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Effects of Information Quality on Signaling through Sovereign Debt Issuance

Hyungseok Joo* Yoon-Jin Lee† Young-Ro Yoon‡

Abstract

This paper develops a sovereign debt model proposing that a debt issuance can be a credible signaling channel between a sovereign government and foreign creditors. The government has private information regarding the future economy. The one with a good economic outlook would like to find a credible way to disclose it to obtain a high bond price. Foreign creditors are interested in inferring the government's private information to assess sovereign default risk precisely. The government's private information is imperfect, so the precision of information matters. We study how the interaction of the prior, the signal, and its precision affects the equilibrium and the resulting welfare. We propose a unique separating equilibrium where a government with a good economic outlook issues a smaller amount of bonds, even though its default risk is low, than one with a bad economic outlook. As the information becomes more precise, the signaling cost for a government with a good economic outlook increases. Interestingly, unless the prior is very pessimistic, a highly precise signal harms it, because a resulting strong signaling motive drives it to reduce bond issuance excessively *(paradox of highly precise information)*.

Keywords: Sovereign debt; Quality of information; Signaling; Incomplete information

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1 Introduction

Sovereign bonds are the means by which sovereign governments borrow from the international financial market. A government may issue debt for intertemporal consumption smoothing (Arellano (2008), Eaton and Gersovitz (1981)). Alternatively, it may do so despite economic surpluses to signal that its economy is strong enough to repay debt, which can facilitate foreign investment in the domestic private sector by influencing foreign creditors' expectations (Phan (2017), Atkins and Wigglesworth (2014), Cifuentes et al. (2002)). Regardless of the motive of sovereign debt issuance, foreign creditors' main interest is whether the issued debt will be repaid or not. The primary concern in negotiating borrowing terms is the bond price, which is inversely related to sovereign default risk - the issuing country's perceived inability to repay debts in the future.

Hence, foreign creditors have an incentive to gather information regarding the sovereign's future economy. However, the sovereign is likely to have more accurate information than foreign creditors regarding its own future economy. A government with information indicating a good economic outlook may be inclined to disclose this to foreign creditors to obtain a higher bond price. However, as it is private information, a government with a bad economic outlook can announce false information to mislead foreign creditors. Hence, a government with good prospects has an interest in finding a credible way to deliver its information to foreign creditors. Foreign creditors are also interested in inferring the government's private information to assess default risk more precisely and charge an appropriate bond price.

This situation can be analyzed in the framework of a signaling game. A government with either a good or bad economic outlook is a sender who has private information. The foreign creditor is a receiver. What matters is whether the government has a signaling device through which its private information can be disclosed credibly. Our argument is that if the government has private information about the future economy, it should be reflected in the amount of bonds he chooses. Then, foreign creditors can infer the government's private information from the amount of bonds issued. This, in turn, may induce the government to make use of bond issuance as an opportunity to disclose private information. In this paper, we propose that bond issuance can be a credible signaling channel between a government and foreign creditor.

Nowadays, more attention is being paid to the signaling aspect of fiscal policy. As an example, in the midst of the COVID-19 pandemic in 2021, UK finance minister Rishi Sunak emphasized a return to sound public finance, alluding to a more favorable economic outlook than previously anticipated. He was very keen on fiscal tightening even though the IMF, World Bank, and OECD still recommended expansionary fiscal policy to combat the pandemic-related recession. Creditors responded to newly issued bonds with a low borrowing cost, confirming Sunak's optimism.[1] The speech by German Chancellor Angela Merkel, emphasizing the need for fiscal austerity to signal the government's credibility on honoring debt during the European debt crisis, is another well known

[1] Underpinning what Sunak called "a new age of optimism" were Office for Budget Responsibility forecasts showing that the UK economy would settle 2 percent below its pre-pandemic path in the medium term rather than the 3 percent it forecast in March, implying less long-term pandemic damage. Gilts were on course for their biggest one-day rally since March 2020 after the government slashed its planned debt sales by almost £60bn, a much larger cut than markets had anticipated. Financial Times, 16-Sep-2021, 28-Oct-2021

example that features the signaling role of fiscal policy.[2]

Given this context, we develop a simple sovereign debt model. We consider a small open economy with a government that issues non-contingent sovereign bonds to risk-neutral foreign creditors. It cannot commit to repaying debt at the time the bonds are issued. Whether it defaults depends on solvency determined by the output level in the next period and default cost. The government has private information regarding whether the future economy will be good or bad. Although informative, that information is imperfect. Hence, even the government itself is uncertain about the future state of the economy. As it is the government's private information, the foreign creditor cannot observe it. However, the foreign creditor knows the precision of the government's information. In this set-up, the government determines the amount of bonds. The foreign creditor observes the amount of bonds, attempts to infer the government's private information, and charges a bond price.

Our results can be summarized as follows. A government (henceforth, G) with information indicating that the future economy is likely to be good (bad) is denoted by type h (l). First, as a benchmark, we consider the case where G's type is known. In equilibrium, the bond price for type h is higher than that for type l, which is intuitive because type h (l) is less (more) likely to default. The interesting result is that type h issues a smaller amount of bonds than type l, despite being less likely to default. This result arises from two forces. Given a high bond price, type h can borrow more by issuing a relatively small amount of bonds. Moreover, type h is concerned about repaying his debt next period, so a large debt can be a burden. On the other hand, type l is offered a low bond price and must issue a relatively large amount of bonds. Moreover, type l is likelier to default, in which case he does not need to repay debt. Hence, type l is less concerned about issuing a large amount of bonds than type h. Therefore, decreasing the amount of bonds is more costly to type l. As the information becomes more precise, type h (l) issues a smaller (larger) amount of bonds while being offered a higher (lower) bond price. More precise information always makes type h better off and type l worse off. That is, in the complete information case, the ex-ante utility is monotone with respect to the precision of signal.

Next, we consider the case in which G's type is private information. We first show the existence of both separating and pooling equilibria. A unique signaling equilibrium that satisfies Cho and Krep's intuitive criterion is a separating equilibrium, implying that the amount of bonds can work as a credible signaling device through which G's type is revealed. Type h, recognizing that issuing fewer bonds is more costly to type l, differentiates himself by issuing an amount of bonds too small for type l to adopt. The bond price for each type is identical to the complete information case because each type is perfectly revealed in this equilibrium. Type l issues the same amount of bonds as the complete information case. As his type is revealed in a separating equilibrium, type l has no incentive to deviate from the optimal amount of bonds given the bond price for type l. On the other hand, although the bond price for type h is identical in both cases, type h issues a smaller amount of bonds than in the complete information case. This implies that type h pays a signaling cost of decreasing the amount of bonds substantially enough that type l cannot mimic him.

As the private information becomes more precise, a stronger signaling incentive causes type h

[2] The Wall Street Journal, 12 July 2011

to decrease his bond issuance more. That is, more precise information makes type h's signaling more costly, resulting in a greater loss in welfare compared to the complete information case. The noteworthy result is that, unlike in the complete information case, the ex-ante utility of type h can be non-monotone with respect to the precision of information. Unless the prior belief for the true state is very pessimistic, more precise information helps type h only up to a certain point, after which greater precision harms him. We name this the *paradox of highly precise information.* This adverse effect of signal precision becomes more severe as the prior becomes more optimistic. Under a more optimistic prior, type h has a stronger signaling motive as he becomes more confident in his signal. However, since the signal indicates the same state as the prior, the impact of his signal becomes smaller. This result proposes that participating in costly signaling with a signal of limited impact can be harmful. On the other hand, if the prior is very pessimistic, a more precise signal of type h always increases his welfare. Given that the signal indicates the opposite of the prior belief, the impact of the signal is significant. The signal is more valuable as it is more precise. In this case, more aggressive signaling of issuing fewer bonds is always beneficial.

There are two additional implications. Lower current period consumption is carried from issuing fewer bonds, which can be a negative consequence of costly signaling. Our model demonstrates a dynamic where welfare improvement through consumption smoothing is distorted by a signaling incentive. We also propose that the gap in bond price between the two types can change even without a change in the market's perception of the sovereign's prospect or other underlying fundamentals. What plays a role is the prior belief. When the initial prior is more optimistic, it decreases. If the optimism recedes and pessimism kicks in, it increases.

The contribution of this paper is to propose that sovereign bond issuance itself can be a credible device disclosing a government's private information. In particular, we assume that the government has noisy information about the future economy. In the literature, the government's private information is often assumed to perfect. However, it is more plausible to assume that the government's private information, although informative, could be wrong. Hence, in our setting, the precision of information is important. In particular, the value of the signal is contingent on the prior belief for the state of the next period. We can study how the interaction of the prior, the signal, and its precision affects the amount of bonds, the bond price, and the resulting welfare in equilibrium. The literature that emphasizes the causal link from fiscal austerity to macroeconomic performance has long been analyzed. In particular, the expansionary fiscal austerity hypothesis emphasizes that fiscal tightening in the current period can stimulate GDP or investment by stabilizing public finances or signaling future tax cuts in an intertemporal setting.[3] On the other hand, we emphasize the reverse causality in that agents with private information of a good economic outlook can use fiscal tightening to lower borrowing costs.[4]

The paper is structured as follows. Section 2 is a literature review. Section 3 presents a model. In section 4, as a benchmark, we consider the complete information case where the sovereign's type is known, while section 5 considers the incomplete information case where the sovereign's

[3] Giavazzi and Pagano (1990), Alesina and Perrotti (1997), and Alesina and Ardagna (1998) find that fiscal consolidations are sometime associated with rapid output growth.

[4] We employ typical signaling-model feature in that fiscal tightening is assumed not to affect the future economy. This is similar to Spencer's model, where the years of education, which is the signaling device, does not affect the productivity of signal sender.

type is unknown. We derive a separating equilibrium in section 5.1 and a pooling equilibrium in section 5.2. In section 5.3, we adopt the intuitive criterion to refine the equilibrium. In the remaining subsections, we analyze the equilibrium and resulting welfare by comparing those with the complete information case. Section 6 is discussion. Section 7 concludes.

2 Related literature

Among the papers on debt markets, there are a small number of papers on the topic of the signaling aspect of fiscal policy, especially fiscal austerity. (Dellas & Niepelt (2021), Gibert (2022), Mihm (2018), and Mihm (2016)). These papers, including ours, commonly propose that a sovereign perceived as creditworthy issues fewer bonds and gets a higher price for them. This is a contrast to the standard sovereign debt model where an increase in perceived creditworthiness correlates a higher bond price and an issuance of more bonds.[5]

Our paper is closely related to Dellas and Niepelt (2021) and Gibert (2022) in that they propose that financial austerity can be motivated by a desire to signal a low probability of default. In Dellas and Niepelt (2021), each country's private information is a default cost. They show that, for a country with high default cost, fiscal austerity and investment that reduce current period consumption can signal its willingness to repay debt. In Gibert (2022), the country's type depends on a minimum level of consumption the country sets. She derives the condition for a separating equilibrium where issuing fewer bonds works as a signaling device for a country with a high reservation level of consumption.[6] Our model considers a different scenario where the government is privately informed about the economic outlook. In particular, we focus on the role of precision of information by assuming that the government's private information is noisy. This enables us to study the effect of precision of information on the signaling motive, the equilibrium behavior in issuing bonds, and the resulting welfare.

Mihm (2016) and Mihm (2018) also consider settings where the sovereign has private information about the state of economy and the sovereign's decision on bond issuance discloses it. Those models provide rationales for the observation of very small bond price differentials between countries despite differences in their debt issuance behavior before the European debt crisis. In Mihm (2016), the sovereign's private information is perfect and a pooling equilibrium that exhibits mispricing of risk is adopted to explain it.[7] In Mihm (2018), the default probability is the government's private information, and the government can have a biased perception of its own type: a risky government can perceive itself to be a safe type by mistake and vice versa. The investors take into account this biased perception, so they interpret a safe (risky) government's borrowing behavior as a possible behavior of a risky (safe) government. This results in a narrow bond price spread for large differences in debt issuance. We incorporate imperfectness of information, recognized by both the sovereign

[5] Dellas & Niepelt (2021), pp. 699.

[6] Using the concept 'undefeated equilibrium (Mailath et al., 1993), Gibert (2022) also proposes that whether the equilibrium is separating or pooling depends on how much reliable public information regarding the country's type is available in the market. It also provides empirical analysis supporting this theoretical proposition.

[7] In Mihm (2016), an economy's output level in the next period is θy, where θ denotes the economic fundamental and y is a stochastic endowment with uniform distribution. Here, $\theta \in \{\theta_H, \theta_L\}$, which is the sovereign's private information.

and creditors, into the model. The bond price is characterized as a function of the prior and the precision of the signal. We show that bond price differentials can change even without a change in the market's perception of the sovereign's underlying state. This provides another possible rationale for what was observed in the European debt crisis.

Our paper is also related to Perez (2017) and Catão et al. (2017) in that the signaling role of bond issuance is studied in different settings. Perez (2017) focuses on the choice of sovereign debt maturity, not the amount of bonds, in a setting where the government's private information is his willingness to repay debt. He proposes a pooling equilibrium in which all choose a shorter maturity structure relative to the debt structure in a perfect information setting. Safe borrowers do so because long term debt pools more risk than short term debt. In this equilibrium, risky borrowers are able to access debt at higher bond prices by mimicking safe borrowers. Catão et al. (2017) investigate sharp decouplings in cross-country sovereign bond yields during European debt crises. They consider a setting in which the sovereign country privately knows the volatility of fiscal revenue shocks, while his fundamentals are public information. They provide numerical solutions suggesting that a fundamentally strong country takes a pooling strategy of not issuing new debt regardless of whether the revenue shock is negative or positive. Meanwhile, a fundamentally weak country takes a separating strategy of issuing new debt under a negative revenue shock but not under a positive shock. Hence, debt issuance has a signaling role.

There is a branch of literature that considers a setting where default only depends on solvency, as is assumed in our model. Gibert (2022), mentioned above, rules out strategic default. In Perez (2017), the borrower exogenously chooses whether to repay or default on its debt depending on an exogenous shock. Haldane et al. (2005) attempt to build a model that captures key features of sovereign debt restructuring and study the conditions under which collective action clauses (CACs) can be beneficial in debt restructuring. In their model, debtors default because they are unable, rather than unwilling, to repay debt. Crosignani (2021), who studies how sovereign debt capacity is related to the capitalization of domestic banks, is another recent paper that assumes away strategic default.[8]

Although our model is not about whether to repay sovereign debt or default, it is broadly related to the models on strategic default in that debt issuance is incorporated in a signaling game model. Among previous studies, Phan (2017), Sandleris (2008), and Cole et al. (1995) show that the decision on debt repayment and default can be a signaling instrument for a privately informed government. In Phan (2017) and Sandleris (2008), the government is privately informed about the domestic economy's fundamentals and the debt is repaid to deliver information that the economy's fundamentals are good. In Cole et al. (1995), the government can be a patient or myopic type; the former uses settlement of old debt as a signaling device to reveal its willingness to repay future loans. Our model differs from those on the role of debt issuance. In those previous works, debt issuance itself has no signaling role, but is undertaken only to create a credible future signaling option. In our model, the action of debt issuance itself has a signaling role in that the government's type is

[8] In addition, Grossman and Van Huyck (1988) investigate excusable sovereign default that can be justified by exogenous bad economic conditions. They propose that default determined by insolvency due to a shock beyond the sovereign's control, rather than due to strategic choice, results in less adverse reputational effects, implying a lower cost of default.

revealed through the amount of bonds. Moreover, in the previous works, the government's private information is about something that has already been realized. In our paper, the government's private information is about what will be realized in the future and is imperfect, so the effect of precision of information can be analyzed.

