frac{P_{F,t}}{P_t}$. We can then write (2.12) as:

$$C_{H,t}^* = (1 - \mathrm{w}_C^*) \left(\frac{1}{S_t}\right)^{-\mu_C^*} C_t^*, \tag{2.13}$$

where $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ are the terms of trade (ToT). Finally, total exports per capita is defined as $EX_t \equiv C_{H,t}^*$.

2.1.3 Constrained Consumers

Constrained consumers have no income from monopolistically competitive retail firms and must consume out of wage income. Their nominal consumption is given by:

$$P_tC_t^C = P_tW_t. \tag{2.14}$$

Constrained consumers choose C_t^C and N_t^C to maximize an analogous utility function to (2.2) but subject to (2.14). The first order conditions can be written as:

$$\frac{U_{N,t}^C}{U_{C,t}^C} = -\left(C_t^C\right)^{\sigma}\left(N_t^C\right)^{\varphi} = \frac{U_{N,t}^R}{U_{C,t}^R} = -W_t, \tag{2.15}$$

which has the same form as eq. (2.6) for the Ricardian consumers, but as we shall discuss further below C_t^C and N_t^C are not generally the same as C_t^R and N_t^R.

With both Ricardian and constrained households, aggregate consumption and hours supplied are given by:

$$C_t = \lambda C_t^R + (1-\lambda)C_t^C, \tag{2.16}$$

$$N_t = \lambda N_t^R + (1-\lambda)N_t^C. \tag{2.17}$$

2.2 Firms

There are wholesale and retail firms. The former act in perfect competition producing a homogeneous final good, whereas the latter are monopolistic competitive that produce differentiated intermediate goods.

2.2.1 Wholesale Sector

Wholesale firms hire labor N_t to produce homogeneous output Y_t^W using the standard labor-augmenting constant returns to scale production technology:

$$Y_t = F(N_t, A_t) = A_t N_t. \tag{2.18}$$

Profit maximization implies:

$$P_t W_t = P_t^W F_{N,t} = P_t^W \frac{Y_t}{N_t} \quad \Rightarrow W_t = MC_t\left(\frac{P_{H,t}}{P_t}\right)\frac{Y_t}{N_t}, \tag{2.19}$$

where $MC_t \equiv \frac{P_t^W}{P_{H,t}}$ is real marginal cost in units of domestic retail output.

2.2.2 Retail Sector

A retail firm m converts an amount of wholesale output $Y_t^W(m)$ into a differentiated good of amount $Y_t(m) - F(m)$, where $F(m) = F$ are fixed costs assumed to be equal across retail firms. The retail differentiated goods are combined into the final good Y_t using a CES-aggregator production technology:

$$Y_t \equiv \left[\int_0^1 Y_t(m)^{\frac{\varsigma-1}{\varsigma}} dm \right]^{\frac{\varsigma}{\varsigma-1}}. \tag{2.20}$$

The CES technology implies demand schedules for each intermediate input j given by:

$$Y_t(m) = \left[\frac{P_{H,t}(m)}{P_{H,t}} \right]^{-\varsigma} Y_t. \tag{2.21}$$

Nominal profits of the retail firm m can be written as:

$$\Omega_t(m) = P_{H,t}(m)[Y_t(m) - F - MC_t Y_t(m)], \tag{2.22}$$

where real marginal cost MC_t is common to all retail firms.

Following Calvo (1983), in every period each retail firm m faces a fixed probability $1 - \xi$ of being able to optimally set their price to $P^0_{H,t}(m)$. If the price is not re-optimized, then it is held fixed. Using (2.22), the objective of a retail producer m at time t is to choose $P^0_{H,t}(m)$ to maximize discounted real profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{H,t+k}} Y_{t+k}(m) \left[P^0_{H,t}(m) - P_{H,t+k} MC_{t+k} \right] \tag{2.23}$$

subject to

$$Y_{t+k}(m) = \left(\frac{P^0_{H,t}(m)}{P_{H,t+k}} \right)^{-\zeta} Y_{t+k}, \tag{2.24}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$. This leads to the usual optimal price condition and aggregate law of motion for aggregate inflation:

$$\frac{P^0_{H,t}}{P_t} = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}\right)^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}\right)^{\zeta-1} Y_{t+k}}, \tag{2.25}$$

$$1 = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta-1} + (1-\xi) \left(\frac{P^0_{H,t}}{P_{H,t}}\right)^{1-\zeta}, \tag{2.26}$$

where the m index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history. Aggregate output Y_t is given by:

$$Y_t = \frac{A_t N_t - F}{\Delta_t}, \tag{2.27}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(m)}{P_{H,t}}\right)^{-\varsigma} dm \geq 1$ is the degree of price dispersion of retail goods which can be shown to follow the dynamic process:

$$\Delta_t = \xi \Pi^{\zeta}_{H,t-1,t} \Delta_{t-1} + (1-\xi) \left(\frac{P^0_{H,t}}{P_{H,t}}\right)^{-\zeta}. \tag{2.28}$$

2.3 Output Market Clearing

Output market clearing for retail firm m is:

$$Y_t(m) = C_{H,t}(m) + C^*_{H_t}(m).$$

Aggregating yields the following resource constraint:

$$Y_t = C_{H,t} + C^*_{H_t} = C_{H,t} + EX_t, \tag{2.29}$$

and using the demand conditions (2.10) and (2.13) yields:

$$Y_t = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t + (1-\mathrm{w}^*_C) \left(\frac{1}{S_t}\right)^{-\mu^*_C} C^*_t. \tag{2.30}$$

2.4 Monetary Policy

The nominal interest rate R_t is a policy variable given by a standard Taylor-type rule:[8]

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \mathbb{E}_t\left[\theta_\pi \log\left(\frac{\Pi_{t,t+1}}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right)\right], \tag{2.31}$$

where $\rho_r, \theta_\pi, \theta_y \geq 0$. We focus on forward-looking rules as many central banks target forecasted inflation in practice due to the observed time delay in the transmission mechanism of monetary policy.[9]

2.5 Foreign Bond Accumulation

In nominal terms and measured in the home country currency, foreign bond holdings evolve according to:

$$P_t^{B^*}\mathcal{E}_t B_{F,t}^* = \mathcal{E}_t B_{F,t-1}^* + P_t TB_t,$$

where the nominal trade balance $P_t TB_t = P_{H,t}Y_t - P_t C_t$ is the difference between domestic output and private consumption. Defining $B_{F,t} \equiv \frac{\mathcal{E}_t B_{F,t}^*}{P_t}$ to be the stock of foreign bonds in home country consumption units, it follows that

$$P_t^{B^*} B_{F,t} = \frac{\Pi_{t-1,t}^{\mathcal{E}}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t, \tag{2.32}$$

where $\Pi_{t-1,t}^{\mathcal{E}} \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$ is the (gross) nominal depreciation of the SOE currency.

2.6 Equilibrium of Small Open Economies with Risk Sharing

Up to now we have modeled the SOE in an environment consisting of the RoW, which from its own viewpoint is closed. We now amend the environment to consist of a continuum of $i \in [0,1]$ identical open economies of which the 'home' economy is just one. We assume there is international risk-sharing in this version of the model so the risk premium is zero. The first-order conditions (2.4) and (2.5) lead to the standard risk-sharing condition:

$$C_t^R = (C_t^R)^i Q_{i,t}^{\frac{1}{\sigma}} \tag{2.33}$$

[8] This is in 'implementable' form as proposed by Schmitt-Grohe and Uribe (2007). The conventional Taylor rule replaces the output level relative to its steady state $\frac{Y_t}{Y}$ with the output gap $\frac{Y_t}{Y_t^n}$ where Y_t^n is the natural rate, i.e., the level of output that would have prevailed if all prices were perfectly flexible.

[9] For further discussion, see Batini and Haldane (1999) and McKnight and Mihailov (2015).

where $Q_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$ is the home country (or SOE) vis-à-vis country (or SOE) i bilateral real exchange rate, with $\mathcal{E}_{i,t}$ now the corresponding bilateral nominal exchange rate between these two countries (both identical SOEs). Naturally, the risk-sharing only applies to Ricardian and not constrained households. Corresponding to using (2.33), we now have

$$\begin{aligned} C^*_{H,t} &= \int_0^1 C^i_{H,t} di = (1-\mathrm{w}^*_C) \int_0^1 \left(\frac{P_{H,t}}{P_t Q_{i,t}}\right)^{-\mu^*_C} C^i_t di \\ &= (1-\mathrm{w}^*_C) \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} Q_t^{\mu_C - \frac{1}{\sigma}} \left[\lambda C^R_t + (1-\lambda) C^C_t\right] \end{aligned} \tag{2.34}$$

in a symmetric equilibrium with $\mu_C = \mu^*_C$, $\lambda_i = \lambda$, and $Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$.

2.7 Equilibrium

An equilibrium is defined in the model variables given the conditions outlined above together with the interest rate rule (2.31) and three structural exogenous shock processes A_t, C^*_t and R^*_t, which are assumed to follow stochastic AR(1) processes. Appendix A of the online appendix provides a summary of this equilibrium.

2.8 Extended Medium-Scale Model

As a robustness check on the analytical results, we run numerical computations of stability and determinacy for an extended medium-scale LAMP model that includes capital and investment spending, government spending, incomplete asset markets, and positive trend inflation. Appendix B of the online appendix summarizes this model version.

3 Stability and Determinacy Analysis

The model is linearized around a non-stochastic steady state where net inflation is zero, i.e., $\Pi = 1$, and prices $P = P_H = P_F = P^* = 1$. Then by definition the steady state terms of trade and real exchange rate are $\varepsilon = Q = 1$. For the LAMP aspects of the model we also impose an equitable outcome $C^R = C^C$ and $N^R = N^C$ which from (2.22) can be achieved by assuming that free entry drives profits to zero in an equilibrium in the steady state with $\frac{F}{Y} = (1 - MC) = \frac{1}{\zeta}$.[10] Since the focus of this section is on (local) stability

[10] As we discuss later in the paper, alternatively a subsidy scheme for the optimal equitable allocation in Proposition 6 in Section 4.1 can support this outcome.

and equilibrium determinacy, we consider the deterministic perfect foresight case with all shocks set equal to zero. In what follows, all lower-case variables in this section denote percentage deviations from the steady state.

We can describe the non-policy aspects of the model using a New Keynesian Phillips Curve (NKPC) and an intertemporal IS curve, both expressed in terms of consumption by Ricardian consumers:[11]

$$\pi_{H,t} = \beta\pi_{H,t+1} + \Psi\Upsilon c_t^R, \tag{3.1}$$

$$c_t^R = c_{t+1}^R - \frac{\mathrm{w}_C}{\sigma}\left(r_t - \pi_{H,t+1}\right), \tag{3.2}$$

$$y_t = \Xi c_t^R, \tag{3.3}$$

where the parameters are defined as:

$$\Upsilon \equiv \frac{\sigma(1-\mathrm{w}_C)}{\mathrm{w}_C} + \frac{\lambda(\varphi+\sigma)\left[\mathrm{w}_C\varphi + \sigma\left(1+\frac{1}{\zeta}\right)\right] + \varphi(1-\mathrm{w}_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}, \tag{3.4}$$

$$\Xi \equiv \mathrm{w}_C\lambda + (1-\mathrm{w}_C)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right] + \mathrm{w}_C(1-\lambda)\frac{(1+\varphi)}{\varphi+\sigma}\left(\Upsilon - \frac{\sigma}{\mathrm{w}_C} + \sigma\right), \tag{3.5}$$

$\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} > 0$, and $\omega \equiv \mathrm{w}_C(\mu_C - 1/\sigma) + \mu_C^* = \mu_C(1+\mathrm{w}_C) - \mathrm{w}_C/\sigma > 0$, if $\mu_C = \mu_C^*$.

The threshold for the proportion of Ricardian households λ below which the inverted aggregate demand logic (IADL) occurs is the point at which Υ changes sign. From (3.4) this is given by

$$\lambda = \lambda^* = \frac{\varphi\left[\mathrm{w}_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\varphi\left[\mathrm{w}_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right] + (\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)}. \tag{3.6}$$

Then replacing $\lambda^* = \lambda^*(\mathrm{w}_C)$ we have the following result:

Proposition 1. ***(IADL threshold)*** *The threshold below which IADL occurs, $\lambda^* = \lambda^*(w_C)$,*

[11] See appendix C.1 for derivation of the minimum state-space representation of the model. Alternative NKPC and IS expressions, written in terms of total consumption in deviations from baseline allocations, and hence in standard output gap terms, are discussed in Section 4.

increases with w_C and therefore decreases with trade openness $1 - w_C$

Proof: See appendix C.3.

Consequently, trade openness *decreases the possibility of IADL*.[12] To understand the IADL, notice that we can write Ricardian labour supply as: $n_t^R = \frac{1}{\varphi}\left(\Upsilon - \frac{\sigma}{\mathrm{w}_C}\right)c_t^R$, which implies that hours fall in consumption for Ricardian households provided $\Upsilon < \frac{\sigma}{\mathrm{w}_C}$. When asset market participation is low, the profit channel dominates the wage effect, and increases in the real interest rate $r_t - \pi_{t+1} = \mathrm{w}_C\left(r_t - \pi_{H,t+1}\right)$ can have an expansionary effect on output y_t. For example, in the closed economy ($\mathrm{w}_C = 1$) it follows from (3.5) that $\Xi^{\mathrm{w}_C=1} = \frac{\Upsilon(1+1/\zeta)}{\varphi+\sigma(1+1/\zeta)} < 0$ under IADL. From (3.2) and (3.3), a rise in the real interest rate increases output by reducing Ricardian consumption c_t^R, exerting upward pressure on inflation from the NKPC. This contrasts with the standard aggregate demand logic (SADL) where both output and consumption respond negatively to real interest rate rises. In open economies, it follows from (3.4) and (3.5) that $\Xi > 0$ when $\Upsilon > 0$, so that c_t^R always increases in y_t under SADL. However, under IADL, c_t^R can either increase or decrease in y_t depending on the degree of LAMP.

The parameter Υ is a function of λ and the other model parameters w_C, φ, σ, and ω, but is independent of the monetary policy rule. For this, we initially assume the policymaker follows a simple inertial rule of the form:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1}, \tag{3.7}$$

where $\rho_r \geq 0$ is the degree of interest rate inertia and $\theta_\pi \geq 0$ is the inflation response coefficient. Note that the integral rule with $\rho_r = 1$ yields a price-level (Wicksellian) rule. The interest-rate rule (3.7) can be expressed as:

$$r_t = \rho_r r_{t-1} - \frac{\sigma(1-\mathrm{w}_C)\theta_\pi}{\mathrm{w}_C} c_t^R + \frac{\sigma(1-\mathrm{w}_C)\theta_\pi}{\mathrm{w}_C} c_{t+1}^R + \theta_\pi \pi_{H,t+1}. \tag{3.8}$$

Equations (3.1), (3.2) and (3.8) imply the minimal state-space representation of the model:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \quad \mathbf{z}_t = \begin{bmatrix} c_t^R & \pi_{H,t} & r_{t-1} \end{bmatrix}'. \tag{3.9}$$

[12] This result is consistent with the findings of Boerma (2014) and Buffie and Zanna (2017).

3.1 Determinacy Analysis

We start by examining the stability properties of the model for the policy rule (3.7).

Proposition 2. ***(Role of interest-rate inertia)*** *For the standard SADL case $\lambda > \lambda^*$, interest rate inertia increases the policy space for θ_π for which there is equilibrium determinacy. An equilibrium exists for all $\lambda \in (\lambda^*, 1]$ with an appropriate choice of θ_π. Under IADL, there exists some value of $\lambda \in [0, \lambda^*)$ for which a unique stable equilibrium exists. Interest rate inertia in this case reduces the policy space for θ_π, and for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)}{\Psi w_C} < \Upsilon < -\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C}$$

then a unique equilibrium does not exist for $\theta_\pi > 0$.

Proposition 3. ***(Role of trade openness)*** *For the standard SADL case $\lambda > \lambda^*$, trade openness $1-w_C$ decreases the policy space for θ_π for which there is equilibrium determinacy. Under IADL, the policy space for θ_π increases with $1-w_C$ for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C} < \Upsilon < 0.$$

The results given in propositions 2 and 3 follow from the necessary and sufficient conditions for equilibrium determinacy:[13]

Case IA: If $\Upsilon > 0$: $\frac{1}{1-\mathrm{w}_C} < \theta_\pi < \Gamma_1$ and $\rho_r > \left(\frac{\mathrm{w}_C}{1-\mathrm{w}_C}\right)\left[\frac{\Psi \mathrm{w}_C \Upsilon}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)}\right]$, where

$$\Gamma_1 \equiv (1+\rho_r)\left[1 + \frac{2(1+\beta)\sigma \mathrm{w}_C}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)}\right];$$

Case IB: If $\Upsilon > 0$: $1-\rho_r < \theta_\pi < \min\left\{\frac{1}{1-\mathrm{w}_C}, \Gamma_1\right\}$ and one of the following inequalities is satisfied:

$$\left|-1 - \frac{1}{\beta} - \frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma\left[1-(1-\mathrm{w}_C)\theta_\pi\right]}\right| > 3, \tag{3.10}$$

$$1 - \beta + \frac{\rho_r}{1-(1-\mathrm{w}_C)\theta_\pi}\left[\frac{\rho_r(1-\beta)}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} + \beta - \frac{1}{\beta} + \frac{\Psi \mathrm{w}_C \Upsilon}{\sigma}\left(1 + \frac{\theta_\pi - 1}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]}\right)\right] > 0; \tag{3.11}$$

[13] See appendix C.4 for the derivation of these conditions.

Case IIA: If $\Upsilon < -\frac{2\sigma(1+\beta)}{\Psi \mathrm{w}_C}$: and one of the following is satisfied:

(i) $1-\rho_r < \theta_\pi < \min\left\{\frac{1}{1-\mathrm{w}_C},\ \Gamma_1\right\}$,

(ii) $\Gamma_1 < \theta_\pi < 1-\rho_r$, $(1-\rho_r)(1+\beta)\sigma\mathrm{w}_C + \left[2\sigma(1+\beta)+\Psi\mathrm{w}_C\Upsilon\right]\rho_r > 0$ and one of the inequalities given by (3.10)–(3.11) holds,

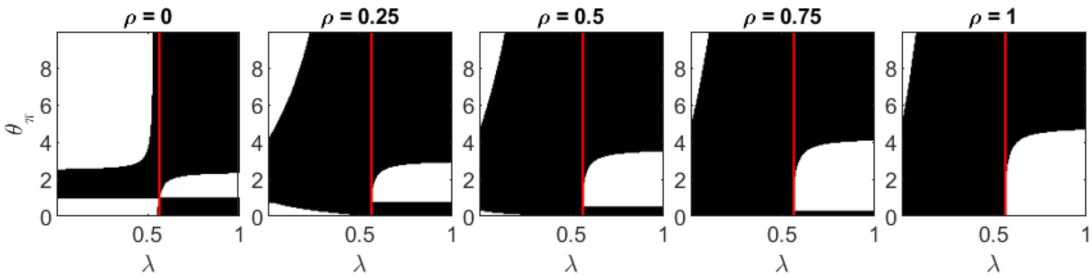
(iii) $\frac{1}{1-\mathrm{w}_C} < \theta_\pi < \Gamma_1,\ \rho_r > \left(\frac{\mathrm{w}_C}{1-\mathrm{w}_C}\right)\left[\frac{\Psi\mathrm{w}_C\Upsilon}{\Psi\mathrm{w}_C\Upsilon+2\sigma(1+\beta)}\right]$, and one of the inequalities given by (3.10)–(3.11) holds;

Case IIB: If $-\frac{2\sigma(1+\beta)}{\Psi\mathrm{w}_C} < \Upsilon < -\frac{2\sigma(1+\beta)(1-\mathrm{w}_C)}{\Psi\mathrm{w}_C}$: $\theta_\pi < 1-\rho_r$, and one of the inequalities given by (3.10)–(3.11) is satisfied;

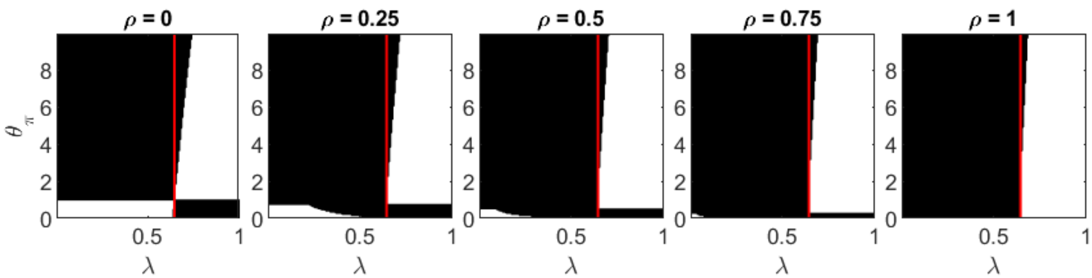
Case IIC: If $-\frac{2\sigma(1+\beta)(1-\mathrm{w}_C)}{\Psi\mathrm{w}_C} < \Upsilon < 0$: one of the inequalities given by (3.10)–(3.11) is satisfied, and either $\theta_\pi < 1-\rho_r$ or $\theta_\pi > \max\left\{\frac{1}{1-\mathrm{w}_C}, \Gamma_1\right\}$.

In the absence of interest rate inertia ($\rho_r = 0$), the Taylor principle ($\theta_\pi > 1$), which implies an 'active' policy feedback to future inflation, is a necessary condition for determinacy in the SADL case ($\Upsilon > 0$). In contrast, for the IADL case ($\Upsilon < 0$) a 'passive' policy stance ($\theta_\pi < 1$), or the *inverted Taylor principle*, is consistent with determinacy for closed economies. The determinacy conditions indicate that increasing interest rate inertia *increases* the range of determinacy under SADL (Case I), while it *reduces* the determinate policy space under IADL (Case II). Trade openness has contrasting effects. By reducing the upper bound Γ_1 on the inflation response coefficient, the determinacy region shrinks in open SADL economies (Case I). However, the region of determinacy can actually increase under IADL (Case IIC), as the economy becomes more open. For sufficiently low values of λ, determinacy arises under the Taylor principle provided the inflation response coefficient is set sufficiently high $\theta_\pi > \max\left\{\frac{1}{1-\mathrm{w}_C}, \Gamma_1\right\}$.

The above results are illustrated in Figure 1 for a standard quarterly parameterization. We set the discount factor $\beta = 0.99$, the CRRA coefficient $\sigma = 2$, $\zeta = 7$, implying a markup of 16 percent, and the real marginal cost elasticity of inflation $\Psi = 0.086$, consistent with an average price duration of one year. The open economy parameters are set with home bias $\mathrm{w}_C = 0.6$ and an elasticity of substitution $\mu_C = 0.62$ in line with the estimates of Boehm et al. (2019). By inspection, while policy inertia has a stabilizing effect on the SADL economy, trade openness has a destabilizing effect. Under IADL, determinacy can also arise under the Taylor principle, as openness exerts a stabilizing effect, whereas policy



(a) Open economy



(b) Closed economy

Figure 1: Determinacy regions (white areas) for the baseline LAMP model. $\Psi = 0.086$, $\varphi = 2$, $\sigma = 2$, $\beta = 0.99$, $\zeta = 7$, $\mathrm{w}_C = 0.6$, and $\mu_C = 0.62$ for the open economy in the top panel and $\mathrm{w}_C = 1$ for the closed in the bottom panel. The red vertical line gives λ^* below which IADL holds.

inertia now destabilizes the IADL economy.

