Discussion Papers in Economics

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DP 13/23

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ISSN: 1749-5075

Optimal Liquidity Provision and Interest Rate Rules: A Tale of Two Frictions*

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November 22, 2023

Abstract

We study central bank liquidity provisions to the banking sector in a DSGE model estimated for the Euro Area with financial frictions on the supply and demand side of credit. We show that liquidity provisions, as in the ECB's recent Long Term Refinancing Operations, can be welfare-enhancing or welfare-reducing when both these financial frictions exist. They relax the banks' leverage constraint and induce banks to provide more credit. This reduces the credit spread facing firms and increases investment, but this comes at the cost of implementing the liquidity policy. We compute a welfare optimized liquidity rule for the central bank responding to output, inflation and the interest rate spread that can increase welfare in comparison with the case of no liquidity provision. Crucially, this result is conditional on a high level of central bank monitoring of the its loanable funds to banks.

JEL Classification: C11; E44; E52; E58; E61
Keywords: Liquidity Injections; Monetary Policy; Financial Frictions; DSGE Model; Bayesian Estimation; Welfare-Optimized Rules.

*A previous version of this paper was presented at a workshop at the Bank of Portugal, June 2023, the CEF Annual Conference, Nice, July 2023 and the MMF Annual Conference, Portsmouth, September 2023. We acknowledge comments by participants at all these events.

Contents

1 Introduction

Since the onset of the Great Recession and the COVID-19 pandemic, the European Central Bank (ECB), among other major central banks, has supported the banking system with liquidity, the key scope of these direct funding programmes being the stabilization of economic activity through a credit expansion.[1] This paper assesses the effectiveness of such a liquidity policies and explores whether a welfare-improving liquidity rule exists.

We analyse the ECB's liquidity provisions, the Long Term Refinancing Operations (LTROs), in a DSGE model, built and estimated for the Euro Area with, financial frictions on both supply and demand side of credit. We develop the two friction setting (alluded to in the title) by combining the two seminal works of Gertler and Kiyotaki (2010) with Bernanke, Gertler, and Gilchrist (1999a) (henceforth GK and BGG respectively). Framing liquidity injections as in Gertler, Kiyotaki, and Queralto (2012a), we identify the three forces after a liquidity injection: of the supply side of credit, liquidity loosens the incentive compatibility constraint of the banks stemming from the GK friction and stimulates lending. At the same time on the demand side increased liquidity can mitigate the BGG friction by reducing the interest rate spread, or worsen the friction by increasing the probability of default. Whether the policy is welfare-enhancing depend on which of these two effects dominate and whether the demand and supply effects together outweigh the central bank cost of implementing the policy. Crucially, we show that that this in turn depends on the monitoring ability of the central bank. We identify the importance of a central bank liquidity monitoring parameter, which by reducing the GK friction, makes the liquidity injection more effective. We show that there exists a threshold value for this parameter that makes the expansionary effect of the liquidity policy on investment outweigh the implementation costs.

The main contribution of the paper is then the design of liquidity and a monetary Taylor-type rules in an empirical setting that are welfare maximizing. We compute a simple and implementable welfare-maximizing liquidity rule that responds to output, inflation and credit spread deviations from its steady state equilibrium. Alongside the liquidity rule, we employ a standard nominal interest rate rule. Our specification then is similar to an 'empirical' Taylor monetary rule assumed in estimated New Keynesian models, such as Smets and Wouters (2007), but applied to liquidity policy. In our estimated model we find a combination of liquidity and interest rate rule response parameters that maximize expected welfare of households in the face of exogenous uncertainty driven by the estimated shock processes. This we compare with the outcome with only the optimized monetary rule but no liquidity rule.

[1] ECB provided about EUR 450 bn of liquidity until 2019 together with the PELTRO during the pandemic. Recently, the Swiss National Bank (SNB) also provided a CHF 56 billion liquidity to Credit Suisse before it was merged with UBS.

Our methodological approach is as follows: first we estimate our model in the absence of the liquidity rule which is our baseline specification. We then set our model's parameters to the estimated ones, add the liquidity rule and search for the rule response parameter values for both rules that maximize expected inter-temporal household welfare. We allow for the effects of a negative liquidity injection which is simply lending from the banks to the central bank similar to the reserves (or the deposit facility in the ECB framework). We extend the penalty function approach of Deak, Levine, and Pham (2023) to optimize welfare subject to an approximate zero lower bound constraint on the nominal interest rate alongside an upper bound unitary constraint on the liquidity-loan ratio. Finally, we stimulate and compare the estimated model with and without the optimized liquidity rule in terms of impulse responses to selected shocks.

Whilst we consider optimized rules responding to exogenous uncertainty, in order to gain more insights into how liquidity affects our economy we also perform a steady state analysis of the non-stochastic environment. We show that as liquidity increases from zero to its limit, loosening the lending spread and providing more credit to the non financial firms is a potentially welfare-enhancing effect despite the increase in the probability of default of the firms due to the higher credit they receive.

In the last part of the paper we provide a brief stability analysis of the conditions that guarantee the uniqueness of equilibrium. We do that for the case of the liquidity rule and also the nominal interest rate rule. We show how the liquidity rule, but also the presence of spread deviations in the policy rate rule, can alter the equilibrium stability properties of our financial frictions model. In particular, under certain parameter configurations for the spread deviation the equilibrium may be indeterminate even when the interest rate rule is one that satisfies the Taylor principle. This further emphasizes the possible drawbacks of adding the liquidity rule to the standard monetary policy framework.

Related Literature.

Macroeconomic models with financial frictions have populated a substantial fraction of the macro literature after the Great Recession. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) were among the first studies to incorporate financial frictions in dynamic macroeconomic models. Building on these, BGG and GK popularized the financial friction literature with their seminal papers. Gertler and Kiyotaki (2010) introduce a costly enforcement problem, in an otherwise standard real business cycle model, which restricts the ability to arbitrage across deposit, and credit markets.[2] Bernanke, Gertler, and Gilchrist (1999b) enrich the canonical New Keynesian model with a friction due to asymmetric information between banks and firms resulting to the financial accelerator mechanism.[3] Prominent works in this literature also include Eggertsson and Woodford

[2]For a similar financial frictions setting see also Gertler and Karadi (2011), Gertler, Kiyotaki, and Queralto (2012b) and Sims and Wu (2021) among others.

[3]For similar approaches see Christiano, Motto, and Rostagno (2014) Carlstrom and Fuerst (1997) and

(2003), Curdia and Woodford (2011), Gertler and Karadi (2011). For a comprehensive literature review on the developments of models with financial frictions see Gertler and Gilchrist (2018).

Related to this paper's research questions, there are studies in the literature assessing the effectiveness of liquidity injections, using models with financial frictions. Cahn, Matheron, and Sahuc (2017) and Bocola (2016) study how liquidity injections affect the banking sector and the macroeconomy incorporating financial frictions following the bank-household friction introduced Gertler and Karadi (2011). Our paper's distinguishing feature lies on the interaction of the two financial frictions on the supply and the demand side of credit, which, to the best of our knowledge, has not been used to study a similar question. The two frictions approach is capable of exploring the tightening or loosening of the frictions between households, firms and the banking sector due to the central bank liquidity injections. A two-frictions model relatively close to ours without the unconventional monetary policy component is Rannenberg (2016). He shows that the model matches the data relatively better and outperforms both a BGG and a GK-type model.