3 Model

We consider a small open economy with a government (henceforth G) and a unit mass of identical households. There are two periods, $t = 0, 1$. The objective of G is to maximize the utility of the representative household over two periods, which takes the form

$$\underset{c_0, c_1}{Max}\ E_0 \left[\sqrt{c_0} + \beta \sqrt{c_1}\right]$$

where $0 < \beta \leq 1$ is the discount factor and c_t is the consumption in period t. For simplicity, we assume that $\beta = 1$. We consider $u(c_t) = \sqrt{c_t}$ as an example of a period utility function that is strictly increasing, strictly concave, and twice differentiable. If G intends to borrow money for c_0, he issues one-period bonds to a unit mass of foreign creditors (henceforth C) who are assumed to be risk-neutral. The creditor market is assumed to be competitive, so in equilibrium the creditors' expected profit is zero.

Let z_t be an exogenous endowment for this economy in period t. We assume that $z_0 = 0$ and the endowment in $t = 1$ is stochastic. We set $z_0 = 0$ as a simple case in which G must issue bonds for consumption in $t = 0$.[9] The true state of $t = 1$ is $w \in \{H, L\}$, and these two states are mutually exclusive. If $w = H$ (L), the output level is z_H (z_L) where $z_H > z_L = 0$. The prior belief for each state is $\Pr(w = H) = \theta$ and $\Pr(w = L) = 1 - \theta$, where $\theta \in (0, 1)$. If θ is close to 0 (1), it denotes the case where the prior is pessimistic (optimistic). Before issuing bonds, G can observe an informative signal $s \in \{h, l\}$ correlated with the true state of $t = 1$. The signal s partially reveals the true state in the following manner: $\Pr(s = h|w = H) = \Pr(s = l|w = L) = p$ and $\Pr(s = h|w = L) = \Pr(s = l|w = H) = 1 - p$ where $p \in \left(\frac{1}{2}, 1\right)$. That is, given the true state, the probability that the signal s is correct is p. Hence, p measures the precision of signal s. As $p > \frac{1}{2}$, s is an informative signal about the unknown true state. However, as $p < 1$, it is imperfect and so can be wrong as well. The signal s is G's private information, while p is public information.[10] Hence, creditors face uncertainty not only over the true state w but also over G's signal s.

The revenue from selling b units of bonds in $t = 0$ at price $0 < g \leq 1$ is gb. So, $c_0 = gb$. Here, a higher bond price g means that G can borrow money on better terms. G is supposed to repay debt b in $t = 1$. We assume that z_w is observable, so G's solvency is known to C. Default is costly in that the output level falls by the fraction $\lambda \in [0, 1]$ if debt is not fully paid. For example, if $w = H$, each type can consider an option of strategic default although he is able to pay his debt because

[9] Thus, the case in which the sovereign issues bonds despite economic surpluses to send a signal of positive economic conditions to the market is not considered in our model.

[10] The quality of the sovereign's information would be closely related to the quality of institutions. Institutional quality (IQ) from the International Country Risk Guide dataset, which includes investment profile, corruption, law and order, and bureaucratic quality, is an example of data which investors could use to measure the quality of institutions with the goal of assessing the precision of the sovereign's information. (Frankel et al. 2013).

$b_s \leq z_H$ in our setting. In that case, $c_1 = z_H - b_s$ if type s pays his debt and $c_1 = (1-\lambda) z_H$ if he defaults strategically. As long as $\lambda \geq \frac{b_s}{z_H}$, type s pays his debt without strategic default. We let $\lambda = 1$, the case where default cost is most costly. Then, the condition that $\lambda \geq \frac{b_s}{z_H}$ is always satisfied, so no type defaults strategically when $w = H$. If $w = L$, G must default because he is unable to repay his debt, i.e., $z_L = 0 \leq b_s$.

If we denote the probability of default by $\Pr(d)$, C's expected profit is

$$
\begin{aligned}
E\pi &= (1-\Pr(d))(-gb+b) + \Pr(d)(-gb) \\
&= -gb + (1-\Pr(d))\, b
\end{aligned}
$$

Given signal s, G's objective function to determine the optimal amount of bonds b_s is

$$V(b_s) = \sqrt{g_s b_s} + \Pr(H|s)\sqrt{z_H - b_s}$$

G's action is to decide the amount of bonds to issue, b. This decision should be based on his private signal $s \in \{h, l\}$. In particular, if $s = h$, G has an incentive to disclose his signal because he is able to obtain a higher price by revealing that he is less likely to default. It would then be interesting to ask whether G can reveal his private signal through the amount of his bond issuance. If there exists a separating equilibrium in which the amount of bonds depends on the private signal $s \in \{h, l\}$, C is able to infer G's private signal perfectly by observing b. C is interested in inferring G's private signal related to the true state w. If C infers G's type s from observing b, C is able to update his belief for $w \in \{H, L\}$ and have more precise information about the probability of default. This is essential for C in deciding the optimal bond price.

Based on this motivation, we adopt the framework of a signaling game. G is a sender whose type is $s \in \{h, l\}$ and C is a receiver who is interested in inferring G's type s by observing b. Note that C wants to infer G's type because C wants to have updated information regarding $\Pr(d)$. If $b > z_H$, then $\Pr(d) = 1$ and no bond market exists because $E\pi = -gb < 0$. On the other hand, if $b < z_L$, then $\Pr(d) = 0$ and $g = 1$ from $E\pi = -gb + b = 0$. In both cases, C has no reason to be interested in G's type because $\Pr(d)$ is known. Hence, C is interested in updating $\Pr(d)$ when $z_L \leq b \leq z_H$. In our setting where $\lambda = 1$ and $z_L = 0$, whether a default occurs depends on whether the state $w = L$ is realized, i.e., $\Pr(d) = \Pr(w = L)$. As $s \in \{h, l\}$ provides meaningful information with regard to $\Pr(w)$, C is willing to infer G's type. This is compatible with type h's incentive to disclose his type to borrow money on better terms of a higher bond price.

The timing of the game is as follows:

1) Nature chooses the true state $w \in \{H, L\}$ and G's type $s \in \{h, l\}$.

In $t = 0$, given $\theta \in (0, 1)$,

2) G observes s and updates his belief for the true state w. He then decides the amount of bonds to issue, $b \geq 0$.

3) C observes b. Then, he updates his belief for G's type and thus the true state and probability of default. Then, a bond price g is decided accordingly.

4) In the bond market, b units of bonds are traded at price g.

In $t = 1$,

5) The true state w and the corresponding z_w are realized. If $w = H$, G repays his debt b and if $w = L$, he defaults.

This model uses the Perfect Bayesian equilibrium concept. Let $\lambda(s|b)$ be C's posterior belief for G's type $s \in \{h, l\}$ given b. Then, the three elements, i) G's strategy to decide b, ii) the belief $\lambda(s|b)$, and iii) C's strategy to decide g, constitute a Perfect Bayesian equilibrium if G's expected utility is maximized given belief $\lambda(s|b)$. The belief $\lambda(s|b)$ should be consistent with Bayes' law whenever possible.

4 When G's type is known: complete information case

In this section, as a benchmark, we consider the case in which G's type $s \in \{h, l\}$ is public information. In the following, we denote by b_s^c each type's optimal amount of bonds under the complete information case. For each type, the unit bond price, g_s, is determined from

$$E\pi = \Pr(H|s)\,(b - g_s b) + \Pr(L|s)\,(-g_s b)$$

where $\Pr(w|s)$ is C's posterior belief for the true state w given s. The creditors' market is assumed to be competitive, so $E\pi = 0$. Then,

$$g_s = \frac{\Pr(H|s)}{\Pr(H|s) + \Pr(L|s)} = \Pr(H|s)$$

Given g_s, b_s^c is derived from

$$\underset{b_s}{Max}\ V_s(b_s) = \sqrt{g_s b_s} + \Pr(H|s)\sqrt{z - b_s}$$

and

$$b_s^c = \frac{z}{1 + \frac{\Pr(H|s)^2}{g_s}} = \frac{z}{1 + \Pr(H|s)}$$

Proposition 1
Suppose that G's type $s \in \{h, l\}$ is known.
1) Bond price: $g_h = \Pr(H|h)$ and $g_l = \Pr(H|l)$. So, $g_h > g_l$.
2) Amount of bonds: $b_h^c = \frac{z}{1+g_h}$ and $b_l^c = \frac{z}{1+g_l}$. So, $b_h^c < b_l^c$.
Here, $\Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ *and* $\Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$.

Proof of Proposition 1
In the appendix.

A higher bond price means better borrowing terms. It is intuitive that the bond price for type h is higher than that for type l because type h (l) is less (more) likely to default. The interesting result is that type h issues fewer bonds than type l despite being less likely to default. There are two forces that yield this result. There is no need for type h to issue a large amount of bonds because he receives a high bond price. In fact, although $b_h^c < b_l^c$, his consumption level at $t = 0$ is greater than that of type l, i.e., $c_0^h = g_h b_h^c > g_l b_l^c = c_0^l$. Moreover, issuing a large amount of bonds is not always beneficial to type h because his signal says it is likely that the true state is $w = H$, and in that case he has to repay his debt. Similarly, type l is willing to issue a large amount of bonds because he receives a low bond price. Moreover, he is likely to default and in that case does not need to repay debt. Hence, type l is less concerned about issuing a large amount of bonds than type h. However, there is an optimal amount of bonds even for type l because his signal is imperfect, so it can turn out that $w = H$, in which case he must repay debt.

In our model, the bond price and the optimal amount of bonds are described as functions of the precision, p, of the signal. Thus, we can analyze how the equilibrium and welfare change as the information quality changes.

Corollary 1
1) $\frac{\partial b_h^c}{\partial p} < 0$ and $\frac{\partial g_h}{\partial p} > 0$.
2) $\frac{\partial b_l^c}{\partial p} > 0$ and $\frac{\partial g_l}{\partial p} < 0$.

Proof of Corollary 1
In the appendix.

As the signal becomes more precise, type h issues a smaller amount of bonds while being offered a higher bond price. On the other hand, type l issues a greater amount of bonds while being offered a lower bond price. For type h, as p increases, it is less likely that he defaults. The creditor C, who knows this, offers a higher price, which induces type h to issue a smaller amount of bonds. This is beneficial to type h who is likely to repay debt. For type l, as p increases, it is more likely that he defaults. The creditor C, who knows this, offers a lower price, which induces type l to issue more bonds to compensate. In addition, the lower likelihood of repaying his debt makes him less concerned about issuing a larger amount of bonds. Therefore, as the precision of the signal increases, the gaps between the amount of borrowing and the bond price for each type increase.

In turn, information quality affects welfare in terms of ex-ante utility.

Corollary 2
$\frac{\partial V_h\left(b_h^c\right)}{\partial p} > 0$, $\frac{\partial V_l\left(b_l^c\right)}{\partial p} < 0$

Proof of Corollary 2
In the appendix.

As the signal becomes more precise, type h becomes better off and type l becomes worse off. Note the result that having more precise information is always beneficial to type h, which seems intuitive. However, this is true when G's type $s \in \{h, l\}$ is known, but not true if it is private information, which will be shown in the following sections.

The effects of the prior on the equilibrium and welfare are as follows.

Corollary 3

$\frac{\partial b_s^c}{\partial \theta} < 0$, $\frac{\partial g_s}{\partial \theta} > 0$, and $\frac{\partial V_s(b_s^c)}{\partial \theta} > 0$.

Proof of Corollary 3
In the appendix.

Greater θ, implying a more optimistic prior, indicates lower default risk. Hence, regardless of the signal, the bond price increases, the corresponding amount of bonds decreases, and G becomes better off.

5 When G's type s is private information: Signaling equilibrium

Now, we study the case in which G's signal $s \in \{h, l\}$ is private information. Our main goal is to check the existence of a separating equilibrium in which the amount of bonds can be a credible signaling device through which G's private signal s is disclosed. We also consider the existence of a pooling equilibrium in which the amount of bonds does not take a signaling role. In the following, we denote by b_s^* the optimal amount of bonds when the type is private information.

Before we proceed, consider the indifference curves (IC_s) for type s, which represents G's expected utility $V(b, g) = \sqrt{gb} + \Pr(H|s)\sqrt{z - b}$, defined in the space of (b, g).

Lemma 1
1) G's expected utility $V(b, g)$ represents strictly convex preference.
2) Two indifference curves of types $s \in \{h, l\}$ intersect with each other only once. (Single crossing property)
3) IC_l is steeper than IC_h at (b, g) under which $MRS_s < 0$ for $s \in \{h, l\}$.

Proof of Lemma 1
In the appendix.

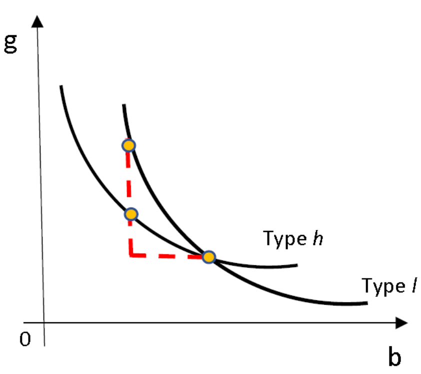


Figure 1: Indifference curve of each type

Note that $\frac{\partial V(g,b)}{\partial g} > 0$, and $\frac{\partial V(g,b)}{\partial b} \gtreqless 0 \Longleftrightarrow b \lesseqgtr \frac{gz}{\Pr(H|s)^2 + g} \equiv \bar{b}_s$. So, it should be that $b_s \leq \bar{b}_s$, under which $MRS_s \leq 0$.[11] In that case, IC_l is steeper than IC_h. This implies that, for the same

[11] Type s has no reason to choose b at which $\frac{\partial V(g,b)}{\partial b} < 0$. Later, it is shown that the equilibrium is always derived at b_s where $\left.\frac{\partial V(g,b)}{\partial b}\right|_{b_s} \geq 0$ and therefore $MRS_s \leq 0$. So, the case in which $MRS_s > 0$ does not matter in our analysis.

amount of decrease in bonds, type l requires more compensation in terms of higher bond prices than type h. That is, reducing bond issuance is more costly to type l than to type h. His signal $s = l$ says that he is likely to default and, if that occurs, he does not need to repay debt. Hence, he must be less concerned about issuing a large amount of bonds than type h. In turn, he needs more compensation in exchange for issuing fewer bonds.

5.1 Separating equilibrium

We consider a separating strategy $b_h^* \neq b_l^*$.

5.1.1 Creditor's response

In the following, $\lambda(s|b)$ denotes C's posterior belief for G's type s given b. If b_s^* is observed when $b_h^* \neq b_l^*$, C updates his belief accordingly, i.e., $\lambda(s|b_s^*) = 1$. If an off-the-equilibrium path $b \neq b_s^*$ is observed, any arbitrary belief for G's type can be assigned. Here, we assume that if $b \neq b_s^*$, C believes that G is type l, i.e., $\lambda(s = h|b \neq b_s^*) = 0$ and $\lambda(s = l|b \neq b_s^*) = 1$. This is the belief under which each type is least likely to deviate from a separating strategy. If type h deviates from b_h^* under this belief, he is believed to be type l and ends up with bad borrowing terms. In the case of type l, under this belief, his deviation must be only to b_h^* for a reason explained below. Considering this off-the-equilibrium path belief results in the largest set of separating equilibria.