Under a price-level targeting ($\rho_r = 1$), determinacy is easily achieved in the SADL case. For instance, consider the closed-economy version of the model ($\mathrm{w}_C = 1$). Case IB simplifies to $0 < \theta_\pi < 2 + \frac{4\sigma(1+\beta)}{\Psi\Upsilon}$, where the upper bound is non-binding within the empirically-relevant interval $\theta_\pi \in [0, 10]$, except for λ very close to the threshold λ^*.[14] In contrast, determinacy is only possible under IADL from Case IIA:

$$\left(1 - \frac{1}{1 + \frac{2\sigma(1+\beta)}{\Psi}\frac{1+\varphi}{\sigma+\varphi}}\right)\lambda^* < \lambda < \lambda^*.$$

For the baseline parameter values, there exists a very small interval of λ for which determinacy is possible ($0.986\lambda^* < \lambda < \lambda^*$). As highlighted in Figure 1, the determinacy region

[14] With our baseline parameter values, the upper bound on θ_π lies in the interval [2, 10] for $\lambda \in [0.644, 0.684]$.

is barely visible under a price-level rule for both closed and open IADL economies. This is in stark contrast to the case of no rule-of-thumb consumers ($\lambda = 1$), where determinacy is easily induced.[15]

For some intuition, first consider a sunspot-induced increase in inflationary expectations in a closed economy. For the SADL case, the Taylor principle induces a rise in the real interest rate, resulting in a fall in consumption and output. This exerts downward pressure on real marginal cost, which lowers inflation from the NKPC, contradicting the initial inflationary expectations. Similar to Bullard and Mitra (2007), interest-rate inertia helps to enlarge the determinacy region, as the long-run nominal interest-rate is $1/1-\rho_r$ times more responsive to permanent changes in inflation compared to the non-inertial case. Under a price-level rule, any increase in inflation results in a rise in both the nominal and real interest rate. For any $\theta_\pi > 0$, the Taylor principle is always satisfied and indeterminacy is not possible.

In IADL economies, Ricardian consumption falls but output rises in response to a higher real interest rate. Consequently, real marginal cost increases and the initial inflationary belief becomes self-fulfilling under the Taylor principle. In this case, a passive policy response by letting the real interest fall in response to higher expected inflation, leads to lower demand and deflation from the NKPC, contradicting the initial inflationary expectations. However, interest rate inertia reduces the determinacy region under the inverted Taylor principle, which becomes nearly impossible under a Wicksellian rule.

In open economies, first note that the next-period consumer-price inflation rate depends on both the rate of future domestic price inflation and changes in the terms of trade:

$$\pi_{t+1} = \pi_{H,t+1} + (1 - \mathrm{w}_C)(s_{t+1} - s_t) = \pi_{H,t+1} + \sigma \left(\frac{1 - \mathrm{w}_C}{\mathrm{w}_C} \right) \left(c^R_{t+1} - c^R_t \right).$$

For the SADL case, a real interest rate rise results in an expected deterioration in the terms of trade $s_{t+1} - s_t > 0$. Consequently, indeterminacy can arise under the Taylor principle provided the upward pressure on consumer-price inflation, generated by the adjustments in the terms of trade, is sufficiently strong to offset the reduction in domestic-price inflation generated from lower domestic demand. As the degree of trade openness $1 - \mathrm{w}_C$ increases, the economy becomes more prone to indeterminacy. However, in stark contrast to closed

[15] For example, there exists a unique stable equilibrium in the closed economy after setting $\lambda = 1$ iff $0 < \theta_\pi < 2\left[1 + \frac{2\sigma(1+\beta)}{\Psi(\sigma+\varphi)}\right]$.

economies, determinacy can be consistent with the Taylor principle under IADL. While rises in the real interest rate now result in an increase in domestic-price inflation, the upward pressure exerted on consumer-price inflation can be more than offset via a reduction in Ricardian consumption ($c^R_{t+1} - c^R_t < 0$) arising from the adjustment in the terms of trade.

Below we examine the robustness of these findings using several variants of the policy rule (3.8) commonly found in the literature.

3.2 Domestic-Price Inflation Targeting

We now consider the determinacy implications of rule-of-thumb consumers under a domestic price inflation rule with policy inertia:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{H,t+1}, \tag{3.12}$$

where setting $\rho_r = 1$ yields a domestic-price-level rule.

Proposition 4. ***(Domestic-price inflation)*** *For the standard SADL case $\lambda > \lambda^*$, interest rate inertia increases the policy space for θ_π for which there is equilibrium determinacy. Under IADL, interest rate inertia decreases the determinate policy space for θ_π. The effect of trade openness is ambiguous. However, for a standard range of parameter values, trade openness enlarges the determinate policy space under SADL and reduces it under IADL.*

Proof: See appendix C.5.

Under a domestic-price inflation rule, the role of trade openness can be reversed. For the SADL case, the upper bound on the inflation response coefficient is now given by:

$$\Gamma_1^{PPI} \equiv (1 + \rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi \mathrm{w}_C \Upsilon}\right],$$

which can either increase or decrease with trade openness $1 - \mathrm{w}_C$ depending on the value of $\lambda > \lambda^*$.[16] For the IADL case, the large determinacy region that arises under the Taylor principle in open economies is no longer available if domestic-price inflation is targeted.[17]

Interest-rate inertia has similar implications for determinacy regardless of the choice of

[16] Close to the IADL threshold, λ^*, the upper-bound Γ_1^{PPI} is decreasing with $1 - \mathrm{w}_C$ and in the case of no rule-of-thumb consumers ($\lambda = 1$), it is increasing with $1 - \mathrm{w}_C$ provided $1 - \sigma\mu_C > 0$.

[17] Plots of the determinacy regions are shown in Figure 8 in appendix C.5.

inflation target. For example, consider a domestic price-level rule by setting $\rho_r = 1$ in (3.12). The necessary and sufficient condition for equilibrium determinacy is given by:

$$0 < \theta_\pi < 2 + \frac{4\sigma(1+\beta)}{\Psi \mathrm{w}_C \Upsilon}.$$

Therefore, determinacy is impossible in IADL economies provided $-2\sigma(1+\beta)/\Psi\mathrm{w}_C < \Upsilon < 0$, which using the baseline parameter values suggests $\lambda < 0.63$ for a closed economy and $\lambda < 0.55$ with $\mathrm{w}_C = 0.6$. In both cases, this threshold is a value approximately 0.01 below λ^*, emphasizing the narrowness of the region for which determinacy is possible.[18]

3.3 Output Stabilization

We now consider the determinacy implications of a policy response to contemporaneous output (or the output gap). Since y_t is linear in c_t^R, the Taylor rule can be expressed as:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1} + \theta_y \Xi c_t^R, \tag{3.13}$$

where $\theta_y \geq 0$ is the output response coefficient and Ξ is given by (3.5).

Proposition 5. *For the standard SADL case $\lambda > \lambda^*$, a policy response to output $\theta_y > 0$ increases the policy space for θ_π for which there is equilibrium determinacy. Under IADL, there exists some values of $\lambda \in [0, \lambda^*)$ for which the Taylor principle is restored. However, in this case, the equilibrium is indeterminate regardless of the value of θ_π if $\theta_y < \overline{\theta}_y$, where $\overline{\theta}_y$ is increasing with interest rate inertia and decreasing with trade openness.*

Proof: See appendix C.6.

Under SADL, both closed and open economies are less prone to indeterminacy with a policy response to output. Since $\Xi > 0$ with $\Upsilon > 0$, it follows that the slope $\frac{(1-\beta)\Xi}{\Psi\Upsilon}$ of the long-run NKPC is positive and the generalized (or long-run) version of the Taylor principle is given by:

$$\theta_\pi + \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y > 1 - \rho_r. \tag{3.14}$$

Increasing ρ_r results in a parallel inward shift of the long-run Taylor principle on the plane

[18] As shown in appendix C.7, similar conclusions are obtained under a contemporaneous-looking interest-rate rule: $r_t = \rho_r r_{t-1} + \theta_\pi \pi_t$.

$(y_t,\, \pi_t)$, and the upper bound Γ_1^y on the inflation response coefficient is increasing in θ_y:

$$\Gamma_1^y \equiv (1+\rho_r)\left[1+\frac{2(1+\beta)\sigma \mathrm{w}_C}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)}\right] + \frac{\mathrm{w}_C(1+\beta)\Xi}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)}\theta_y.$$

With a policy response to output, the IADL breaks down in closed economies. Since $\Xi < 0$ with $\Upsilon < 0$ (after setting $\mathrm{w}_C = 1$), determinacy can only arise under the inverted Taylor principle when $\theta_y = 0$. Instead, determinacy requires the central bank to follow the generalized Taylor principle (3.14) and place a sufficiently large weight on output: $\theta_y > \left(\frac{\theta_\pi - 1 - \rho_r}{1+\beta}\right)\frac{\Psi\Upsilon}{\Xi} - \frac{2\sigma(1+\rho_r)}{\Xi}$. However, as illustrated in Figure 3(b), determinacy requires θ_y to be large suggesting that indeterminacy is likely to arise in a closed economy when $\Upsilon < 0$ for empirically realistic output responses $\theta_y \in [0, 2]$. Moreover, since the lower bound on θ_y is increasing in ρ_r, policy inertia further undermines the ability of a policy response to output to help restore the Taylor principle when $\lambda < \lambda^*$.

In open economies Ξ can be positive or negative under $\Upsilon < 0$. This switch is clearly shown in Figure 2 by setting $\lambda = 0.2, 0.5$, since $\Xi > 0$ for any $\lambda < 0.3$ under the baseline parameter values. Thus, for low levels of λ, the IADL is maintained and determinacy arises under the inverted Taylor principle. Figure 3(a) highlights the role of trade openness and policy inertia under $\Upsilon < 0$ when $\Xi < 0$. By inspection, openness not only improves the determinacy properties of the IADL economy by lowering λ^*, but for the case $\lambda < \lambda^*$, the determinate policy space is also much larger in the open economy.

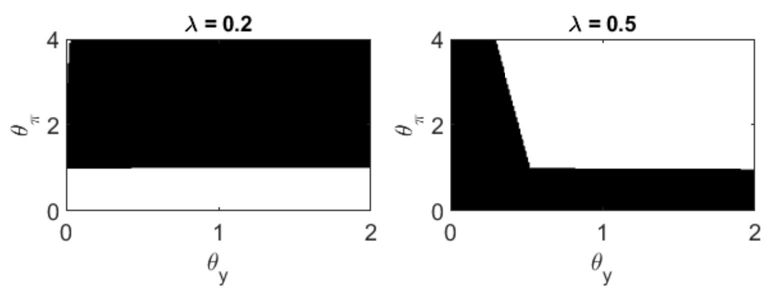
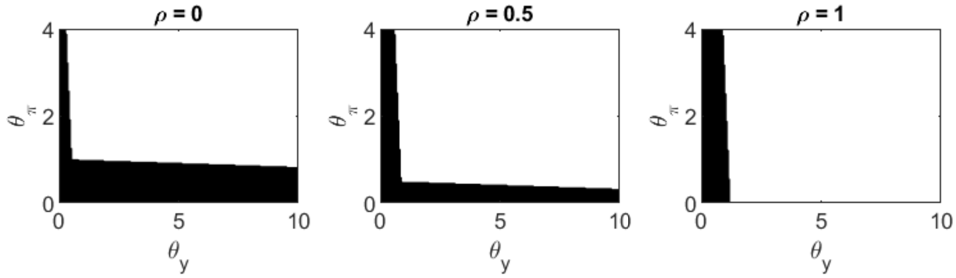
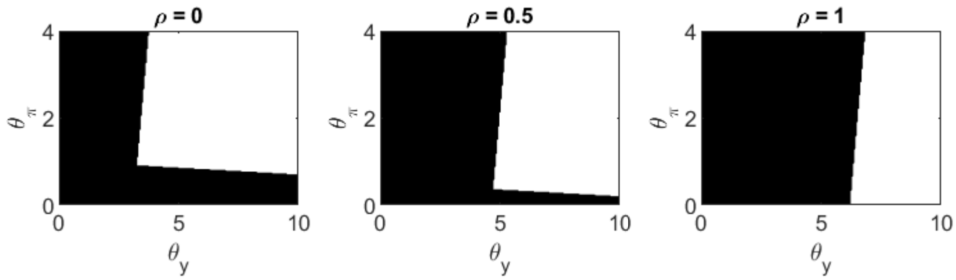


Figure 2: Determinacy regions (white areas) under IADL for the baseline LAMP model. Parameter values are $\lambda = 0.2, 0.5$, $\Psi = 0.086$, $\varphi = 2$, $\sigma = 2$, $\beta = 0.99$, $\zeta = 7$, $\mu_C = 0.62$, and $\mathrm{w}_C = 0.6$.



(a) Open economy



(b) Closed economy

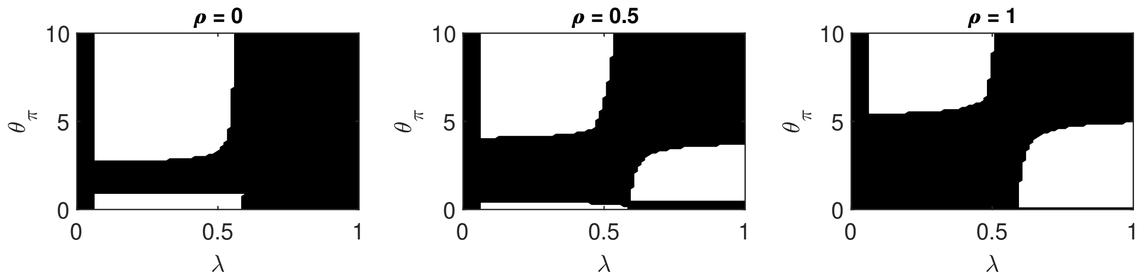
Figure 3: Determinacy regions (white areas) under IADL for the baseline LAMP model. Parameterization is $\lambda = 0.5$, $\Psi = 0.086$, $\varphi = 2$, $\sigma = 2$, $\beta = 0.99$, $\zeta = 7$, $\mu_C = 0.62$, and $\mathrm{w}_C = 0.6$ for the open economy in the top panel and $\mathrm{w}_C = 1$ for the closed in the bottom panel.

3.4 Exchange Rate Stabilization

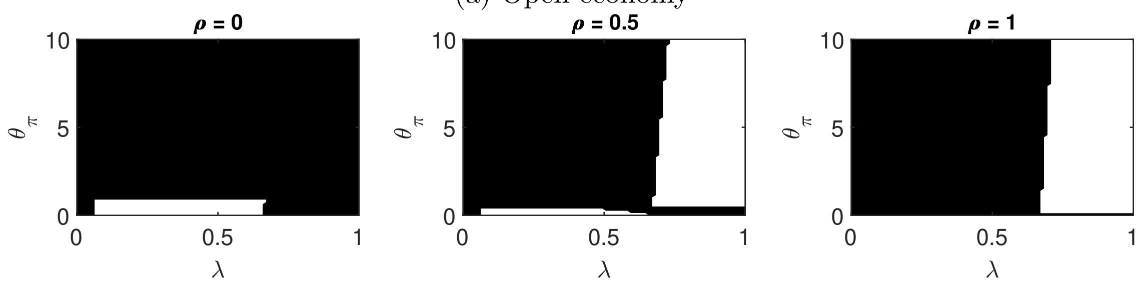
We can show the results are also robust to a modified interest-rate rule that incorporates a policy response to either the real exchange rate q_t or changes in the nominal exchange rate $\Pi^{\mathcal{E}}_{t-1,t}$. Because the UIP condition holds, the exchange rate depreciation is equal to the lagged interest rate. Choosing the weight on this term is therefore equivalent to choosing ρ_r, leaving the results unchanged. For the real exchange rate, it is straightforward to show that under complete asset markets $q_t = \sigma \left(c_t^R - c_t^{R,*} \right)$. It follows directly that the determinacy conditions are equivalent to those with a policy response to output.

3.5 Determinacy Analysis in the Medium-Scale Model

Similar conclusions to the baseline model are obtained for the medium-scale LAMP model discussed in Section 2.8. For the inertial feedback rule (3.7), Figure 4 depicts the effect of adjusting the proportion of rule-of-thumb consumers, $1 - \lambda$, using the parameter values



(a) Open economy



(b) Closed economy

Figure 4: Determinacy regions (white areas) for the medium-scale LAMP model. Parameterization is given in Table 1 of appendix B.6. $\mathrm{w}_C = \mathrm{w}_I = 0.6$ for the open economy in the top panel and $\mathrm{w}_C = \mathrm{w}_I = 1$ for the closed in the bottom panel.

summarized in Table 1 of appendix B.6. In the absence of inertia, determinacy is not possible in the SADL case. However, by increasing the value of ρ_r, determinacy easily arises under the Taylor principle, although the region is relatively smaller for open economies.

For the IADL case, determinacy requires the inverted Taylor principle in the closed economy, which shrinks as ρ_r increases and completely disappears under a price-level targeting rule. This is in stark contrast to the case of no rule-of-thumb consumers, where it is well known that by increasing the degree of interest rate inertia (see, e.g., Duffy and Xiao, 2011) or adopting a Wicksellian rule (see, e.g., McKnight, 2018) leads to significant determinacy gains in NK models with capital and investment. Similar to the baseline (labor-only) LAMP model, the Taylor principle can achieve determinacy in open IADL economies provided λ is sufficiently small and θ_π is sufficiently greater than 1.

3.6 Zero Lower Bound Considerations

Suppose that the interest rate is subject to a zero lower bound (ZLB) such that:

$$r_t + \bar{r} = \max\left\{0, \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}\right\}. \tag{3.15}$$

The presence of a ZLB can alter the determinacy properties of the model and introduces the possibility of both dynamic and steady-state indeterminacy.[19] Consider the following intuition for a sunspot shock induced by the ZLB. The expectation that the ZLB will bind in the future is equivalent to the expectation that for some period the nominal interest rate will be elevated above the level otherwise set by the policy rule. The higher future interest rate will have a deflationary effect and induce a cut in the interest rate today. If either the fall in inflation or the response of current monetary policy is large enough, then the interest rate can reach zero and the ZLB episode would be self-fulfilling.

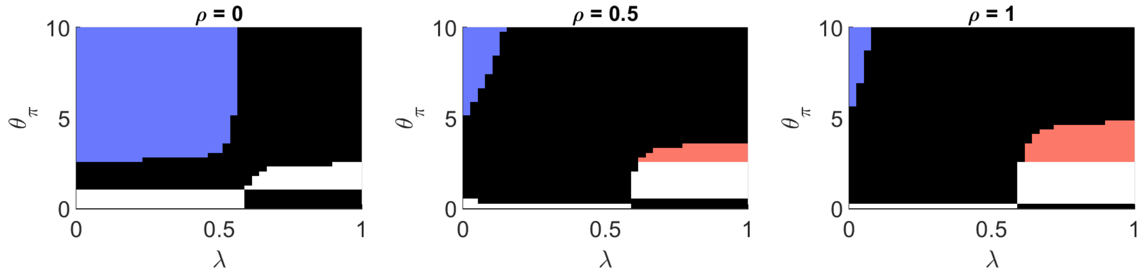
Such a sunspot shock can be contemporaneous; even if agents expect to be away from the ZLB in the following period, an expectation of the ZLB binding in the current period can be self-fulfilling. If indeterminacy is possible when agents expect to be away from the ZLB in the next period, then the model is always indeterminate irregardless of whether there is a horizon, T, after which agents expect to escape the ZLB. However, if indeterminacy is shown to be possible when agents expect to be away from the ZLB after some number of periods $T > 1$, then it is not sufficient proof that indeterminacy is possible when $T \leq 1$.[20] Using the tests discussed in Holden (2019), we check the determinacy properties of the model for different horizons, beyond which the ZLB is not expected to bind.[21] A detailed discussion of the tests is provided in appendix C.8.

While a full check of all the necessary and sufficient conditions is too computationally expensive for large values of T, we can check some sufficient conditions with a horizon $T = 200$, which is equivalent to agents expecting to have escaped the ZLB within 50 years. This exercise reveals that when a determinate policy rule is available under IADL,

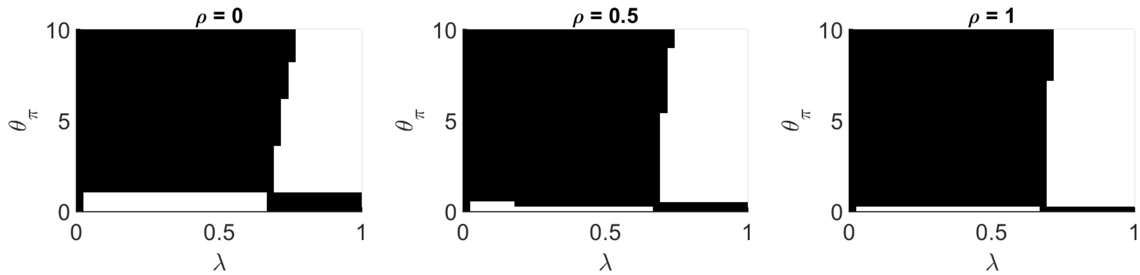
[19] It is easily verified that two deterministic steady states exist in the standard NK model with a ZLB; one when the nominal interest rate is at zero and inflation is below target, and the second with a positive interest rate and inflation on target. See Benhabib and Uribe (2002) and Fernández-Villaverde et al. (2015) for a detailed analysis of dynamic indeterminacy under a ZLB.

[20] This is discussed in detail by Holden (2019) who outlines the necessary and sufficient conditions for determinacy in an otherwise linear model with a ZLB.

[21] In principle, T could be set large enough that the risk of the ZLB binding at this future point should not plausibly affect current inflation.



(a) Open economy



(b) Closed economy

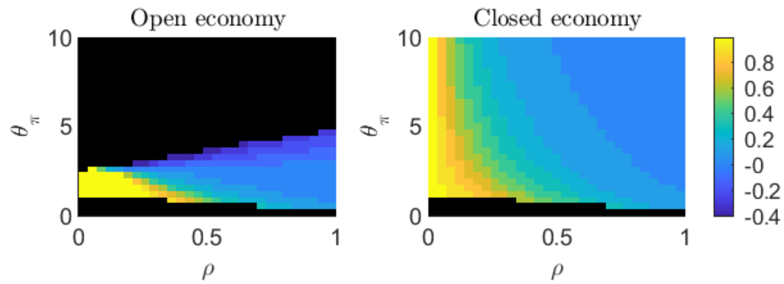
Figure 5: Uniqueness results for the baseline LAMP model with a ZLB. The black areas represent indeterminacy in the linear model, the white areas indicate there is always a unique equilibrium conditional on agents expecting to be away from the ZLB in 20 quarters. Uniqueness can only be guaranteed in the red areas when the economy escapes the ZLB in the following period. In the blue areas, self-fulfilling ZLB episodes are always possible.

uniqueness is always guaranteed except for high values of $\theta_\pi > \max\left\{\frac{1}{1-\mathrm{W}_c}, \Gamma_1\right\}$. However, we cannot rule out multiplicity under SADL except in the absence of interest rate inertia.[22] Restricting our analysis to a shorter horizon $T = 20$, such that agents expect the economy to have escaped the ZLB in 5 years, allows us to check the full set of necessary and sufficient conditions by employing the recursive test proposed in Tsatsomeros and Li (2000).[23]

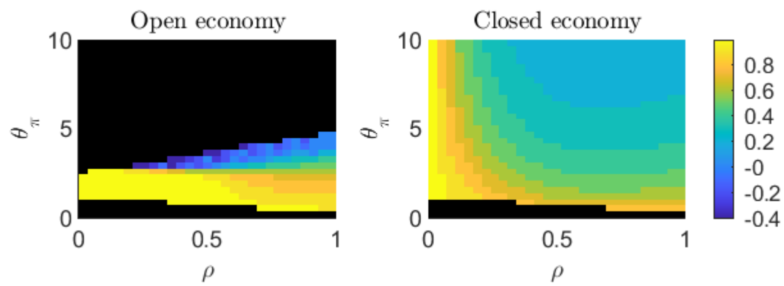
Figure 5 shows the results of these tests. For open IADL economies, the determinate blue region arising from a sufficiently large inflation response θ_π suffers from the risk of equilibrium multiplicity. Here, the nominal interest rate responds negatively to a positive contemporaneous monetary policy shock and self-fulfilling ZLB episodes are always possible

[22] It turns out that uniqueness under SADL with no policy inertia is a knife-edge result that does not hold in the medium-scale model.

[23] We rely on the implementation of these tests in the dynareOBC toolkit (see `https://github.com/tholden/dynareOBC`) as described in Holden (2019).