While studies on optimal setting of monetary policy populated the literature in the past, studies on optimal macroprudential rules are a more recent strand of the literature. For example, Angelini, Neri, and Panetta (2014) and Ferrero, Harrison, and Nelson (2022) study the optimal interaction of monetary and macroprudential policy. Our focus is on optimal liquidity and its interaction with interest rate rules. However our optimal simple rules framework can be extended to macroprudential policy and would complement these studies.

Finally, Sims and Wu (2021), Gertler and Karadi (2013) and Tsiaras (2023) among others, study the effectiveness of Quantitative Easing in models with GK financial frictions. Although this is of a similar nature to our liquidity injections related questions, their mechanism relaxes the banking friction in a different way. Specifically, although liquidity injections work by providing credit to the baking sector, relaxing the bank constraints from their liability side asset position, Quantitative Easing effectiveness lies on the exchange of banks asset with risk-free reserves.

Road-Map

The outline of the paper is as follows. Section 2 describes the financial frictions component of the model together with the liquidity rule framework. Section 3 presents the estimation results of the model, the data used and the measures of fit. The sections that follow then present the main results. We first perform a steady state analysis in Section 4 to understand the model's behaviour for all the possible values of the liquidity provision. Sections 5 and 6 describe the general framework of welfare-optimized simple monetary and liquidity rules and a 'delegation game' that imposes an approximate zero-lower-bound on

others.

the nominal interest rate and an upper bound on the liquidity-loan ratio in an equilibrium. Section 7 sets out the welfare optimizing monetary and liquidity rules. This section also performs a determinacy analysis and compares impulse responses to shocks with and without the liquidity rule. The last section concludes.

2 The Financial Frictions Model

The model combines the the banks-firms asymmetric information framework of BGG and the banks-households limited commitment problem of GK. This setting is incorporated into an New Keynesian model with monopolistic competition, sticky prices and sticky wages similar to Smets and Wouters (2007). In this Section we describe only the financial frictions component of the model. Appendix A outlines the NK part of the model.

The financial frictions economy is populated by a continuum of financial intermediaries owned by households. Following BGG, there is a continuum of entrepreneurs that own the non-financial firms. A monetary authority and the treasury complete this part of the economy. There is a moral hazard problem between the households and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an incentive constraint to the model to be followed by the banks. The second financial friction originates from a firm-bank problem. Entrepreneurs at every period receive an idiosyncratic shock that change the value of their assets. Low values of the shock can lead to default on their credit. Finally, the central bank performs its conventional monetary policy under a Taylor rule, but can also provide liquidity following our liquidity rule.

2.1 The Debt-Contracting Problem and the Entrepreneur-Bank Friction

At every period there is a fixed mass of intermediaries indexed by e. Each period, a fraction $1 - \sigma_E$ of entrepreneurs, exit and give retained earnings to their household. An equal number of new entrepreneurs enter at the same time. They begin with a start-up fund of ξ_E given to them by their household. The entrepreneur e (the non-financial firm) seeks loans $L_{e,t}$ to bridge the gap between its net worth $N^E_{e,t}$[4] and the expenditure on new capital $Q_t K_{e,t}$, all end-of-period and expressed in real terms. Thus

$$L_{e,t} = Q_t K_{e,t} - N^E_{e,t} \tag{1}$$

[4]The entrepreneur's net worth $N^E_{e,t}$ has a counterpart for bank b, $N^B_{b,t}$, introduced for the second financial friction in Section 2.2.

where the entrepreneur's *real* net worth accumulates according to

$$N_{e,t}^{E} = R_t^K Q_{t-1} K_{e,t-1} - \frac{R_{t-1}^L}{\Pi_t} L_{e,t-1}$$

where R_t^K is the gross real return on capital as in the NK model and R_t^L is the *nominal* loan rate to be decided in the contract.

Each entrepreneur determines the utilization rate of capital, u_t, and provides an effective amount of capital to the firms for production, a cost of $a(u_t)K_t$, getting bank the rental rate of capital r_t^K. $a(u_t)$ satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. (See (A.15) in Appendix A). At the end of the production schedule, the capital is being resold to capital goods producers at price Q_t. Then the gross real return on capital is defined as:

$$R_t^K = \frac{r_t^K u_t - a(u_t) + (1-\delta)Q_t}{Q_{t-1}} \tag{2}$$

In each period an idiosyncratic capital quality shock, ψ_t results in a return $R_t^K \psi_t$ which is the entrepreneur's private information. Following BGG, we assume that ψ_t has a unit-mean log normal distribution that is independently drawn across time and across entrepreneurs. Specifically, $\log(\psi) \sim \mathcal{N}\left(-\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right)$. With the mean set to $-\frac{\sigma_\psi^2}{2}$, $\mathbb{E}[\psi] = 1$. σ_ψ is the period t standard deviation of $\log(\psi)$. Similarly to Christiano et al. (2014) we label σ_ψ, the cross-sectional dispersion in ψ, the risk shock and we allow it to vary stochastically over time.

Default in period t occurs when net worth becomes negative, i.e., when $N_{E,e,t} < 0$ and shock falls below a threshold $\bar{\psi}_t$ given by

$$\bar{\psi}_t = \frac{R_{t-1}^L L_{e,t-1}}{\Pi_t R_t^K Q_{t-1} K_{e,t-1}} \tag{3}$$

With the idiosyncratic shock, ψ_t drawn from a density $f(\psi_t)$ with a lower bound ψ_{min}, the probability of default is then given by

$$p(\bar{\psi}_t) = \int_{\psi_{min}}^{\bar{\psi}_t} f(\psi) d\psi$$

In the event of default the bank receives the assets of the firm and pays a proportion μ of monitoring costs to observe the realized return. Otherwise the bank receives the full payment on its loans, $R_t^L L_{e,t}/\Pi_{t+1}$ where R_t^L is the agreed loan rate at time t.

Appendix 2.1 sets out the incentive compatibility constraint, the optimal contract for the risk-neutral entrepreneur (the firm), aggregation over old and new entrepreneurs and banks and the choice of density function for ψ. The main results are

$$\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R^B_{t+1}] \tag{4}$$

which replaces (A.19) in the core NK model (where $R^B_t = R_t$, the ex post real interest rate for bonds) set out in Appendix A, where the *premium on external finance*, $\rho(\bar{\psi}_{t+1})$ is given by

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]}. \tag{5}$$

where functions Γ', Γ, G and G' are given in Appendix 2.1.

2.2 Banks and the Bank-Household Friction

At every period period there is a fixed mass of intermediaries (banks) indexed by b. Each bank allocates its funds to credit $L_{b,t} = L_{e,t}$. It funds its operations by receiving deposit from households $D_{b,t}$, emergency funding (LTRO) from the central bank $M_{b,t}$, expressed in real terms, and also by raising equity $N_{b,t}$. Each period, a fraction $1 - \sigma_B$ of bankers, exit and give retain earnings to their household. An equal number of new bankers enter at the same time. They begin with a start up fund fraction of assets, ξ_B, given to them by their household.