> **Remark**: *Off-the-equilibrium path belief for a separating strategy*
> *If $b \neq b_s^*$, C believes that G is type l, i.e., $\lambda(s = h|b \neq b_s^*) = 0$ and $\lambda(s = l|b \neq b_s^*) = 1$.*

C's expected profit is

$$E\pi = \mu_H(b)(b - gb) + \mu_L(b)(-gb)$$

where $\mu_w(b)$ is C's posterior belief for the true state after observing b. Note that $\Pr(d) = \Pr(w = L)$. Given $b = b_h^*$ (b_l^*), C believes that $s = h$ ($s = l$). Hence, $\mu_H(b_h^*) = \Pr(H|h)$ and $\mu_H(b_l^*) = \Pr(H|l)$. If $b \neq b_s^*$, $\mu_H(b \neq b_s^*) = \Pr(H|l)$ due to the belief for the off-the-equilibrium path that $\lambda(l|b \neq b_h^*) = 1$. Then, the bond price contingent on b is determined as follows.

$$g(b_h^*) = \Pr(H|h) = \frac{p\theta}{p\theta + (1-p)(1-\theta)}$$

$$g(b_l^*) = g(b \neq b_s^*) = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta + p(1-\theta)}$$

5.1.2 Type $s = l$'s choice

If he issues b_l^* following a given separating strategy, C's belief is that $\lambda(l|b_l^*) = 1$ and so C offers a bond price $g(b_l^*) = \Pr(H|l)$. Then,

$$\underset{b_l}{Max}\ V_l(b_l) = \sqrt{g_l b_l} + \Pr(H|l)\sqrt{z - b_l}$$

and the optimal amount of bonds that type l issues is

$$b_l^* = \frac{z}{1+\Pr(H|l)}$$

The corresponding utility is

$$V_l\left(b_l^*\right) = \sqrt{z}\sqrt{\Pr(H|l)\left(\Pr(H|l)+1\right)}$$

If he deviates from a given separating strategy, i.e., $b_l \neq b_l^*$, it should be $b_l = b_h^*$. The reasoning is as follows. When he deviates from b_l^*, as long as $b \neq b_h^*$, $\lambda(l|b \neq b_h^*) = 1$ and faces a bond price $g = \Pr(H|l)$. As b_l^* is the optimal amount of bonds given $g = \Pr(H|l)$, he has no reason to deviate from b_l^* as long as he is still believed to be type l even after deviation. In other words, if he deviates from b_l^*, he does so in order to appear to be type h, so that $b = b_h^*$.

If $b_l = b_h^*$, as $\lambda(h|b_h^*) = 1$, the bond price $g\left(b_h^*\right) = \Pr(H|h)$ is given. In this case, the value function is

$$\begin{aligned} V_l\left(b_h^*\right) &= \sqrt{g_h b_h^*} + \Pr(H|l)\sqrt{z-b_h^*} \\ &= \sqrt{\Pr(H|h)b_h^*} + \Pr(H|l)\sqrt{z-b_h^*} \end{aligned}$$

Type l has no incentive to deviate from $b_l = b_l^*$ as long as $V_l\left(b_l^*\right) \geq V_l\left(b_h^*\right)$.

5.1.3 Type $s = h$'s choice

If he follows the given separating strategy $b_h = b_h^*$, $\lambda(h|b_h^*) = 1$ and $g\left(b_h^*\right) = \Pr(H|h)$. The corresponding value function is then

$$V_h\left(b_h^*\right) = \sqrt{\Pr(H|h)b_h^*} + \Pr(H|h)\sqrt{z-b_h^*}$$

If he deviates from b_h^*, $\lambda(l|b \neq b_h^*) = 1$ and C offers $g = \Pr(H|l)$. Hence, if he chooses $b_h \neq b_h^*$, his choice should be the optimal amount of bonds given $g = \Pr(H|l)$. If we denote this by b_h^{**},

$$b_h^{**} = \frac{z}{1+\frac{\Pr(H|s)^2}{g_s}} = \frac{z}{1+\frac{\Pr(H|h)^2}{\Pr(H|l)}}$$

Hence, the value function when he deviates from b_h^* is

$$\begin{aligned} V_h\left(b_h^{**}\right) &= \sqrt{\Pr(H|l)b_h^{**}} + \Pr(H|h)\sqrt{z-b_h^{**}} \\ &= \sqrt{\left(\Pr(H|l)+\Pr(H|h)^2\right)z} \end{aligned}$$

Type $s = h$ does not deviate from b_h^* as long as $V_h\left(b_h^*\right) \geq V_h\left(b_h^{**}\right)$.

5.1.4 Existence of a separating equilibrium

The existence of a separating equilibrium depends on whether there exists b_h^* for which both i) $V_l\left(b_l^*\right) \geq V_l\left(b_h^*\right)$ and ii) $V_h\left(b_h^*\right) \geq V_h\left(b_h^{**}\right)$ are satisfied.

Proposition 2 (Separating equilibrium)
1) Amount of bonds: $b_h^* \in [b_3, b_1]$ *and* $b_l^* = \frac{z}{1+\Pr(H|l)}$. *Here,* $b_h^* < b_l^*$.
2) Bond price: $g = \Pr(H|h)$ *if* $b = [b_3, b_1]$ *and* $g = \Pr(H|l)$ *if* $b = b_l^*$.
3) C's belief: $\lambda(h|b \in [b_3, b_1]) = 1$, $\Pr(l|b = b_l^*) = 1$, *and* $\lambda(l|b \notin [b_3, b_1]) = \lambda(l|b \neq b_l^*) = 1$.
Here,
$b_3 = \frac{z}{A^2+A}\left(\frac{2\sqrt{A^2+B}\left(\sqrt{A^2+B}-\sqrt{A(A-B)}\right)}{A+1} - B\right)$,
$b_1 = \frac{z}{A+B^2}\left(2B^2 + B - 2B\frac{\sqrt{A(A-B)}+B\sqrt{B(B+1)}}{A+B^2}\sqrt{B+B^2}\right)$
where $A = \Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ *and* $B = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$.

Proof of Proposition 2
In the appendix.

5.2 Pooling equilibrium

We next consider a pooling strategy $b^* = b_h^* = b_l^*$.

5.2.1 Creditor's response

If b^* is observed, no information is updated regarding G's type. Hence, C's posterior belief for type s is

$$\lambda(s|b^*) = \lambda(s) = \sum_{w\in\{H,L\}} \Pr\left(s|w\right)\Pr(w)$$

Then, $\lambda(h|b^*) = p\theta + (1-p)(1-\theta)$ and $\lambda(l|b^*) = (1-p)\theta + p(1-\theta)$. Accordingly, from $\mu_w(b) = \Pr(w|h)\lambda(h) + \Pr(w|l)\lambda(l)$ where $\mu_w(b)$ is C's posterior belief for the state w after observing b,

$$\mu_H(b^*) = \theta \text{ and } \mu_L(b^*) = 1-\theta$$

The inference of G's type is essential for having updated information regarding the true state. As no information is updated regarding G's type, the posterior beliefs for true state are identical to the prior beliefs.

If an off-the-equilibrium path $b \neq b^*$ is observed, any arbitrary C's belief for G's type can be assigned. Here, we assume that if $b \neq b^*$, C believes that G's type is $s = l$, i.e., $\lambda(h|b \neq b^*) = 0$ and $\lambda(l|b \neq b^*) = 1$. This is the belief under which each type is least likely to deviate from a pooling strategy because no type wants to appear to be type l and receive bad borrowing terms.

Remark: *Off-the-equilibrium path belief for a pooling strategy*
If an off-the-equilibrium path $b\,(\neq b^*)$ is observed, C believes that G's type is $s = l$, i.e., $\lambda(h|b \neq b^*) = 0$ and $\lambda(l|b \neq b^*) = 1$

If $b = b^*$ is observed, C's expected profit is

$$E\pi = \theta\left(b^* - gb^*\right) + (1-\theta)\left(-gb^*\right) = b^*\left(-g+\theta\right)$$

On the other hand, if $b \neq b^*$ is observed, $\mu_H\left(b \neq b^*\right) = \Pr\left(H|l\right) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$ and $\mu_L\left(b \neq b^*\right) = \Pr\left(L|l\right) = \frac{p(1-\theta)}{p(1-\theta)+(1-p)\theta}$ because $\lambda(l|b \neq b^*) = 1$. Then,

$$E\pi = \Pr\left(H|l\right)\left(b-gb\right) + \Pr\left(L|l\right)\left(-gb\right)$$

Then, the bond price contingent on b is determined as follows.

$$g\left(b^*\right) = \theta \quad \text{and} \quad g\left(b \neq b^*\right) = \Pr\left(H|l\right)$$

5.2.2 Type $s = l$'s choice

If type l follows a given pooling strategy b^*, given $g\left(b^*\right) = \theta$, type l's utility is

$$\begin{aligned} V_l\left(b^*\right) &= \sqrt{gb^*} + \Pr(H|l)\sqrt{z-b^*} \\ &= \sqrt{\theta b^*} + \Pr(H|l)\sqrt{z-b^*} \end{aligned}$$

If he deviates from b^*, the bond price offered by C is $g\left(b \neq b^*\right) = \Pr\left(H|l\right)$ and the corresponding optimal amount of bonds is $\dot{b}_l = \frac{z}{1+\Pr(H|l)}$.[12] Then, his utility is

$$\begin{aligned} V_l\left(b_l \neq b^*\right) &= \sqrt{g\dot{b}_l} + \Pr(H|l)\sqrt{z-\dot{b}_l} \\ &= \sqrt{\Pr\left(H|l\right)\left(1+\Pr\left(H|l\right)\right)z} \end{aligned}$$

Type l does not deviate from b^* as long as $V_l\left(b^*\right) \geq V_l\left(b_l \neq b^*\right)$.

5.2.3 Type $s = h$'s choice

If type h follows a pooling strategy b^*, given $g\left(b^*\right) = \theta$, his utility is

$$\begin{aligned} V_h\left(b^*\right) &= \sqrt{gb^*} + \Pr(H|h)\sqrt{z-b^*} \\ &= \sqrt{\theta b^*} + \Pr(H|h)\sqrt{z-b^*} \end{aligned}$$

If he deviates from b^*, given $g\left(b \neq b^*\right) = \Pr\left(H|l\right)$, the corresponding optimal amount of bonds type h should issue is

$$\dot{b}_h = \frac{z}{1+\frac{\Pr(H|h)^2}{\Pr(H|l)}}$$

[12] Refer to Proposition 1, which proposes the optimal amount of bonds given bond price.

Then, his utility is

$$\begin{aligned} V_h\left(b_h \neq b^*\right) &= \sqrt{g\dot{b}_h} + \Pr(H|h)\sqrt{z - \dot{b}_h} \\ &= \sqrt{\left(\Pr(H|h)^2 + \Pr\left(H|l\right)\right) z} \end{aligned}$$

He does not deviate from b^* as long as $V_h\left(b^*\right) \geq V_h\left(b_h \neq b^*\right)$.

5.2.4 Existence of a pooling equilibrium

The existence of a pooling equilibrium depends on whether there exists b^* under which both i) $V_l\left(b^*\right) \geq V_l\left(b_l \neq b^*\right)$ and ii) $V_h\left(b^*\right) \geq V_h\left(b_h \neq b^*\right)$ hold.

Proposition 3 (Pooling equilibrium)
1) Amount of bonds: If $B-\theta+B^2 < 0$, $b^* = [b_5, b_8]$ *and if* $B-\theta+B^2 > 0$, $b \in [b_5, \min\{b_6, b_8\}]$.[13]
2) Bond price: $g = \theta$.
3) C's belief: $\lambda(h|b^*) = p\theta + (1-p)\left(1-\theta\right)$, $\Pr(l|b^*) = \left(1-p\right)\theta + p\left(1-\theta\right)$, and $\Pr(l|b \neq b^*) = 1$.
Here,
$b_5 = \frac{z}{\theta+B^2}\left(2B^2 + B - 2B\frac{\sqrt{B^2+B}}{\theta+B^2}\left(B\sqrt{B\left(B+1\right)} + \sqrt{\theta\left(\theta-B\right)}\right)\right)$,
$b_6 = \frac{z}{\theta+B^2}\left(2B^2 + B - 2B\frac{\sqrt{B^2+B}}{\theta+B^2}\left(B\sqrt{B\left(B+1\right)} - \sqrt{\theta\left(\theta-B\right)}\right)\right)$,
$b_8 = \frac{z}{\theta+A^2}\left(2A^2 + B + 2A\frac{\sqrt{A^2+B}}{\theta+A^2}\sqrt{\theta\left(\theta-B\right)} - A\sqrt{\left(B+A^2\right)}\right)$
where $A = \Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ *and* $B = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$.

Proof of Proposition 3
In the appendix.

5.3 Equilibrium refinement

In this section, for the multiple separating and pooling equilibria derived in previous sections, we use the intuitive criterion by Cho and Kreps (1987) to refine the equilibrium by ruling out those based on irrational beliefs for action off the equilibrium path.

First, we consider the separating equilibria where, for type h, the bond price is $g_h = \Pr(H|h)$ and the amount of bonds is $b_h^* \in [b_3, b_1]$. If we recall the complete information case, type h issues $b_h = \frac{z}{1+\Pr(H|h)}$ given $g_h = \Pr(H|h)$. That is, given $g = \Pr(H|h)$, $b = \frac{z}{1+\Pr(H|h)}$ maximizes type h's ex-ante utility function $V_h\left(b_h\right)$. Here, $V_h\left(b_h\right)$ is a concave function in b_h and $b_h = [b_3, b_1] < \frac{z}{1+\Pr(H|h)}$. Hence, for $b_h \in [b_3, b_1]$, a smaller amount of bonds hurts type h and $V_h\left(b\right)$ increases as b_h is greater. Therefore, type h has no reason to issue $b_h \in [b_3, b_1)$. As long as he is identified as type h, he issues b_h that maximizes his utility given $g = \Pr(H|h)$. In this case, that is b_1, the least-costly amount of bonds he must issue to differentiate himself from type l. Hence, for $b_h \in [b_3, b_1)$, type h has an incentive to deviate to b_1.

[13] Here, given θ, $\exists p^*$ such that if $p \in (\frac{1}{2}, p^*)$, $b_6 > b_8$ and if $p \in (p^*, 1)$, $b_6 < b_8$. As $\theta \nearrow 1$, $p^* \nearrow 1$.