(a) Lagged interest rate in policy rule



(b) Lagged shadow rate in policy rule

Figure 6: The role of trade openness and policy inertia for the baseline model under a ZLB setting $\lambda = 1$. The black areas represent indeterminacy in the linear model, the high values in yellow correspond to better determinacy properties and low values in blue to worse properties. Values >1 truncated to 1, and values < -0.4 to 0.4.

due to the aggressiveness of the policy rule. The red area shows the region in the parameter space for which indeterminacy arises from the ZLB in open SADL economies. Although not quite as severe as the blue region of the IADL economy, multiple equilibria arises unless the economy is expected to be away from the ZLB in the following period. Consequently, the determinate policy space shrinks in open SADL economies with policy inertia, where multiple equilibria can occur as a result of future news of the ZLB binding.

While it does not seem that policy inertia has much of an impact on the indeterminacy stemming from the ZLB, note that the lag of the interest rate in (3.15) will be zero when at the ZLB. Any price-level information stored is therefore lost when the ZLB is binding. We can retain this if we include a shadow interest rate r_t^* in the interest rate rule:

$$\begin{aligned} r_t + \bar{r} &= \max\{0, r_t^* + \bar{r}\} \\ r_t^* &= \rho_r r_{t-1}^* + \theta_\pi \pi_{t+1}. \end{aligned} \tag{3.16}$$

Under this policy rule, determinacy is restored to the red regions highlighted in Figure 5.

We can look further at how policy inertia and trade openness affect the determinacy properties of the model under a ZLB using other indicative statistics. Figure 6 shows the minimum determinant of a principal sub-matrix of M, where M is a 5×5 matrix containing impulse response functions to a positive monetary policy news shocks at different horizons up to $T = 5$. When this determinant is positive, uniqueness is guaranteed (up to $T = 5$).[24]

We focus on the SADL case and set $\lambda = 1$.[25] For the interest-rate rule (3.15), except for small values of ρ_r, higher policy inertia worsens the determinacy properties of both the closed and open economy versions of the model. However, by including the lagged shadow rate (3.16), policy inertia tends to improve the determinacy conditions, except for a small interval of θ_π in the open economy.

Consider the following intuition. As previously discussed, the presence of self-fulfilling ZLB episodes depends on the current impact of future monetary policy news shocks. Policy inertia can have two competing effects in this regard. On one hand, policy inertia increases the persistence of monetary policy shocks, implying the ZLB binding is more contractionary in the presence of inertia, increasing the risk of sunspots. On the other hand, under inertia, a change in the interest rate will move long-term interest rates, thus having a larger impact on current inflation through the expectation channel. As in the case of a shadow rate rule, the higher inflation expectations under policy inertia offsets the contractionary effect of news of future ZLB episodes.

4 Optimal Monetary Policy

As is standard in the NK literature, in order to derive analytical results we define an approximate linear-quadratic optimal policy problem. We follow Galí and Monacelli (2005), among others, and restrict our welfare analysis to the special case where $\sigma = \mu_C = \mu_C^* = 1$.

[24] To understand why, consider that the determinant of M can be thought of as equivalent to a measure of volume. The sign of the determinant gives information on the positivity of the response of monetary policy to news shocks at different horizons. Recall from the earlier intuition of a ZLB-induced sunspot shock that a self-fulfilling ZLB episode relies on a negative response of monetary policy to a positive monetary policy shock at some horizon.

[25] Trade openness and policy inertia do not affect the outcomes under IADL, since we always have uniqueness under IADL unless we are in the blue regions highlighted in Figure 5, when we always have multiplicity.

Assumption 1. ***(Restricted parameterization)*** *We hereafter assume: (i) log utility in consumption ($\sigma = 1$); (ii) unit elasticity of substitution between home and foreign goods ($\mu_C = 1$); (iii) unit elasticity of substitution between goods produced in the RoW ($\mu_C^* = 1$); (iv) no fixed costs ($F = 0$) so without subsidies, the steady state is not equitable.*

In our model there are three market distortions. In addition to market power arising from monopolistic competition and relative price dispersion arising from nominal price stickiness, the terms of trade can be influenced to the benefit of domestic consumers. Moreover, with LAMP, there is an additional behavioral distortion which creates inequality across household types.[26]

4.1 The Distorted, Efficient and Equitable Allocations

We define three optimal allocations.

Definition 1. ***(Welfare-relevant output gap)*** *We consider three definitions of the welfare-relevant output gap: (i) the flexi-price (decentralized) equilibrium; (ii) the efficient (social planner) allocation; and (iii) the equitable (social planner) allocation. The stochastic and time-varying levels of output associated with these forms are denoted by Y_t^n (the natural level), Y_t^e (the efficient level), and $\bar{Y}_t$ (the equitable level).*

We show below how allocations (ii) and (iii) can be decentralized by coordinated fiscal and monetary policy to enhance (i).

4.1.1 The Decentralized Flexi-Price Equilibrium

The model conditions outlined in Section 2 are adjusted to allow flexible price setting. This replaces the optimal price setting condition with a constant mark-up giving a fixed real marginal cost $MC_t = 1 - \frac{1}{\zeta}$. With $\mu_C = 1$, we can express the conditions characterizing the open economy aspects as:[27]

$$\frac{P_{H,t}}{P_t} = S_t^{1-\mathrm{w}_C}, \tag{4.1}$$

$$C_t^{\mathrm{w}_C}(C_t^R)^{1-\mathrm{w}_C} = Y_t^{\mathrm{w}_C}(Y_t^*)^{1-\mathrm{w}_C}, \tag{4.2}$$

$$Y_t = C_t S_t^{1-\mathrm{w}_C}. \tag{4.3}$$

[26] This distortion arises from three sources preventing constrained households from (i) owning domestic shares, (ii) owning foreign shares, and (iii) trading in international state-contingent securities.

[27] Where equation (4.2) follows from the assumption that there are no constrained consumers in the RoW.

We subsequently refer to this equilibrium as the economy's *natural rate allocation.*

4.1.2 The Social Planner's Problem and the Efficient Allocation

Given exogenous processes for A_t and Y_t^*, the social planner's problem for the SOE with LAMP is to choose C_t^i and N_t^i for $i = C, R$ to maximize aggregate utility $\lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C)$ subject to the resource constraint (4.2) with C_t, N_t, and Y_t given by equations (2.16)–(2.18). Since the first-order conditions for the optimal choice of hours of labor supply lead to an equivalent expression for both agent types, $N_t^R = N_t^C = \mu_3 + \mu_4 A_t$, the relevant first-order conditions are those for consumption, C_t^R and C_t^C:

$$\lambda U_{C_t^R} + \mu_1(1-\mathrm{w}_C)C_t^{\mathrm{w}_C}(C_t^R)^{-\mathrm{w}_C} - \mu_2\lambda = 0, \tag{4.4}$$

$$U_{C_t^C} - \mu_2 = 0, \tag{4.5}$$

where μ_i, $i = 1, 2, 3, 4$, are the Lagrange multipliers. It follows from the FOCs that the efficient allocation is not also an equitable allocation (see Section 4.1.3 below). This contrasts with the closed economy case ($w_C = 1$), where the efficient allocation is also equitable, as in Bilbiie (2008).

4.1.3 The Social Planner's Problem and the Equitable Allocation

The optimal equitable allocation with $C_t = C_t^R = C_t^C$ and $N_t = N_t^R = N_t^C$ follows from optimizing the same aggregate utility function but subject to the following constraint:

$$C_t = (A_t N_t)^{\mathrm{w}_C}(Y_t^*)^{1-\mathrm{w}_C}. \tag{4.6}$$

The FOCs imply:

$$C_t N_t^{\varphi} = \mathrm{w}_C \frac{C_t}{N_t} \Rightarrow N_t^R = N_t^C = N_t = (\mathrm{w}_C)^{\frac{1}{1+\varphi}}. \tag{4.7}$$

This is the baseline about which the first-order solution of the model and the second-order approximation of the welfare criterion are conducted.

How then can the decentralized flexi-price SOE with LAMP support this optimal and equitable allocation? We seek two tax instruments, a firm subsidy τ_f and a household subsidy τ_h financed out of lump-sum taxation, such that:

$$W_t(1-\tau_f) = -\frac{U_{N^i,t}}{U_{C^i,t}}; \qquad i = R, C, \tag{4.8}$$

$$C_t^C = W_t(1+\tau_h)N_t^C. \tag{4.9}$$

From the flexi-price decentralized equilibrium this requires tax subsidies that satisfy:

$$\mathrm{w}_C(1-\tau_f) = 1 - \frac{1}{\zeta}, \tag{4.10}$$

$$1+\tau_h = \frac{1}{\mathrm{w}_C}. \tag{4.11}$$

The next proposition directly follows.

Proposition 6. ***(Optimal Subsidies for an Equitable Allocation)*** *Given the flexi-price equilibrium* (2.16), (2.17), *and* (4.3), *the social optimum is not an equitable allocation. An optimal equitable flexi-price allocation is sustained following* (4.10) *and* (4.11) *which determine tax subsidies for the firm* τ_f *and household* τ_h. *These subsidies are financed by lump-sum taxes, introduced in the budget constraint for Ricardian households* (2.3).

Note that the LAMP dimension, via λ, does not appear in (4.10) and (4.11). The optimal employment subsidy paid to the firm is influenced (negatively) by the degree of trade openness, $(1-\mathrm{w}_C)$, in addition to its standard (positive) dependence on the inverse of the markup, $(1-1/\zeta)$. By contrast, the optimal wage subsidy paid to all households is positively related to the degree of trade openness. These results generalize the results of Bilbiie (2008) for the closed LAMP economy ($\mathrm{w}_C = 1$), where no household subsidy is required, and Galí and Monacelli (2005) for the open economy case without LAMP ($\lambda = 1$).

4.1.4 Discussion

Consider the three allocations derived above. The decentralized equilibrium is a distorted allocation, neither efficient nor equitable (indeed, 'laissez-faire'). The efficient allocation is not equitable; it should use only the employment subsidy, as in Galí and Monacelli (2005) and Bilbiie (2008), but unlike the closed LAMP economy, it is now not enough to attain equality across agent types. In open economies, the equitable allocation is not efficient; it adds the wage subsidy to the C-types, but this is akin to an unemployment benefit and, by introducing a labour market distortion, implies an efficiency and equality tradeoff.

Why is the efficient allocation not also equitable? The key difference is the imposition of equality in the resource constraint (4.6), while (4.2) implies an inequality across agent types due both to foreign profit ownership and risk insurance available to R-types in the open economy.

Another result that we establish here is that the optimal equitable hours of work (4.7) depends on the degree of trade openness: the more open is the economy, the less R and C agents work. The result arises because of the same risk-sharing condition (4.2) across Ricardian consumers in the SOE and the RoW. Our interpretation is linked to the role of the open-economy dimension in risk-sharing seen clearly here: the benefit of foreign profits and international risk-sharing, originally going only to the Ricardian types in the LAMP SOE, now gets shared between both types via the redistribution that makes the allocation equitable. The more open an economy, the wider the range of risk-sharing.

4.2 The Optimal Policy Problem

The optimal policy problem consists of minimizing the second-order approximation to social welfare loss, given the constraints embodied in the model economy, summarized by the intertemporal IS equation (NKIS) and the NKPC of Section 3. For the remainder of the optimal policy analysis we choose the steady state of the determinacy analysis of Section 3 corresponding to the optimal equitable allocation. As is standard in the literature, we rewrite these equations in terms of the output gap x_t and the natural rate of interest r_t^n:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\delta}\left(r_t - \mathbb{E}_t \pi_{H,t+1} - r_t^n\right), \tag{4.12}$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \kappa x_t, \tag{4.13}$$

where

$$\delta = \left(1 - \frac{1-\lambda}{\lambda}\varphi\right) \mathrm{w}_C; \qquad \kappa \equiv \Psi\Delta > 0; \qquad \Psi \equiv \tfrac{(1-\xi);(1-\beta\xi)}{\xi} > 0;$$
$$\Delta \equiv 1 + \left[1 - (1-\lambda)\,\mathrm{w}_C\right] \tfrac{\varphi}{\lambda} > 1, \tag{4.14}$$

with ξ denoting Calvo price stickinesss and $\frac{\partial \Delta}{\partial w_C}, \frac{\partial \Delta}{\partial \lambda} < 0$.[28] Recall from Section 3 that the sign (hence, IADL) and slope of the NKIS curve, $-1/\delta$, are affected both by λ and w_C.

In light of Proposition 6 we now choose our social welfare criterion and by implication the welfare-relevant output gap.

Assumption 2. ***(Social welfare criterion)*** *Our social welfare criterion is a second-order approximation of the sum of the Ricardian and constrained households utility weighted by their mass in the region of the optimal equitable flexible-price allocation $\bar{Y}_t$ with a welfare-relevant output gap $x_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$ supported by the subsidy scheme of Proposition 6.*

The form of this welfare criterion is given by the following proposition:

Proposition 7. ***(Social welfare loss with a flexi-price equilibrium)*** *For the non-linear model of Section 2 and welfare-relevant output gap x_t, given Proposition 6, the micro-founded social welfare loss criterion for the LAMP SOE is approximated as:*

$$\Omega_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_{H,t}^2 + \varpi x_t^2) - \Lambda_x x_t \right], \tag{4.15}$$

where $\varpi = \varpi(\lambda) = \frac{\Psi(1+\varphi)}{\zeta \lambda}$, $\Psi \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$, $\Lambda_x = \frac{(1-w_C)(1-\lambda)\varphi}{\lambda}$.

Proof: See Appendix D.1.

The linear term in x_t captures the fact that any marginal increase in the output gap relative to its steady state value has a positive first-order effect on social welfare, since output is below its efficient level at that steady state.

We now turn to the policy implications of our results with respect to the central bank operating first under discretion, and then under commitment, before deriving the corresponding targeting rules and the inflation outcomes.

4.3 Discretionary Equilibrium and Implied Inflation Dynamics

Under discretion, in each period t the monetary authority chooses output and inflation according to the following optimization problem:

$$\min_{x_t, \pi_{H,t}} \quad \pi_{H,t}^2 + \varpi x_t^2 - \Lambda_x x_t \qquad \text{s.t.} \qquad \pi_{H,t} = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{H,t+1}, \tag{4.16}$$

[28]This is shown with further detail in appendix D.7.

where $\kappa = \kappa(\lambda, w_C) \equiv \Psi\Delta(\lambda, w_C)$ with $\Delta = \Delta(\lambda, w_C)$ as defined by (4.14), $\tilde{y}_t \equiv y_t - y_t^n$, where y_t^n is the above flexi-price equilibrium (natural level) in log-linear deviation and is a function of the three exogenous shock processes: $a_t = \log(A_t/A)$, $c_t^* = \log(C_t^*/C^*)$, and $r_t^* = \log(R_t^*/R^*)$. The form of this function has no bearing on the optimal policy analysis for discretion and commitment.

For our welfare-relevant output gap, $x_t \equiv y_t - \bar{y}_t = y_t - y_t^n + y_t^n - \bar{y}_t = \tilde{y}_t + y_t^n - \bar{y}_t$. Hence we can write the constraint in (4.16) as:

$$\pi_{H,t} = \kappa x_t + \beta \mathbb{E}_t \pi_{H,t+1} + u_t, \tag{4.17}$$

where $u_t \equiv \kappa(\bar{y}_t - y_t^n)$ is a cost-push shock process, which will be zero with the *restored* optimal equitable flexi-price equilibrium $x_t = 0$. Following much of the literature, the disturbance is assumed to evolve as an exogenous AR(1) stochastic process: $u_t = \rho_u u_{t-1} + \epsilon_{u,t}$, where $\rho_u \in [0, 1)$ and $\{\epsilon_{u,t}\}$ is a white-noise innovation with constant variance σ_u^2.

In the discretionary equilibrium, the inflation expectation $\mathbb{E}_t \pi_{H,t+1}$ is taken as given since there are no endogenous state variables and therefore is a function of future output gaps which cannot be contemporaneously influenced by the policymaker.

Proposition 8. ***(Targeting rule and inflation under discretion)*** *Under discretion, a higher degree of trade openness and a lower degree of LAMP require an increased degree of aggressiveness of the optimal targeting rule:*

$$x_t = -\frac{\kappa}{\varpi} \pi_{H,t} + \frac{\Lambda_x}{\varpi}, \tag{4.18}$$

with a domestic inflation stabilization rule given by:

$$\pi_{H,t} = \frac{\kappa \Lambda_x}{\kappa^2 + (1 - \beta)\varpi} + \frac{\varpi}{\kappa^2 + (1 - \beta\rho_u)\varpi} u_t. \tag{4.19}$$

Proof: See appendix D.5.

The usual interpretation of (4.18) is that when facing inflationary pressure arising from a cost-push shock, the optimal monetary policy response is to generate a negative output gap to dampen the rise in inflation. However, via the composite parameter κ/ϖ, the interaction of trade openness and LAMP imply competing influences with regard to the optimal degree of 'aggressiveness' of the targeting rule under discretion. Our interpretation is that trade openness requires a more aggressive response to inflation by the central bank since the more open the economy, the less x_t depends on domestic-price inflation, so the central bank needs to be more aggressive. By contrast, LAMP contributes to lowering the aggressiveness since the higher the LAMP, the more x_t (via the spending of total current income of C types, without any saving or insurance options available to them) depends on domestic-price inflation, so the central bank needs to be less aggressive. These results stress the point requiring optimal monetary policy to strike the right balance between the opposite influences of trade openness and LAMP on the degree of central bank aggressiveness in the targeting rule.

In (4.19), $\Lambda_x > 0$, and therefore discretion leads to a positive steady state domestic inflation rate, or an 'inflationary bias' as well established in the literature. Further inspection reveals that the inflationary bias increases in both the degree of LAMP and trade openness.[29] The second term in (4.19) is a stabilization term specifying how domestic-price inflation responds positively to a cost-push shock. Notice this term is a function of the loss function weight on output gap deviations, ϖ, and the slope of the NKPC, κ. As has already been established, ϖ and κ both depend on the population share of constrained consumers and κ also on the degree of trade openness. Further analysis reveals that either higher trade openness $(1 - w_C)$ or a higher degree of LAMP $(1 - \lambda)$ leads to a more aggressive inflation response to cost-push shocks.[30]

4.4 Commitment Equilibrium and Implied Price-Level Dynamics

We next discuss optimal monetary policy assuming commitment with full credibility. Under credible commitment, the central bank implements a policy plan announced at $t = 0$. Formally, the central bank now is assumed to choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ in order to:

$$\min_{\{x_t,\pi_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_{H,t}^2 + \varpi x_t^2 - \Lambda_x x_t\right), \tag{4.20}$$

subject to a sequence of NKPC constraints:

$$\text{s.t.} \quad \pi_{H,t} = \kappa x_t + \beta \mathbb{E}_t \pi_{H,t+1} + u_t, \text{ for } t = 0, .., \infty. \tag{4.21}$$

[29] See appendix D.7.1.

[30] This is shown in appendix D.7.

Proposition 9. ***(Targeting rule and price level under commitment)*** *Under commitment, a higher degree of trade openness and a lower degree of LAMP require an increased degree of aggressiveness of the optimal targeting rule:*

$$x_t = -\frac{\kappa}{\varpi}\widehat{p}_{H,t} + \frac{\Lambda_x}{\varpi}, \tag{4.22}$$

and a price level rule for $\widehat{p}_{H,t} \equiv p_{H,t} - p_{H,-1}$ *and output gap dynamics* x_t *given by, respectively:*

$$\widehat{p}_{H,t} = \gamma\widehat{p}_{H,t-1} + \frac{\gamma}{1-\gamma\beta\rho_u}u_t + \frac{\gamma}{1-\gamma\beta}\frac{\kappa\Lambda_x}{\varpi}, \tag{4.23}$$

$$x_t = \gamma x_{t-1} - \tfrac{\kappa\gamma}{\varpi(1-\gamma\beta\rho_u)}u_t,\ t = 1,2,...,\quad x_0 = -\tfrac{\kappa\gamma}{\varpi(1-\gamma\beta\rho_u)}u_0 + \tfrac{(1-\gamma)\Lambda_x}{\varpi},\ t = 0, \tag{4.24}$$

where $\gamma \equiv \frac{1-\sqrt{1-4\beta a^2}}{2\beta a} \in (0,1)$ *and* $a \equiv \frac{\varpi}{\varpi(1+\beta)+\kappa^2} \in (0,1)$.

Proof: See appendix D.6.

$\widehat{p}_{H,t}$ is the relative deviation between the price level and an 'implicit target' given by the price level prevailing in the period just before the central bank committed and chose its optimal plan. Our interpretation is analogous to the case of discretion,[31] but now, with commitment, steady-state inflation is zero, not positive. The results in (4.23) and (4.24) in Proposition 9 can be viewed as targeting rules that the central bank must follow period by period, in a way parallel to the case of discretion, in order to implement the optimal monetary policy under credible commitment.

The deterministic path for (4.23) displays a positive jump at the beginning and then follows a path given by $\gamma^{t+1}\frac{\gamma\kappa\Lambda_x}{(1-\gamma\beta)\varpi}$ at time t relative to a constant steady state. Hence with commitment the long-run domestic inflation rate is zero. Note that the stabilization component of the price-level rule is very different from that under discretion with an inflation rule.

4.5 Implementation of Optimal Policy with Interest Rate Rules

Optimal monetary policy up to now has been expressed in terms of targeting rules (4.18) for discretion and (4.22) for commitment. The question now is whether these rules can be implemented in the form of nominal interest-rate rules of the kind studied in Section 3. To

[31]This is shown in appendix D.7.

address this question we combine the targeting rules with the NKIS equation (4.12), which we write as:

$$r_t = r_t^n + \mathbb{E}_t \pi_{H,t+1} + \frac{\delta}{\mathrm{w}_C} \left(\mathbb{E}_t x_{t-1} - x_t \right). \tag{4.25}$$

4.5.1 The Interest Rate Rule for the Case of Discretion

From (4.18) and (4.19) we have:

$$x_t = -\frac{\kappa}{\varpi} \pi_{H,t} + \bar{x}, \tag{4.26}$$

$$\pi_{H,t} = \frac{\varpi}{\kappa^2 + (1 - \beta\rho_u)\varpi} u_t + \bar{\pi}_H, \tag{4.27}$$

where $\bar{x}$ and $\bar{\pi}_H$ are steady state values in deviation form about the original steady state. Hence:

$$\mathbb{E}_t x_{t+1} - x_t = \frac{\kappa(1-\rho_u)}{\varpi\rho_u} \mathbb{E}_t (\pi_{H,t+1} - \pi_{H,t}). \tag{4.28}$$

From (4.25) and (4.28), we arrive at the following nominal interest rate rule:

$$r_t = r_t^n + \left(1 + \frac{\delta}{\mathrm{w}_C} \frac{\kappa(1-\rho_u)}{\varpi\rho_u} \right) \mathbb{E}_t \pi_{H,t+1} - \left(\frac{\delta}{\mathrm{w}_C} \frac{\kappa(1-\rho_u)}{\varpi\rho_u} \right) \pi_{H,t}. \tag{4.29}$$

There are two points to make about this rule. First, it depends on the persistence of the AR(1) shock process $\rho_u \in [0, 1]$ and, second, it follows that there must be some parameter values for which the conditions for saddle-path stability found in Section 3 fail. It then follows that, in general, the discretionary optimal monetary policy cannot be implemented as a nominal interest rate rule, a result that carries over from the closed-economy case of Bilbiie (2008).