From the above specification, it follows that the bank's balance sheet is:

$$L_{b,t} = N^B_{b,t} + D_{b,t} + M_{b,t} \tag{6}$$

The bank's net worth evolves as the difference between interest income and interest expenses. Net worth of the bank accumulates according to :

$$N^B_{b,t} = R^B_t L_{b,t-1} - R_t D_{b,t-1} - R^M_t M_{b,t-1} \tag{7}$$

where $R^M_{b,t}$ is the gross real interest rate paid for emergency funding. To understand this dynamic problem better we can substitute for $D_{b,t}$ from (6) and rewrite (7) as

$$N^B_{b,t} = R_t N_{b,t-1} + (R^B_t - R_t)L_{b,t-1} - (R^M_t - R_t)M_{b,t-1} \tag{8}$$

R^M_t the interest rate of the emergency funding (LTRO) defined endogenously in the model as will be shown momentarily.

Banks exit with probability $1 - \sigma_B$ per period and therefore survive for $j - 1$ periods and exit in the j^{th} period with probability $(1 - \sigma_B)\sigma_B^{j-1}$. Given the fact that bank pays dividends only when it exists, the banker's objective is to maximize expected discounted terminal wealth

$$V_{b,t}^{B} = \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma_B)\sigma_B^{j-1} \Lambda_{t,t+j} N_{b,t+j}^{B} \tag{9}$$

subject to an incentive constraint for lenders (households) to be willing to supply funds to the banker. $\Lambda_{t,t+j} = \beta^j \frac{\Lambda_{C,t+j}}{\Lambda_{C,t}}$ is the j-period ahead stochastic discount factor used (for j=1) in the household optimization problem (see Appendix A.1).

As in Gertler and Karadi (2011) there is an endogenous constraint on the banks ability to borrow. A banker after collecting deposits from households and liquidity from the central bank may divert a fraction of these funds. This occurs when the bank's value from diverting is higher than its franchise value. It is assumed that the bank can abscond a fraction $\theta \in [0,1]$ of the loans. They can also abscond a fraction $(1-\omega)\theta \in [0,1]$ of the central bank's liquidity. Liquidity is assumed to be more difficult to divert due to the monitoring ability of the central bank of its own loanable funds. We assume that the central bank can have additional means in order to retrieve its assets, or the loan has been extra collateralized.[5] At the same time, under this assumption, liquidity injections serve to relax the incentive constraint of banks. With $\omega = 1$ the bank can divert no liquidity and with $\omega = 0$ can divert the total of it. Parameter ω is a key driver for our results as it will become clear shortly since it can change the welfare outcomes. In the results section we analyse two cases: a "medium central bank monitoring ability" when $\omega = 0.5$ and a "high central bank monitoring" when $\omega = 0.9$. Higher central bank monitoring ensures that the funds provided to the banking sector as liquidity relax the incentive constraint of the banking sector and crowd out deposits, therefore relaxing the financial friction between depositors and the banks.

In case of the absconding of its funds the creditors can force the intermediary into bankruptcy at the beginning of the next period. A constraint therefore sets a limit to the bankers borrowing from either the depositors or the central bank. For the banks' creditors to continue providing funds to the bank, the following incentive constraint must always hold:

$$V_{b,t}^{B} \geqslant \theta[L_{b,t} - \omega M_{b,t}] \tag{10}$$

The bank's value must be greater or at least equal to the value of its divertable assets. When this constraint holds bankers have no incentive to steal from their creditors.

The detailed solution to the banker's problem is presented in Appendix C. In this Section we present the key equilibrium conditions of the bank's problem. Combining the optimality conditions with the banker's incentive constraint yields a central equation of

[5] This assumption is followed also by papers with the same mechanism on the liquidity injections in the literature, see Boehl, Goy, and Strobel (2021), Cahn et al. (2017).

the model: The leverage constraint of the bank:

$$L_{b,t} = \phi_t N_{b,t}^B + \omega M_{b,t}. \tag{11}$$

2.2.1 Aggregation

At the aggregate level the banking sector balance sheet is:

$$L_t = N_t^B + M_t + D_t$$

At the aggregate level net worth is the sum of existing (old) bankers and new bankers:

$$N_t^B = N_{o,t}^B + N_{n,t}^B$$

Net worth of existing bankers equals earnings on assets held in the previous period net cost of deposit finance, multiplied by a fraction σ_B, the probability that they survive until the current period:

$$N_{o,t}^B = \sigma_B \{R_t^B L_{t-1} - R_t D_{t-1} - R_t^M M_{t-1}\}$$

Since new bankers cannot operate without any net worth, we assume that the family transfers to each one the fraction $\xi_B/(1-\sigma_B)$ of the total value assets of exiting bankers. This implies:

$$N_{n,t}^B = \xi_B R_t^B L_{t-1}$$

Equation (11) constrains the financial intermediary's leverage and, owing to this, excess returns are generated. At the aggregate level, from Appendix C, ϕ_t^B is the maximum adjusted leverage ratio of the bank:

$$\phi_t^B = \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^B - R_{t+1})]} = \frac{L_t - \omega M_t}{N_t^B}.$$

where the shadow price of the bank's net worth is given by

$$\Omega_t = 1 - \sigma_B + \sigma_B \theta \phi_t^B.$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits and on the excess value of bank assets. As the credit spread $R_t^B - R_t$ increases, banks' franchise value V_t increases and the probability of a bank diverting its funds declines. On the other hand, as the proportion of assets that a bank can divert, θ, increases the constraint binds more.

Importantly, the maximum adjusted leverage ratio does not depend on any individual bank characteristics, therefore the heterogeneity in the bankers' holdings and net worth, does not affect aggregate dynamics. Hence, it is straightforward to express individual financial sector variables in aggregate form.

Finally, from the first order conditions yields the arbitrage condition between the lending and liquidity returns. This endogenously determines the liquidity interest rate R_t^M according to:

$$\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^M - R_{t+1})] = \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^B - R_{t+1})].$$

The excess cost to a bank of liquidity credit relative to deposits equals to the credit spread multiplied by the monitoring ability of the central bank to the liquidity provided to the bankers ω. In particular $\omega = 0$ implies $R_t^M = R_t$ and $\omega = 1$ implies $R_t^M = R_t^B$ are respectively solutions.

According to this equation, to make banks indifferent between liquidity and deposits at the margin, the central bank should set R_t^M to make the excess cost of liquidity equal to the fraction ω of the excess value of assets. From the incentive constraint of the bank (10), a unit of liquidity relaxes the constraint of the banks and therefore permits a bank to expand assets by a greater amount than a unit of deposits, it is willing to pay a higher cost for this form of credit. In this way, as in Gertler and Kiyotaki (2010), the model generates an endogenously determined penalty rate for liquidity.

In steady state, the liquidity interest rate will be a weighted average of the lending rate R_t^B and the deposit interest rate R_t (i.e., $R^M = \omega R^B + (1-\omega)R$). In the quantitative analysis of the paper we allow for a zero lower bound in the nominal interest rate which essentially defines the real interest deposit rate. Therefore, indirectly, there is a zero lower bound on the liquidity returns. The relationship between the nominal and the real liquidity rate is $R_t^M = \frac{R_{m,t-1}}{\Pi_t}$, where Π_t is the gross inflation rate.