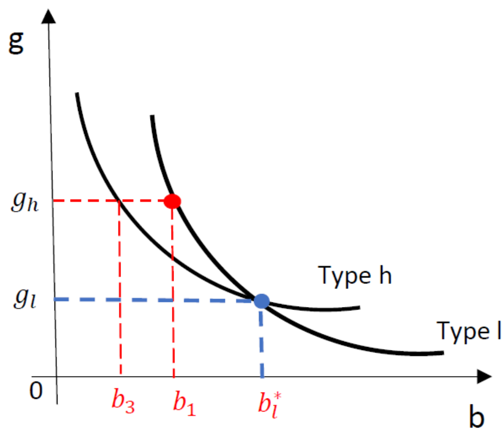


Figure 2 : Intuitive criterion and Separating equilibrium

Our separating equilibrium where $b_h^* \in [b_3, b_1]$ is based on the off-the-equilibrium path belief that $\Pr(s = l|b \neq b_h^*) = 1$. However, the reasoning described above says that, for $b_h^* \in [b_3, b_1)$, if C observes $b_1 \notin b_h^*$, the correct belief should be that it is type h who issued b_1, i.e., $\Pr(s = h|b_1 \notin [b_3, b_1)) = 1$, because type l does not become better off by choosing b_1 than b_l^*, and so has no incentive to deviate from b_l^*. In this sense, given $b_h^* \in [b_3, b_1)$, the belief that $\Pr(s = l|b \neq b_h^*) = 1$ is not rational. Then, the separating equilibrium where $b_h^* \in [b_3, b_1)$, based on this irrational belief, does not satisfy the intuitive criterion. A unique separating equilibrium that satisfies the intuitive criterion is $b_h^* = b_1$.

Corollary 4

The separating equilibrium, which satisfies the intuitive criterion, is the one where $b_h^* = b_1$ and $b_l^* = \frac{z}{1+\Pr(H|l)}$. Here, $b_h^* < b_l^*$.

Next, we consider pooling equilibria. To check the stability of pooling equilibria, consider a scenario in which C charges a bond price $g = \Pr(H|h)$ if $b \neq b^*$ is observed. Given that scenario, we attempt to check whether there exists $b \neq b^*$ for which the following two conditions are satisfied. Here, $V_s\left(b \neq b^*\right)$ is the utility of type s when he deviates from a pooling equilibrium strategy by selecting $b\left(\neq b^*\right)$. $V_s\left(b^*\right)$ denotes the utility of type s under the pooling equilibrium.

$$V_h\left(b \neq b^*\right) = \sqrt{\Pr(H|h)b} + \Pr(H|h)\sqrt{z-b} \geq \sqrt{\theta b^*} + \Pr(H|h)\sqrt{z-b^*} = V_h\left(b^*\right)$$

$$V_l\left(b^*\right) = \sqrt{\theta b^*} + \Pr(H|l)\sqrt{z-b^*} \geq \sqrt{\Pr(H|h)b} + \Pr(H|l)\sqrt{z-b} = V_l\left(b \neq b^*\right)$$

Why do we consider this scenario? If there exists $b\left(\neq b^*\right)$ that satisfies both conditions, type h deviates from b^*, while type l has no incentive to do so. We consider these two conditions because our pooling equilibrium is based on the off-the-equilibrium path belief that $s = l$ if $b\left(\neq b^*\right)$ is observed. This scenario is constructed to check whether there is a case in which type h selects $b_h\left(\neq b^*\right)$, while type l still selects b^*. If such a case exists, the off-the-equilibrium path belief that $\lambda(s = l|b \neq b^*) = 1$ is not rational because the correct belief should be that it is type h who issued $b\left(\neq b^*\right)$. That is, given that there exists $b_h\left(\neq b^*\right)$ under which the above conditions are satisfied, C is able to identify type h and charges $g = \Pr(H|h)$. Then, type h, knowing this, deviates from b^*, and this pooling equilibrium collapses.

Corollary 5

The pooling equilibrium does not satisfy the intuitive criterion.

The diagram of the indifference curves (IC_s) for type s, defined in the space of (b, g), is very effective in proving this.

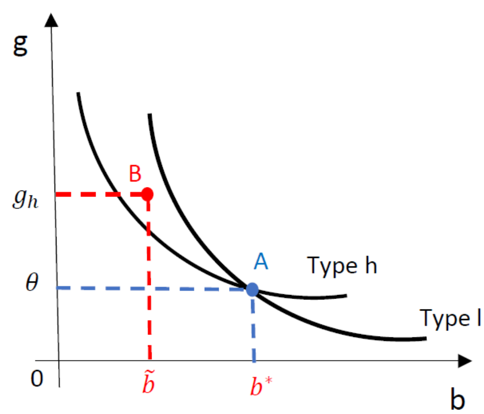


Figure 3 : Intuitive criterion and Pooling equilibrium

Given a pooling equilibrium point A, $(b, g) = (b^*, \theta)$, consider the point B where $b = \tilde{b}\,(\neq b^*)$. Type l never selects point B because that deviation results in lower utility than point A. For type h, there is room for him to choose $\tilde{b}$ depending on the bond price g. Then, if G issues $\tilde{b}$, C should correctly update his belief that the one who issued $\tilde{b}$ must be type h, and charge a price $g = \Pr(H|h)$. At the point B where $(b, g) = \left(\tilde{b}, \Pr(H|h)\right)$, type h attains a greater utility than in the pooling equilibrium. Hence, only type h deviates from the pooling equilibrium. As the off-the-equilibrium path belief that $\lambda(s = l|b \neq b^*) = 1$ is not rational, the pooling equilibrium does not satisfy the intuitive criterion.

Finally, the unique equilibrium, which satisfies the intuitive criterion, can be characterized as follows.

Proposition 4

The following separating equilibrium is a unique signaling equilibrium that satisfies the intuitive criterion.

1) Amount of bonds: $b_h^* = b_1$ *and* $b_l^* = \frac{z}{1+\Pr(H|l)}$. *Here,* $b_h^* < b_l^*$.

2) Bond price: $g = \begin{cases} \Pr(H|h) & \text{if } b = b_1 \\ \Pr(H|l) & \text{if } b = \frac{z}{1+\Pr(H|l)} \end{cases}$. *Hence,* $g_h > g_l$.

3) C's belief: $\lambda(h|b = b_1) = 1$, $\Pr(l|b = \frac{z}{1+\Pr(H|l)}) = 1$, *and* $\Pr(l|b \neq b_1) = 1$ *(Off-the-equilibrium path belief).*

Here, $b_1 = \frac{z}{A+B^2}\left(2B^2 + B - 2B\frac{\sqrt{A(A-B)}+B\sqrt{B(B+1)}}{A+B^2}\sqrt{B+B^2}\right)$

where $A = \Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ *and* $B = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$.

In this equilibrium, each type is disclosed through the amount of bonds issued. This suggests that the amount of bonds can work as a credible signaling channel between G and C. Note that $b_h^*(p, \theta) < b_l^*(p, \theta)$ for $p \in \left(\frac{1}{2}, 1\right)$ and $\theta \in (0, 1)$, indicating that, given p and θ, type h can differentiate himself from type l by issuing a smaller amount of bonds.

This can occur because issuing a smaller amount of bonds is more costly for type l. Type l is less concerned than type h about issuing a large amount of bonds because he is less likely to repay his debt. Type h, who is less likely to default, is concerned about repaying his debt. Type h, recognizing this, decreases the amount of bonds enough that type l cannot mimic him, resulting in a separating equilibrium.

The creditor C can identify G's type $s \in \{h, l\}$ by observing the amount of bonds. After updating his belief regarding the probability of default, C offers a bond price accordingly. The bond price is $g = \Pr(H|h)$ for $b = b_1$ and $g = \Pr(H|l)$ for $b = b_l^*$. Since his type is disclosed, each type is charged a bond price identical to that in the complete information case.

5.4 Amount of bonds

We first compare the unique signaling equilibrium with the complete information case. Refer to propositions 1 and 4. If we compare the amount of bonds each type issues in each equilibrium,

$$b_h^* < b_h^c \text{ and } b_l^* = b_l^c$$

The key result is that the amount of bonds type h issues in the signaling equilibrium is smaller than in the complete information case. Considering that the bond price for type h is identical in both cases, this implies that type h pays a signaling cost of issuing fewer bonds than the optimal amount given the bond price. That cost is due to a need to decrease the amount of bonds enough that type l cannot mimic him. On the other hand, type l issues the same amount of bonds regardless of whether his type is known or not. In a separating equilibrium, type l cannot decrease the amount of bonds as much as type h does. Then, type l, who knows that his type is revealed regardless of his choice of amount of bonds, has no reason to deviate from his optimal amount of bonds given $g = \Pr(H|l)$. Hence, $b_l^* = b_l^c$.

We then analyze the effect of the prior belief, θ, and the precision, p, of the signal on the equilibrium. We only consider type h because type l's amount of bonds is identical regardless of whether his type is known. The prior θ influences the impact of private signal s indicating a good state. One way to measure it is $v(h) \equiv \Pr(H|h) - \Pr(H)$, from which we can evaluate how much the signal $s = h$ contributes to the belief that the true state is $w = H$.

$$v(h) = \theta(\theta - 1)\frac{2p - 1}{p + \theta - 2p\theta - 1}$$

and $\frac{\partial v(h)}{\partial \theta} = \frac{(2p-1)}{(-p-\theta+2p\theta+1)^2}\left(-\theta^2(2p-1) + \theta(2p-2) + 1 - p\right)$. Let $f(\theta) = -\theta^2(2p-1) + \theta(2p-2) + 1 - p$. Then, $f(\theta)$ is maximized at $\theta = \frac{1}{2p-1}(p-1) < 0$. So, for $\theta \in (0, 1)$, $f(\theta)$ is decreasing. Therefore,

$$\frac{\partial v(h)}{\partial \theta} < 0.$$

indicating that the value of $s = h$ increases (decreases) as the prior belief is more pessimistic (optimistic). In other words, a signal indicating the opposite state to the prior has greater impact. As the prior is more pessimistic, type h, who has a signal of greater impact, decreases the amount of bonds more due to his stronger signaling motive. Therefore, given p, as θ is smaller, the distortion in the amount of bonds, i.e., $b_h^c - b_h^*$, is greater. Figure 4 demonstrates the case in which $z = 1$.

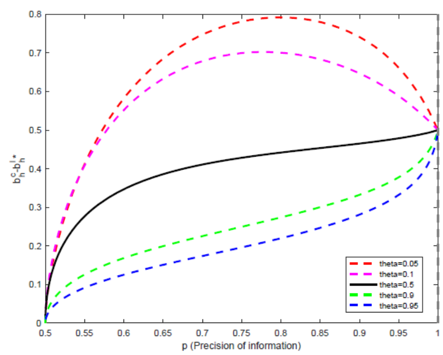


Figure 4 (i) $b_h^c - b_h^*$

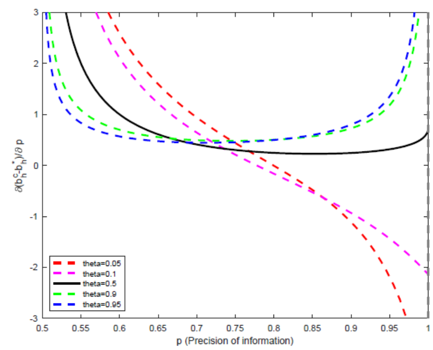


Figure 4 (ii) $\frac{\partial\left(b_h^c - b_h^*\right)}{\partial p}$

We then analyze the effect of the precision, p, of the signal on the equilibrium given the prior belief θ. The effect of the change in p on $b_h^c - b_h^*$ varies depending on the prior. If the prior is very pessimistic (θ close to 0), $b_h^c - b_h^*$ is non-monotone with respect to p with concavity. Accordingly, the marginal deviation with respect to p, i.e., $\frac{\partial\left(b_h^c - b_h^*\right)}{\partial p}$, is monotone decreasing. If not, $b_h^c - b_h^*$ is monotone increasing. It should be noted that, in that case, the marginal deviation is non-monotone with convexity with respect to p. The observation that the marginal deviation increases after a certain level of p suggests an important clue regarding the adverse effect of p on welfare, as is shown in the next section.

For b_h^* and b_h^c,

$$\frac{\partial b_h^*}{\partial p} < 0, \ \frac{\partial b_h^c}{\partial p} < 0, \ \lim_{p \to 1/2} b_h^* = \lim_{p \to 1/2} b_h^c = \frac{z}{1+\theta}$$

$$\lim_{p \to 1} b_h^* = 0, \ \lim_{p \to 1} b_h^c = \frac{z}{2}$$

As his signal becomes less informative, type h is less confident in its accuracy, so he has a smaller incentive to signal his type. In the extreme case where his signal is not informative at all, there is no need for him to engage in costly signaling. So, the amount of bonds type h issues when his type is unknown is identical to that of the complete information case, i.e., $\lim_{p \to 1/2} b_h^* = \lim_{p \to 1/2} b_h^c$. On the other hand, as his signal becomes more informative, type h has a stronger incentive to reveal his type, and so issues a smaller amount of bonds. Note that $\lim_{p \to 1} b_h^* < \lim_{p \to 1} b_h^c$, which is due to a signaling cost.

5.5 Welfare

Since information quality affects the amount of bonds, it must affect welfare as well. In the following, $V_h\left(b_h^*\right)$ denotes type h's ex-ante utility in the signaling equilibrium and $V_h\left(b_h^c\right)$ denotes that in the complete information case. Here, $V_h\left(b_h^c\right) = \sqrt{(\Pr(H|h)+1)\Pr(H|h)}$ and $V_h\left(b_h^*\right) = \sqrt{\Pr(H|h)b_h^*} + \Pr(H|h)\sqrt{1-b_h^*}$.

We can observe that

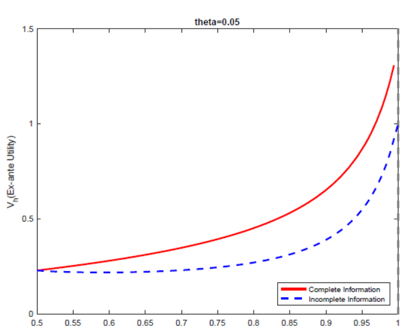
$$V_h\left(b_h^*\right) < V_h\left(b_h^c\right) \text{ for } p \in \left(\frac{1}{2}, 1\right) \text{ and } \theta \in (0,1)$$

Compared to the perfect information case, type h is worse off due to costly signaling. The loss in his ex-ante utility compared to the complete information case is $\Delta V_h(p,\theta) = V_h\left(b_h^c\right) - V_h\left(b_h^*\right)$, and it can be checked that

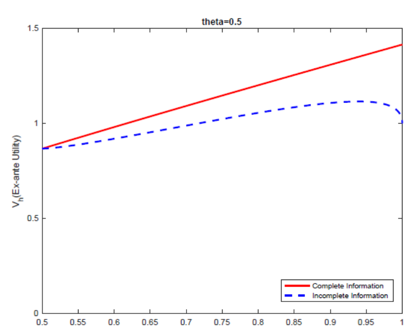
$$\frac{\partial\left(\Delta V_h(p,\theta)\right)}{\partial p} > 0 \text{ for all } \theta$$

As p increases, $\Delta V_h(p,\theta)$ increases due to a stronger signaling incentive. That is, the more precise signal results in a greater signaling cost. Recall that, unless the prior belief is very pessimistic, type h's signaling motive is not monotone with respect to p. The marginal deviation increases for a relatively high p (Figure 4 (ii)), indicating that substantial deviation from the complete information case occurs in the case of a highly precise signal. This suggests that a highly precise signal may hurt type h in terms of ex-ante utility.