4.5.2 The Interest Rate Rule for the Case of Commitment

However, for the commitment case, optimal policy can be implemented as a Taylor-type rule. To see this we combine the NKIS curve (4.25) and the NKPC (4.17), and find:[32]

$$\pi_{H,t} + \frac{\varpi}{\kappa} x_t = \frac{\mathrm{w}_C}{\delta\kappa} (\kappa^2 + \varpi)(r_t - r_t^n) + u_t + \left[\beta + \frac{\mathrm{w}_C}{\delta\kappa} (\kappa^2 + \varpi) \right] \mathbb{E}_t \pi_{H,t+1} + \frac{\kappa^2 + \varpi}{\kappa} \mathbb{E}_t x_{t+1}$$

[32] This follows the logic of Woodford (2003), Chapter 7, for the closed economy without LAMP.

from which we arrive at the interest rate rule:

$$r_t = r_t^n + \frac{\delta\kappa}{\mathrm{w}_C(\kappa^2+\varpi)}u_t + \left(1+\frac{\beta\delta\kappa}{\mathrm{w}_C(\kappa^2+\varpi)}\right)\mathbb{E}_t\pi_{H,t+1} + \frac{\delta}{\mathrm{w}_C}\mathbb{E}_t x_{t+1} + \frac{\varpi}{\mathrm{w}_C(\kappa^2+\varpi)}x_{t-1}. \tag{4.30}$$

Then, following the reasoning behind Woodford (2003) and Bilbiie (2008), the rule (4.30) implements the optimal commitment solution as a unique saddlepath stable RE solution for the open LAMP economy. An important feature of (4.30) is that it is robust in the sense that its stability properties hold for any shock process for u_t unlike the rule (4.29) for the case of discretion. Thus, for the open LAMP economy, two benefits from commitment emerge. First, welfare gains are enhanced by the avoidance of an inflation bias in the steady state; and, second, the targeting rule under commitment can be implemented as a saddle-path stable RE robust interest rate rule, a result that carries over from the closed economy and non-LAMP cases in the literature.

Finally, we note that the optimal robust interest rate rule (4.30) is one *without* policy inertia. How can this feature be reconciled with the emphasis on "make-up" strategies and their extreme form, price-level or nominal interest rate targeting rules discussed in the Introduction and analyzed in Section 3? Following Woodford (2003), Levine, McAdam and Pearlman (2008), and Deak et al. (2020), inertia in the optimal policy rule arises from penalizing the variance of the nominal interest rate in the objective of the central bank. The weight on this penalty can be chosen to achieve a form of the ZLB constraint that imposes a given low probability of hitting the ZLB, as opposed to the absolute constraint considered in Section 3.6. Then a welfare-optimized interest rate rule can be found computationally with respect to the feedback parameters in the rule and the penalty weight.[33]

5 Concluding Remarks

This paper examines the role of limited asset market participation and trade openness in the design of monetary policy. These features are empirically-relevant and important considerations for policy.

Our first key contribution relates to the equilibrium determinacy of commonly employed

[33]See Mirfatah et al. (2021) who carry out such an exercise in an estimated SOE model. They find that optimized rules have very high degrees of policy inertia and indeed can be closely mimicked by price-level or nominal interest rate rules. Their computational findings complement our analytical results.

interest-rate rules. We challenge the conventional wisdom that policy inertia and price-level targeting reduce the likelihood of indeterminacy. For IADL economies, determinacy is undermined if the central bank reacts aggressively to interest rate inertia. In contrast, we find that trade openness, which typically exacerbates indeterminacy in standard models, exerts a stabilizing effect in IADL economies. This highlights an important caveat concerning the potential benefits of "make-up" strategies for central banks regardless if the nominal interest rate is close to the zero lower bound or not.

Our second key contribution concerns optimal monetary policy. We first show that the equivalence between the efficient and equitable equilibrium allocation in closed economies with LAMP breaks down in open economies. Focusing on the equitable allocation we derive targeting rules under optimal discretion and commitment and highlight the competing forces that trade openness and LAMP exert on their aggressiveness. Our results stress the point that the central bank has to strike the right balance between these opposing influences. Finally, we derived implementable optimal interest rate rules, and showed that commitment is superior to discretion for two reasons. First, welfare gains are enhanced by the avoidance of an inflation bias in the steady state; and, second, the targeting rule under commitment can be implemented as a saddle-path stable, robust interest rate rule.

Our paper has some limitations, which should be explored in future research. One such avenue is to generalize our SOE model with LAMP to a corresponding two-country analysis. Another extension is to replace complete exchange rate pass-through with incomplete pass-through. A third dimension is to introduce some form of bounded rationality.

References

Aguiar, M. A., Bils, M. and Boar, C. (2020), Who Are the Hand-to-Mouth?, NBER Working Papers 26643, National Bureau of Economic Research, Inc.

Aguiar, M. and Bils, M. (2015), 'Has Consumption Inequality Mirrored Income Inequality?', *American Economic Review* **105**(9), 2725–56.

Batini, N. and Haldane, A. G. (1999), Forward-Looking Rules for Monetary Policy, *in* J. B. Taylor, ed., 'Monetary Policy Rules', Chicago: University of Chicago Press, chapter 4, pp. 157–202.

Benhabib, J., S. S.-G. and Uribe, M. (2002), 'Avoiding Liquidity Traps', *Journal of Political Economy* **110**, 535–563.

Benigno, P. and Woodford, M. (2004), Optimal Monetary and Fiscal policy: A Linear-Quadratic Approach. in M. Gertler and K. Rogoff (eds.), NBER Macroeconomics Annual, 2003, pp 271–333.

Bernanke, B. S. (2017), Monetary Policy in a New Era. Brookings Institution.

Bilbiie, F. (2008), 'Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic', *Journal of Economic Theory* **140**(1), 162–196.

Bilbiie, F. O. (2020), 'The New Keynesian cross', *Journal of Monetary Economics* **114**, 90–108.

Boerma, J. (2014), Openness and the (Inverted) Aggregate Demand Logic. De Nederlandsche Bank NV, Working Paper No. 436.

Buffie, E. and Zanna, L. (2017), 'Limited Asset Markets Participation and Determinacy in the Open Economy ', *Macroeconomic Dynamics* .

Bullard, J. and Mitra, K. (2007), 'Determinacy, Learnability, and Monetary Policy Inertia', *Journal of Money, Credit and Banking* **39**(5), 1177–1212.

Calvo, G. (1983), 'Staggered Prices in a Utility-Maximizing Framework', *Journal of Monetary Economics* **12**(3), 383–398.

Carlstrom, C. T. and Fuerst, S. (2002), Monetary Policy Rules and Stability: Inflation

Targeting versus Price-Level Targeting. Federal Reserve Bank of Cleveland, Economic Commentary.

Cottle, R. W., Pang, J.-S. and Stone, R. E. (2009), *The Linear Complementarity Problem*, 2nd edn, Society for Industrial and Applied Mathematics, 3600 Market Street, 6th Floor, Philadelphia, PA 19104-2688 USA.

Deak, S., Levine, P. and Pham, S. T. (2020), Mandates and Monetary Rules a New Keynesian Framework. School of Economics Discussion Papers 0120, School of Economics, University of Surrey.

Debortoli, D. and Galí, J. (2017), Monetary Policy with Heterogeneous Agents: Insights from TANK models, Economics Working Papers 1686, Department of Economics and Business, Universitat Pompeu Fabra.

Dib, A., Mendicinon, C. and Zhang, Y. (2013), 'Price-Level Targeting Rules and Financial Shocks: The Case of Canada', *Economic Modelling* **30**, 941–953.

Duffy, J. and Xiao, W. (2011), 'Investment and Monetary Policy: Learning and Determinacy of Equilibrium', *Journal of Money, Credit and Banking* **43**(5), 959–992.

Fernández-Villaverde, J., Gordon, G., Guerrón-Quintana, P. and Rubio-Ramírez, J. F. (2015), 'Nonlinear adventures at the zero lower bound', *Journal of Economic Dynamics and Control* **57**, 182–204.

Galí, J. (2015), *Monetary Policy, Inflation and the Business Cycle*, 2nd edition edn, Princeton University Press.

Galí, J., Lopez-Salido, J. D. and Valles, J. (2004), 'Rule-of-Thumb Consumers and the Design of Interest Rate Rules', *Journal of Money, Credit and Banking* **36**(4), 739–763.

Galí, J. and Monacelli, T. (2005), 'Monetary Policy and Exchange Rate Volatility in a Small Open Economy', *Review of Economic Studies* **72**(3), 707–734.

Gaspar, V., Smets, F. and Vestin, D. (2007), Is Time Ripe for Price Level Path Stability. European Central Bank Working Paper Series No. 818.

Giannoni, M. P. (2014), 'Optimal Interest-Rate Rules and Inflation Stabilization versus Price-Level Stabilization', *Journal of Economic Dynamics and Control* **41**(C), 110–129.

Holden, T. (2016), Existence and uniqueness of solutions to dynamic models with occasionally binding constraints. EconStor Preprints 130142, ZBW - German National Library of Economics.

Holden, T. D. (2019), Existence and Uniqueness of Solutions to Dynamic Models with Occasionally Binding Constraints, EconStor Preprints 144570, ZBW - Leibniz Information Centre for Economics.

Kaplan, G., Violante, G. L. and Weidner, J. (2014), 'The Wealthy Hand-to-Mouth', *Brookings Papers on Economic Activity* **45**(1 (Spring), 77–153.

Levine, P., McAdam, P. and Pearlman, J. (2008), 'Quantifying and Sustaining Welfare Gains from Monetary Commitment', *Journal of Monetary Economics* **55**(7), 1253–1276.

Levine, P., Pearlman, J. and Pierse, R. (2008), 'Linear-Quadratic Approximation, Efficiency and Target-Implementability', *Journal of Economic Dynamics and Control* **32**, 3315–3349.

Mankiw, N. G. (2000), 'The Savers-Spenders Theory of Fiscal Policy', *American Economic Review* **90**(2), 120–125.

McKnight, S. (2018), 'Investment and Forward-Looking Monetary Policy: A Wicksellian Solution to The Problem of Indeterminacy', *Macroeconomic Dynamics* **22**(5), 1345–1369.

McKnight, S. and Mihailov, A. (2015), 'Do Real Balance Effects Invalidate the Taylor Principle in Closed and Open Economies?', *Economica* **82**(328), 938–975.

Mirfatah, M., Gabriel, V. J. and Levine, P. (2021), Imperfect Exchange Rate Pass-through: Empirical Evidence and Monetary Policy Implications. School of Economics Discussion Papers 0321, School of Economics, University of Surrey.

Powell, J. H. (2020), New Economic Challenges and the Fed's Monetary Policy Review. Speech given at "Navigating the Decade Ahead: Implications for Monetary Policy," an economic policy symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole.

Schmitt-Grohe, S. and Uribe, M. (2007), 'Optimal Simple and Implementable Monetary and Fiscal Rules', *Journal of Monetary Economics* **54**(6), 1702–1725.

Svensson, L. E. O. (2020), Monetary Policy Strategies for the Federal Reserve. National Bureau of Economic Research Working Paper No. 26657.

Tsatsomeros, M. J. and Li, L. (2000), 'A Recursive Test for P-Matrices', *BIT Numerical Mathematics* **40**, 410–414.

Vestin, D. (2006), 'Price Level Targeting Versus Inflation Targeting', *Journal of Monetary Economics* **53**, 1361–1376.

Walsh, C. (2003), Implications of a Changing Economic Structure for the Strategy of Monetary Policy, *in* 'Monetary Policy and Uncertainty: Adapting to a Changing Economy', Kansas City: Federal Reserve Bank of Kansas City.

Woodford, M. (2003), *Interest and Prices. Foundations of a Theory of Monetary Policy*, Princeton University Press.

Technical Appendix: Derivations and Proofs (for online publication)

A Equilibrium Conditions for Baseline LAMP Model

A.1 Households: Aggregate Consumption and Labor

$$C_t = \lambda C_t^C + (1-\lambda) C_t^R$$
$$N_t = \lambda N_t^C + (1-\lambda) N_t^R$$
$$\frac{(N_t^C)^{\varphi}}{(C_t^C)^{-\sigma}} = W_t$$
$$\frac{(N_t^R)^{\varphi}}{(C_t^R)^{-\sigma}} = W_t$$
$$C_t^C = W_t N_t^C$$
$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \right] R_t$$
$$1 = R_t^* \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \Pi_{t,t+1}^{\mathcal{E}} \right]$$

A.2 Households: Consumption, Investment and Export Demand

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{A.1}$$

$$\frac{P_t}{P_{H,t}} = \left(\mathrm{w}_C + (1-\mathrm{w}_C) S_t^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \tag{A.2}$$

$$\frac{P_t}{P_{F,t}} = \left(\mathrm{w}_C S_t^{\mu_C - 1} + (1-\mathrm{w}_C) \right)^{\frac{1}{1-\mu_C}} \tag{A.3}$$

$$\Pi_{t-1,t} = \left[\mathrm{w}_C \left(\Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-\mathrm{w}_C) \left(\Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{A.4}$$

$$C_{H,t} = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \tag{A.5}$$

$$C_{F,t} = (1-\mathrm{w}_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \tag{A.6}$$

$$C^*_{H,t} = (1 - \mathrm{w}^*_C) \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\mu^*_C} C^*_t \tag{A.7}$$

A.3 Firms

$$W_t = \frac{Y_t}{N_t} MC_t \frac{P_{H,t}}{P_t} \tag{A.8}$$

$$\begin{aligned}
Y^W_t &= A_t N_t \\
Y_t &= \frac{Y^W_t - F}{\Delta_t} \\
1 &= \xi \left(\Pi_{H,t-1,t} \right)^{\zeta - 1} + (1 - \xi) \left(\frac{JJ_t}{J_t} \right)^{1-\zeta} \\
\frac{P^0_{H,t}}{P_{H,t}} &= \frac{J_t}{JJ_t} \\
\Delta_t &= \xi \left(\Pi_{H,t-1,t} \right)^{\zeta} \Delta_{t-1} + (1 - \xi) \left(\frac{JJ_t}{J_t} \right)^{-\zeta} \\
JJ_t &= \frac{\zeta}{\zeta - 1} Y_t MS_t MC_t + \xi \mathbb{E}_t \left[\Lambda^R_{t,t+1} \left(\Pi_{H,t,t+1} \right)^{\zeta} JJ_{t+1} \right] \\
J_t &= \frac{P_{H,t}}{P_t} Y_t + \xi \mathbb{E}_t \left[\Lambda^R_{t,t+1} \frac{\left(\Pi_{H,t,t+1} \right)^{\zeta}}{\Pi_{t,t+1}} J_{t+1} \right] \\
MC_t &= \frac{P^W_t}{P_{H,t}}
\end{aligned}$$

A.4 Market Clearing

$$\begin{aligned}
Y_t &= C_{H,t} + C^*_{H_t} \\
TB_t &= \frac{P_{H,t}}{P_t} Y_t - C_t \\
P^{B*}_t B_{F,t} &= \frac{\Pi^{\mathcal{E}}_{t-1,t}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t
\end{aligned}$$

A.5 Deterministic Zero-Growth Steady State

In a non-zero-net inflation steady state given $B_F = \bar{B}_F$, $\Pi = \Pi_H = \Pi_F = \Pi^*$, with appropriate choice of units such that $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P^O}{P} = 1$ we have:

$$\Pi^{\mathcal{E}} = \frac{\Pi}{\Pi^*}$$
$$N = \overline{N}$$
$$N^C = 1$$
$$N^R = \frac{N - \lambda N^C}{1 - \lambda}$$
$$S = 1$$
$$\Lambda^R = \beta$$
$$R = \frac{\Pi}{\beta}$$
$$R^* = \frac{\Pi^*}{\beta^*}$$
$$\frac{JJ}{J} = \left(\frac{1 - \xi \left(\Pi\right)^{\zeta-1}}{1 - \xi} \right)^{\frac{1}{1-\zeta}}$$
$$MC = \frac{JJ}{J} \frac{\zeta - 1}{\zeta} \frac{1 - \xi\beta \left(\Pi_H\right)^{\zeta}}{1 - \xi\beta \left(\Pi\right)^{\zeta-1}}$$
$$\Delta = \frac{(1 - \xi) \left(\frac{JJ}{J}\right)^{-\zeta}}{1 - \xi \left(\Pi_H\right)^{\zeta}}$$
$$C^C = N^C W$$
$$Y^W = N$$
$$Y = \frac{Y^W - F}{\Delta}$$
$$W = \frac{Y}{N} MC$$
$$B_F = \bar{B}_F$$
$$TB = \left(\frac{1}{R^*} - \frac{\Pi^S}{\Pi} \right) B_F$$
$$C = Y - TB$$
$$C_H = \mathrm{w}_C C$$

$$C_F = (1 - \mathrm{w}_C)C$$

$$N^R = \frac{1}{1-\lambda}N - \frac{\lambda}{1-\lambda}N^C$$

$$C^R = \frac{1}{1-\lambda}C - \frac{\lambda}{1-\lambda}C^C$$

$$\frac{P_H^0}{P_H} = \frac{J}{JJ}$$

$$JJ = \frac{\frac{\zeta}{\zeta-1}YMC}{1-\xi\beta\left(\Pi\right)^{\zeta}}$$

$$J = \frac{Y}{1-\xi\beta\left(\Pi\right)^{\zeta-1}}$$

$$EX = Y - C_H$$

$$C_H^* = EX$$

B Equilibrium Conditions for Medium-Scale LAMP Model

B.1 Households: Aggregate Consumption and Labor

$$C_t = \lambda C_t^C + (1-\lambda)C_t^R$$

$$N_t = \lambda N_t^C + (1-\lambda)N_t^R$$

$$\frac{(N_t^C)^{\varphi}}{(C_t^C)^{-\sigma}} = W_t$$

$$\frac{(N_t^R)^{\varphi}}{(C_t^R)^{-\sigma}} = W_t$$

$$C_t^C = W_t N_t^C$$

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}}\right]R_t$$

$$1 = R_t^* \phi\left(\frac{\mathcal{E}_t B_{F,t}^*}{P_{H,t}Y_t}\right)\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}}\Pi_{t,t+1}^{\mathcal{E}}\right]$$

B.2 Households: Consumption, Investment and Export Demand

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{B.1}$$

$$\frac{P_t}{P_{H,t}} = \left(\mathrm{w}_C + (1-\mathrm{w}_C) S_t^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}} \tag{B.2}$$

$$\frac{P_t}{P_{F,t}} = \left(\mathrm{w}_C S_t^{\mu_C - 1} + (1-\mathrm{w}_C)\right)^{\frac{1}{1-\mu_C}} \tag{B.3}$$

$$\frac{P_t^I}{P_{H,t}} = \left(\mathrm{w}_I + (1-\mathrm{w}_I) S_t^{1-\mu_I}\right)^{\frac{1}{1-\mu_I}} \tag{B.4}$$

$$\frac{P_t^I}{P_{F,t}} = \left(\mathrm{w}_I S_t^{\mu_I - 1} + (1-\mathrm{w}_I)\right)^{\frac{1}{1-\mu_I}} \tag{B.5}$$

$$\Pi_{t-1,t} = \left[\mathrm{w}_C \left(\Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}}\right)^{1-\mu_C} + (1-\mathrm{w}_C) \left(\Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}}\right)^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}} \tag{B.6}$$

$$C_{H,t} = \mathrm{w}_C \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t \tag{B.7}$$

$$C_{F,t} = (1-\mathrm{w}_C) \left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t \tag{B.8}$$

$$C_{H,t}^* = (1-\mathrm{w}_C^*) \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\mu_C^*} C_t^* \tag{B.9}$$

$$I_{H,t} = \mathrm{w}_I \left(\frac{P_{H,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{B.10}$$

$$I_{F,t} = (1-\mathrm{w}_I) \left(\frac{P_{F,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{B.11}$$

$$I_{H,t}^* = (1-\mathrm{w}_I^*) \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\mu_I^*} I_t^* \tag{B.12}$$

B.3 Firms

$$Z_t = \alpha \frac{Y_t}{K_{t-1}} MC_t \frac{P_{H,t}}{P_t} \tag{B.13}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t} MC_t \frac{P_{H,t}}{P_t} \tag{B.14}$$

$$R_t^k = \frac{Z_t + (1-\delta_K)Q_t}{Q_{t-1}}$$
$$Y_t^W = (A_t N_t)^{1-\alpha} K_{t-1}^{\alpha}$$
$$Y_t = \frac{Y_t^W - F}{\Delta_t}$$
$$1 = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta-1} + (1-\xi)\left(\frac{JJ_t}{J_t}\right)^{1-\zeta}$$
$$\Delta_t = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{JJ_t}{J_t}\right)^{-\zeta}$$
$$\frac{P_{H,t}^0}{P_{H,t}} = \frac{J_t}{JJ_t}$$
$$JJ_t = \frac{\zeta}{\zeta-1} Y_t MS_t MC_t + \xi \mathbb{E}_t\left[\Lambda_{t,t+1}^R \left(\Pi_{H,t,t+1}\right)^{\zeta} JJ_{t+1}\right]$$
$$J_t = \frac{P_{H,t}}{P_t} Y_t + \xi \mathbb{E}_t\left[\Lambda_{t,t+1}^R \frac{\left(\Pi_{H,t,t+1}\right)^{\zeta}}{\Pi_{t,t+1}} J_{t+1}\right]$$
$$MC_t = \frac{P_t^W}{P_{H,t}}$$
$$K_t = (1-\delta_K)K_{t-1} + (1-\mathcal{S}(X_t))I_t$$
$$Q_t(1-\mathcal{S}(X_t) - X_t\mathcal{S}'(X_t)) + \mathbb{E}_t\left[\Lambda_{t,t+1}^R\, Q_{t+1}\mathcal{S}'(X_{t+1})\frac{I_{t+1}^2}{I_t^2}\right] = \frac{P_t^I}{P_t}$$

B.4 Market Clearing

$$Y_t = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* + G_t$$
$$EX_t = C_{H,t}^* + I_{H,t}^*$$
$$TB_t = \frac{P_{H,t}}{P_t} Y_t - C_t - \frac{P_{I,t}}{P_t} I_t - \frac{P_{H,t}}{P_t} G_t$$
$$P_t^{B^*} B_{F,t} = \frac{\Pi_{t-1,t}^{\mathcal{E}}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t$$

B.5 Deterministic Zero-Growth Steady State

In a non-zero-net inflation steady state and constant Q_t given $\Pi = \Pi_H = \Pi_F = \Pi^*$, with appropriate choice of units such that $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P^I}{P} = \frac{P^O}{P} = 1$ we have:

$$Q = 1$$

$$\Pi^{\mathcal{E}} = \frac{\Pi}{\Pi^*}$$

$$N = \overline{N}$$

$$N^C = 1$$

$$N^R = \frac{N - \lambda N^C}{1 - \lambda}$$

$$S = 1$$

$$\Lambda^R = \beta$$

$$Q = 1$$

$$R = \frac{\Pi}{\beta}$$

$$R^* = \frac{\Pi^*}{\beta^*}$$

$$\phi = \frac{R}{R^* \Pi^{\mathcal{E}}} = \frac{\Pi}{\Pi^* \Pi^{\mathcal{E}}} \frac{\beta^*}{\beta} = \frac{\beta^*}{\beta}$$

$$R^k = R$$

$$\frac{JJ}{J} = \left(\frac{1 - \xi \left(\Pi\right)^{\zeta - 1}}{1 - \xi} \right)^{\frac{1}{1-\zeta}}$$

$$MC = \frac{JJ}{J} \frac{\zeta - 1}{\zeta} \frac{1 - \xi\beta \left(\Pi_H\right)^{\zeta}}{1 - \xi\beta \left(\Pi\right)^{\zeta - 1}}$$

$$\Delta = \frac{(1 - \xi) \left(\frac{JJ}{J}\right)^{-\zeta}}{1 - \xi \left(\Pi_H\right)^{\zeta}}$$

$$\frac{N}{K} = \left(\frac{\frac{R^K}{\Pi} - (1 - \delta_K)}{MC\alpha \left(1 - \tau^k\right)} \right)^{1/(1-\alpha)}$$

$$W = (1 - \alpha) \left(\frac{N}{K} \right)^{\alpha} MC$$

$$C^C = N^C W$$

$$K = \frac{N}{\frac{N}{K}}$$
$$Y^W = K^\alpha N^{1-\alpha}$$
$$I = \delta_K K$$
$$Y = \frac{Y^W - F}{\Delta}$$
$$B_F = -\frac{Y \log(\phi)}{\phi_B} \geq 0 \text{ iff } \beta \geq \beta^*$$
$$TB = \left(\frac{1}{\phi R^*} - \frac{\Pi^S}{\Pi}\right) B_F$$
$$G = g_y Y$$
$$C = Y - I - G - TB$$
$$C_H = \mathrm{w}_C C$$
$$C_F = (1 - \mathrm{w}_C)C$$
$$N^R = \frac{1}{1-\lambda} N - \frac{\lambda}{1-\lambda} N^C$$
$$I_H = \mathrm{w}_I I$$
$$I_F = (1 - \mathrm{w}_I)I$$
$$C^R = \frac{1}{1-\lambda} C - \frac{\lambda}{1-\lambda} C^C$$
$$\frac{P_H^0}{P_H} = \frac{J}{JJ}$$
$$JJ = \frac{\frac{\zeta}{\zeta-1} YMC}{1 - \xi\beta\left(\Pi\right)^{\zeta}}$$
$$J = \frac{Y}{1 - \xi\beta\left(\Pi\right)^{\zeta-1}}$$
$$EX = Y - C_H - I_H - G$$
$$C_H^* = EX_C = EX_C(1) = cs_{exp} EX$$
$$I_H^* = EX_I = EX_I(1) = is_{exp} EX$$

B.6 Calibration of the Medium-Scale LAMP Model

The baseline calibration of the medium-scale LAMP model is given in Table 1.