2.3 The Central Bank

The central bank can make use of two policy tools. Firstly, it adjusts the policy rate (the nominal interest rate) according to a Taylor monetary rule. Secondly, it supplies liquidity to the banking sector.

The relative increase in the liquidity of the banking sector is determined endogenously following the liquidity rule specified later. The effectiveness of the policy comes primarily from its ability to ease the financial constraints of banks. When balance sheet constraints are tight and excess returns are positive, central bank liquidity injections loosen the incentive constraint of the banks and allow them to extend new lending to non-financial corporations. The easier credit conditions increase the value of capital and banks' net worth, through the financial accelerator mechanism, increasing further the banks' net worth and easing the financial constraint.

Following Gertler et al. (2012a), liquidity injections involve efficiency costs for the central bank: in particular, the central bank liquidity consumes resources of $\Psi_t(M_t)$, where the function Ψ_t is increasing in the quantity of liquidity provided to the banking sector. These costs could be thought as administrative costs of raising new funds through government debt or any inefficiency the central bank faces in order to provide liquidity to the banks such as identifying which banks is mostly beneficial to receive the liquidity. The function is assumed to be a quadratic function of liquidity M_t governed by the penalty parameters (τ_1, τ_2):

$$\Psi_t(M_t) = \tau_1 M_t + \tau_2 M_t^2$$

It is assumed also that the central bank turns over any profits to the treasury and receives transfers to cover any losses.

2.3.1 The Monetary Rule

The central bank sets the policy interest rate according to a Taylor Rule responding to inflation and output deviations from their steady state and also to deviations in the lending spread, in the same fashion with the liquidity rule.

The nominal interest rate, $R_{n,t}$ is given by the following standard Taylor-type rule

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\Big[\theta_{\pi,r}\log\left(\frac{\Pi_t}{\Pi}\right) \\ &+ \theta_{y,r}\log\left(\frac{Y_t}{Y}\right) + \theta_{dy,r}\log\left(\frac{Y_t}{Y_{t-1}}\right)\Big] + \epsilon_{MPS,t}\,; \quad \rho_r \in [0,1) \end{aligned} \tag{12}$$

Unlike rules studied in the NK literature which respond to the output gap and therefore a flexi-price version of the model, this rule makes no such demands on the policymaker and rational agents; it only requires knowledge of the model itself and its deterministic steady state. Schmitt-Grohe and Uribe (2007) refer to such rules as 'implementable'. $\epsilon_{MPS,t}$ is a monetary policy shock which is included for estimation but not for the welfare-optimized rules.

The link between nominal and real interest rates is given by the following Fisher relation for the ex post real interest rate for bonds purchased in period $t-1$:

$$R_t = \frac{R_{n,t-1}}{\Pi_t}.$$

A similar relation holds for the interest rate on liquidity.

$$R_t^M = \frac{R_{m,t-1}}{\Pi_t}.$$

2.3.2 The Liquidity Rule

Liquidity is provided by the central bank to the banking sector according to the rule $\chi_{m,t}$, defined as the fraction of the total bank assets financed through LTRO where $\chi_{m,t}$ =

$\frac{M_t}{L_t} \leqslant 1$. We calibrate $\chi_m = 0.1$ to match data from the LTROs period that turns out to be close to the value that maximizes household welfare in the non-stochastic steady state (see Section (4)). The liquidity rule $\chi_{m,t}$ responds, similarly to a policy rate Taylor rule above, to the variables' deviations form their steady state levels. The variables we choose are again: output and inflation, but with the addition of the lending spread.

Therefore the rule reads as follows:

$$\begin{aligned} log\left(\frac{\chi_{m,t}}{\chi_m}\right) &= \rho_\chi \log\left(\frac{\chi_{m,t-1}}{\chi_m}\right) + (1-\rho_\chi)\Big[\theta_{\pi,\chi}\log\left(\frac{\Pi_t}{\Pi}\right) + \theta_{y,\chi}\log\left(\frac{Y_t}{Y}\right) \\ &+ \theta_{dy,\chi}\log\left(\frac{Y_t}{Y_{t-1}}\right) + \theta_{sp,\chi}\log\left(\frac{R^K_{t+1} - R_{t+1}}{R^K - R}\right)\Big]; \quad \rho_\chi \in [0,1) \end{aligned} \tag{13}$$

Eliminating the persistence term and responses to output and inflation changes the rule collapses to the same liquidity rule introduced in Gertler and Karadi (2011) where central bank liquidity responds only to spread deviations.

2.4 The Government Budget and Economy Resource Constraints

Government collects lump sum taxes T_t to finance its public expenditures G_t. The central bank receives interest rate payments $R_{m,t}M_{t-1}$ from liquidity provision which comes at a cost $\Psi(M_t)$. The consolidated government budget constraint of the government and the central bank is therefore:

$$G_t + \Psi_t(M_t) = T_t + R_{m,t}M_{t-1} \tag{14}$$

and the economy's resource constraint is:

$$Y_t = C_t + C_{E,t} + G_t + \Psi_t(M_t) + I_t + \mu G(\bar{\psi}_t))R^K_t Q_{t-1}K_{t-1} + \alpha(u_t)K_{t-1}$$

where $C_{E,t}$ is the consumption of exiting entrepreneurs and which incorporates contract monitoring costs and capacity utilization from the core NK model.

2.5 Welfare

In order to rank alternative policies we use a welfare-based criterion based on the inter-temporal household expected utility with internal habit[6]

$$\Omega_t \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau U(C_{t+\tau}, C_{t+\tau-1}, H_{t+\tau}) \tag{15}$$

[6] Internal as opposed to external habit ensures that financial frictions are welfare-reducing.

which we can write in recursive form as:

$$\Omega_t = U_t + \beta \mathbb{E}_t \Omega_{t+1}. \tag{16}$$

2.6 Structural Shocks

The model is closed with eight exogenous AR(1) shock processes to technology, government spending, the real marginal cost (the latter being interpreted as a mark-up shock), the marginal rate of substitution, an investment shock, a risk premium shock, a shock to monetary policy and a risk shock. Therefore we have eight first order autoregressive processes for the variables $\{A_t, G_t, MS_t, MRSS_t, IS_t, RPS_t, MPS_t, \sigma_{\psi,t}\}$.

3 Data and Estimation

We estimate our model on quarterly Euro-area data from 1991Q1 to 2018Q4 using Bayesian techniques.[7] We use a total of eight observables in the estimation matching the number of exogenous AR(1) processes.. As is standard in the estimation of medium scale models, we include the real per capita growth rates of GDP, consumption, and investment, real wage growth, a measure of labour hours, the GDP deflator, and the ECB's policy rate. In order to take into account the unconventional monetary policy of the Euro Area we make use of the shadow rate by Wu and Xia (2016). This is available from 2004 onwards. For the time period 1991-2004 we use the policy rate of the ECB. Finally, we include the lending spread of the EA economy which is defined as the average lending rate minus the deposit rate.

In our model's estimation we do not use data for liquidity injections and we estimate the model as if liquidity injections were absent. We do this in order to perform our normative exercises for various liquidity rules and see which maximizes welfare.

Following the literature, some parameters of the model are calibrated to conventional values and also to match some Euro Area long term averages.