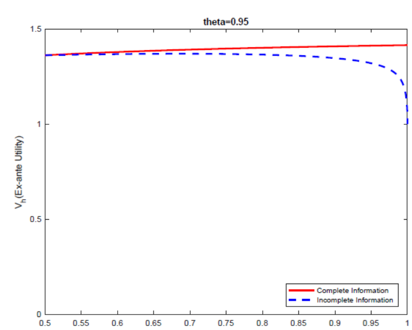
Consider $V_h\left(b_h^*\right)$. If the prior belief is not very pessimistic, it can be checked that $\left.\frac{\partial V_h\left(b_h^*\right)}{\partial p}\right|_{p=\frac{1}{2}+\varepsilon} > 0$ and $\left.\frac{\partial V_h\left(b_h^*\right)}{\partial p}\right|_{p=1-\varepsilon} < 0$, implying that there exists a critical value $p^* \in \left(\frac{1}{2}, 1\right)$ such that if $p \in \left(\frac{1}{2}, p^*\right)$, $\frac{\partial V_h\left(b_h^*\right)}{\partial p} > 0$ and if $p \in (p^*, 1)$, $\frac{\partial V_h\left(b_h^*\right)}{\partial p} < 0$. That is, the ex-ante utility of type h is non-monotone with respect to p. This is a critical difference from the complete information case, where a more precise signal is always beneficial to type h. (Corollary 2)



(i) $\theta = 0.05$



(ii) $\theta = 0.5$



(iii) $\theta = 0.95$

Figure 5 : $V_h\left(b_h^*\right)$ and $V_h\left(b_h^c\right)$

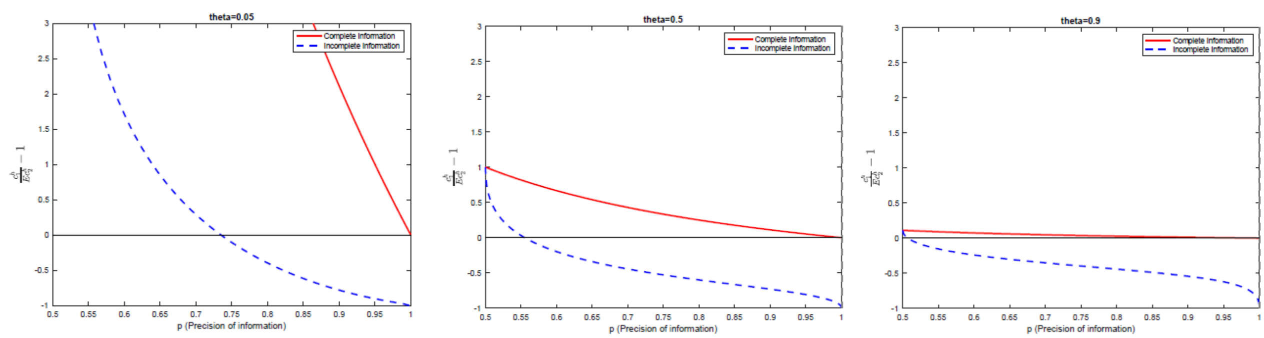
Figures 5 (i), (ii), and (iii) demonstrate $V_h(b_h^*)$ and $V_h(b_h^c)$ of the case where $z = 1$. Unless the prior is very pessimistic (Figure 5 (ii) and (iii)), there exists a critical value of p such that if the signal is highly precise, $V_h(b_h^*)$ decreases as p increases. In this case, type h's strong incentive to differentiate himself from type l, due to his highly precise signal, drives him to reduce the amount of bonds excessively. Even though it is his optimal choice, he becomes worse off than the case where his signal is less precise. That is, signal precision above a certain level hurts rather than helps, a phenomenon we call the paradox of highly precise information.

This adverse effect of precise signal becomes more severe as the prior becomes more optimistic, i.e., $\theta \longrightarrow 1$. While the welfare enhancing effect of costly signaling is very limited, there is a marked decrease in $V_h(b_h^*)$ for a highly precise signal. Under a more optimistic prior, since the signal indicates the same state as the prior, the impact of his signal becomes less significant. Our result implies that participating in costly signaling with a signal of limited impact can be harmful. This adverse effect becomes more severe as the prior is more optimistic and so the impact of signal $s = h$ becomes less significant.

On the other hand, if the prior is very pessimistic (Figure 5 (i)), a more precise signal of $s = h$ always increases $V_h(b_h^*)$. Given that the signal indicates the opposite state to what the prior says, the impact of the signal is significant. Then, although more aggressive signaling occurs with a more precise signal, it does not hurt. Participating in costly signaling in that case is always beneficial.

5.6 Consumption smoothing

In the model, the consumption in both periods is determined by the amount of bonds issued in $t = 0$. Hence, we can also analyze the effect of the prior and the precision of the signal on consumption smoothing.



When $\theta = 0.05$ When $\theta = 0.5$ When $\theta = 0.9$

Figure 6 : $\frac{c_0^h(p)}{Ec_1^h(p)} - 1$

Consider the complete information case. For all $p \in \left(\frac{1}{2}, 1\right)$ and $\theta \in (0, 1)$, i) $c_0^h(p) > Ec_1^h(p)$ and ii) $\frac{c_0^h(p)}{Ec_1^h(p)} \longrightarrow 1$ as $p \longrightarrow 1$ where c_0^h is the consumption in $t = 0$ and Ec_1^h is the expected consumption in $t = 1$. A more precise signal yields more balanced consumption between the two periods. This always improves welfare as is indicated in Figure 5. On the other hand, if G's type is unknown, this channel of welfare improvement through consumption smoothing is distorted. As the signal is more precise, the proportion of c_0^h decreases substantially and consumption becomes more skewed toward the next period. Balanced consumption between the two periods is realized at p not substantially high. If the prior is very pessimistic, a certain level of p is necessary for backloading of consumption to occur. As the prior becomes more optimistic, the value of p at which consumption between the two periods is balanced decreases. Under a more optimistic prior, type h is more confident that the true state is $w = H$, so backloading of consumption starts to happen with a less precise signal.

5.7 Prior and bond price

In the signaling equilibrium, C can identify each type by observing the amount of bonds. We denote by $\Delta g_s^* = g_h^* - g_l^*$ the gap in bond price for each type. From Proposition 4,

$$\Delta g_s^* = \theta(\theta - 1)\frac{2p-1}{(-p-\theta+2p\theta)(-p-\theta+2p\theta+1)}$$

Corollary 6
1) Δg_s^* is concave and symmetric across $\theta = 1/2$.
2) $\Delta g_s^*(\theta = 0) = \Delta g_s^*(\theta = 1) = 0$.
3) For all θ and p, $\frac{\partial(\Delta g_s^*)}{\partial p} > 0$.

Proof of Corollary 6
In the appendix.

The corresponding diagrams are as follows.

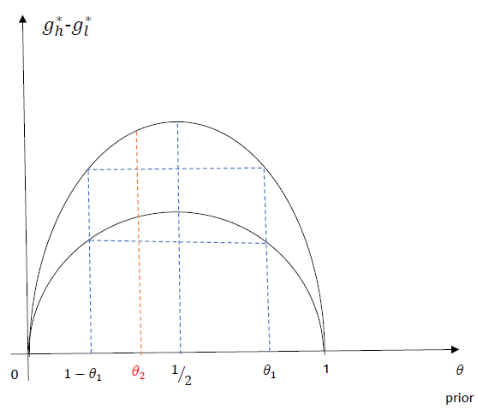


Figure 7 : $\Delta g_s^* = g_h^* - g_l^*$

We denote by $\theta_1 (> \frac{1}{2})$ the initial prior and by θ_2 the new prior. As $\theta_1 > \frac{1}{2}$, we consider the case in which the initial prior is relatively optimistic. First, given p and θ_1, for $\theta_2 \in (1-\theta_1, \theta_1)$,

$\Delta g_s^*(\theta_2) > \Delta g_s^*(\theta_1)$. Second, given p, as θ_1 is greater, $\Delta g_s^*(\theta_1)$ decreases and the parameter set of θ_2 under which $\Delta g_s^*(\theta_2) > \Delta g_s^*(\theta_1)$ increases. Lastly, given θ, $\Delta g_s^*(\theta)$ is greater as p is greater.

The implications are as follows. Type h (l) represents the sovereign identified as the one of good (bad) prospect. When the initial prior is more optimistic, the gap in bond price between the two types, Δg_s^*, is smaller. If the optimism recedes and pessimism kicks in, Δg_s^* increases. Unless the pessimism is extreme, Δg_s^* under the less optimistic prior is greater than that under the more optimistic prior. As the initial prior is more optimistic, greater Δg_s^* is more likely to be observed.

Note that, as it is a separating equilibrium, the market differentiates the types and recognizes the underlying differences between types. This result suggests that Δg_s^* can change even without a change in the market's perception of the sovereign's prospect or other underlying fundamentals. Here, what plays a role is the prior belief that the market shares. This result is not only from the incompleteness of information, but largely from the imperfectness of information. Even under the complete information case, as long as the signal is noisy, the same result is derived because the bond prices are identical. Our model suggests that the same property can be observed even in the incomplete information case by characterizing the separating equilibrium.

6 Discussion

6.1 Strategic default

In the main setting, the default cost is $\lambda = 1$, the most costly case where strategic default does not occur. In this section, we discuss the condition under which our signaling equilibrium is maintained even when $\lambda < 1$. Suppose that the default cost is $\lambda \in (0, 1)$, which is known to G and C. In $t = 1$, if the realized state is $w = H$, $b \leq z_H = z$ in our setting. Then, G can consider strategic default although he is able to pay. If G pays his debt, $c_1 = z - b$ and if he defaults strategically, $c_1 = (1 - \lambda) z$. From $z - b \geq (1 - \lambda) z$, G pays his debt if $b \leq \lambda z$ and defaults strategically if $b > \lambda z$. If the realized true state is $w = L$, G cannot but default due to insolvency, i.e., $z_L = 0 \leq b$.

In $t = 0$, G knows that, if $w = H$, his utility will be $u(gb) + u((1 - \lambda) z)$ for $b > \lambda z$, and $u(gb) + u(z - b)$ for $b \leq \lambda z$. If $w = L$, it is $u(gb)$. Note that G knows the values of λ and z. So, his optimization problem in $t = 0$ is

$$\begin{aligned} \underset{b}{Max}\ V(b) &= \sqrt{gb} + \Pr(H|s)\sqrt{(1-\lambda) z} \\ s.t.\ b &\in (\lambda z, z] \end{aligned} \tag{1}$$

and

$$\begin{aligned} \underset{b}{Max}\ V(b) &= \sqrt{gb} + \Pr(H|s)\sqrt{z - b} \\ s.t.\ b &\in (0, \lambda z] \end{aligned} \tag{2}$$

We first find the optimal amount of b and the corresponding indirect utility for each case. We then compare those to find the optimal b in the range of $(0, z]$.

Here, what matters is the bond price, g. From $E\pi = (1 - \Pr(d))(-gb + b) + \Pr(d)(-gb) = 0$,

$g = 1 - \Pr(d)$. In $t = 0$, given b, g reflects the probability of default. C knows that, if $w = L$ in $t = 1$, G must default. C knows the values of λ and z. If $b > \lambda z$, C knows that G will default strategically in $t = 1$ even when $w = H$ and therefore always default. As $\Pr(d) = 1$, $g = 0$, implying that C is not willing to buy bonds at all. On the other hand, if $b \in (0, \lambda z]$, C knows that G will repay b when $w = H$, and therefore G defaults only if $w = L$, i.e., $\Pr(d) = \Pr(w = L)$. Hence, under the default cost $\lambda \in (0, 1)$, G's problem leads to (2). The case in which $b \in (\lambda z, z]$ is excluded because in that case bonds are not traded.

In (2), the objective function is identical to that of our current model, but the constraint is different. In our model, i) $b_h^c < b_l^c = \frac{z}{1+\Pr(H|l)}$ and ii) in the signaling equilibrium, $b_h^* < b_h^c$ and $b_l^* = b_l^c$. Therefore, as long as $b_l^* = b_l^c \leq \lambda z$, the current results derived under $\lambda = 1$ are not affected by the constraint in (2) that $b \in (0, \lambda z]$.

From $b_l^* = b_l^c = \frac{z}{1+\Pr(H|l)} \leq \lambda z$, as long as $\lambda \in \left[\frac{1}{1+\Pr(H|l)}, 1\right)$, the equilibrium is identical to that under $\lambda = 1$. If we let $\lambda^* = \frac{1}{1+\Pr(H|l)}$,

$$\frac{\partial \lambda^*}{\partial p} > 0 \text{ and } \frac{\partial \lambda^*}{\partial \theta} < 0$$

As p decreases or θ increases, the parameter set of λ for which $\lambda \in [\lambda^*, 1]$ increases. Therefore, as the signal is less precise or the prior is more optimistic, even for a relatively low default cost, the equilibrium identical to that of the most costly default case is more likely to occur.

Consider the case in which our equilibrium under $\lambda = 1$ is constrained by the constrain that $b \in (0, \lambda z]$, i.e., $\lambda z < b_l^c$. Then, $b_l^c = b_l^* = \lambda z$. In the case of type h, i) there is no change if $b_h^c \leq \lambda z$, ii) $(b_h^*, b_h^c) = (b_h^*, \lambda z)$ if $b_h^* \leq \lambda z < b_h^c$, and iii) $(b_h^*, b_h^c) = (\lambda z, \lambda z)$ if $\lambda z < b_h^*$.

6.2 Implications

In this subsection, we introduce real examples that highlight two key implications of our model. However, we are not claiming that these patterns can be explained only by signaling motives. We propose one possible mechanism that can be behind these dynamics.

6.2.1 The role of prior information

The first implication is about the effect of the prior belief on the differential in sovereign bond prices. We propose that the differential in the bond prices of the two types, Δg_s^*, can change due to a change in the prior belief. As the prior is more optimistic, Δg_s^*, is smaller. When the prior belief becomes more pessimistic, Δg_s^* increases. This is consistent with what was observed in the recent European debt crisis.

Before the crisis, there was optimism that the establishment of the EU would bring economic prosperity and stable financial markets to member countries. After the 2008-2009 global financial crisis, however, a pessimistic view on European economies emerged due to the Greek debt crisis and the contagion effect on some southern European countries.

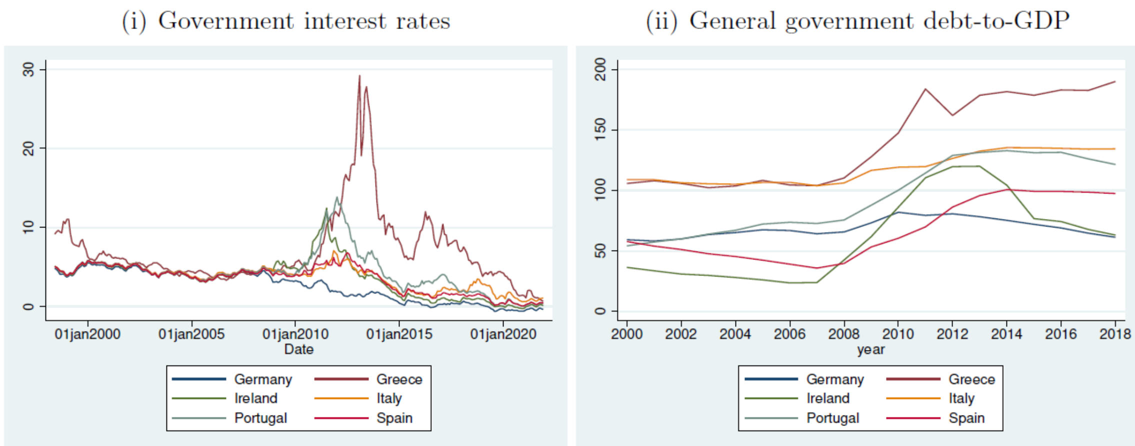


Figure 8 : European debt crisis

Before the economic crisis, the difference in the long-term interest rates of sovereign debt between periphery EU countries and Germany, of which the bond is regarded as risk-free, was very small even though there were large differences in the sovereigns' borrowing behaviors. The decoupling of long-term interest rates occurred when pessimism started to prevail with the onset of crisis. Our model provides a rationale for changes in bond price differentials even without a change in the market's perception of the sovereign's prospect or underlying fundamentals. Given large differences in the sovereigns' debt issuance before the crisis, the market may have recognized the differences between sovereigns. We propose that narrow gap of bond prices can occur even in such a case if optimism strongly dominates.