Parameter	Description	Value
$\beta\ (=\beta^*)$	Home (foreign) discount factor	0.99
$\Pi\ (=\Pi^*)$	Steady-state home (foreign) inflation rate	1.005
σ	Relative risk aversion	2
φ	Inverse of the Frisch elasticity of labor supply	2
α	Cost share of capital	0.3
δ_K	Depreciation rate of capital	0.025
ϕ_I	Investment adjustment costs	10
ξ	Degree of price stickiness	0.75
$1-\lambda$	Degree of LAMP	$\lambda \in [0,1]$
$\mu_C\ (=\mu_C^*)$, $\mu_I\ (=\mu_I^*)$	Elasticity of substitution between home and foreign goods	0.62
$1-\mathrm{w}_C$, $1-\mathrm{w}_I$	Degree of trade openness	0.4
ϕ_B	Bond adjustment costs	0.001
ζ	Elasticity of substitution of differentiated goods	7
g_Y	Government spending share of output	0.1

Table 1: Parameter Values Used in the Numerical Analysis of Determinacy

C Equilibrium Determinacy: Derivations and Proofs

C.1 Derivation of the Minimal Form of the Dynamic System

The model is linearized around a zero-growth, zero-inflation steady state so $\Pi = 1$ and prices $P = P_H = P_F = P^* = 1$. Then by definition the steady state terms of trade and real exchange rate are $\mathcal{E} = Q = 1$. As discussed in the main text, we assume an equitable steady state.[34] All lower-case variables denote percentage deviations from the steady state. All shocks are set equal to zero.

Aggregate demand:

$$y_t = \mathrm{w}_C c_t + (1-\mathrm{w}_C)(c_t^R + \omega s_t), \tag{C.1}$$

where $\omega \equiv \frac{\mathrm{w}_C(\sigma\mu_C - 1) + \sigma\mu_C^*}{\sigma} = \frac{\sigma\mu_C(1+\mathrm{w}_C) - \mathrm{w}_C}{\sigma}$, if $\mu_C = \mu_C^*$.

[34] Therefore, we need either the zero-profit condition, $F/Y = 1/\zeta$, or the subsidy scheme that supports the welfare-relevant choice of the output gap x_t.

Aggregate supply:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \Psi mc_t, \tag{C.2}$$

$$mc_t = w_t + (1-\mathrm{w}_C)s_t, \tag{C.3}$$

$$y_t = \lambda\left(1+\frac{1}{\zeta}\right)n_t^R + (1-\lambda)\left(1+\frac{1}{\zeta}\right)n_t^C, \tag{C.4}$$

where $\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} > 0$ and

$$\pi_t = \pi_{H,t} + (1-\mathrm{w}_C)(s_t - s_{t-1}). \tag{C.5}$$

Household optimality conditions:

$$w_t = \varphi n_t^R + \sigma c_t^R, \tag{C.6}$$

$$w_t = \varphi n_t^C + \sigma c_t^C, \tag{C.7}$$

$$c_t = \lambda c_t^R + (1-\lambda)c_t^C, \tag{C.8}$$

$$c_t^C = w_t + n_t^C, \tag{C.9}$$

$$s_t = \frac{\sigma}{\mathrm{w}_C}c_t^R, \tag{C.10}$$

$$c_t^R = c_{t+1}^R - \frac{1}{\sigma}(r_t - \pi_{t+1}). \tag{C.11}$$

Combining (C.5), (C.10), and (C.11), we obtain:

$$\begin{aligned}\pi_t &= \pi_{H,t} + (1-\mathrm{w}_C)\frac{\sigma}{\mathrm{w}_C}(c_t^R - c_{t-1}^R)\\ &= \pi_{H,t} + (1-\mathrm{w}_C)\frac{1}{\mathrm{w}_C}(r_{t-1} - \pi_t).\end{aligned} \tag{C.12}$$

It follows that

$$r_t - \pi_{t+1} = r_t - \pi_{H,t+1} - (1-\mathrm{w}_C)\frac{1}{\mathrm{w}_C}(r_t - \pi_{t+1})$$

from which

$$r_t - \pi_{t+1} = \mathrm{w}_C(r_t - \pi_{H,t+1}). \tag{C.13}$$

The intertemporal IS equation (C.11) can be expressed as:

$$c_{t+1}^{R} + \frac{\mathrm{w}_C}{\sigma}\pi_{H,t+1} - \frac{\mathrm{w}_C}{\sigma}r_t = c_t^R. \tag{C.14}$$

Using (C.6) to eliminate w_t and (C.10) to eliminate s_t, equations (C.3), (C.7), and (C.9) become:

$$mc_t = \varphi n_t^R + \frac{\sigma}{\mathrm{w}_C}c_t^R, \tag{C.15}$$

$$\varphi n_t^C = \varphi n_t^R + \sigma c_t^R - \sigma c_t^C, \tag{C.16}$$

$$c_t^C = \varphi n_t^R + \sigma c_t^R + n_t^C. \tag{C.17}$$

Using (C.8) and (C.10) to eliminate c_t and s_t from (C.1) yields:

$$y_t = w_C \lambda c_t^R + \mathrm{w}_C(1-\lambda)c_t^C + (1-\mathrm{w}_C)\left[1 + \frac{\omega\sigma}{\mathrm{w}_C}\right]c_t^R, \tag{C.18}$$

and rearranging (C.4)

$$n_t^R = \frac{1}{\lambda\left(1+\frac{1}{\zeta}\right)}y_t - \frac{(1-\lambda)}{\lambda}n_t^C. \tag{C.19}$$

Combining (C.16) and (C.17) gives:

$$n_t^C = \frac{\varphi(1-\sigma)}{\varphi+\sigma}n_t^R + \frac{\sigma(1-\sigma)}{\varphi+\sigma}c_t^R. \tag{C.20}$$

Combining (C.17), (C.18), and (C.20) yields:

$$y_t = \mathrm{w}_C\lambda c_t^R + \mathrm{w}_C(1-\lambda)\frac{(1+\varphi)}{\varphi+\sigma}\left[\varphi n_t^R + \sigma c_t^R\right] + (1-\mathrm{w}_C)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right]c_t^R. \tag{C.21}$$

Combining (C.19), (C.20), and (C.21) gives:

$$n_t^R = \left[\frac{\mathrm{w}_C\lambda(\varphi+\sigma) + (1-\mathrm{w}_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right] + (1-\lambda)\sigma\left[\mathrm{w}_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[\mathrm{w}_C(1+\varphi)+(\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}\right]c_t^R. \tag{C.22}$$

Finally, combining (C.2), (C.15), and (C.22) results in

$$\beta\pi_{H,t+1} = \pi_{H,t} - \Psi\Upsilon c_t^R, \tag{C.23}$$

where

$$\Upsilon = \frac{\sigma}{\mathrm{w}_C} + \frac{\varphi\lambda \mathrm{w}_C(\varphi+\sigma) + \varphi(1-\mathrm{w}_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right] + \varphi\sigma(1-\lambda)\left[\mathrm{w}_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[\mathrm{w}_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]},$$

$$\Rightarrow \Upsilon = \frac{\sigma(1-\mathrm{w}_C)}{\mathrm{w}_C} + \frac{\lambda(\varphi+\sigma)\left[\mathrm{w}_C\varphi + \sigma\left(1+\frac{1}{\zeta}\right)\right] + \varphi(1-\mathrm{w}_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{\mathrm{w}_C}\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}.$$

The interest-rate rule is given by:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1}. \tag{C.24}$$

The dynamic system given by (C.14), (C.23), and (C.24) can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_3}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \ \pi_{H,t} \ r_{t-1}\right]',$$

$$\mathbf{A_3} \equiv \begin{bmatrix} 1 - \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\mathrm{w}_C(\theta_\pi-1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\rho_r \mathrm{w}_C}{\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi \mathrm{w}_C \Upsilon \theta_\pi}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\mathrm{w}_C\theta_\pi}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]} \end{bmatrix}.$$

C.2 The NKIS and NKPC Equations when $\sigma = 1$

Setting $\sigma = 1$ implies $n_t^C = 0$ from (C.20) and thus $n_t^R = \frac{1}{\lambda} y_t$ from (C.19). Therefore:

$$y_t = \lambda c_t^R + (1-\lambda)c_t^C = \lambda c_t^R + (1-\lambda)w_t = \lambda c_t^R + (1-\lambda)(\varphi n_t^R + c_t^R) = \lambda c_t^R + (1-\lambda)(\varphi \frac{1}{\lambda} y_t + c_t^R),$$

and hence:

$$c_t^R = \left(1 - \frac{1-\lambda}{\lambda}\varphi\right) y_t \equiv \delta y_t, \tag{C.25}$$

and from (C.14):

$$y_t = y_{t+1} - \frac{\mathrm{w}_C}{\delta}\left(r_t - \pi_{H,t+1}\right). \tag{C.26}$$

In the flexi-price case, $\pi_{H,t} = 0$, and hence:

$$y_t^n = y_{t+1}^n - \frac{\mathrm{w}_C}{\delta} r_t^n. \tag{C.27}$$

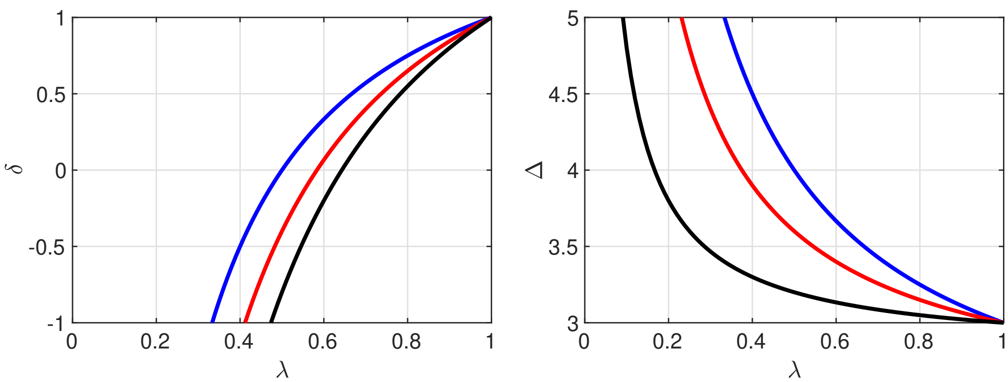


Figure 7: Role of λ and openness on supply and demand curves for $w_C = 0.5$ (blue), 0.7 (red), 0.9 (black). Left panel: NKIS (δ). Right panel: NKPC (Δ)

Finally, in terms of the output gap $x_t = y_t - y_t^n$ we have

$$x_t = x_{t+1} - \frac{\mathrm{w}_C}{\delta}\left(r_t - \pi_{H,t+1} - r_t^n\right). \tag{C.28}$$

For any given w_C and inverse Frisch elasticity φ, when gradually increasing the degree of LAMP from nil (at $\lambda = 1$), at some point the sign of the NKIS curve becomes positive (as δ becomes negative). The intuition is that the more open the economy or the higher the degree of LAMP, the less domestic output depends on the domestic real interest rate. The latter is because constrained consumers spend their current income *irrespective* of the interest rate.

The role of trade openness and LAMP in the NKPC, operating via the composite parameter Δ, is illustrated graphically in the right panel of Figure 7. Observe that the slope, but now not the sign, of the NKPC is affected by λ and w_C, so that the output gap exerts greater influence on domestic inflation in the SOE with LAMP than in the RANK SOE. The intuition is that the more open the economy, the more domestic inflation depends on the domestic output gap due to the aggregate demand effect of increased spending on imports; and the higher the degree of LAMP, the more domestic inflation depends on the domestic output gap due to a greater share of constrained households consuming all current income, thus strengthening the link between output and inflation.

C.3 Proof of Proposition 1

Differentiating (3.6) we obtain

$$\frac{d\lambda^*}{d\mathrm{w}_C} = \frac{\varphi(1+\varphi)(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)}{\left[\varphi\left[\mathrm{w}_C(1+\varphi)+(\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]+(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)\right]^2}.$$

Hence $\frac{d\lambda^*}{d\mathrm{w}_c} > 0$ from which the proposition is proved. □

C.4 Proof of Propositions 2 and 3

The minimum state-space representation of the model is $\mathbf{z}_{t+1} = \mathbf{A_3}\mathbf{z}_t$ where $\mathbf{A_3}$ is given by (C.1). The three eigenvalues of $\mathbf{A_3}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\Psi \mathrm{w}_C \Upsilon \rho_r}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]}$, and $a_0 = -\frac{\rho_r}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1+a_2+a_1+a_0 < 0$, $-1+a_2-a_1+a_0 > 0$, Case II: $1+a_2+a_1+a_0 > 0$, $-1+a_2-a_1+a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$.

For Case I, the two inequalities reduce to:

$$\frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1 + \rho_r)}{\beta\sigma\left[1-(1-\mathrm{w}_C)\theta_\pi\right]} < 0, \tag{C.29}$$

$$-2(1+\beta)\left(1+\frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]}\right) + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1 - \rho_r)}{\sigma\left[1-(1-\mathrm{w}_C)\theta_\pi\right]} > 0. \tag{C.30}$$

First assume that $\Upsilon > 0$. Condition (C.29) requires either (i) $0 < \theta_\pi < \min\left\{1-\rho_r, \frac{1}{1-\mathrm{w}_C}\right\}$ or (ii) $\max\left\{1-\rho_r, \frac{1}{1-\mathrm{w}_C}\right\} < \theta_\pi$. By inspection, (C.30) can never be satisfied under (i). For (ii), first note that the lower-bound $1-\rho_r$ is redundant since $1-\rho_r < \frac{1}{1-\mathrm{w}_C}$ and (C.30) requires:

$$\theta_\pi < \frac{(1+\rho_r)\left[\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)\right]}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)}. \tag{C.31}$$

For the determinacy region to be non-empty requires $\left(\frac{\mathrm{w}_C}{1-\mathrm{w}_C}\right)\left[\frac{\Psi \mathrm{w}_C \Upsilon}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)}\right] < \rho_r$. Now assume that $\Upsilon < 0$. Condition (C.29) requires that $1-\rho_r < \theta_\pi < \frac{1}{1-\mathrm{w}_C}$ and (C.30) requires

$\theta_\pi\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)\right] > (1+\rho_r)\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)\right]$. The latter can only be satisfied if $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)<0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)<0$, which requires an additional upper-bound on θ_π given by (C.31).

For Case II, the first two inequalities reduce to:

$$\frac{\Psi \text{w}_C \Upsilon(\theta_\pi-1+\rho_r)}{\beta\sigma\left[1-(1-\text{w}_C)\theta_\pi\right]}>0, \tag{C.32}$$

$$-2(1+\beta)\left(1+\frac{\rho_r}{\left[1-(1-\text{w}_C)\theta_\pi\right]}\right)+\frac{\Psi \text{w}_C \Upsilon(\theta_\pi-1-\rho_r)}{\sigma\left[1-(1-\text{w}_C)\theta_\pi\right]}<0. \tag{C.33}$$

First assume that $\Upsilon>0$. Equation (C.32) requires that $1-\rho_r<\theta_\pi<\frac{1}{1-\text{w}_C}$ and (C.33) requires the upper-bound on θ_π given by (C.31). The remaining inequalities give (3.10) and (3.11). Now assume that $\Upsilon<0$. Condition (C.32) requires either (i) $0<\theta_\pi<\min\left\{1-\rho_r,\frac{1}{1-\text{w}_C}\right\}$ or (ii) $\max\left\{1-\rho_r,\frac{1}{1-\text{w}_C}\right\}<\theta_\pi$. For (i), first note that $1-\rho_r<\frac{1}{1-\text{w}_C}$, and (C.33) requires $\theta_\pi\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)\right]<(1+\rho_r)\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)\right]$, which is always satisfied if $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)<0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)>0$. If $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)>0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)>0$, the upper-bound (C.31) is redundant. If $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)<0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)<0$, the following lower-bound on θ_π is needed:

$$\theta_\pi>\frac{(1+\rho_r)\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)\right]}{\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)}. \tag{C.34}$$

In this case, for the region to be non-empty requires $(1-\rho_r)(1+\beta)\sigma \text{w}_C+\left[2\sigma(1+\beta)+\text{w}_C\Psi\Upsilon\right]\rho_r>0$. The remaining inequalities give (3.10) and (3.11). For (ii), the lower-bound $1-\rho_r$ is redundant and (C.33) requires $\theta_\pi\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)\right]>(1+\rho_r)\left[\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)\right]$, which can never be satisfied if $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)<0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)>0$. If $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)>0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)>0$, an additional lower-bound on θ_π given by (C.33) is required. If $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)(1-\text{w}_C)<0$ and $\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)<0$, requires the upper-bound on θ_π given by (C.31). For the determinacy region to be non-empty requires $\left(\frac{\text{w}_C}{1-\text{w}_C}\right)\left[\frac{\Psi \text{w}_C \Upsilon}{\Psi \text{w}_C \Upsilon+2\sigma(1+\beta)}\right]<\rho_r$. The remaining inequalities give (3.10) and (3.11). This completes the proof. $\square$

C.5 Determinacy Conditions under a Domestic-Price Inflation Feedback Rule

Proposition 10. ***(Domestic-Price Inflation)*** *Under a domestic-price inflation rule with interest-rate inertia, the necessary and sufficient conditions for equilibrium determinacy are:*

$$\textit{Case I:}\quad \textit{If}\ \ \Upsilon > 0 : \max\{0, 1-\rho_r\} < \theta_\pi < (1+\rho_r)\left[1+\frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right],$$

and one of the following inequalities is satisfied:

$$\left|-1-\frac{1}{\beta}-\rho_r+\frac{\Psi w_C \Upsilon(\theta_\pi-1)}{\beta\sigma}\right| > 3, \tag{C.35}$$

$$1-\beta+\rho_r\left[\frac{\rho_r(1-\beta)}{\beta}+\beta-\frac{1}{\beta}+\frac{\Psi w_C \Upsilon}{\sigma}\left(1+\frac{\theta_\pi-1}{\beta}\right)\right] > 0. \tag{C.36}$$

$$\textit{Case IIA:}\ \ \Upsilon < 0, \quad \textit{and} \quad \max\{0, 1-\rho_r\} < \theta_\pi < (1+\rho_r)\left[1+\frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right].$$

$$\textit{Case IIB:}\ \ \Upsilon < 0, \quad (1+\rho_r)\left[1+\frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right] < \theta_\pi < 1-\rho_r,$$

and one of the inequalities given by (C.35) *and* (C.36) *is satisfied.*

Proof. The dynamic system is given by:

$$c^R_{t+1}+\frac{\mathrm{w}_C}{\sigma}\pi_{H,t+1}-\frac{\mathrm{w}_C}{\sigma}r_t = c^R_t, \tag{C.37}$$

$$\beta\pi_{H,t+1} = \pi_{H,t}-\Psi\Upsilon c^R_t, \tag{C.38}$$

$$r_t = \rho_r r_{t-1}+\theta_\pi \pi_{H,t+1}. \tag{C.39}$$

This can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_4}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c^R_t\ \pi_{H,t}\ r_{t-1}\right]',$$

$$\mathbf{A_4} \equiv \begin{bmatrix} 1 - \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma} & \frac{\mathrm{w}_C(\theta_\pi - 1)}{\beta\sigma} & \frac{\rho_r \mathrm{w}_C}{\sigma} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi\Upsilon\theta_\pi}{\beta} & \frac{\theta_\pi}{\beta} & \rho_r \end{bmatrix}.$$

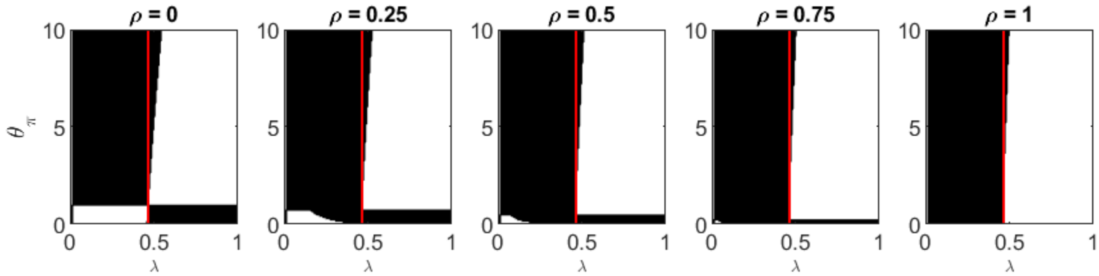
The three eigenvalues of $\mathbf{A_4}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \rho_r + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta} + \frac{\Psi \mathrm{w}_C \Upsilon \rho_r}{\beta\sigma}$, and $a_0 = -\frac{\rho_r}{\beta}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1+a_2+a_1+a_0 < 0$, $-1+a_2-a_1+a_0 > 0$, Case II: $1+a_2+a_1+a_0 > 0$, $-1+a_2-a_1+a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$. For Case I, the second inequality can never be satisfied with $\Upsilon > 0$. With $\Upsilon < 0$, the first inequality of Case I requires $1 - \rho_r < \theta_\pi$ and the second inequality yields $\theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi \mathrm{w}_C \Upsilon}\right]$. For Case II, the first inequality requires either $\Upsilon > 0$ and $\theta_\pi > 1 - \rho_r$, or $\Upsilon < 0$ and $\theta_\pi < 1 - \rho_r$. With $\Upsilon > 0$, the second inequality is automatically satisfied if $\theta_\pi < 1 + \rho_r$. Otherwise, the following upper-bound $\theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi \mathrm{w}_C \Upsilon}\right]$ is additionally required. With $\Upsilon < 0$, the second inequality yields $\theta_\pi > (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi \mathrm{w}_C \Upsilon}\right]$. The remaining inequalities of Case II give (C.35) and (C.36). This completes the proof. □

The analytical conditions indicate that increasing policy inertia enlarges the region of determinacy under Case I and IIA, and shrinks the determinacy region under Case IIB. For a standard range of parameter values, Case IIA only holds for a small range of $\lambda < \lambda^*$ and the combined impact of increased inertia on the bounds of Cases IIA and IIB results in an overall reduced policy space. The effect of openness is ambiguous from these conditions, however from numerical results, we find that openness appears to enlarge the determinate policy space under SADL and shrink it under IADL for standard parameter. Determinacy regions are shown in Figure 8.

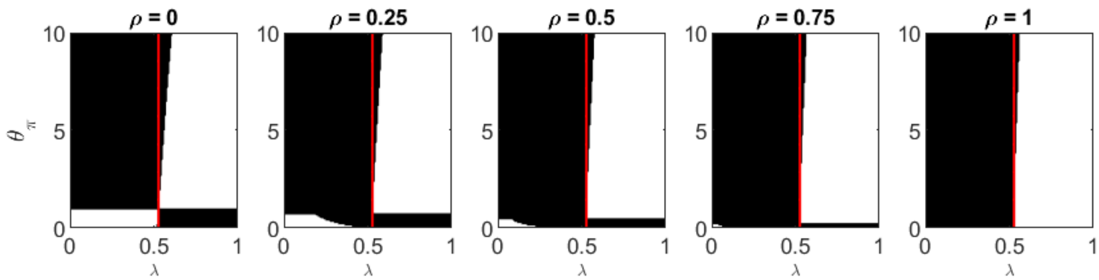
C.6 Determinacy Conditions with a Policy Response to Output

Proposition 11. ***(Output gap targeting)*** *If the interest-rate rule reacts to future consumer-price inflation rule and contemporaneous output with interest-rate inertia, the necessary and sufficient conditions for equilibrium determinacy are:*

Case IA: $\max\left\{\frac{1}{1-w_C}, 1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y\right\} < \theta_\pi < \Gamma_1$ *and* $\rho_r > \left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C \Upsilon - (1-w_C)(1+\beta)\Xi\theta_y}{\Psi w_C \Upsilon + 2\sigma(1+\beta)}\right]$.