3.1 Calibration

The model's calibration is performed in order to match Euro Area stylized facts and is divided into conventional and banking parameters and it is show in Table 1. It follows broadly the calibration of the updated version of the New Area-Wide Model (NAWM), (Christoffel, Coenen, and Warne (2008), Coenen, Karadi, Schmidt, and Warne (2018)), the DSGE model of the ECB.

Banking parameter values are chosen in order to match specific Euro Area banking characteristics namely the banks' average leverage, lending spread and the bankers' plan-

[7] For a detailed analysis see An and Schorfheide (2007).

ning horizon. There are three parameters that characterise the behaviour of the banking sector in the model. This is the absconding rate θ, the fraction of entering bankers initial capital fund ξ_B , and the steady-state value of the survival rate, σ_B. We calibrate these parameters to match certain steady-state moments in the data and the moments reported in Coenen et al. (2018). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well as non-financial corporations, with weights equal to their share of assets in total assets between 1999Q1 and 2014Q4 according to the euro area sectoral accounts. The steady-state spread of the lending rate over the risk-free rate, $R_t^L - R_t$ is set to 1.656 percentage points on an annualised basis at the steady state, which is the average spread between the long-term cost of private-sector borrowing and the deposit rate for our sample period 1991Q1 to 2018Q4. The banks' planning horizon is set equal to 5 years. This moment targeting exercise leads to $\theta = 0.290, \xi_B = 0.005$ and $\sigma_B = 0.942$. These parameters are also in line with the related studies in the literature. Finally, we set the monitoring parameter that the central bank has on its loanable funds to the banking sector, ω, to values from 50% - 90%. A value 50% targets a steady state bond spread to half to this of the lending spread in line with Gertler and Kiyotaki (2010). In the following sections we experiment with this parameter to see its impact on our welfare results.

Entrepreneur specific parameters μ, ξ_E, σ_E, are the monitoring costs, their entry start up fund and their lifetime duration respectively. We calibrate the monitoring costs in order to match an annual probability of default of 3%. This is in line with Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015) where they calibrate a similar BGG-type model for the Euro Area. This target leads to monitoring costs, μ equal to 0.21 annually as assumed similarly by Christiano et al. (2014), Rannenberg (2016) and Clerc et al. (2015). Also we set the leverage in non-financial firm sector to 2, following Rannenberg (2016); this leads to a continuity probability of the entrepreneurs equal to $\sigma_E = 0.978$. Entrepreneurs' start up fund ξ_E is set such that the external finance premium as defined by (B.9) is close to the lending spread in equilibrium. Finally the idiosyncratic dispersion of the entrepreneurs σ_ψ is set to 0.27, very close to the estimate of Christiano et al. (2014) of 0.26.

The values for the share of capital α and the depreciation rate δ are chosen to be 0.33 and 0.025 respectively following the estimation results of Christoffel et al. (2008). Similarly, the value of β is assigned to 0.998, chosen to be consistent with the balanced growth relationship

$$\beta = \frac{\Pi}{R_n(1+g)^{-\sigma_c}} \tag{17}$$

with σ_c estimated and long term equilibrium values for growth (g), the inflation target (Π) taken directly from the Euro Area data as averages of our data sample 1991Q1 to 2018Q4. Finally, the government spending as a fraction of the GDP is set to 18% also following other studies for the Euro Area.

We calibrate the steady state such that the central bank provides some liquidity. The data average in the Euro area before 2010 is about 10% of the total banks' assets and therefore we set χ_m in the steady state equal to 0.1. Regarding the cost of the central bank intervention, we follow Gertler et al. (2012a) and we calibrate the parameters that govern the cost such as they provide an annualised liquidity cost of 10 basis points in steady state. We set τ_1 and τ_2 equal to 0.000255 and 0.0025 respectively.

Parameter	Description	Value
A. Preferences		
β	Discount factor	0.998
B. Technology		
α	Capital share	0.670
δ	Depreciation rate	0.025
C. Banks		
θ	Banker's absconding rate	0.290
σ_B	Exit probability: bankers	0.942
ξ_B	Entry start up fund fraction: bankers	0.005
ω	Absconding fraction for LTRO	0.500
D. Entrepreneurs		
ξ_E	Entry start up fund: entrepreneurs	0.005
σ_E	Exit probability: entrepreneurs	0.978
σ_ψ	Entrepreneur's idiosyncratic dispersion	0.2712
μ	Monitoring costs	0.2092
E. Liquidity Injections		
τ_1	Credit cost	0.000255
τ_2	Credit cost	0.0025
χ_m	Steady state liquidity level	0.1
F. Long Term Equilibrium		
$\bar{A}$	Steady state technology	1.000
Π	Gross inflation objective	1.005
g	Steady state growth	0.003
$\frac{G}{Y}$	Gov. spending over GDP	0.180

Table 1: Calibrated Parameter Values

3.2 Estimation

We estimate the rest of the parameters using Bayesian techniques. We use as many observables as shocks in the model which is consistent with the perfect information assumption.[8]

[8]Most DSGE models are still solved and/or estimated on the assumption that agents are simply provided with perfect information (henceforth PI) regarding the states including the exogenous processes, effectively

We treat our observable variables in order to match their data counterparts. Specifically for output, inflation, consumption and real wages we match the logarithmic of first differences of stationarized variables. For labour hours we use proportional deviations. The net interest rate in the model is matched with that for the data and the credit spread remain unchanged. Finally for inflation we match the logarithm of the gross rate. This implies the following measurement equations:

$$\begin{aligned}
\text{Real GDP growth} &= \log\left(\frac{Y_t}{Y_{t-1}}(1+g)\right) \\
\text{Real consumption growth} &= \log\left(\frac{C_t}{C_{t-1}}(1+g)\right) \\
\text{Real investment growth} &= \log\left(\frac{I_t}{I_{t-1}}(1+g)\right) \\
\text{Real wage growth} &= \log\left(\frac{W_t}{W_{t-1}}(1+g)\right) \\
\text{Labour hours} &= \frac{H_{d,t}-H_d}{H_d} \\
\text{Shadow Net interest rate} &= R_{n,t}-1 \text{ if } > 0; \text{the Wu-Xia Shadow Rate otherwise} \\
\text{Inflation} &= \log(\Pi_t) \\
\text{Lending Spread} &= R_{k,t}-R_t
\end{aligned}$$

We satisfy a balanced growth path by accounting for a deterministic trend in the growth rate g in our measurement equations for non-stationary data for Y_t, C_t, I_t and W_t which are stationarized in the model setup. We set the growth rate to the average quarterly growth for the Euro Area for the time interval we study which is $g = 0.366\%$.

Table 2 show our priors and posterior estimates. The posterior distributions of the parameters have been estimated using the random-walk Metropolis sampler, taking into account the system priors. The estimation results are based on a Markov chain with 100000 draws. The priors for the parameters of the real economy are set in line with Smets and Wouters (2007).