In particular, as the prior becomes more optimistic, the probability of default decreases and therefore the bond price for each type increases. Our result that Δg_s^* decreases in that case implies that the bond price of type l increases more rapidly than that of type h. Likewise, as the prior becomes more pessimistic, the bond price for each type decreases. The result that Δg_s^* increases implies that the bond price of type l decreases more rapidly than that of type h. These imply that the bond price of type l is more sensitive than that of type h to a change in the prior. This is consistent with the substantial decoupling of interest rates of some vulnerable countries hit hard by the economic crisis.

6.2.2 The role of precision of signal

Our model's another proposition is that the more precise signal of a good future economy a country has, the more it tends to implement debt reduction, that is, fiscal tightening, accompanied by a lower interest rate when everything else is constant. This is consistent with what was observed in Chile's case. Chile introduced a structural budget institution in 2000 and codified it legally in 2006, aiming to stabilize the macroeconomic environment and the structural budget deficit. One innovation of structural budget reform was to give responsibility for forecasting to independent expert commissions, insulated from political pressures and politicians' wishful thinking (Frankel et al. (2013)). As a result, Chile's GDP growth rate forecasts have not been subject to over-optimism, unlike other emerging economies.

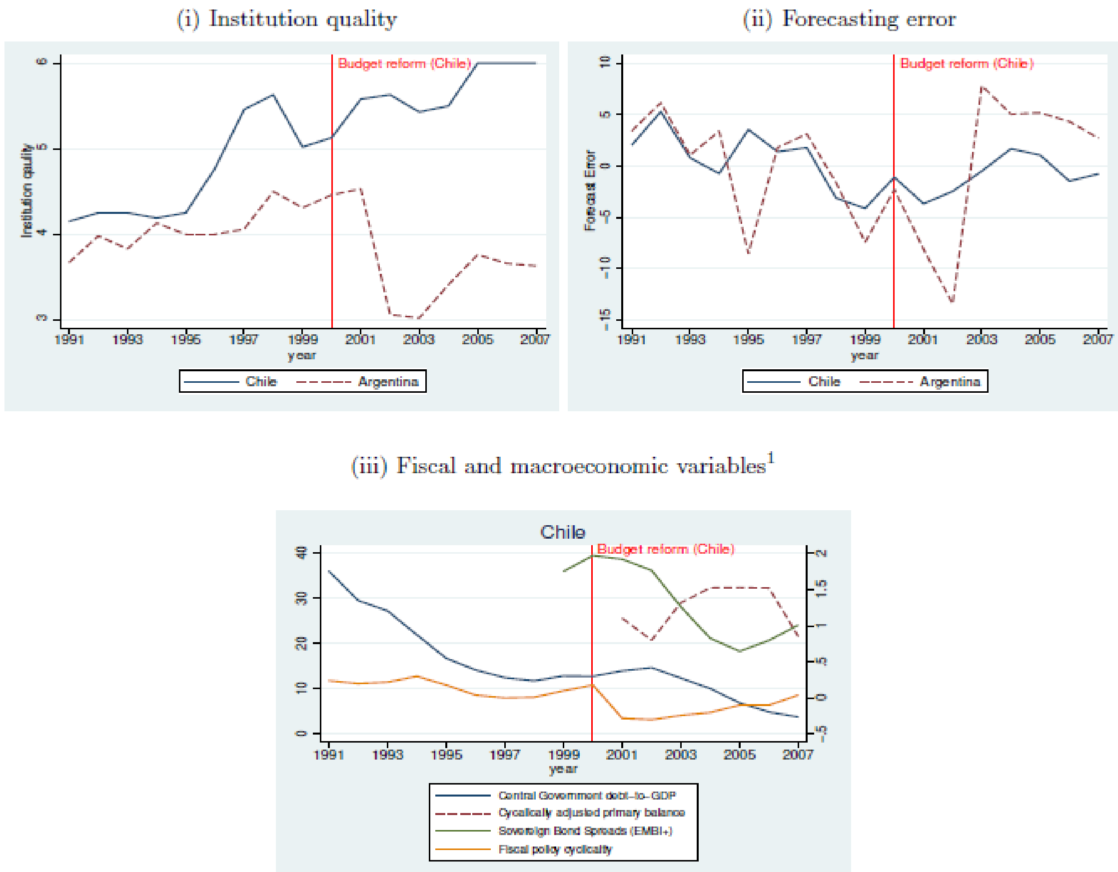


Figure 9 : Chile's case

Figure 9 shows the general picture of signal precision on forecasting as well as fiscal outcomes.[14] Central government Debt-to-GDP is shown on the left axis. Cyclically adjusted primary balance, Sovereign bond spreads (EMBI+) and fiscal policy cyclicality is shown on the right. Panel (i) shows the divergent path of institutional quality between Chile and Argentina around 2000 when Chile implemented budget reform, but there was no such a reform in Argentina. Institutional quality is based on International Country Risk Guide (ICRG) reports.[15] Panel (ii) shows the improved precision in its forecasts of one-year-ahead GDP growth rates based on the IMF World Economic Outlook for Chile after its 2000 budget reform. By comparison, Argentina had more volatile forecast errors on future GDP growth around the same periods.

With this structural change in Chile around 2000 related to the precision of its signal for its future economic outlook, we examine how the government's fiscal policy changes especially in boom periods from 2003 to 2007. First of all, panel (iii) shows that the central government debt-to-GDP ratio after 2000 is much lower than its pre-2000 level of debt. This fiscal discipline has made an important contribution to the decrease in Chile's country risk as observed in sovereign bond spreads

[14] Source: International Country Risk Guide (ICRG), World Economic Outlook and International Financial Statistics (IMF), Frankel et al. (2013), Rodriguez et al. (2007).

[15] Following Frankel et al. (2013), institutional quality is a normalized index that ranges between 0 (lowest institutional quality) and 1 (highest institutional quality). The index is calculated as the average of four components: investment profile, corruption, law and order, and bureaucratic quality.

(EMBI+) with US interest rates, especially after 2000.[16] The cyclically adjusted primary balance, i.e., the fiscal stance of government independent of business cycles, improved after the 2000 reform, indicating the Chilean government's intention to maintain fiscal discipline. Over a longer horizon, the cyclicality of government spending, measured as the correlation between GDP and government spending, turned from procyclical to countercyclical around the year 2000.[17] This indicates that government saves during good times by issuing fewer bonds, decreasing spending, or increasing taxes. In sum, in Chile's case, when a government had precise information, it implemented tighter fiscal policy, saved more of windfalls, and reduced the debt level which was rewarded with lower borrowing costs. This is consistent with our proposition.

6.3 Possible extension

6.3.1 Probability of default and the amount of bonds

Recall that the bond price is $g = 1 - \Pr(d)$ where $\Pr(d) = \Pr(w = L)$. The realization of the true state $w \in \{L, H\}$ is independent of b. Hence, the amount of bonds, b, is related to g only in that creditors update $\Pr(d)$ by identifying G's type from observing b. Therefore, b is linked with $\Pr(d)$ and g only through the channel of information inference, which could be a limit of our setting. It would be more convincing if b is linked with $\Pr(d)$ and g through the market disciplining mechanism as well where $\Pr(d)$ is increasing and g is decreasing in b. The model where the output level z is continuous is more appealing for incorporating that aspect. Suppose that the output level $z \in [0, Z]$ is drawn from the distribution $F(z)$. Then, given b, the creditor's expected profit function is

$$\begin{aligned} E\pi &= \int_0^b (-gb)\, dF(z) + \int_b^Z (-gb + b)\, dF(z) \\ &= -gb + b(1 - F(b)) \end{aligned}$$

So, from $E\pi = 0$, the bond price is $g = 1 - F(b)$. Solvency is a function of b, i.e., $\Pr(d) = F(b)$ and $\frac{\partial \Pr(d)}{\partial b} = f(b) > 0$. So, $\Pr(d)$ is increasing in b, and therefore the bond price, $g = 1 - \Pr(d)$, is decreasing in b. Checking the signaling equilibrium under this setting would be a complementary analysis to ours, and awaits future work.

6.3.2 Two signaling stages

What would happen if we consider another signaling stage with additional private information? Let us consider the repayment-as-signal mechanism suitable for the case where realized GDP in $t = 1$ is private information as well and a further benefit arises from disclosing it through the action of

[16] Rodriguez, Tokman, and Vega (2007)

[17] The cyclicality of Chilean fiscal policy is from Frankel et al. (2013). They measure the cyclical components using the Hodrick–Prescott Filter. A positive (negative) correlation indicates procyclical (countercyclical) fiscal policy. Real government expenditure is defined as central government expenditure and net lending deflated by the GDP deflator. Country correlations between the cyclical components of real government expenditure and real GDP (i.e., Corr(G, GDP)) are calculated as 20-year rolling windows for the period 1960–2009. Frankel et al. (2013) show that average correlation of the cyclical components of real government expenditure and real GDP in Chile was 0.27 (pro-cyclical fiscal policy) in 1960–1999 but -0.64 (counter-cyclical fiscal policy) in 2000–2009.

repayment.[18] In $t = 1$, given b determined in $t = 0$ and the realized state, G repays the debt as long as $w = H$ because of additional gain. So, whether G repays the debt or not in $t = 1$ only depends on the state. That is, G's choice of b in $t = 0$ does not affect G's choice in $t = 1$. Next, in $t = 0$, there is no updating via backward induction because G's choice $t = 1$ is not a function of G's choice in $t = 0$, b. Therefore, G's and C's problems in $t = 0$ are almost identical to those of our current setting. The only difference is that the additional gain from repayment should be added in G's objective function. If we denote by a the gain from repayment, the utility function in $t = 0$ is $V(b) = \sqrt{gb} + \Pr(H|s)\sqrt{(z+a) - b}$. It can be checked that, in equilibrium, $b_s^c(z+a) > b_s^c(z)$ and $b_s^*(z+a) > b_s^*(z)$. Hence, G issues more bonds with another stage of repayment-as-signal. However, the results are qualitatively identical to those of the current model.

Above reasoning is due to a our setting in which the probability of default, $\Pr(d)$, is not a function of b. If we consider a setting in which b affects $\Pr(d)$ like the one of the previous subsection, the effects of another signaling stage on the equilibrium are not decisive. In $t = 1$, solvency $\Pr(d) = F(b)$ and the bond price $g = 1 - F(b)$ are a function of b. In $t = 0$, all of these should be considered in G's decision on b through backward induction. Then, unlike our current setting, an incentive to decrease bond issuance may emerge. Due to the gain from additional signaling through repayment, default is more costly than the case without signaling through repayment. Therefore, more bond issuance becomes risky and less bond issuance could be beneficial under certain conditions. There may also be an incentive to increase bond issuance due to additional gain from repayment. If a greater amount of debt is repaid, it would signal the realization of a greater level of GDP and induce greater gain. Hence, more bond issuance could be beneficial as well. To explore the net effect, a thorough analysis is needed.

6.3.3 Partial repayment of debt

The standard model of sovereign default assumes that the debtor country repudiates all its outstanding debt on default (e.g. Eaton and Gersovits (1981) or Arellano (2008)). In order to purely focus on the effect of the signaling motive on the interest rate, we follow this assumption of no partial default. If partial repayment of debt through haircut is considered, it may affect the amount of bonds and the bond price. From the perspective of creditors, partial repayment means a reduction in loss when the sovereign defaults because they can recover part of loan. This leads to a higher bond pice. As a response, the sovereign is willing to issue more bonds. However, this results in a higher default risk and lower bond price, and therefore less sovereign debt issuance. Therefore, the net effect of partial payment is ambiguous. We need detailed analysis to have clear insight on this. In recent years, a number of papers have attempted to generate a realistic partial-default feature, i.e., positive recovered debt payment after default. (Cruces and Trebesch, 2013).[19] This literature focus on the debt restructuring negotiation procedure following sovereign default through which

[18] This is a setting used in Phan (2017).

[19] Yue (2010) introduces one period of Nash bargaining to generate an empirically relevant haircut. Benjamin and Wright (2013) investigate the multi-period debt renegotiation process to get a lengthy duration of financial exclusion with a haircut. Asonuma and Joo (2020) highlight the importance of foreign creditor's fundamental in generating a lengthy duration of renegotiation and a haircut. Arellano et al. (2022) provide partial defaults with lengthy default episodes featuring high interest rates and a high level of debt.

the equilibrium recovery rate and haircut are derived. The effects of partial repayment on bond issuance under incomplete information are not analyzed in those papers.

7 Conclusion

In this paper, using a sovereign debt model, we study whether debt issuance can be a credible signaling channel between a sovereign government and foreign creditors when the government has imperfect private information regarding economic outlook. This is related to solvency, so creditors are interested in inferring the government's private information to assess its default risk. A government with a good economic outlook is willing to disclose it to obtain a high bond price. We propose that the amount of bonds can work as a credible signaling device. The unique signaling equilibrium, that satisfies the intuitive criterion, is a separating equilibrium where a government with a good economic outlook issues fewer bonds, even though its default risk is lower, than one with a bad economic outlook. This occurs because reducing bond issuance is more costly to a government with a bad economic outlook. The amount of bonds issued by a government with a good economic outlook is smaller than that in the complete information case, which confirms that it pays a signaling cost. This signaling cost increases as its information becomes more precise. In particular, its signaling motive can be non-monotone with respect to the quality of information. Unless the prior for the next period economy is very pessimistic, precise information triggers a strong signaling motive, driving it to reduce bond issuance excessively. As a result, interestingly, it can become worse off than the case in which its signal is less precise. That is, a precise signal above a certain level hurts rather than helps it. This adverse effect of precise information becomes more severe as the prior becomes more optimistic. Our result implies that participating in costly signaling with a signal of limited impact, in our case one indicating the same state as the prior, can be harmful. On the other hand, if the prior is very pessimistic, then it is always better off with more precise information even though it signals more aggressively by reducing the number of bonds.

In the model, information asymmetry is associated with the sovereign's information indicating the state of its economy in the next period, i.e., whether the sovereign G observes a signal $s = h$ or l. If the creditors C have more information of G's type, type h has a smaller signaling incentive. This implies that, in reality, the signaling premium would be dissipated as the sovereign's information becomes more open and information asymmetries between the market and the sovereign decrease. In particular, the transparency of unfavorable information is essential. In the model, type h needs to be involved in costly signaling, mainly due to type l's willingness to mislead the market by behaving like type h. This suggests that the consensus that the sovereign discloses even unfavorable information without distortion is essential. If the market is suspicious about the transparency of unfavorable information, it is inevitable for a sovereign with favorable information to do costly signaling, which otherwise would be unnecessary. More transparency for unfavorable information is rewarded by a lower signaling premium for favorable information.