(a) Open economy



(b) Closed economy

Figure 8: Determinacy regions for small model with domestic-price inflation targeting. Parameter values are $\Psi = 0.086$, $\varphi = 2$, $\sigma = 2$, $\zeta = 7$, $\beta = 0.99$. $\mathrm{w}_C = 0.6$ and $\mu_C = 0.62$ for the open economy in the top panel and $\mathrm{w}_C = 1$ for the closed in the bottom panel. The red vertical line gives λ^* below which IADL holds.

Case IB: $\max\left\{0, 1-\rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y\right\} < \theta_\pi < \min\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$ *and one of the following inequalities is satisfied:*

$$\left|-1-\frac{1}{\beta}-\frac{\rho_r}{[1-(1-w_C)\theta_\pi]}+\frac{\Psi w_C\Upsilon(\theta_\pi-1)}{\beta\sigma\left[1-(1-w_C)\theta_\pi\right]}-\frac{w_C\Xi\theta_y}{\sigma\left[1-(1-w_C)\theta_\pi\right]}\right|>3, \tag{C.40}$$

$$1-\beta+\frac{\rho_r}{1-(1-w_C)\theta_\pi}\left[\frac{\rho_r(1-\beta)}{\beta[1-(1-w_C)\theta_\pi]}+\beta-\frac{1}{\beta}+\frac{\Psi w_C\Upsilon}{\sigma}\left(1+\frac{\theta_\pi-1}{\beta[1-(1-w_C)\theta_\pi]}\right)\right]$$
$$+\frac{w_c\Xi\theta_y}{1-(1-w_C)\theta_\pi}\left[\frac{1-\rho_r-(1-w_C)\theta_\pi}{\sigma\left[1-(1-w_C)\theta_\pi\right]}\right]>0, \tag{C.41}$$

where

$$\Gamma_1 \equiv (1+\rho_r)\left[1+\frac{2(1+\beta)\sigma w_C}{\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)}\right]+\frac{w_C(1+\beta)\Xi}{\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)}\theta_y.$$

Case IIA: $\Upsilon < \frac{\sigma(1-w_C)}{w_C}-\frac{(\varphi+\sigma)}{w_C(1-\lambda)(1+\varphi)}\left[w_C\lambda+\frac{\sigma\mu_C(1+w_C)(1-w_C)}{w_C}\right]$ *and one of the following:*

(i) $1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y<\theta_\pi<\frac{1}{1-w_C}$ *and* $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(ii) $0<\theta_\pi<\min\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$ *and one of the inequalities given by* (C.40)–(C.41) *is satisfied.*

(iii) $\theta_\pi>\max\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$ *and one of the inequalities given by* (C.40)–(C.41) *is satisfied.*

Case IIB: $\frac{\sigma(1-w_C)}{w_C}-\frac{(\varphi+\sigma)}{w_C(1-\lambda)(1+\varphi)}\left[w_C\lambda+\frac{\sigma\mu_C(1+w_C)(1-w_C)}{w_C}\right]<\Upsilon<0$ *and one of the following:*

(i) $1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y<\theta_\pi<\frac{1}{1-w_C}$, $\rho_r>-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y-\frac{w_C}{1-w_C}$ *and* $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(ii) $\frac{1}{1-w_C}<\theta_\pi<1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y$, $\rho_r<-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y-\frac{w_C}{1-w_C}$ *and* $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(iii) $0<\theta_\pi<\min\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$ *and one of the inequalities given by* (C.40)–(C.41) *is satisfied.*

(iv) $\theta_\pi>\max\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$ *and one of the inequalities given by* (C.40)–(C.41) *is satisfied.*

Proof. The dynamic system is given by:

$$c^R_{t+1}+\frac{\mathrm{w}_C}{\sigma}\pi_{H,t+1}-\frac{\mathrm{w}_C}{\sigma}r_t=c^R_t, \tag{C.42}$$

$$\beta\pi_{H,t+1}=\pi_{H,t}-\Psi\Upsilon c^R_t, \tag{C.43}$$

$$r_t-\frac{\sigma(1-\mathrm{w}_C)\theta_\pi}{\mathrm{w}_C}c^R_{t+1}-\theta_\pi\pi_{H,t+1}=\rho_r r_{t-1}+\left[\Xi\theta_y-\frac{\sigma(1-\mathrm{w}_C)\theta_\pi}{\mathrm{w}_C}\right]c^R_t. \tag{C.44}$$

This can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_5}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \ \pi_{H,t} \ r_{t-1}\right]',$$

$$\mathbf{A_5} \equiv \begin{bmatrix} 1 - \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\mathrm{w}_C \Xi \theta_y}{\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\mathrm{w}_C(\theta_\pi - 1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\rho_r \mathrm{w}_C}{\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi \mathrm{w}_C \Upsilon \theta_\pi}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\Xi\theta_y}{1-(1-\mathrm{w}_C)\theta_\pi} & \frac{\mathrm{w}_C\theta_\pi}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} & \frac{\rho_r}{1-(1-\mathrm{w}_C)\theta_\pi} \end{bmatrix}.$$

The three eigenvalues of $\mathbf{A_5}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \frac{\rho_r}{1-(1-\mathrm{w}_C)\theta_\pi} + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} - \frac{\mathrm{w}_C \Xi \theta_y}{\sigma[1-(1-\mathrm{w}_C)\theta_\pi]}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\Psi \mathrm{w}_C \Upsilon \rho_r}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]} + \frac{\mathrm{w}_C \Xi \theta_y}{\beta\sigma[1-(1-\mathrm{w}_C)\theta_\pi]}$, and $a_0 = -\frac{\rho_r}{\beta[1-(1-\mathrm{w}_C)\theta_\pi]}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1 + a_2 + a_1 + a_0 < 0$, $-1 + a_2 - a_1 + a_0 > 0$, Case II: $1 + a_2 + a_1 + a_0 > 0$, $-1 + a_2 - a_1 + a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$.

For Case I, the two inequalities reduce to:

$$\frac{\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y}{1 - (1-\mathrm{w}_C)\theta_\pi} < 0, \tag{C.45}$$

$$-2(1+\beta)\left(1 + \frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]}\right) + \frac{\Psi \mathrm{w}_C \Upsilon(\theta_\pi - 1 - \rho_r) - \mathrm{w}_C(1+\beta)\Xi\theta_y}{\sigma\left[1-(1-\mathrm{w}_C)\theta_\pi\right]} > 0. \tag{C.46}$$

First assume that $\Upsilon > 0$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y > 0$, conditions (C.45) and (C.46) require:

$$\frac{1}{1-\mathrm{w}_C} < \theta_\pi < \frac{(1+\rho_r)\left[\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)\right]}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)} + \frac{\mathrm{w}_C(1+\beta)\Xi}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)(1-\mathrm{w}_C)}\theta_y. \tag{C.47}$$

For the determinacy region to be non-empty requires $\left(\frac{\mathrm{w}_C}{1-\mathrm{w}_C}\right)\left[\frac{\Psi \mathrm{w}_C \Upsilon - (1-\mathrm{w}_C)(1+\beta)\Xi\theta_y}{\Psi \mathrm{w}_C \Upsilon + 2\sigma(1+\beta)}\right] < \rho_r$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y < 0$, condition (C.46) can never be satisfied since $\Xi > 0$. Now assume that $\Upsilon < 0$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y > 0$, condition (C.45) requires:

$$\Xi > 0 \Leftrightarrow \Upsilon > \frac{\sigma(1-\mathrm{w}_C)}{\mathrm{w}_C} - \frac{(\varphi + \sigma)}{\mathrm{w}_C(1-\lambda)(1+\varphi)}\left[\mathrm{w}_C\lambda + \frac{\sigma\mu_C(1+\mathrm{w}_C)(1-\mathrm{w}_C)}{\mathrm{w}_C}\right] \tag{C.48}$$

and

$$\frac{1}{1-\mathrm{w}_C} < \theta_\pi < 1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y. \tag{C.49}$$

For the determinacy region to be non-empty $\rho_r < -\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y - \frac{\mathrm{w}_C}{1-\mathrm{w}_C}$, while condition (C.46) requires:

$$\theta_y > \frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma(1-\mathrm{w}_C)}{\mathrm{w}_C}\right] - \frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma}{\mathrm{w}_C}\right). \tag{C.50}$$

If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y < 0$, condition (C.45) requires that $1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y < \theta_\pi < \frac{1}{1-\mathrm{w}_C}$ and (C.46) requires (C.50) if $\Xi < 0$. Otherwise

$$\theta_y < \frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma(1-\mathrm{w}_C)}{\mathrm{w}_C}\right] - \frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma}{\mathrm{w}_C}\right) \tag{C.51}$$

and $\rho_r > -\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y - \frac{\mathrm{w}_C}{1-\mathrm{w}_C}$ for the determinacy region to be non-empty.

For Case II, the first two inequalities reduce to:

$$\frac{\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y}{1-(1-\mathrm{w}_C)\theta_\pi} > 0, \tag{C.52}$$

$$-2(1+\beta)\left(1 + \frac{\rho_r}{[1-(1-\mathrm{w}_C)\theta_\pi]}\right) + \frac{\Psi\mathrm{w}_C\Upsilon(\theta_\pi - 1 - \rho_r) - \mathrm{w}_C(1+\beta)\Xi\theta_y}{\sigma\left[1-(1-\mathrm{w}_C)\theta_\pi\right]} < 0. \tag{C.53}$$

First assume that $\Upsilon > 0$. Equation (C.52) requires that $1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y < \theta_\pi < \frac{1}{1-\mathrm{w}_C}$ and (C.53) requires the upper-bound on θ_π given by (C.47). The remaining inequalities give (C.40) and (C.41). Now assume that $\Upsilon < 0$. Condition (C.52) requires either (i) $0 < \theta_\pi < \min\left\{1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y, \frac{1}{1-\mathrm{w}_C}\right\}$ or (ii) $\max\left\{1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y, \frac{1}{1-\mathrm{w}_C}\right\} < \theta_\pi$. For (i), (C.53) is always satisfied if $\Xi > 0$. Otherwise, the upper-bound on θ_y given by (C.49) is needed. The remaining inequalities give (C.40) and (C.41). For (ii), condition (C.53) requires (C.51) if $\Xi > 0$ and (C.50) if $\Xi < 0$. The remaining inequalities give (C.40) and (C.41). This completes the proof. $\square$

C.7 Determinacy Conditions under a Contemporaneous-Looking Feedback Rule

Proposition 12. ***(Current-looking rule)*** *If the interest-rate rule reacts to current-looking CPI inflation rule with interest-rate inertia, the necessary and sufficient conditions for equilibrium determinacy are:*

Case I: $\Upsilon > 0$, $\theta_\pi > \max\{0, 1-\rho_r\}$, *and one of the following inequalities is satisfied:*

$$-1-\frac{1}{\beta}-\rho_r-\theta_\pi(1-w_C)-\frac{\Psi w_C \Upsilon}{\beta\sigma} < -3, \tag{C.54}$$

$$\begin{aligned}[\theta_\pi(1-w_C)+\rho_r]\left[[\theta_\pi(1-w_C)+\rho_r]\frac{(1-\beta)}{\beta}+\beta-\frac{1}{\beta}-\frac{\Psi w_C\Upsilon}{\beta\sigma}\right]& \\ +1-\beta+\frac{\Psi w_C \Upsilon\left(\theta_\pi+\rho_r\right)}{\sigma} > 0.&\end{aligned} \tag{C.55}$$

Case IIA: $\Upsilon < 0$, $\theta_\pi > \max\{0, 1-\rho_r\}$, *and* $\Upsilon < -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$.

Case IIB: $\Upsilon < 0$, $0 < \theta_\pi < 1-\rho_r$, $\Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$, *and either* (C.55) *or the following inequality*

$$\left|-1-\frac{1}{\beta}-\rho_r-\theta_\pi(1-w_C)-\frac{\Psi w_C \Upsilon}{\beta\sigma}\right| < -3 \tag{C.56}$$

is satisfied.

Proof. The dynamic system is given by:

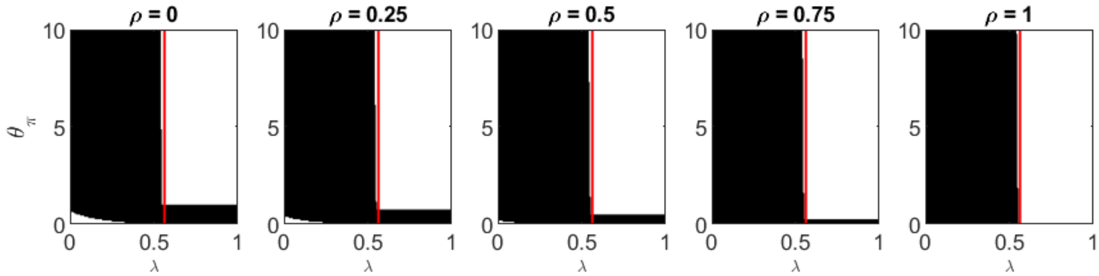
$$c^R_{t+1}+\frac{\mathrm{w}_C}{\sigma}\pi_{H,t+1}-\frac{\mathrm{w}_C}{\sigma}r_t = c^R_t, \tag{C.57}$$

$$\beta\pi_{H,t+1} = \pi_{H,t}-\Psi\Upsilon c^R_t, \tag{C.58}$$

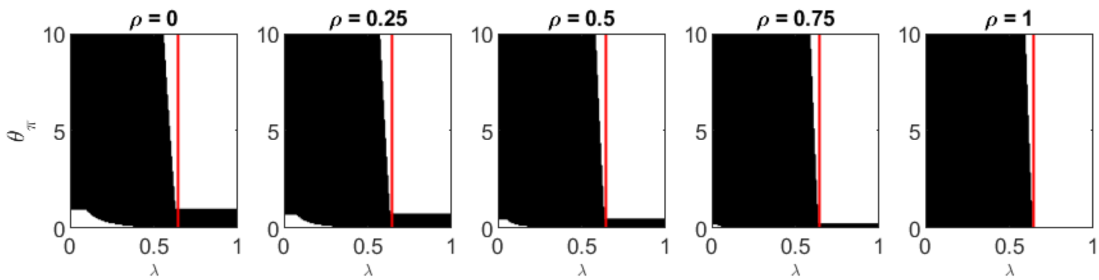
$$r_t = \rho_r r_{t-1}+\theta_\pi\pi_t = \left[\rho_r+\theta_\pi(1-\mathrm{w}_C)\right]r_{t-1}+\mathrm{w}_C\theta_\pi\pi_{H,t}. \tag{C.59}$$

This can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_6}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c^R_t \ \pi_{H,t} \ r_{t-1}\right]',$$



(a) Open economy



(b) Closed economy

Figure 9: Determinacy regions for small model with current-CPI inflation feedback rule. Parametrization is $\Psi = 0.086$, $\varphi = 2$, $\sigma = 2$, $\zeta = 7$, $\beta = 0.99$. $\mathrm{w}_C = 0.6$ and $\mu_C = 0.62$ for the open economy in the top panel and $\mathrm{w}_C = 1$ for the closed in the bottom panel. The red vertical line gives λ^* below which IADL holds.

$$\mathbf{A_6} \equiv \begin{bmatrix} 1 + \frac{\Psi \mathrm{w}_C \Upsilon}{\beta\sigma} & \frac{\mathrm{w}_C}{\sigma}\left(\mathrm{w}_C\theta_\pi - \frac{1}{\beta}\right) & \frac{\mathrm{w}_c}{\sigma}[\rho_r + \theta_\pi(1-\mathrm{w}_C)] \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ 0 & \mathrm{w}_C\theta_\pi & \rho_r + \theta_\pi(1-\mathrm{w}_C) \end{bmatrix}.$$

The three eigenvalues of $\mathbf{A_6}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \rho_r - \theta_\pi(1-\mathrm{w}_C) - \frac{\Psi \mathrm{w}_C \Upsilon}{\beta\sigma}$, $a_1 = \frac{1}{\beta} + \frac{\theta_\pi(1-\mathrm{w}_C)(1+\beta)}{\beta} + \frac{\Psi \mathrm{w}_C \Upsilon \theta_\pi}{\beta\sigma} + \rho_r\left(1 + \frac{1}{\beta} + \frac{\Psi \mathrm{w}_C \Upsilon}{\beta\sigma}\right)$, and $a_0 = -\frac{\theta_\pi(1-\mathrm{w}_C)}{\beta} - \frac{\rho_r}{\beta}$. With one predetermined variable r_t, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case 1: $1+a_2+a_1+a_0 < 0$, $-1+a_2-a_1+a_0 > 0$, Case 2: $1+a_2+a_1+a_0 > 0$, $-1+a_2-a_1+a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$. For Case I, the second inequality can never be satisfied with $\Upsilon > 0$. With $\Upsilon < 0$, the first inequality of Case I requires $\theta_\pi > 1 - \rho_r$ and the second inequality yields reduces to

$\Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-\mathrm{W}_C)]}{\Psi \mathrm{W}_C(1+\rho_r+\theta_\pi)}$. For Case II, the two inequalities are satisfied if $\theta_\pi > 1-\rho_r$, provided $\Upsilon > 0$. The remaining inequalities give (C.54) and (C.55). If $\Upsilon < 0$, the first inequality requires $0 < \theta_\pi < 1-\rho_r$ and the second inequality $\Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-\mathrm{W}_C)]}{\Psi \mathrm{W}_C(1+\rho_r+\theta_\pi)}$. The remaining inequalities give (C.55) and (C.56). This completes the proof. □

The analytical conditions generate similar conclusions to a forward-looking domestic price inflation rule. As shown in Figure 9, policy inertia shrinks the determinate policy space, whereas trade openness enlarges the determinate policy space under SADL and shrinks it under IADL.

C.8 Determinacy Analysis at the ZLB

This subsection overviews the environment and tests used to study the determinacy properties of the model with a zero lower bound (ZLB) on the nominal interest rate. The necessary and sufficient conditions are discussed in detail in Holden (2019). First, note that the interest-rate rule with a ZLB can be written as:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1} + \eta_t, \tag{C.60}$$

where η_t is a partially anticipated add-factor defined as:

$$\eta_t \equiv \max\{0, \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}\} - \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}. \tag{C.61}$$

Because η_t is partially predictable it can be considered as a monetary policy news shock; information that the ZLB will bind in k periods ahead is equivalent to news that $\eta_{t+k} > 0$.

Starting with a path for r_t, ignoring the ZLB up to horizon T, the problem of computing the sequence of η_t to impose the ZLB can be characterized as a linear complimentarity problem (LCP). This is convenient because it is a well-studied problem in the mathematics literature and so we can use existing tests to check the uniqueness and determinacy properties of a particular interest-rate rule (see Holden, 2019). Let vector $q \equiv [q_1, \cdots, q_T]'$ be the path of $r_t + \bar{r}$ ignoring the bound up to horizon T, and let M be a $T \times T$ matrix where the nth column gives the values of $[r_1, \cdots, r_T]$ conditional on an anticipated news shock, η_n of size 1 in period $t = n$. Given the otherwise linearity of the model, conditional on a path ignoring the bound q, and sequence of news shocks $\eta \equiv [\eta_1, \cdots, \eta_T]'$, the path of the interest rate is given by:

$$r + \bar{r} = q + M\eta, \tag{C.62}$$

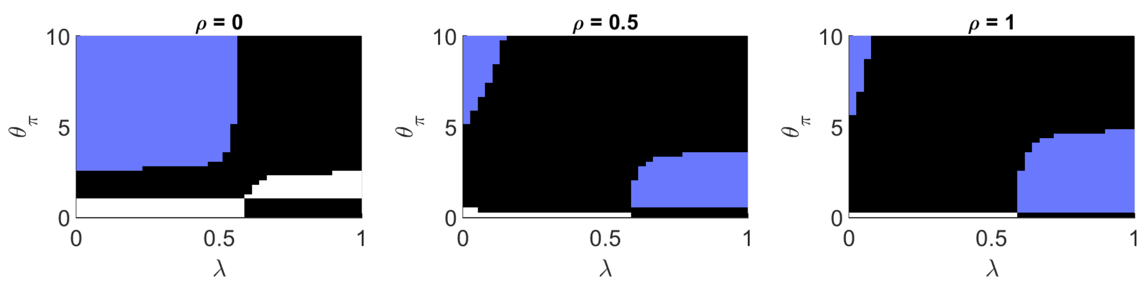
where $r \equiv [r_1, \cdots, r_T]'$. M and q are readily solved using the linear model without a ZLB. The LCP(q, M) is to solve the vector η to satisfy the following constraints:

$$\eta \geq 0, \tag{C.63}$$

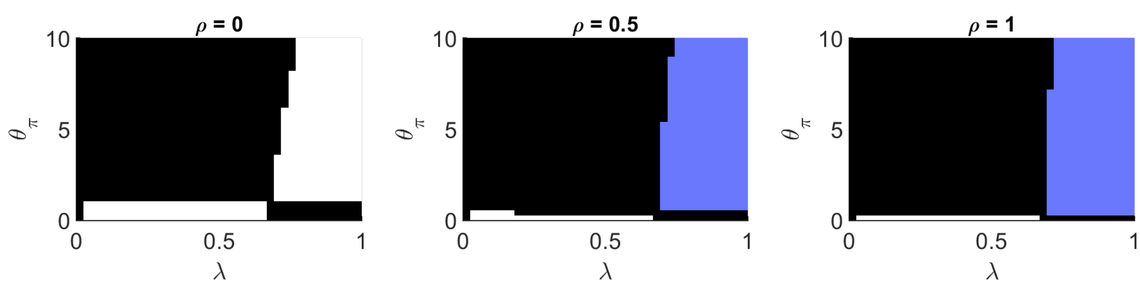
$$q + M\eta \geq 0, \tag{C.64}$$

$$y'(q + M\eta) = 0. \tag{C.65}$$

The above conditions are that news shocks must always be positive (C.63), the ZLB must not be violated (C.64), and the complimentary slackness condition (C.65) which requires



(a) Open economy



(b) Closed economy

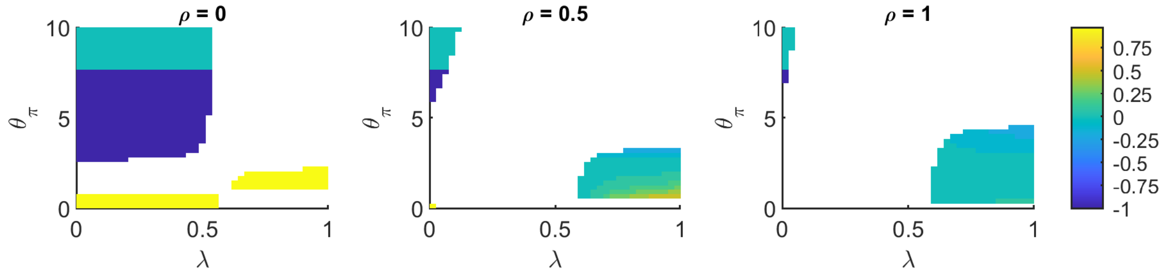
Figure 10: Initial tests for multiplicity. The black area represents indeterminacy in the baseline linear LAMP model, the white area indicates there is always a unique equilibrium conditional on households expecting to be away from the ZLB in 200 quarters. Multiplicity cannot be ruled out for the blue area.

that news shocks are only non-zero when the ZLB is binding. To determine whether there are multiple equilibria or explosiveness (infeasibility) requires checking the properties of matrix M.