Our estimates are close to those from Coenen et al. (2018), who estimate a SW model variant with financial frictions in the banking sector for the Euro Area. Most notably, we find very similar values for the monetary rule component of the model. Both estimates for the inflation coefficient, θ_π, are above 2.5, which seems to be a Euro Area characteristic in

as an endowment. If we drop this implausible assumption we must consider a signal extraction problem under imperfect information (II) for the agents in the model. Fortunately we can retain the PI solution if we restrict ourselves to a class of models which are 'A-invertible' meaning that agents can infer the structural shocks from the information set assumed to be that of the econometrician. Levine, Pearlman, Wright, and Yang (2023) provide an A-invertibility condition that generalizes the "Poor Man's Invertibility Condition" of Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007). Using this we can show that in the model of this paper with eight shock processes and eight observables is indeed A-invertible. It follows that II and PI solutions coincide and the standard information assumption is valid in our model.

comparison with the literature on US data where these values are usually less than two. Similarities of the two models continue for the rest of the interest rate rule and the real economy parameter estimates.

			Prior		Posterior
Parameter	Description	Dist.	Mean	Std	Mean
A. Preferences					
σ_c	Relative risk aversion	$\mathcal{N}$	1.50	0.375	1.8037
ψ	Inverse Frisch elasticity	$\mathcal{N}$	2.00	0.750	1.3906
χ	Internal Habit formation	B	0.50	0.100	0.4788
ϕ_X	Adjustment costs	$\mathcal{N}$	2.00	0.750	2.0345
B. Wage and price set.					
ξ_p	Calvo scheme: prices	B	0.50	0.100	0.5193
ξ_w	Calvo scheme: wages	B	0.50	0.100	0.6065
γ_p	Indexation: prices	B	0.50	0.100	0.3610
γ_w	Indexation: wages	B	0.50	0.100	0.4823
C. Interest-rate rule					
ρ_r	Interest-rate smoothing	B	0.75	0.100	0.7139
θ_π	Response to inflation	$\mathcal{N}$	2.00	0.250	2.6101
θ_y	Response to output gap	$\mathcal{N}$	0.12	0.050	0.0530
$\theta_{\Delta y}$	Response to $\Delta(Y_{gap})$	$\mathcal{N}$	0.12	0.050	0.2049
D. Autocorr. parameters					
ρ_A	Technology	B	0.50	0.200	0.9520
ρ_G	Gov. spending	B	0.50	0.200	0.8409
ρ_{MCS}	Marginal cost	B	0.50	0.200	0.8905
ρ_{MRSS}	Marginal rate of subst.	B	0.50	0.200	0.9565
ρ_{MPS}	Monetary policy	B	0.50	0.200	0.3579
ρ_{RPS}	Risk premium	B	0.50	0.200	0.9836
ρ_{IS}	Investment	B	0.50	0.200	0.9825
ρ_{RS}	Risk	B	0.50	0.200	0.9669
E. Shock parameters					
σ_A	Technology	Γ^{-1}	0.001	0.020	0.0064
σ_G	Gov. spending	Γ^{-1}	0.001	0.020	0.0227
σ_{MCS}	Marginal cost	Γ^{-1}	0.001	0.020	0.0068
σ_{MRSS}	Marginal rate of subst.	Γ^{-1}	0.001	0.020	0.0196
σ_{MPS}	Monetary policy	Γ^{-1}	0.001	0.020	0.0019
σ_{RPS}	Risk premium	Γ^{-1}	0.001	0.020	0.0008
σ_{IS}	Investment	Γ^{-1}	0.001	0.020	0.0295
σ_{RS}	Risk	Γ^{-1}	0.001	0.020	0.0490

Table 2: Estimation results for the model. Notes: $\mathcal{N}$ stands for the Normal distribution, B for the Betta and Γ^{-1} for the inverted Gamma distribution.

3.3 Validation

The last step in our estimation exercise is the model's validation with the first two moments of the data counterparts of the observable variables we use. Table 3 shows the results.

Our estimated model produces long run averages close to the data counterparts. It is noteworthy that we do not include any data on net worth and loans in our estimation. Nevertheless, the model provides long run averages close to the real values of the two variables. Our estimation method is likelihood based where the basis of inference is the full range of empirical inferences implied by the model rather than second-moment based estimation as in GMM or SMM based on a subset of moments. Nevertheless, our standard deviation estimates of the chosen observables fit reasonably well in line with the data. Exceptions to this are the investment growth and the variable for net worth which again we do not use in our estimation procedure.

Variable	Mean		Std	
	Data	Model	Data	Model
Output Growth	0.0037	0.0037	0.0058	0.0081
Consumption Growth	0.0032	0.0037	0.0047	0.0050
Investment Growth	0.0032	0.0037	0.0170	0.0340
Wage Growth	0.0024	0.0037	0.0038	0.0053
Labour supply	0.0000	0.0000	0.0277	0.0313
Inflation	0.0076	0.0076	0.0062	0.0050
Shadow rate	0.0121	0.0155	0.0125	0.0089
Lending spread	0.0041	0.0086	0.0010	0.0054
Net Worth*	0.0048	0.0037	0.0800	0.0489
Loans*	0.0064	0.0037	0.0116	0.0065

Table 3: Model vs Data Moment Comparison. Variables with (*) are not included in the observable variables for the estimation.

4 Steady State Analysis: The Effect of Central Bank Monitoring

We first provide a detailed analysis on the two effects of a liquidity injection arising from the two frictions in our model. An increase in liquidity benefits the economy and is potentially welfare improving under two conditions. Firstly, its level needs to be high enough in order to loosen the GK friction and overcome the adverse effects of the BGG friction. Secondly, given that liquidity penalty costs are increasing in the value of liquidity, liquidity provision must be strong enough to additionally overcome the welfare reducing penalty costs.

We first focus on the steady state of the model and perform an exercise of the model's response to changes in the liquidity provision volume. The notion of liquidity provision here is a general one: we assume that χ_m,the ratio of liquidity to the total banks' asset, is deterministic and can vary in the grid [0, 0.9].[9] When the ratio is non-negative, then liquidity is provided by the central bank authority to the banks. We compute the steady state equilibrium for each input of the grid and report how our welfare measure responds along with some macro variables of interest and the liquidity costs. First we ignore the liquidity costs by imposing $\Phi = 0$ and then we introduce the latter. Figures 1 and 2 compare these two cases.

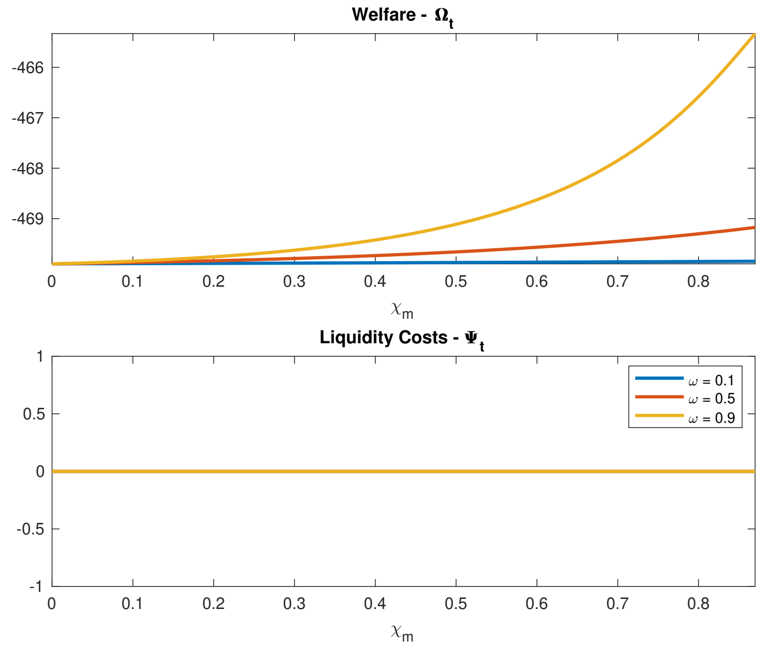

Figure 1: Welfare conditional on central bank liquidity: zero liquidity costs

[9]We notice that for value of χ_m above 0.9 deposits turn negative and therefore we do not include the values above 0.9.