It would be useful to test the implications of our model on real world data. However, this task is challenging because this model is based on private information. For example, the signaling premium is associated with type h's deviation from the complete information case. Hence, identifying the sovereign with good economic prospect is required. Sovereign's own prospect for the future economy is unobservable information practically difficult to be detected even later. This private information is also noisy, so realized outcomes are not a perfect proxy. For this reason, it is difficult to know whether sovereigns' bond issuance decisions were based on either good or bad prospects. Even when it is detected later, it had to be private information at the time it was used. Moreover, because the signaling premium is based on deviation from the complete information case, we would also need to find a comparison period in which sovereigns' prospects were public rather than private information. For these reasons, measuring the signaling premium would require a careful and sophisticated empirical strategy. This applies to other testable implications as well.

8 Appendix

8.1 Proof of Proposition 1

Given $g_s = \Pr(H|s)$, b_s^c, is determined from

$$\underset{b_s}{Max}\ V(b_s) = \sqrt{g_s b_s} + \Pr(H|s)\sqrt{z - b_s}$$

FOC and SOC are respectively

$$\frac{\partial V(b)}{\partial b_s} = \frac{g_s}{2\sqrt{g_s b_s}} - \Pr(H|s)\frac{1}{2\sqrt{z-b_s}} = 0$$

and $\frac{\partial^2 V(b)}{\partial b^2} = -\frac{g_s^2}{4}(g_s b_s)^{-\frac{3}{2}} - \frac{\Pr(H|s)}{4}(z-b_s)^{-\frac{3}{2}} \leq 0$. From FOC,

$$b_s^c = \frac{z}{1 + \frac{\Pr(H|s)^2}{g_s}} = \frac{z}{1 + \Pr(H|s)} = \frac{z}{1+g_s}$$

From $g_s = \Pr(H|s) = \frac{\Pr(s|H)\Pr(H)}{\sum_{w\in\{H,L\}}\Pr(s|w)\Pr(w)}$,

$$g_h = \frac{p\theta}{p\theta + (1-p)(1-\theta)} \text{ and } g_l = \frac{(1-p)\theta}{(1-p)\theta + p(1-\theta)}$$

and

$$g_h - g_l = \theta(\theta-1)\frac{2p-1}{(-p-\theta+2p\theta)(\theta(2p-1)+(1-p))}$$

Here, $-p-\theta+2p\theta = \theta(2p-1)-p$. So, if $\theta \gtreqless \frac{p}{2p-1}$, $-p-\theta+2p\theta \gtreqless 0$. As $\frac{p}{2p-1} - 1 = \frac{1-p}{2p-1} > 0$, for all $\theta \in (0,1)$, $-p-\theta+2p\theta < 0$. So,

$$g_h > g_l$$

Then, accordingly,

$$b_h^c = \frac{z}{1+g_h} < \frac{z}{1+g_l} = b_l^c$$

■

8.2 Proof of Corollary 1

1)
$\frac{\partial b_h^c}{\partial p} = z\theta\frac{\theta-1}{(-p-\theta+3p\theta+1)^2} < 0$. $\frac{\partial g_h}{\partial p} = \theta\frac{1-\theta}{(-p-\theta+2p\theta+1)^2} > 0$

2)
$\frac{\partial b_l^c}{\partial p} = z\theta\frac{1-\theta}{(-p-2\theta+3p\theta)^2} > 0$, $\frac{\partial g_l}{\partial p} = \theta\frac{\theta-1}{(-p-\theta+2p\theta)^2} < 0$

■

8.3 Proof of Corollary 2

$$\begin{aligned} V_h\left(b_h^c\right) &= \sqrt{g_h b_h^c} + \Pr(H|h)\sqrt{z-b_h^c} \\ &= \sqrt{pz\frac{\theta}{1-p-\theta+3p\theta}}\left(\frac{1-p-\theta+3p\theta}{1-p-\theta+2p\theta}\right) \end{aligned}$$

$$\frac{\partial V_h\left(b_h^c\right)}{\partial p} = \frac{(1-\theta)\left(1-p-\theta+4p\theta\right)\sqrt{pz\frac{\theta}{1-p-\theta+3p\theta}}}{2p\left(-p-\theta+2p\theta+1\right)^2} > 0$$

where $1-p-\theta+4p\theta = \theta\left(2p-1\right)+\left(1-p\right)+2p\theta > 0$.

$$\begin{aligned} V_l\left(b_l^c\right) &= \sqrt{g_l b_l^c} + \Pr(H|l)\sqrt{z-b_l^c} \\ &= \sqrt{\frac{z\theta(1-p)}{p+2\theta-3p\theta}}\left(\frac{3p\theta-p-2\theta}{2p\theta-p-\theta}\right) \end{aligned}$$

$$\frac{\partial V_l\left(b_l^c\right)}{\partial p} = -\frac{(1-\theta)\left(p+3\theta-4p\theta\right)\sqrt{z\theta\frac{1-p}{p+2\theta-3p\theta}}}{2\left(1-p\right)\left(-p-\theta+2p\theta\right)^2} < 0$$

Here, $p+3\theta-4p\theta = p\left(1-\theta\right)+3\theta(1-p) > 0$ and $p+2\theta-3p\theta = p(1-\theta)+2\theta(1-p) > 0$.

■

8.4 Proof of Corollary 3

1)

$\frac{\partial b_h^c}{\partial \theta} = pz\frac{p-1}{(-p-\theta+3p\theta+1)^2} < 0$, $\frac{\partial b_l^c}{\partial \theta} = pz\frac{p-1}{(-p-2\theta+3p\theta)^2} < 0$.

2)

$\frac{\partial g_h}{\partial \theta} = p\frac{1-p}{(-p-\theta+2p\theta+1)^2} > 0$, $\frac{\partial g_l}{\partial \theta} = p\frac{1-p}{(-p-\theta+2p\theta)^2} > 0$.

3)

$\frac{\partial V_h\left(b_h^c\right)}{\partial \theta} = \frac{(1-p)(1-p-\theta+4p\theta)\sqrt{pz\frac{\theta}{1-p-\theta+3p\theta}}}{2\theta(-p-\theta+2p\theta+1)^2} > 0$ where $1-p-\theta+4p\theta = \theta\left(2p-1\right)+(1-p)+2p\theta > 0$.

4)

$\frac{\partial V_l\left(b_l^c\right)}{\partial \theta} = \frac{p(p+3\theta-4p\theta)\sqrt{z\theta\frac{1-p}{p+2\theta-3p\theta}}}{2\theta(-p-\theta+2p\theta)^2} > 0$ where $p+3\theta-4p\theta = p\left(1-\theta\right)+3\theta(1-p) > 0$.

■

8.5 Proof of Lemma 1

1)

Given G's expected utility $V\left(b,g\right) = \sqrt{gb}+\Pr(H|s)\sqrt{z-b}$, $\frac{\partial V(g,b)}{\partial b} = \frac{1}{2}\frac{\sqrt{z-b}\sqrt{bg}-b\Pr(H|s)}{b\sqrt{z-b}}$ and

$\frac{\partial V(b,g)}{\partial g} = \frac{1}{2}\frac{b}{\sqrt{bg}}$. The corresponding Hessian Matrix is

$$H = \begin{pmatrix} \frac{\partial^2 V(g,b)}{\partial b^2} & \frac{\partial^2 V(g,b)}{\partial b \partial g} \\ \frac{\partial^2 V(g,b)}{\partial g \partial b} & \frac{\partial^2 V(g,b)}{\partial g^2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4}\frac{(z-b)^{\frac{3}{2}}\sqrt{bg}+b^2 \Pr(H|s)}{b^2\left(\sqrt{z-b}\right)^3} & \frac{1}{4\sqrt{bg}} \\ \frac{1}{4}\frac{\sqrt{bg}}{bg} & -\frac{1}{4}\frac{b^2}{\left(\sqrt{bg}\right)^3} \end{pmatrix}$$

For $b, g > 0$, $\frac{\partial^2 V(g,b)}{\partial b^2} < 0$, $\frac{\partial^2 V(g,b)}{\partial g^2} < 0$, and $\det H = \frac{1}{16}\frac{b^3 g \Pr(H|s)}{bg\left(\sqrt{z-b}\right)^3\left(\sqrt{bg}\right)^3} > 0$, indicating that H is negative definite. Then, $V(b,g)$ is strictly concave, so $V(b,g)$ represents strictly convex preference.

2)

Given $V(b,g)$, the marginal rate of substitution for type s is

$$MRS_s = -\frac{\frac{\partial V(g,b)}{\partial b}}{\frac{\partial V(g,b)}{\partial g}} = \Pr(H|s)\sqrt{\frac{bg}{(z-b)}} - g$$

Then, $MRS_s \lesseqgtr 0 \Longleftrightarrow b \lesseqgtr \frac{gz}{\Pr(H|s)^2+g} \equiv \bar{b}_s$. As $\Pr(H|h) > \Pr(H|l)$, $\bar{b}_h < \bar{b}_l$. So, given (b,g), there is no case in which $MRS_h < 0$ and $MRS_l > 0$.

First, for (b,g) at which $MRS_h < 0$ (and $MRS_l < 0$ as well), $|MRS|_s = g - \Pr(H|s)\sqrt{\frac{bg}{(z-b)}}$. Then, $|MRS|_{s=h} < |MRS|_{s=l}$ from $\Pr(H|h) > \Pr(H|l)$. So, IC_l is steeper than IC_h. Second, for (b,g) at which $MRS_s > 0$, $MRS_h > MRS_l$. So, IC_h is steeper than IC_l. Given b and g at which $MRS_h > 0$ and $MRS_l < 0$, the curves intersect once. From these, the single crossing property is confirmed.

3)

Note that $\frac{\partial V(g,b)}{\partial g} > 0$, and $\frac{\partial V(g,b)}{\partial b} \gtreqless 0 \Longleftrightarrow b \lesseqgtr \bar{b}_s$. So, type s's choice b_s should be $b_s \leq \bar{b}_s$, under which $MRS_s \leq 0$ and IC_l is steeper than IC_h. ■

8.6 Proof of Proposition 2

The existence of a separating equilibrium depends on whether or not the conditions, under which both i) $V_l(b_l^*) \geq V_l(b_h^*)$ and ii) $V_h(b_h^*) \geq V_h(b_h^{**})$ are satisfied, exist. In the following, for simple notation, we let $\Pr(H|h) = A$ and $\Pr(H|l) = B$.

1) Type l

Type l does not deviate from a separating strategy if

$$V_l(b_h^*) = \sqrt{Ab_h^*} + B\sqrt{z - b_h^*} \leq \sqrt{z}\sqrt{B(B+1)} = V_l(b_l^*)$$

$V_l(b_l^*)$ in the RHS is a constant. As for $V_l(b_h^*)$ in the LHS,

i)

$\frac{\partial V_l(b_h^*)}{\partial b_h^*} = -\frac{1}{2}\frac{-\sqrt{z-b_h^*}\sqrt{Ab_h^*}+Bb_h^*}{b\sqrt{z-b_h^*}} = 0 \Longrightarrow b_h^* = \frac{A}{A+B^2}z$ and $\frac{\partial^2 V_l(b_h^*)}{\partial b^2} = -\frac{1}{4}\frac{(z-b_h^*)^{\frac{3}{2}}\sqrt{Ab_h^*}+B\left(b_h^*\right)^2}{\left(b_h^*\right)^2\left(\sqrt{z-b_h^*}\right)^3} < 0$.

Also, $V_l(b_h^*)|_{b_h^*=A\frac{z}{A+B^2}} = \sqrt{(A+B^2)z}$.

ii)
$\left(V_l\left(b_h^*\right)|_{b_h^*=A\frac{z}{A+B^2}} \right)^2 - \left(V_l\left(b_l^*\right)\right)^2 = z\left(A-B\right) > 0$. And $\left(V_l\left(b_l^*\right)\right)^2 - \left(V_l\left(b_h^*=0\right)\right)^2 = Bz > 0$.
These imply that,

i) $V_l\left(b_h^*\right)$ is a concave function of b_h^* that attains its maximum value at $b_h^* = \frac{A}{A+B^2}z$ where $0 < \frac{A}{A+B^2}z < z$.

ii) $V_l\left(b_h^*\right)|_{b_h^*=0} < V_l\left(b_l^*\right) < V_l\left(b_h^*\right)|_{b_h^*=A\frac{z}{A+B^2}}$ for $p \in \left(\frac{1}{2}, 1\right)$.

Then, $\exists b_1$ s.t. if $b_h^* \in (0, b_1]$, $V_l\left(b_h^*\right) \leq V_l\left(b_l^*\right)$.

Lemma A.1.
The value of b_h^* for which type l does not deviate from a separating strategy: $\exists b_1$ s.t. if $b_h^* \in (0, b_1]$, $V_l\left(b_h^*\right) \leq V_l\left(b_l^*\right)$.

2) Type h

Type h does not deviate from a separating strategy if

$$V_h\left(b_h^*\right) = \sqrt{Ab} + A\sqrt{z-b} \geq \sqrt{\left(A^2+B\right)z} = V_h\left(b_h^{**}\right)$$

$V_h\left(b_h^{**}\right)$ in the RHS is a constant. As for $V_h\left(b_h^*\right)$ in the LHS,

i) $\frac{\partial V_h\left(b_h^*\right)}{\partial b_h^*} = \frac{1}{2}\frac{\sqrt{z-b_h^*}\sqrt{Ab_h^*}-Ab_h^*}{b\sqrt{z-b_h^*}} = 0 \Longrightarrow b_h^* = \frac{z}{A+1}$ and $\frac{\partial^2 V_h\left(b_h^*\right)}{\partial b_h^{*2}} = -\frac{1}{4}\frac{\left(z-b_h^*\right)^{\frac{3}{2}}\sqrt{Ab_h^*}+Ab_h^{*2}}{b_h^{*2}\left(\sqrt{z-b_h^*}\right)^3} < 0$.

ii) $V_h\left(b_h^*=0\right) = A\sqrt{z}$ and $V_h\left(b_h^{**}\right)^2 - V_h\left(b_h^*=0\right)^2 = Bz > 0$. Also, $V_h\left(b_h^*\right)|_{b_h^*=\frac{z}{A+1}} = \sqrt{A\left(A+1\right)z}$ and $\left(V_h\left(b_h^*\right)|_{b_h^*=\frac{z}{A+1}} \right)^2 - V_h\left(b_h^{**}\right)^2 = z\left(A-B\right) > 0$.

These imply that

i) $V_h\left(b_h^*\right)$ is a concave function of b_h^* that attains its maximum value at $b_h^* = \frac{z}{p+1}$ $(< z)$.

ii) $V_h\left(b_h^*\right)|_{b_h^*=0} < V_h\left(b_h^{**}\right) < V_h\left(b_h^*\right)|_{b_h^*=\frac{z}{A+1}}$.

Then, $\exists b_3$ s.t. if $b \geq b_3$, $V_h\left(b_h^{**}\right) \leq V_h\left(b_h^*\right)$.

Lemma A.2.
The value of b_h^* for which type h does not deviate from a separating strategy: $\exists b_3$ s.t. if $b \geq b_3$, $V_h\left(b_h^{**}\right) \leq V_h\left(b_h^*\right)$.