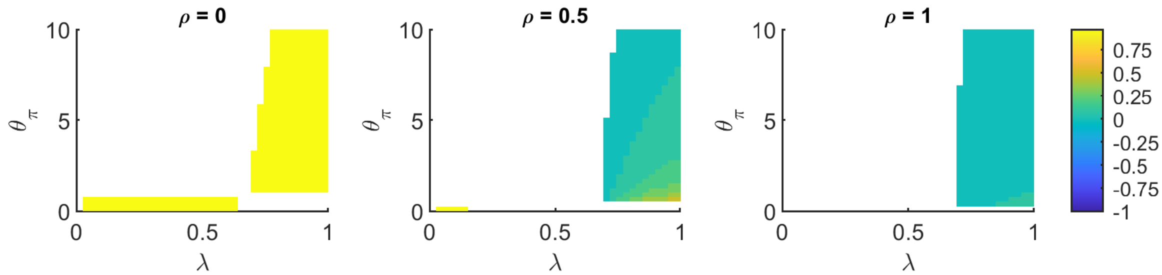
Cottle, Pang and Stone (2009, ch. 3) show that uniqueness is guaranteed if, for all $z \in \mathbb{R}^{T\times 1}$ with $z \neq 0$, there exists $t \in \{1, ..., T\}$ such that $z_t \left(Mz\right)_t > 0$. Following the notation of Cottle et al. (2009) and Holden (2019), we refer to a matrix satisfying this condition as a **P**-matrix. This is a particular definition of positivity and to gain some intuition, if M is a **P**-matrix in our model, then monetary policy shocks must increase nominal interest rates. This is consistent with the above description of a self-fulfilling ZLB episode which relies on news shocks lowering nominal rates. A full test to determine whether M is a **P**-matrix may be infeasible for a large horizon T, however it is possible to check other necessary and sufficient conditions. For example, M is definitely a **P**-matrix if it is symmetric positive definite and it is definitely not a **P**-matrix if it has any complex eigenvalues outside the interval $\left(-\pi + \frac{\pi}{T}, \pi - \frac{\pi}{T},\right)$ (see Holden, 2019).[35] Figure 10 shows the results of the initial checks where, as before, the black area represents calibrations leading to indeterminacy in the baseline linear LAMP model. The white area now represents calibrations for which uniqueness is guaranteed providing the economy is expected to be away from the ZLB in 200 quarters. These initial checks show that when a determinate policy rule is available under IADL, uniqueness is always guaranteed except for high values of $\theta_\pi > \max\left\{\frac{1}{1-w_c}, \Gamma_1\right\}$. Under SADL, we cannot rule out multiplicity except when interest rate inertia is absent.

As an alternative, we can look to other indicative statistics such as the minimum determinant of a principal sub-matrix of M. When this is positive, M is a **P**-matrix. This is useful as it is a continuous measure and so allows us to gain insight as to whether a parameter worsens or improves the multiplicity properties of the model. Figure 11 presents the results of this. A visual check of this reveals that policy inertia worsens the problem of multiple equilibria under SADL. Increasing the response of policy to inflation also worsens this problem. The reason being that a more aggressive policy stance will cut the interest rate further if a future contraction is expected. It is this mechanism that can lead to a self-fulfilling ZLB episode.

[35] Refer to Corollaries 4 and 5 in appendix C of Holden (2019) for the necessary and sufficient conditions.



(a) Open economy



(b) Closed economy

Figure 11: Minimum determinant of the 5×5 leading principal sub-matrix of M. A positive values implies M is a **P**-matrix.

D Optimal Policy: Derivations and Proofs

D.1 Proof of Proposition 7

Under LAMP, the SOE government chooses monetary policy to maximize a utilitarian social welfare given by $\lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C)$, where from (2.1) as $\sigma \to 1$

$$U(C_t^i, N_t^i) = \log C_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}, \quad i = R, C. \tag{D.1}$$

To approximate this welfare criterion we implement the standard algorithm of Taylor-series expansion, in particular following the steps in Woodford (2003), Benigno and Woodford (2004), Galí and Monacelli (2005), Bilbiie (2008), and Levine, Pearlman and Pierse (2008).

D.2 Step 1: Taylor Series Expansion

Taking a second-order Taylor linear expansion we get:

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + U_{C^i}\left(C_t^i - \overline{C}_t^i\right) + U_{N^i}\left(N_t^i - \overline{N}_t^i\right) \\ &+ \frac{1}{2}\left[U_{C^iC^i}\left(C_t^i - \overline{C}_t^i\right)^2 + 2\,U_{C^iN^i}\left(C_t^i - \overline{C}_t^i\right)\left(N_t^i - \overline{N}_t^i\right) + U_{N^iN^i}\left(N_t^i - \overline{N}_t^i\right)^2\right] \\ i &= R, C \qquad \text{up to second order terms.} \end{aligned} \tag{D.2}$$

Defining $c_t^i \equiv \frac{C_t^i - \overline{C}_t^i}{\overline{C}_t^i}$ and $n_t^i \equiv \frac{N_t^i - \overline{N}_t^i}{\overline{N}_t^i}$ to be relative deviations about $\overline{C}_t^i$ or $\overline{N}_t^i$, which can be steady states or flexi-price equilibria, (D.2) becomes

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + U_{C^i}\overline{C}_t^i c_t^i + U_{N_t^i}\overline{N}_t^i n_t^i \\ &+ \frac{1}{2}\left[U_{C^iC^i}(\overline{C}_t^i)^2(c_t^i)^2 + 2\,U_{C^iN^i}\overline{C}^i\overline{N}^i c_t^i n_t^i + U_{N^iN^i}(\overline{N}^i)^2(n_t^i)^2\right], \ i = R, C. \end{aligned} \tag{D.3}$$

(D.3) is completely general. Adopting our particular choice of preferences (D.1), we have that $U_{C^iN^i} = 0$, $U_{C^i} = (\overline{C}_t^i)^{-1}$, $U_{C^iC^i} = -(\overline{C}_t^i)^{-2}$, $U_{N^i} = -(\overline{N}_t^i)^{\varphi}$, $U_{N^iN^i} = -\varphi(\overline{N}_t^i)^{\varphi-1}$. Then (D.3) becomes:

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + c_t^i - (\overline{N}_t^i)^{1+\varphi} n_t^i \\ &- \frac{1}{2}\left[(c_t^i)^2 + \varphi(\overline{N_t}^i)^{1+\varphi}(n_t^i)^2\right], \ i = R, C. \end{aligned}$$

Hence the social welfare criterion, wel_t, is given approximately up to second order terms by:

$$\begin{aligned} wel_t &= U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C) \approx U(\overline{C}_t^R, \overline{N}_t^R) + (1-\lambda)U(\overline{C}_t^C, \overline{N}_t^C) \\ &+ \lambda c_t^R + (1-\lambda)c_t^C - \lambda(\overline{N}_t^R)^{1+\varphi} n_t^R + (1-\lambda)(\overline{N}_t^C)^{1+\varphi} n_t^C \\ &- \frac{1}{2}\left[\lambda\left((c_t^R)^2 + \varphi(\overline{N_t}^R)^{1+\varphi}(n_t^R)^2\right) + (1-\lambda)\left((c_t^C)^2 + \varphi(\overline{N_t}^C)^{1+\varphi}(n_t^C)^2\right)\right]. \end{aligned} \tag{D.4}$$

(D.4) holds for our particular choice of household preferences for any baseline $U(\overline{C}_t^i, \overline{N}_t^i)$, about which the Taylor series expansion (or approximation) is based. In our paper this is the distorted equitable steady state. We now choose the optimal equitable flexi-price equilibrium with a welfare-relevant output gap $x_{1,t}$ for which $(\overline{N}_t^R)^{1+\varphi} = (\overline{N}_t^C)^{1+\varphi} = \mathrm{w}_C$. Then (D.4) becomes:

$$\begin{aligned} wel_t &= \lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C) \approx \lambda U(\overline{C}_t^R, \overline{N}_t^R) + (1-\lambda)U(\overline{C}_t^C, \overline{N}_t^C) \\ &+ \lambda c_t^R + (1-\lambda)c_t^C - \mathrm{w}_C\left[\lambda n_t^R + (1-\lambda)n_t^C\right] \\ &- \frac{1}{2}\left[\lambda\left((c_t^R)^2 + \varphi \mathrm{w}_C (n_t^R)^2\right) + (1-\lambda)\left((c_t^C)^2 + \varphi \mathrm{w}_C (n_t^C)^2\right)\right]. \end{aligned} \tag{D.5}$$

D.3 Step 2: Use of the Resource Constraint in Linearized Form

To express this as a quadratic form we now impose the resource constraint which can be expressed for our purposes as:

$$C_t^{\mathrm{w}_C}(C_t^R)^{1-\mathrm{w}_C} = Y_t^{\mathrm{w}_C}(Y_t^*)^{1-\mathrm{w}_C}, \tag{D.6}$$

$$Y_t = \frac{A_t N_t}{\Delta_t}, \tag{D.7}$$

$$N_t = \lambda N_t^R + (1-\lambda)N_t^C, \tag{D.8}$$

$$C_t = \lambda C_t^R + (1-\lambda)C_t^C. \tag{D.9}$$

Denoting any variable Z_t in log-deviation form $\hat{z}_t \equiv \log\left(Z_t/\bar{Z}_t\right)$ and in relative deviation form by $z_t = (Z_t - \bar{Z}_t)/\bar{Z}_t$, a Taylor series expansion gives

$$z_t \approx \widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 \text{ (up to second order).} \tag{D.10}$$

In what follows we take $\bar{Z}_t$ to be the equitable flexi-price equilibrium supported by tax subsidies set out in Proposition 3. Then $x_t = (Y_t - \bar{Y}_t)/\bar{Y}_t$ becomes the output gap.

Taking logs, (D.6) and (D.7) can be written exactly as:

$$\mathrm{w}_C\widehat{c}_t + (1-\mathrm{w}_C)\widehat{c}_t^R = \mathrm{w}_C\widehat{x}_t + \text{ t.i.p.} \tag{D.11}$$

$$\widehat{x}_t = \widehat{n}_t - \widehat{\delta}_t + \text{ t.i.p.} \tag{D.12}$$

where terms independent of policy (t.i.p.) are those only involving shock processes y_t^* and

a_t.

Now consider the linear term in (D.5) which can be written simply as $c_t - \mathrm{w}_C n_t = c_t - \mathrm{w}_C(x_t + \delta_t)$ plus t.i.p.. Using (D.10) we have up to o(2):

$$
\begin{aligned}
c_t - \mathrm{w}_C n_t &= \lambda c_t^R + (1-\lambda)c_t^C - \mathrm{w}_C \left[\lambda n_t^R + (1-\lambda)n_t^C\right] \\
&\approx \lambda \widehat{c}_t^R + (1-\lambda)\widehat{c}_t^C - \mathrm{w}_C \left[\lambda(\widehat{n}_t^R + (1-\lambda)\widehat{n}_t^C\right] \\
&+ \frac{1}{2}\left[\lambda(\widehat{c}_t^R)^2 + (1-\lambda)(\widehat{c}_t^C)^2 - \mathrm{w}_C \left[\lambda((\widehat{n}_t^R)^2 + (1-\lambda)(\widehat{n}_t^C)^2\right]\right].
\end{aligned}
$$

Then (D.5) becomes

$$
wel_t = wel + \widehat{c}_t - \mathrm{w}_C \widehat{n}_t - \frac{1}{2}\left[\lambda\left((1+\varphi)\mathrm{w}_C(n_t^R)^2\right) + (1-\lambda)\left((1+\varphi)\mathrm{w}_C(n_t^C)^2\right)\right]. \quad \text{(D.13)}
$$

Using the exact log-linear resource constraint (D.11) we then have

$$
\begin{aligned}
\widehat{c}_t - \mathrm{w}_C \widehat{x}_t &= (1-\mathrm{w}_C)\left(\widehat{x}_t - \frac{1}{\mathrm{w}_C}\widehat{c}_t^R\right) \\
&= \frac{1-\mathrm{w}_C}{\mathrm{w}_C}(\mathrm{w}_C \widehat{x}_t - \widehat{c}_t^R) \\
&= \frac{1-\mathrm{w}_C}{\mathrm{w}_C}(\mathrm{w}_C \widehat{x}_t - \widehat{c}_t + \widehat{c}_t - \widehat{c}_t^R). \qquad \text{(D.14)}
\end{aligned}
$$

Hence solving for $\widehat{c}_t - \mathrm{w}_C \widehat{x}_t$ we arrive at

$$
\widehat{c}_t - \mathrm{w}_C \widehat{x}_t = (1-\mathrm{w}_C)(\widehat{c}_t - \widehat{c}_t^R). \qquad \text{(D.15)}
$$

To complete the transformation of (D.15) into second-order terms we recall relevant results for the linearization of our model in log-deviation form from Appendix C.1:

$$
\begin{aligned}
\widehat{w}_t &= \varphi \widehat{n}_t^R + \sigma \widehat{c}_t^R, \\
\widehat{w}_t &= \varphi \widehat{n}_t^C + \sigma \widehat{c}_t^C, \\
n_t^C &= \frac{\varphi(1-\sigma)}{\varphi+\sigma} n_t^R + \frac{\sigma(1-\sigma)}{\varphi+\sigma} c_t^R, \\
\widehat{x}_t &= \widehat{n}_t - \widehat{\delta}_t + \text{t.i.p.}.
\end{aligned}
$$

Hence with $\sigma = 1$ we have:

$$\begin{aligned} n_t^C &= \widehat{n}_t^C = 0, \\ \widehat{c}_t^C - \widehat{c}_t^R &= -\varphi(\widehat{n}_t^C - \widehat{n}_t^R) = \varphi \widehat{n}_t^R, \\ \widehat{x}_t &= \lambda \widehat{n}_t^R - \widehat{\delta}_t + \text{t.i.p..} \end{aligned}$$

Using these results we obtain:

$$\begin{aligned} \widehat{c}_t - \mathrm{w}_C \widehat{x}_t &= (1-\mathrm{w}_C)(\widehat{c}_t - \widehat{c}_t^R) \\ &= (1-\mathrm{w}_C)(c_t - c_t^R + o(2)) = (1-\mathrm{w}_C)(1-\lambda)(c_t^C - c_t^R + o(2)) \\ &= (1-\mathrm{w}_C)(1-\lambda)(\varphi n_t^R + o(2)) = \frac{(1-\mathrm{w}_C)(1-\lambda)}{\lambda}\varphi(x_t + o(2)). \end{aligned}$$

In what follows we consider the case where $\frac{(1-\mathrm{w}_C)(1-\lambda)\varphi}{\lambda}$ is small[36] and of the same order as deviations of variables about the baseline flexi-price equilibrium allocation or steady state. For example, even for a small open economy the share of imported consumption goods $(1-\mathrm{w}_C)$ is typically less than 0.3. Further, if the share of RoT consumers $(1-\lambda) < 0.2$ and $\varphi = 2$, then $(1-\mathrm{w}_C)(1-\lambda)\varphi/\lambda < 0.15$. For economies with these features we can then treat $(1-\mathrm{w}_C)(1-\lambda)\varphi/\lambda x_t$ as o(2).[37]

Gathering our results together we can now write (D.13) up to o(2) as:

$$wel_t - wel = \frac{(1-\mathrm{w}_C)(1-\lambda)\varphi}{\lambda} x_t - \mathrm{w}_C \widehat{\delta}_t - \frac{1}{2}\frac{\mathrm{w}_C(1+\varphi)x_t^2}{\lambda}. \tag{D.16}$$

Note that with our distorted equitable steady state, standard derivations lead to an additional linear term $\frac{\Phi\Psi}{\zeta}x_t$ as in the closed economy (see, for example, Galí (2015)).

[36] Notice that this term vanishes in the closed TANK economy (of Bilbiie (2008)), with $\mathrm{w}_C = 1$, as well as in the SOE model without LAMP (of Galí and Monacelli (2005)), with $\lambda = 1$.

[37] This is analogous to the way small distortions in the steady state are incorporated into a quadratic approximation in the literature.

D.4 Step 3: Quadratic Approximation of Dispersion Term

The remaining step is to obtain a quadratic approximation for the price dispersion term $\widehat{\delta}_t$ in (D.16). To do this we use the following results:

$$\Delta_t = \xi \Pi_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{-\zeta}, \tag{D.17}$$

$$\left(\frac{J_t}{JJ_t}\right)^{1-\zeta} = \frac{1-\xi\Pi_{H,t}^{\zeta-1}}{1-\xi}, \tag{D.18}$$

where $\frac{J_t}{JJ_t} = \frac{P^0_{H,t}}{P_{H,t}}$ is the optimal reset price. This results in $\Delta_t = \Delta(\Pi_{H,t})$.

We now use a second order Taylor series expansion **about a zero net inflation** $\Pi = \Pi_H = 1$ to show that

$$\delta_t = \xi\delta_{t-1} + \frac{\xi\zeta}{2(1-\xi)}\pi^2_{H,t}. \tag{D.19}$$

Proof

First write (D.17) and (D.18) as:

$$\begin{aligned}
\Delta_t &= \xi \Pi_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{1-\xi\Pi_{H,t}^{\zeta-1}}{1-\xi}\right)^{-\frac{\zeta}{1-\zeta}} \\
&= \xi \Pi_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi)^{\frac{1}{1-\zeta}}\left(1-\xi\Pi_{H,t}^{\zeta-1}\right)^{\frac{\zeta}{\zeta-1}} \\
&= \xi F(\Pi_{H,t}, \Delta_{t-1}) + (1-\xi)^{\frac{1}{1-\zeta}} G(\Pi_{H,t}).
\end{aligned} \tag{D.20}$$

Next we expand $F(\Pi_{H,t}, \Delta_{t-1})$ and $G(\Pi_{H,t})$ as Taylor series up to second order:

$$\begin{aligned}
F(\Pi_{H,t}, \Delta_{t-1}) &= F(\Pi, \Delta) + F_{\Pi}(\Pi, \Delta)(\Pi_{H,t} - \Pi) + F_{\Delta}(\Pi, \Delta)(\Delta_{t-1} - \Delta) \\
&\quad + \frac{1}{2}\left(\begin{array}{c} F_{\Pi\,\Pi}(\Pi, \Delta)(\Pi_{H,t} - \Pi)^2 + 2F_{\Pi\,\Delta}(\Pi, \Delta)(\Pi_{H,t} - \Pi)(\Delta_{t-1} - \Delta) \\ + F_{\Delta\,\Delta}(\Pi, \Delta))(\Delta_{t-1} - \Delta)^2 \end{array}\right) \\
&\quad + \cdots \\
G(\Pi_{H,t}) &= G(\Pi) + G'(\Pi)(\Pi_{H,t} - \Pi) + \frac{1}{2}G''(\Pi)(\Pi_{H,t} - \Pi)^2 + \cdots
\end{aligned}$$

Subtract $\Delta = \xi F(\Pi,\Delta) + (1-\xi)^{\frac{1}{1-\zeta}} G(\Pi)$ from both sides of (D.20) to give

$$\begin{aligned}
\Delta_t - \Delta &= \xi\Big(F_\Pi(\Pi,\Delta)(\Pi_{H,t}-\Pi) + F_\Delta(\Pi,\Delta)(\Delta_{t-1}-\Delta) \\
&+ \frac{1}{2}\left(F_{\Pi\,\Pi}(\Pi,\Delta)(\Pi_{H,t}-\Pi)^2 + 2F_{\Pi\,\Delta}(\Pi,\Delta)(\Pi_{H,t}-\Pi)(\Delta_{t-1}-\Delta) + F_{\Delta\,\Delta}(\Pi,\Delta))(\Delta_{t-1}-\Delta)^2\right)\Big) \\
&+ (1-\xi)^{\frac{1}{1-\zeta}}\left(G'(\Pi)(\Pi_{H,t}-\Pi) + \frac{1}{2}G''(\Pi)(\Pi_{H,t}-\Pi)^2\right).
\end{aligned}$$

Hence:

$$\begin{aligned}
\delta_t \equiv \frac{\Delta_t-\Delta}{\Delta} &= \xi\Big(F_\Pi(\Pi,\Delta)\Pi\,\pi_{H,t} + F_\Delta(\Pi,\Delta)\Delta\,\delta_{t-1} \\
&+ \frac{1}{2}\left(F_{\Pi\,\Pi}(\Pi,\Delta)\Pi^2\,\pi_{H,t}^2 + 2F_{\Pi\,\Delta}(\Pi,\Delta)\Pi\Delta\,\pi_{H,t}\delta_{t-1}) + F_{\Delta\,\Delta}(\Pi,\Delta))\Delta^2\,\delta_{t-1}^2\right)\Big) \\
&+ (1-\xi)^{\frac{1}{1-\zeta}}\left(G'(\Pi)\Pi\pi_{H,t} + \frac{1}{2}G''^2\pi_{H,t}^2\right), \qquad \text{(D.21)}
\end{aligned}$$

up to second order terms.

From the definitions

$$\begin{aligned}
F(\Pi,\Delta) &\equiv \Pi^\zeta\Delta, \\
G(\Pi) &\equiv \left(1-\xi\Pi^{\zeta-1}\right)^{\frac{\zeta}{\zeta-1}},
\end{aligned}$$

we have

$$\begin{aligned}
F_\Pi(\Pi,\Delta) &= \zeta\Pi^{\zeta-1}\Delta, \\
F_\Delta(\Pi,\Delta) &= \Pi^\zeta, \\
F_{\Pi\Pi}(\Pi,\Delta) &= \zeta(\zeta-1)\Pi^{\zeta-2}\Delta \\
& \quad G'^{\zeta-2}\left(1-\xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}} \\
& \quad G''^2\zeta\Pi^{2(\zeta-1)}\left(1-\xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}-1} - \xi\zeta(\zeta-2)\Pi^{\zeta-3}\left(1-\xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}}.
\end{aligned}$$

About a zero net inflation steady state, $\Pi = \Delta = 1$ and we have:

$$\begin{aligned}
F_\Pi(1,1) &= \zeta, \\
F_{\Pi\Pi}(1,1) &= \zeta(\zeta-1),
\end{aligned}$$

$$G'(1) \quad = \quad -\xi\zeta\,(1-\xi)^{\frac{1}{\zeta-1}}$$
$$G''2\zeta\,(1-\xi)^{\frac{1}{\zeta-1}-1} - \xi\zeta(\zeta-2)\,(1-\xi)^{\frac{1}{\zeta-1}}$$

Hence the terms in $\pi_{H,t}$, $\left(\xi F_{\Pi}(1,1) + (1-\xi)^{\frac{1}{1-\zeta}} G'(1)\Pi\right)\pi_{H,t} = 0$. In other words about a zero net inflation steady state only second order terms in inflation affect dispersion. Then, with a little algebra, (D.19) follows from (D.21) and the derivatives above.

Now we complete the quadratic approximation using (D.19):

$$\sum_{t=0}^{\infty} \beta^t \delta_t = \sum_{\tau=1}^{\infty} \beta^{\tau-1} \delta_{\tau-1} = \beta^{-1} \sum_{t=1}^{\infty} \beta^t \delta_{t-1} = \beta^{-1} (\sum_{t=0}^{\infty} \beta^t \delta_{t-1} - \delta_{-1}). \tag{D.22}$$

Then assuming that prior to the optimization exercise the economy is at its steady state, $\delta_{-1} = 0$, and using (D.22), we have that

$$\sum_{t=0}^{\infty} \beta^t \delta_{t-1} = \beta \sum_{t=0}^{\infty} \beta^t \delta_t \Rightarrow \sum_{t=0}^{\infty} \beta^t (\delta_t - \xi \delta_{t-1}) = (1-\xi\beta) \sum_{t=0}^{\infty} \beta^t \delta_t. \tag{D.23}$$

Hence from (D.19) and (D.23) up to o(2) we have

$$\sum_{t=0}^{\infty} \beta^t \delta_t = \sum_{t=0}^{\infty} \beta^t \widehat{\delta}_t = \frac{\xi\zeta}{2(1-\beta\xi)(1-\xi)} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2. \tag{D.24}$$

We can now write the intertemporal social welfare loss as:

$$\Omega_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[-(1-\mathrm{w}_C)(1-\lambda) x_t + \pi_{H,t}^2 + \frac{\Psi\,(1+\varphi)}{\varsigma\lambda} x_t^2 \right], \tag{D.25}$$

where $\Psi \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$. The terms in x_t in (D.25) can be written as $\frac{\mathrm{w}_C(1+\varphi)}{2\lambda}(x_t - x_t^{bliss})^2$ where x_t has a *bliss point* $x_t = x_t^{bliss} = \frac{(1-\mathrm{w}_C)(1-\lambda)\lambda}{\mathrm{w}_C(1+\varphi)}$. This confirms the non-social-optimality of the optimal equitable allocation emphasized in Proposition 3. The bliss point as a function of λ reaches a maximum of $\frac{(1-\mathrm{w}_C)}{4\mathrm{w}_C(1+\varphi)}$ at $\lambda = \frac{1}{2}$. For typical values $\mathrm{w}_C = 0.7$ and $\varphi = 2$ this gives $x_t^{bliss} = 5\%$.