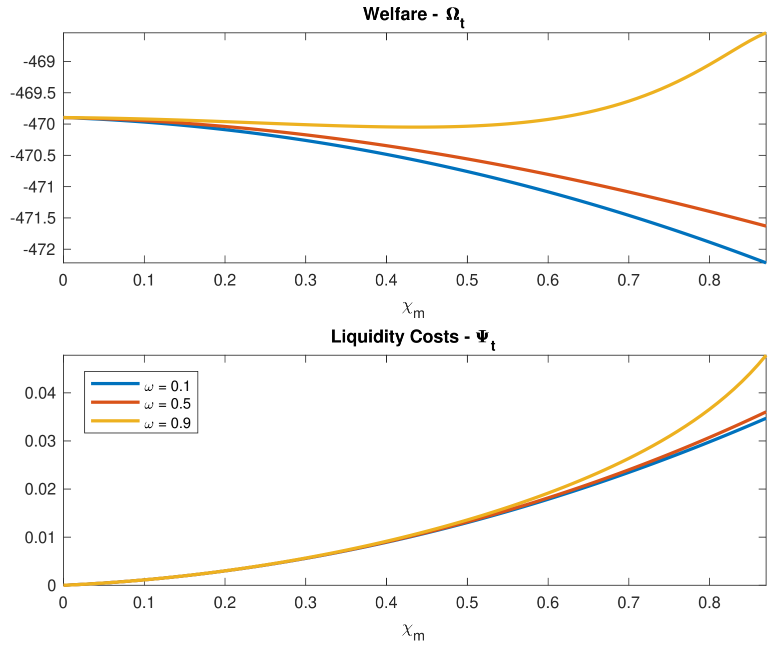

Figure 2: Welfare conditional on central bank liquidity: non-zero liquidity costs

These two figures show the welfare, Ω_t, path as a function of the liquidity ratio χ_m.[10] The figure is plotted for three different values of ω: (0.1,0.5,0.9), the monitoring ability of the central bank to the liquidity funds. We will show that this is an important parameter that can change the efficacy of the liquidity rule. In Figure 2, with non-zero liquidity costs, for low values of the monitoring parameter a deviation of liquidity provision from zero is welfare reducing. The combined mitigation of the GK and BGG frictions cannot then compensate for the penalty costs.

This result flips when we increase banks' monitoring parameter to the higher value, 90%, plotted under the yellow line. Welfare is increasing for higher values of liquidity provision and reaches its maximum at the end of the grid. The reasoning of why this occurs at a high level of monitoring ability is the following. An increase in the monitoring parameter increases the effectiveness of liquidity in the bank's financial constraint as shown by the banks' leverage constraint (11). For low values of monitoring ability, due to the presence of the liquidity costs, the positive effect from the banks' constraint loosening cannot lead to a welfare enhancing effect of liquidity injections. Nevertheless, a higher monitoring value such as the one plotted, increases the effectiveness of the liquidity provision and leads to a loosening of the constraint that is high enough to counteract the

[10]The penalty function is parameterized according to the baseline calibration to have a 10 basis points cost for a $\chi_m = 10\%$. We provide the same figure for an alternative parameterization of the penalty function that yields a 25 and 50 basis points cost in Appendix F. Results remain qualitatively similar. Nevertheless, naturally as the penalty costs would increase, the welfare inducing effects would vanish.

liquidity costs and therefore more liquidity leads to higher welfare.

Figures 3 and 4 show the paths of key non-financial and financial variables, conditional to the liquidity provision, that demonstrate the two forces in the model. Figure 3 shows how liquidity injections increase credit, mitigates the BGG friction leading to an increase in output and investment. When the liquidity fraction turns negative, there are no significant changes in the variables' values so we focus on the domain of $\chi_m > 0$. From Figure 4, as liquidity provision increases, the default probability shifts upwards. At the same time, both spreads, $R^K - R$ and $R^B - R$, fall due to the loosening of the banks' financial constraint. This explains the increase in investment from the liquidity injection which occurs despite the increase in the the external finance premium $\rho(\psi) = R^K/R^B$ and the increase of the default probability because even though R^B and R^K fall, R^B falls more than R^K. These effects shed light on how the two financial frictions interact with each other. Liquidity relaxes the GK constraint and both spreads fall. Despite the increase of the default probability the fall in the spreads leads to an increase in investment thus offsetting the BGG friction. Whether these two effects can be welfare enhancing depends on the monitoring ability of the central bank and the costs of liquidity intervention policy.

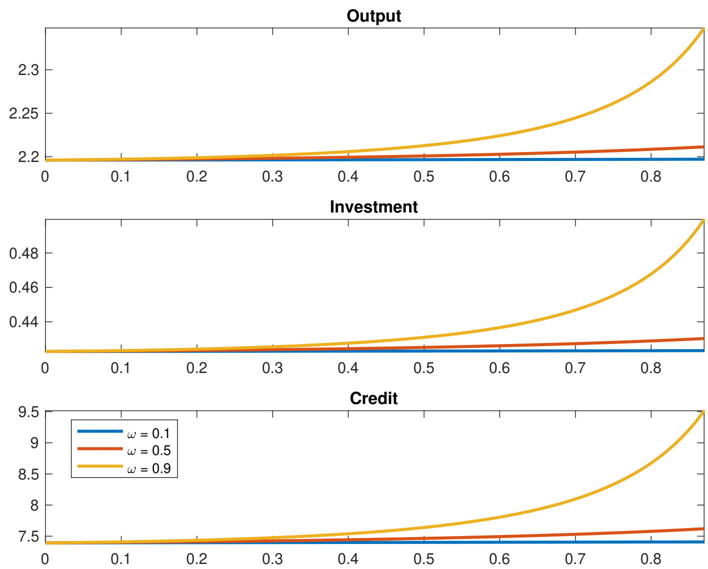

Figure 3: Financial Variables conditional on central bank liquidity χ_m

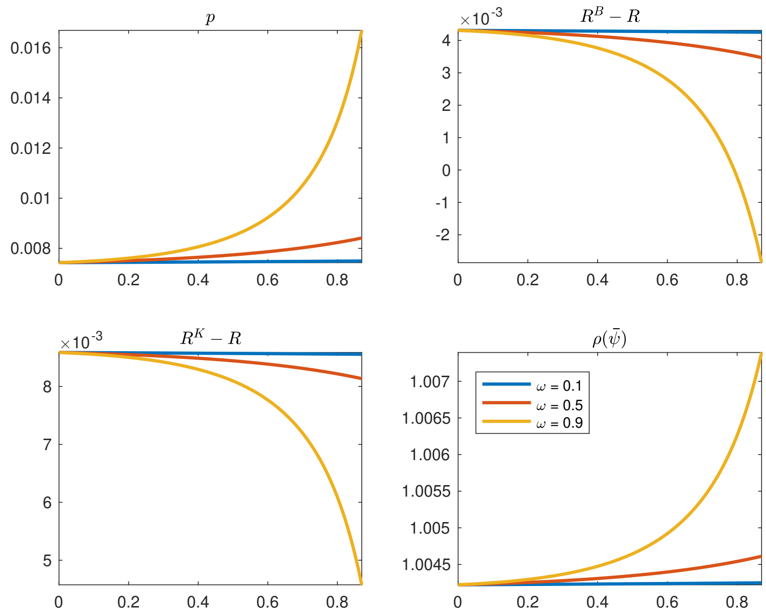

Figure 4: Financial Variables conditional on central bank liquidity χ_m

5 Welfare-Optimal Simple Rules

The concept and computation of optimized simple rules in an estimated model is central to this paper. We first make some general points before turning to the full delegation game and the results. We follow Schmitt-Grohe and Uribe (2007) quite closely, but with some important differences.