The existence of a separating equilibrium depends on whether there exists b_h^* for which both i) $V_l\left(b_l^*\right) \geq V_l\left(b_h^*\right)$ and ii) $V_h\left(b_h^*\right) \geq V_h\left(b_h^{**}\right)$ hold. From Lemmas A.1 and A.2, such b_h^* exists if $b_3 \leq b_1$ where

$$b_1 = \frac{z}{A+B^2}\left(2B^2 + B - 2B\frac{\sqrt{A\left(A-B\right)} + B\sqrt{B\left(B+1\right)}}{A+B^2}\sqrt{B+B^2}\right)$$

$$b_3 = \frac{z}{A^2+A}\left(\frac{2\sqrt{A^2+B}\left(\sqrt{A^2+B} - \sqrt{A\left(A-B\right)}\right)}{A+1} - B\right)$$

Here, b_1 is derived from $V_l\left(b_l^*\right) = V_l\left(b_h^*\right)$ and b_3 is derived from $V_h\left(b_h^*\right) = V_h\left(b_h^{**}\right)$. These are illustrated by the Figures A.1-1 and A.1-2.

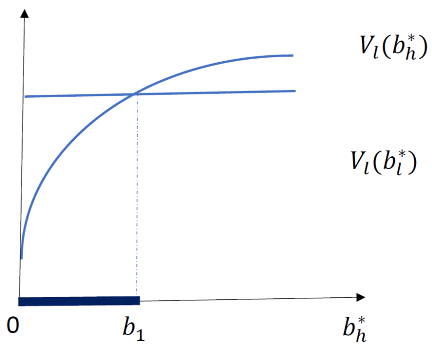


Figure A.1-1

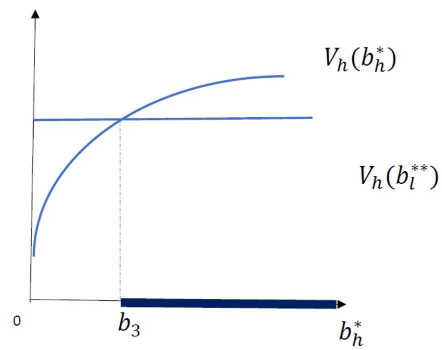


Figure A.1-2

Given $A = \Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ and $B = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$, the computation of $b_1 - b_3$ illustrates that, for $\theta \in (0,1)$ and $p \in \left(\frac{1}{2}, 1\right)$,

$$b_1 > b_3$$

Therefore, for $b_h^* \in [b_3, b_1]$ and $b_l^* = \frac{z}{2-p}$, $V_l\left(b_l^*\right) \geq V_l\left(b_h^*\right)$ and $V_h\left(b_h^*\right) \geq V_h\left(b_h^{**}\right)$. Hence, there exists a separating equilibrium. The corresponding bond price for each type and the beliefs that support this separating equilibrium are described in Proposition 2. ■

8.7 Proof of Proposition 3

The existence of a pooling equilibrium depends on whether or not the conditions, under which both i) $V_l\left(b^*\right) \geq V_l\left(b_l \neq b^*\right)$ and ii) $V_h\left(b^*\right) \geq V_h\left(b_h \neq b^*\right)$ are satisfied, exist. In the following, for simple notation, we let $\Pr(H|h) = A$ and $\Pr(H|l) = B$.

1) Type l

Type l does not deviate from a pooling strategy $b = b^*$ if

$$V_l\left(b^*\right) = \sqrt{\theta b^*} + B\sqrt{z-b^*} \geq \sqrt{\left(B^2+B\right)z} = V_l\left(b_l \neq b^*\right)$$

$V_l\left(b_l \neq b^*\right)$ in the RHS is a constant. For $V_l\left(b^*\right)$ in the LHS,

i)

$\frac{\partial V_l(b^*)}{\partial b^*} = \frac{1}{2}\frac{\sqrt{b^*\theta}\sqrt{z-b^*}-Bb}{b\sqrt{z-b^*}} = 0 \Longrightarrow b^* = z\frac{\theta}{\theta+B^2}$ and $\frac{\partial^2 V_l(b^*)}{\partial (b^*)^2} = -\frac{1}{4}\frac{B(b^*)^2+\sqrt{b^*\theta}(z-b^*)^{\frac{3}{2}}}{(b^*)^2\left(\sqrt{z-b^*}\right)^3} < 0$. Also, $V_l\left(b^*\right)\big|_{b^*=z\frac{\theta}{\theta+B^2}} = \sqrt{\left(\theta+B^2\right)z}$.

ii)

$\left(V_l\left(b\right)\big|_{b=z\frac{\theta}{\theta+B^2}}\right)^2 - \left(V_l\left(b_l \neq b^*\right)\right)^2 = z\left(\theta - B\right) > 0.$

Here,

$\theta - B = \theta - \Pr\left(H|l\right) = \theta\left(1-\theta\right)\frac{2p-1}{p+\theta-2p\theta}$. So, if $\theta \lesseqgtr \frac{p}{2p-1}$, $p+\theta-2p\theta \gtreqless 0$. As $\frac{p}{2p-1} - 1 = \frac{1-p}{2p-1} > 0$, always $\theta < \frac{p}{2p-1} \Longrightarrow p+\theta-2p\theta > 0$. So, $\theta > B$. Also, $\left(V_l\left(b_l \neq b^*\right)\right)^2 - \left(V_l\left(b\right)\big|_{b=0}\right)^2 =$

$Bz > 0$.

iii)

$(V_l\,(b_l \neq b^*))^2 - \left(V_l\,(b)|_{b=z}\right)^2 = z\left(B - \theta + B^2\right)$. Here, $B - \theta + B^2 \gtreqqless 0$. Hence, $(V_l\,(b_l \neq b^*))^2 \gtreqqless \left(V_l\,(b)|_{b=z}\right)^2$

These imply that,

i) $V_l\,(b^*)$ is a concave function of b^* that attains its maximum value at $b^* = z\frac{\theta}{\theta+B^2}$ where $0 < z\frac{\theta}{\theta+B^2} < z$.

ii) $V_l\,(b^*)|_{b^*=0} < V_l\,(b_l \neq b^*) < V_l\,(b^*)|_{b^*=z\frac{\theta}{\theta+B^2}}$.

iii) If $B-\theta+B^2 \leq 0$, $V_l\,(b_l \neq b^*) \leq V_l\,(b^*)|_{b^*=z}$ and if $B-\theta+B^2 > 0$, $V_l\,(b_l \neq b^*) > V_l\,(b^*)|_{b^*=z}$.

Then,

Case 1) If $B - \theta + B^2 \leq 0$, $\exists b_5$ *s.t. for* $b \in [b_5, z]$, $V_l\,(b^*) \geq V_l\,(b_l \neq b^*)$.

Case 2) If $B - \theta + B^2 > 0$, $\exists b_5, b_6$ *s.t. for* $b \in [b_5, b_6]$, $V_l\,(b^*) \geq V_l\,(b_l \neq b^*)$

These can be illustrated by the Figures A.2-1 and A.2-2.

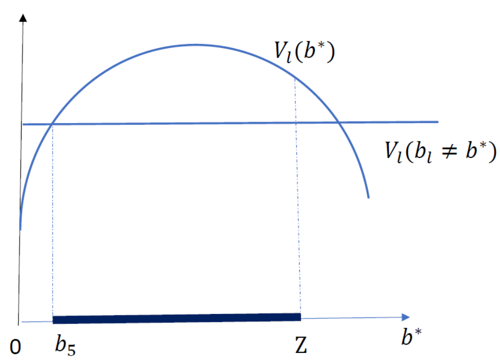


Figure A.2-1

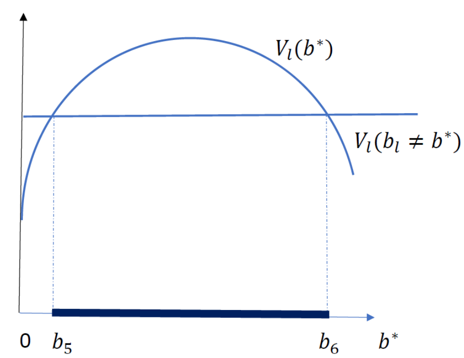


Figure A.2-2

2) Type h

$$V_h\,(b^*) = \sqrt{\theta b^*} + A\sqrt{z - b^*} \geq \sqrt{(A^2 + B)\,z} = V_h\,(b_h \neq b^*)$$

$V_h\,(b_h \neq b^*)$ in the RHS is a constant. As for $V_h\,(b^*)$ in the LHS,

i)

$\frac{\partial V_h(b^*)}{\partial b^*} = -\frac{1}{2}\frac{-\sqrt{b^*\theta}\sqrt{z-b^*}+Ab^*}{b^*\sqrt{z-b^*}} = 0 \Longrightarrow b^* = z\frac{\theta}{\theta+A^2}$. And $\frac{\partial^2 V_l(b^*)}{\partial b^{*2}} = -\frac{1}{4}\frac{A(b^*)^2+\sqrt{b^*\theta}(z-b^*)^{\frac{3}{2}}}{(b^*)^2\left(\sqrt{z-b^*}\right)^3} < 0$. Also, $V_h\,(b^*)|_{b^*=z\frac{\theta}{\theta+A^2}} = \sqrt{(\theta + A^2)\,z}$.

ii)

$\left(V_h\,(b^*)|_{b^*=z\frac{\theta}{\theta+A^2}}\right)^2 - (V_h\,(b_h \neq b^*))^2 = z\,(\theta - B) > 0$ and $\left(V_h\,(b^*)|_{b^*=0}\right)^2 - (V_h\,(b_h \neq b^*))^2 = -Bz < 0$.

iii)

$(V_h\,(b_h \neq b^*))^2 - \left(V_h\,(b^*)|_{b^*=z}\right)^2 = z\left(B - \theta + A^2\right) > 0$.

These imply that,

i) $V_h(b^*)$ is a concave function of b^* that attains its maximum value at $b^* = z\frac{\theta}{\theta+A^2}$ where $0 < z\frac{\theta}{\theta+A^2} < z$.

ii) $V_h(b^*)|_{b^*=0} < V_h(b_h \neq b^*) < V_h(b^*)|_{b^*=z\frac{\theta}{\theta+A^2}}$.

iii) $V_h(b^*)|_{b^*=z} < V_h(b_h \neq b^*)$

Then, $\exists b_7, b_8$ s.t. for $b \in [b_7, b_8]$, $V_h(b^*) \geq V_h(b_h \neq b^*)$. This can be illustrated by the following diagram.

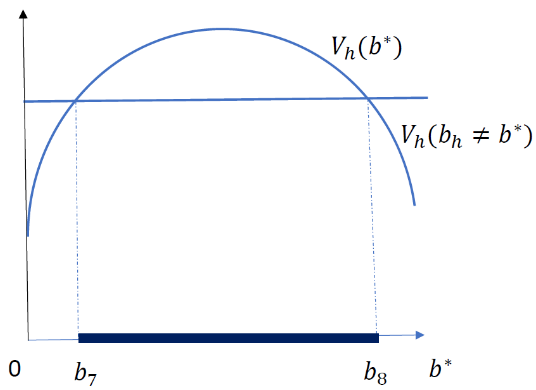


Figure A.3

3) Existence of pooling equilibrium

For type l, from $\sqrt{\theta b} + B\sqrt{z-b} = \sqrt{B(1+B)z}$,

$$b_5 = \frac{1}{\theta+B^2}\left(2B^2z + Bz - 2B\frac{\sqrt{B^2z+Bz}}{\theta+B^2}\left(B\sqrt{Bz(B+1)} + \sqrt{z\theta(\theta-B)}\right)\right)$$

$$b_6 = \frac{1}{\theta+B^2}\left(2B^2z + Bz - 2B\frac{\sqrt{B^2z+Bz}}{\theta+B^2}\left(B\sqrt{Bz(B+1)} - \sqrt{z\theta(\theta-B)}\right)\right)$$

For type h, from $\sqrt{\theta b} + A\sqrt{z-b} = \sqrt{(A^2+B)z}$,

$$b_7 = \frac{1}{\theta+A^2}\left(2A^2z + Bz - 2A\sqrt{A^2z+Bz}\frac{\sqrt{z\theta(\theta-B)}+A\sqrt{z(B+A^2)}}{\theta+A^2}\right)$$

$$b_8 = \frac{1}{\theta+A^2}\left(2A^2z + Bz + 2A\sqrt{A^2z+Bz}\frac{\sqrt{z\theta(\theta-B)}-A\sqrt{z(B+A^2)}}{\theta+A^2}\right)$$

Given $A = \Pr(H|h) = \frac{p\theta}{p\theta+(1-p)(1-\theta)}$ and $B = \Pr(H|l) = \frac{(1-p)\theta}{(1-p)\theta+p(1-\theta)}$, the computation yields that, for $\theta \in (0,1)$ and $p \in \left(\frac{1}{2}, 1\right)$,

$$b_7 < b_5 \text{ and } b_5 < b_8$$

Then, the pooling equilibrium always exists. Specifically, the pooling equilibrium can be characterized as follows.

Case 1) $B - \theta + B^2 < 0$. This is the case where $V_l(b^*) \geq V_l(b_l \neq b^*)$ for $b \in [b_5, z]$. In this case, it is $b \in [b_5, b_8]$.

Case 2) If $B - \theta + B^2 > 0$. This is the case where $V_l(b^*) \geq V_l(b_l \neq b^*)$ for $b \in [b_5, b_6]$. In this case, it is $b \in [b_5, \min\{b_6, b_8\}]$. Here, given θ, $\exists p^*$ such that if $p \in (\frac{1}{2}, p^*)$, $b_6 > b_8$ and if $p \in (p^*, 1)$, $b_6 < b_8$. As $\theta \nearrow 1$, $p^* \nearrow 1$.

The corresponding bond price for each type and the beliefs that support this separating equilibrium are described in Proposition 3. ■

8.8 Proof of Corollary 6

From Proposition 4,

$$\Delta g_s^* = \theta(\theta - 1)\frac{2p - 1}{(-p - \theta + 2p\theta)(-p - \theta + 2p\theta + 1)}$$

First, $\Delta g_s^*(\theta = 0) = \Delta g_s^*(\theta = 1) = 0$. Second,

$$\frac{\partial(\Delta g_s^*)}{\partial \theta} = -(2\theta - 1)\frac{p(1 - p)(2p - 1)}{(-p - \theta + 2p\theta)^2(-p - \theta + 2p\theta + 1)^2}$$

Then, i) Δg_s^* is concave and symmetric across $\theta = 1/2$, ii) $\Delta g_s^*(\theta = 0) = \Delta g_s^*(\theta = 1) = 0$.

Third.

$$\frac{\partial(\Delta g_s^*)}{\partial p} = -\theta(1 - \theta)\frac{\left(8\theta - 8\theta^2 - 2\right)p^2 + \left(8\theta^2 - 8\theta + 2\right)p + \left(2\theta - 2\theta^2 - 1\right)}{(-p - \theta + 2p\theta)^2(-p - \theta + 2p\theta + 1)^2}$$

If we let $f(p,\theta) = \left(8\theta - 8\theta^2 - 2\right)p^2 + \left(8\theta^2 - 8\theta + 2\right)p + \left(2\theta - 2\theta^2 - 1\right)$, i) it is concave because $8\theta - 8\theta^2 - 2 < 0$ for all $\theta \in (0,1)$, and ii) it attains the maximum $-\frac{1}{2}$ at $p = \frac{1}{2}$. So, for all θ and p, $f(p,\theta) < 0$, and therefore $\frac{\partial(\Delta g_s^*)}{\partial p} > 0$. ■

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