D.5 Proof of Proposition 8

Under discretionary policy, in each period t the monetary authority chooses output and inflation according to the following optimization problem:

$$\min_{x_t,\pi_{H,t}} \quad \frac{1}{2}\left[\pi_{H,t}^2 + \varpi x_t^2\right] - \Lambda_x x_t$$

subject to the constraint of the economy embodied in the NKPC with already formed by the private sector, i.e., taken as fixed and given by the policymaker, next-period inflation expectations $\mathbb{E}_t\left[\pi_{H,t+1}\right]$, and with the current-period cost-push shock process u_t already materialized and observed:

$$\pi_{H,t} = \kappa x_t + \beta \mathbb{E}_t\left[\pi_{H,t+1}\right] + u_t, \tag{D.26}$$

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}. \tag{D.27}$$

Writing down the Lagrangian function for this problem and combining its first-order conditions with respect to $\pi_{H,t}$ and x_t results in the targeting rule:

$$x_t = \frac{1}{\varpi}(\Lambda_x - \kappa \pi_{H,t}). \tag{D.28}$$

This is identical to Galí (2015), p. 140, eq. (19), but embodies our richer composite parameters, such as κ and Λ_x, as well as the three measures we introduced for the welfare-relevant output gap, x_t. Substituting (D.28) into (D.26) leads to domestic inflation dynamics given by

$$\pi_{H,t} = \frac{1}{\varpi + \kappa^2}\left(\kappa \Lambda_x + \varpi u_t + \beta \varpi \mathbb{E}_t\left[\pi_{H,t+1}\right]\right). \tag{D.29}$$

Solving forward iteratively, with (D.27) and using the law of iterated expectations gives,

$$\mathbb{E}_t\left[\pi_{H,t+1}\right] = \frac{\kappa \Lambda_x}{\varpi + \kappa^2} + \frac{\varpi \rho_u}{\varpi + \kappa^2} u_t + \frac{\beta \varpi}{\varpi + \kappa^2} \mathbb{E}_t\left[\pi_{H,t+2}\right]$$

and, plugging the above expected inflation term in the initial equation (D.29), one further obtains

$$\pi_{H,t} = \frac{\kappa \Lambda_x}{\varpi + \kappa^2}\left(1 + \frac{\beta \varpi}{\varpi + \kappa^2}\right) + \frac{\varpi}{\varpi + \kappa^2}\left(1 + \frac{\beta \rho_u \varpi}{\varpi + \kappa^2}\right) u_t + \left(\frac{\beta \varpi}{\varpi + \kappa^2}\right)^2 \mathbb{E}_t\left[\pi_{H,t+2}\right]. \tag{D.30}$$

Next, expressing $\pi_{H,t+2}$ from (D.29),

$$\pi_{H,t+2} = \frac{\kappa\Lambda_x}{\varpi+\kappa^2} + \frac{\varpi}{\varpi+\kappa^2}u_{t+2} + \frac{\beta\varpi}{\varpi+\kappa^2}\mathbb{E}_{t+2}\left[\pi_{H,t+3}\right],$$

taking conditional expectations and applying the law of iterated expectations,

$$\mathbb{E}_t\left[\pi_{H,t+2}\right] = \frac{\kappa\Lambda_x}{\varpi+\kappa^2} + \frac{\varpi\rho_u^2}{\varpi+\kappa^2}u_t + \frac{\beta\varpi}{\varpi+\kappa^2}\mathbb{E}_t\left[\pi_{H,t+3}\right]$$

and, plugging $\mathbb{E}_t\left[\pi_{H,t+2}\right]$ back in (D.30)

$$\begin{aligned}\pi_{H,t} &= \frac{\kappa\Lambda_x}{\varpi+\kappa^2}\left(1+\frac{\beta\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^2\right)\\ &\quad +\frac{\varpi}{\varpi+\kappa^2}\left(1+\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}\right)^2\right)u_t+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^3\mathbb{E}_t\left[\pi_{H,t+3}\right].\end{aligned}$$

Up to here, we've got 2 periods ahead. Moving forward to n periods ahead, we get (by analogy)

$$\begin{aligned}\pi_{H,t} &= \frac{\kappa\Lambda_x}{\varpi+\kappa^2}\left(1+\frac{\beta\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^2+\ldots+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^n\right)\\ &\quad +\frac{\varpi}{\varpi+\kappa^2}\left(1+\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}\right)^2+\ldots+\left(\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}\right)^n\right)u_t+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^{n+1}\mathbb{E}_t\left[\pi_{H,t+3}\right].\end{aligned}$$

Taking the limit when $n \longrightarrow \infty$, and noting that, for realistic calibration, $0 < \frac{\beta\varpi}{\varpi+\kappa^2} < 1$, since

$$\begin{aligned}\frac{\beta\varpi}{\varpi+\kappa^2} &= \frac{\beta\frac{\Psi(1+\varphi)}{\zeta\lambda}}{\frac{\Psi(1+\varphi)}{\zeta\lambda}+\Psi\left[1+\frac{\varphi}{\lambda}\left(1-(1-\lambda)\mathrm{w}_C\right)\right]}\\ &= \frac{\beta}{1+\frac{\zeta\lambda}{1+\varphi}\left[1+\frac{\varphi}{\lambda}\left(1-(1-\lambda)\mathrm{w}_C\right)\right]},\end{aligned}$$

with a numerical check, for $\varphi = 2$, $\beta = 0.99$, $\zeta = 6$, $\xi = 0.75$, $w_C = 0.5$, $\lambda = 0.5$, $\frac{\beta}{1+\frac{\zeta\lambda}{1+\varphi}\left[1+\frac{\varphi}{\lambda}(1-(1-\lambda)\mathrm{w}_C)\right]} = 0.198 < 1$, and so

$$\begin{aligned}\pi_{H,t} &= \frac{\kappa\Lambda_x}{\varpi+\kappa^2}\left(1+\frac{\beta\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\varpi}{\varpi+\kappa^2}\right)^2+...\right)\\ &\quad+\frac{\varpi}{\varpi+\kappa^2}\left(1+\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}+\left(\frac{\beta\rho_u\varpi}{\varpi+\kappa^2}\right)^2+...\right)u_t\end{aligned}$$

and using the formula for an infinite sum of a geometric sequence, gives

$$\pi_{H,t} = \frac{\kappa\Lambda_x}{\varpi+\kappa^2}\frac{\varpi+\kappa^2}{\kappa^2+(1-\beta)\,\varpi}+\frac{\varpi}{\varpi+\kappa^2}\frac{\varpi+\kappa^2}{\kappa^2+(1-\beta\rho_u)\,\varpi}u_t$$

and finally

$$\pi_{H,t} = \frac{\kappa\Lambda_x}{\kappa^2+(1-\beta)\varpi}+\frac{\varpi}{\kappa^2+(1-\beta\rho_u)\varpi}u_t \tag{D.31}$$

as in (4.19).

□

D.6 Proof of Proposition 9

The Lagrangian function for the optimization problem under commitment is given by

$$\mathcal{L}^C\left(\{x_t,\pi_t\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty}\right)\equiv$$

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\tfrac{1}{2}\left(\pi_{H,t}^2+\varpi x_t^2\right)-\Lambda_x x_t+\mu_t\left(\pi_{H,t}-\kappa x_t-\beta\pi_{H,t+1}\right)\right]+t.i.p. \tag{D.32}$$

for the welfare-relevant output gap x_t, where $\{\mu_t\}_{t=0}^{\infty}$ is a *sequence* of Lagrange multipliers for $t = 0, 1, 2, ...$, and where the *law of iterated expectations* has been applied to eliminate the conditional expectation that appeared in each constraint. Then, differentiating the Lagrangian function with respect to the decision variables, we obtain the respective FOCs

$$\frac{\partial\mathcal{L}^C\left(\{x_t,\pi_{H,t}\}_{t=0}^{\infty};\{\mu_t\}_{t=0}^{\infty}\right)}{\partial x_t} = \varpi x_t-\kappa\mu_t-\Lambda_x = 0 \Leftrightarrow \mu_t = \frac{\varpi}{\kappa}x_t-\frac{\Lambda_x}{\kappa},$$

$$\frac{\partial \mathcal{L}^C\left(\{x_t, \pi_{H,t}\}_{t=0}^{\infty}; \{\mu_t\}_{t=0}^{\infty}\right)}{\partial \pi_{H,t}} = \pi_{H,t} + \mu_t - \mu_{t-1} = 0 \Leftrightarrow \mu_t = \mu_{t-1} - \pi_{H,t},$$

that must hold for $t = 0, 1, 2, ...$, and where $\mu_{-1} = 0$.[38]

Combining, as we did under discretion earlier, the two FONCs into a single equation by eliminating the Lagrange multiplier, we obtain an optimal policy *price level targeting rule* that parallels (D.28) for the case of discretion:

$$x_t = -\frac{\kappa}{\varpi}\widehat{p}_{H,t} + \frac{\Lambda_x}{\varpi}, \tag{D.33}$$

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\widehat{p}_{H,t} \equiv p_{H,t} - p_{H,-1}$ is the deviation between the (log) price level and an 'implicit target' given by the (log) price level prevailing in the period just before the central bank committed and chose its optimal plan. Substituting (D.33) into the NKPC (4.16), we obtain the following difference equation for the (log) price level as deviation from the 'implicit target':

$$\widehat{p}_{H,t} = a\widehat{p}_{H,t-1} + a\beta\mathbb{E}_t\widehat{p}_{H,t+1} + \frac{a\kappa\Lambda_x}{\varpi} + au_t,$$

where $a \equiv \frac{\varpi}{\varpi(1+\beta)+\kappa^2} \in (0,1)$. The stationary solution to the equation above describes how the price level evolves under optimal policy with commitment:

$$\widehat{p}_{H,t} = \gamma\widehat{p}_{H,t-1} + \frac{\gamma}{1-\gamma\beta\rho_u}u_t + \frac{\gamma}{1-\gamma\beta}\frac{\kappa\Lambda_x}{\varpi},$$

for $t = 0, 1, 2, ...$, where $\gamma \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\beta\gamma} \in (0,1)$. Combining the preceding equation with (D.33) – as in Galí (2015), p. 143, but with our much richer composite parameters, such as κ and Λ_x, and three versions of the welfare-theoretic output x_t – one can obtain (after some algebraic manipulation)

$$x_t = \gamma x_{t-1} - \frac{\kappa\gamma}{\varpi\left(1-\gamma\beta\rho_u\right)}u_t,$$

[38] The last equality results because the inflation FONC corresponding to period -1 is not an effective constraint to the monetary authority when choosing its optimal policy plan in period 0.

for $t = 1, 2, 3, ...$, with the initial condition at $t = 0$ given by:

$$x_{i,0} = -\frac{\kappa\gamma}{\varpi\left(1-\gamma\beta\rho_u\right)}u_0 + \frac{\left(1-\gamma\right)\Lambda_x}{\varpi}.$$

The two last equations specify the corresponding path for output under optimal policy with commitment.

This completes the proof.

□

D.7 Effect of Openness and Asset Market Participation on Optimal Policy

We now examine the effect of openness $1 - \mathrm{w}_C$, and the degree of LAMP $1 - \lambda$, on the classical inflationary bias and the strength of optimal stabilization for both discretion and commitment, as operating via the composite parameters summarized in Table 2.

Policy Feature	Commitment	Discretion
Steady State Inflation	0	$\frac{\kappa\Lambda_x}{\kappa^2+(1-\beta)\varpi} > 0$
Output Gap Stabilization	$x_t - x_{t-1} = -\frac{\kappa}{\varpi}\pi_{H,t}$	$x_t = -\frac{\kappa}{\varpi}\pi_{H,t}$
Price Level or Inflation Stabilization	$p_{H,t} = \gamma p_{H,t-1} + \frac{\gamma}{1-\gamma\beta\rho_u}u_t$	$\pi_{H,t} = \frac{\varpi}{\kappa^2+(1-\beta\rho_u)\varpi}u_t$

Table 2: Summary of Optimal Policy

First, we gather the parameters involving w_C and λ:

$$\begin{aligned}
\varpi(\lambda) &= \frac{\Psi(1+\varphi)}{\zeta\lambda} > 0, && \text{(D.34)}\\
\Lambda_x(\lambda, \mathrm{w}_C) &= \frac{(1-\mathrm{w}_C)(1-\lambda)\varphi}{\lambda} > 0, && \text{(D.35)}\\
\Delta(\lambda, \mathrm{w}_C) &= 1 + \frac{\varphi}{\lambda}\left(1-(1-\lambda)\mathrm{w}_C\right) = \frac{1}{\lambda}(\lambda + \varphi(1-(1-\lambda)\mathrm{w}_C)) > 0, && \text{(D.36)}\\
\kappa(\lambda, \mathrm{w}_C) &= \Psi\Delta(\lambda, \mathrm{w}_C) > 0, && \text{(D.37)}\\
a(\lambda, \mathrm{w}_C) &\equiv \frac{\varpi}{\varpi(1+\beta)+\kappa^2} \in (0,1), && \text{(D.38)}\\
\gamma(\lambda, \mathrm{w}_C) &\equiv \frac{1-\sqrt{1-4\beta a^2}}{2\beta a} \in (0,1). && \text{(D.39)}
\end{aligned}$$

In determining the effects of trade openness and LAMP on the composite parameters in Table 2, most of our results are shown analytically, and these are explicit next. Some of our results have been checked numerically, and these are just reported next. We proceed with the case of discretion, first, and then follow with the case of commitment, in the order of the respective columns in Table 2 from top down to bottom. Second, we begin by clarifying analytically the effects of trade openness and LAMP on the SOE NKPC with LAMP. Differentiating (D.36) with respect to the home bias, we have

$$\frac{\partial \Delta}{\partial \mathrm{w}_C} = -\frac{\varphi(1-\lambda)}{\lambda} < 0, \tag{D.40}$$

that is, the sensitivity of the SOE NKPC with LAMP to the output gap increases with trade openness, $1 - \mathrm{w}_C$, i.e., its curvature becomes less bowed. And differentiating (D.36) now with respect to asset market participation, we obtain:

$$\begin{aligned}\frac{\partial \Delta}{\partial \lambda} &= \frac{\frac{\partial}{\partial \lambda}\left[\lambda + \varphi(1-(1-\lambda)\mathrm{w}_C)\right] \times \lambda - \left[\lambda + \varphi(1-(1-\lambda)\mathrm{w}_C)\right] \times \frac{\partial}{\partial \lambda}\lambda}{\lambda^2} \\ &= -\frac{\varphi\left(1-\mathrm{w}_C\right)}{\lambda^2} < 0,\end{aligned} \tag{D.41}$$

that is, the sensitivity of the SOE NKPC with LAMP to the output gap increases with the degree of LAMP, $1-\lambda$, i.e., its slope becomes steeper.

D.7.1 Signing of Terms under Optimal Discretion

Steady State Inflation The composite parameter expressing steady state inflation under discretion in Table 2:

$$\frac{\kappa \Lambda_x}{\kappa^2 + (1-\beta)\varpi} > 0$$

can be written in terms of structural parameters (by appropriate substitutions):

$$\frac{\left\{1 + \left[1-(1-\lambda)\, w_C\right] \frac{\varphi}{\lambda}\right\} \frac{(1-w_C)(1-\lambda)\varphi}{\lambda}}{\frac{(1-\xi)(1-\beta\xi)}{\xi}\left(1 + \left[1-(1-\lambda)\, w_C\right] \frac{\varphi}{\lambda}\right)^2 + (1-\beta)\frac{(1+\varphi)}{\zeta\lambda}} > 0.$$

Numerical checks for plausible parameter values setting $\varphi = 2$, $\beta = 0.99$, $\zeta = 6$, $\xi = 0.75$, confirm that SS inflation under optimal discretionary policy increases with both the degree of LAMP, $1-\lambda$, and the degree of trade openness, $1 - w_C$.

Output Gap Stabilization The composite parameter, $-\frac{\kappa}{\varpi}$, expressing output gap stabilization under discretion in Table 2, can be written in terms of structural parameters (by appropriate substitutions):

$\frac{\kappa}{\varpi} = \frac{\Psi\Delta}{\frac{\Psi(1+\varphi)}{\zeta\lambda}} = \frac{\zeta\lambda\Delta}{1+\varphi} = \frac{\zeta\lambda\frac{1}{\lambda}(\lambda+\varphi(1-(1-\lambda)\mathrm{W}_C))}{1+\varphi} = \frac{\zeta(\lambda+\varphi(1-(1-\lambda)\mathrm{W}_C))}{1+\varphi} = \frac{\zeta\lambda}{1+\varphi} + \frac{\zeta\varphi}{1+\varphi} - \frac{\zeta\varphi(1-\lambda)w_C}{1+\varphi} = \frac{\zeta\lambda}{1+\varphi} + \frac{\zeta\varphi}{1+\varphi} - \frac{\zeta\varphi w_C}{1+\varphi} + \frac{\zeta\varphi\lambda w_C}{1+\varphi}$ and so $\frac{\partial\frac{\kappa}{\varpi}}{\partial w_C} = -\frac{\zeta\varphi(1-\lambda)}{1+\varphi} < 0$: hence, output gap stabilization under optimal discretionary policy is stronger (in absolute value) for higher degrees of trade openness; and $\frac{\partial\frac{\kappa}{\varpi}}{\partial\lambda} = \frac{\zeta+\zeta\varphi w_C}{1+\varphi} = \frac{\zeta(1+\varphi w_C)}{1+\varphi} > 0$: hence, output gap stabilization under optimal discretionary policy is weaker (in absolute value) for higher degrees of LAMP.

Inflation Stabilization We express the composite parameter in

$$\pi_{H,t} = \frac{\varpi}{\kappa^2 + (1-\beta\rho_u)\varpi} u_t$$

in terms of the underlying structural parameters (by appropriate substitutions):

$$\pi_{H,t} = \frac{\Psi(1+\varphi)}{\Psi^2\left(1+[1-(1-\lambda)\,w_C]\,\frac{\varphi}{\lambda}\right)^2\zeta\lambda + (1-\beta\rho_u)\Psi(1+\varphi)} u_t.$$

We find that the strength of domestic-price inflation stabilization following a cost-push shock under optimal discretion decreases with the degree of trade openness. Formally:

$$\begin{aligned} & \frac{\partial \frac{\Psi(1+\varphi)}{\Psi^2\left(1+[1-(1-\lambda)w_C]\frac{\varphi}{\lambda}\right)^2\zeta\lambda+(1-\beta\rho_u)\Psi(1+\varphi)}}{\partial w_C} \\ = & \frac{\left\{2\Psi^2\,(1-\lambda)\left(\frac{\varphi}{\lambda}\right)^2[1-(1-\lambda)\,w_C]\right\}\zeta\lambda\Psi(1+\varphi)}{\left(\Psi^2\left(1+[1-(1-\lambda)\,w_C]\,\frac{\varphi}{\lambda}\right)^2\zeta\lambda+(1-\beta\rho_u)\Psi(1+\varphi)\right)^2} > 0 \end{aligned}$$

since $0 < (1-\lambda)\,w_C < 1$.

The strength of domestic-price inflation stabilization following a cost-push shock under optimal discretion depending on the degree of LAMP can be analyzed formally too, as follows:

$$\frac{\partial \frac{\Psi(1+\varphi)}{\Psi^2\left(1+[1-(1-\lambda)w_C]\frac{\varphi}{\lambda}\right)^2\zeta\lambda+(1-\beta\rho_u)\Psi(1+\varphi)}}{\partial\lambda}$$

$$= \quad \frac{-\Psi^3\left(1+\varphi\right)\zeta + \Psi(1+\varphi)\varphi^2\zeta\frac{2}{\lambda} - 2\Psi(1+\varphi)\varphi^2\frac{1}{\lambda^2}\zeta w_C - 2\Psi^2(1+\varphi)^2\frac{2-(1-\lambda)^2}{\lambda}\varphi^2\zeta w_C^2}{\left[\Psi^2\left(1+\left[1-(1-\lambda)\,w_C\right]\frac{\varphi}{\lambda}\right)^2\zeta\lambda + (1-\beta\rho_u)\Psi(1+\varphi)\right]^2} < 0 \text{ iff}$$

$$\begin{aligned}
\Psi^3\left(1+\varphi\right)\zeta + 2\Psi(1+\varphi)\varphi^2\frac{1}{\lambda^2}\zeta w_C + 2\Psi^2(1+\varphi)^2\frac{2-(1-\lambda)^2}{\lambda}\varphi^2\zeta w_C^2 &> \Psi(1+\varphi)\varphi^2\zeta\frac{2}{\lambda} \\
\frac{1}{2}\Psi^2\varphi^2\lambda^2 + w_C + \Psi(1+\varphi)\left[2-(1-\lambda)^2\right]\lambda w_C^2 &> 1 \\
\frac{1}{2}\Psi^2\varphi^2\lambda^2 + \Psi(1+\varphi)\left[2-(1-\lambda)^2\right]\lambda w_C^2 &> 1 - w_C \\
\frac{1}{2}\left[\frac{(1-\xi)(1-\beta\xi)}{\xi}\right]^2\varphi^2\lambda^2 + \frac{(1-\xi)(1-\beta\xi)}{\xi}(1+\varphi)\left[2-(1-\lambda)^2\right]\lambda w_C^2 &> 1 - w_C.
\end{aligned}$$

The final inequality has been checked numerically for plausible parameter constellations as earlier, namely, setting $\varphi = 2$, $\beta = 0.99$, $\zeta = 6$, $\xi = 0.75$: it is satisfied for $0 < w_C < 0.94$. This means that for usual degrees of trade openness (but not in a nearly closed economy), the strength of domestic-price inflation stabilization following a cost-push shock under optimal discretion increases with the degree of LAMP.

D.7.2 Signing of Terms under Optimal Commitment

Steady State Inflation Steady state inflation is now 0, so the inflationary bias of discretion vanishes under commitment, due to the anchoring of inflation expectations that has been pointed in the literature.

Output Gap Stabilization

$$x_t - x_{t-1} = -\frac{\kappa}{\varpi}\pi_{H,t}$$

Our results for the case of optimal discretion earlier are valid here again, but now, under optimal commitment, concerning the first difference of the output gap (or its short-run dynamics), $x_t - x_{t-1}$. This is analogous to 'speed-limit' Taylor-type rules, introduced by Walsh (2003), where output growth enters as well, in addition to the output gap or in place of it.

Price Level Stabilization It is clear from the respective expression in Table 2,

$$p_{H,t} = \gamma p_{H,t-1} + \frac{\gamma}{1 - \gamma\beta\rho_u} u_t$$

that the response of the domestic-price level to a cost-push shock

$$\frac{\partial p_{H,t}}{\partial u_t} = \frac{\gamma}{1 - \gamma\beta\rho_u}$$

increases with γ, β, and ρ_u. For the plausible parameter values we used already, i.e., with $\varphi = 2$, $\beta = 0.99$, $\zeta = 6$, $\xi = 0.75$, and now with also $\rho_u = 0.5$, numerical results show that the response of price level stabilization under optimal commitment to a cost-push shock decreases with both trade openness, $1 - w_C$, and the degree of LAMP, $1 - \lambda$.