For optimal policy purposes we remove the monetary policy shock $\log(MPS_t)$ and re-parameterize the monetary and liquidity rules by the following Taylor-type rules

$$
\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_{\pi,r} \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_{y,r} \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy,r} \log\left(\frac{Y_t}{Y_{t-1}}\right) \\
\log\left(\frac{\chi_{m,t}}{\chi_m}\right) &= \rho_\chi \log\left(\frac{\chi_{m,t-1}}{\chi_m}\right) + \alpha_{\pi,\chi} \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_{y,\chi} \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy,\chi} \log\left(\frac{Y_t}{Y_{t-1}}\right) \\
&+ \alpha_{sp,\chi} \log\left(\frac{R^K_{t+1} - R_{t+1}}{R^K - R}\right) \; ; \quad \rho_r, \rho_\chi \in [0,1] \qquad (18)
\end{aligned}
$$

which allows for the possibility of an integral rule with $\rho_r = 1$ and/or $\rho_\chi = 1$. Let $\rho \equiv [\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}, \; \rho_\chi, \alpha_{\pi,\chi}\alpha_{y,\chi}, \alpha_{dy,\chi}]$ be the policy choice of feedback parameters that defines the exact form of the combined monetary and liquidity rules. We restrict ourselves to a class of possible rules that are locally saddle-path stable in the vicinity of the non-stochastic (deterministic) steady state. We denote this sub-set of rules by S; thus $\rho \in S$.

We begin by defining the inter-temporal household welfare at time t in recursive Bellman stationarized form in a symmetric equilibrium form as:

$$\Omega_t = U_t(C_t, C_{t-1}, H_t^s) + \beta_g \mathbb{E}_t [\Omega_{t+1}] \tag{19}$$

where β_g is a growth-adjusted discount factor defined by $\beta_g \equiv \beta(1+g)^{1-\sigma}$.

Optimal monetary policy at time $t = 0$, sets steady-state values for the nominal interest rate instrument $R_{n,t}$, denoted by R_n given initial values for the predetermined variables Z_0, solves the maximization problem :

$$\max_{\rho \in S} \Omega_0(\mathsf{Z}_0, R_n, \rho) \tag{20}$$

In fact the long-run (steady state) gross inflation rate target in the rule which we take to be $\Pi \geqslant 1$ (ruling out a liquidity trap) uniquely pins down the rest of the steady state so we can rewrite (20) as

$$\max_{\rho \in S} \Omega_0(\mathsf{Z}_0, \Pi, \rho) \tag{21}$$

But this is a *conditional* and *time-inconsistent criterion* as the optimized rule at time t becomes

$$\max_{\rho \in S} \Omega_t(\mathsf{Z}_t, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{Z}_t, \Pi) \tag{22}$$

and there emerges an *incentive to re-optimize.*

We remove one source of time-inconsistency by choosing a welfare conditional on being at the steady state $\mathsf{z}_t = \mathsf{z}$, which is policy-invariant, and the choice of Π which is a policy choice.[11]. The optimization problem then becomes

$$\max_{\rho \in S} \Omega(\mathsf{z}, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{z}, \Pi) \tag{23}$$

Since z is policy-invariant so is welfare Ω. In what follows we simply write $\rho = \rho(\Pi)$ which is now the time-less optimized rule. Thus welfare at the steady state is maximized *on average* over all realizations of the shocks driving the exogenous stochastic processes give their deterministic steady states.[12] The optimal ρ^* is computed using a *second-order perturbation solution* [13] But there are *no ZLB considerations* for the nominal interest rate as yet. This leads us to the delegation game.

[11]This follows Schmitt-Grohe and Uribe (2007) and is the *timeless* criterion proposed by Woodford (2003), Chapter 7, based on Levine and Currie (1987)

[12]The maximization of the unconditional welfare under exogenous uncertainty can be compared with the optimal strategy for the board game backgammon whose outcome depends on throws of dice as well as skill. This contrasts with deterministic games such as chess.

[13]This is implemented in a Dynare program that calls a matlab subroutine **fminconc** that finds a constrained minimum of a function of several variables. A general toolkit for any DSGE model set-up is available for this.

6 The Delegation Game

We first consider the optimized interest rate rule with no liquidity injections by the central bank. We introduce ZLB considerations for the nominal interest rate rule following the methodology set out in Deak et al. (2023). We examine the solution of the two-stage delegation game in the estimated model in the case where the choice of response parameters ρ for both monetary and liquidity policy is delegated to a central bank with a 'modified' objective of the form (24) where U_t is household utility and the rule takes the form (18). The equilibrium of this ZLB delegation mandate is solved by backward induction in the following two-stage delegation game.

1. **Stage 1**: The Government (the leader) chooses an acceptable per period probability of hitting the ZLB, a trend inflation rate and designs the optimal loss function in the mandate. The optimal steady state inflation rate consistent with stage 1 is chosen also by the Government.

2. **Stage 2**: The central bank (the follower) receives the mandate in the form of a modified purely stochastic welfare criterion of the form $\Omega_t(\mathsf{Z}, \Pi, \rho)$ of the form (19) with an additional penalty to limit the variance of the nominal interest rate rule. Welfare is then optimized with respect to $\rho \in S$ resulting in an optimized simple rule.

This delegation game is solved by backwards induction as follows:

6.1 Stage 2: The Central Bank's Choice of Rule

Given a steady state inflation rate target, Π, the Central Bank (CB) receives a mandate to implement the rule (18) and to maximize with respect to $\rho \in S$ a modified welfare criterion

$$\begin{aligned} \Omega_t^{mod} &\equiv \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\left(U_{t+\tau} - w_r\left(R_{n,t+\tau} - R_n\right)^2\right)\right] \\ &= \left(U_t - w_r\left(R_{n,t} - R_n\right)^2\right) + \beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{mod}\right] \end{aligned} \tag{24}$$

One can think of this as a mandate with a penalty function $P = w_r\left(R_{n,t} - R_n\right)^2$, penalizing the variance of the nominal interest rate with weight w_r.[14] Both the symmetric and

[14] This closely follows the approximate form of the ZLB constraint of Woodford (2003) and Levine, McAdam, and Pearlman (2008). Following Den Haan and Wind (2012), an alternative mandate that only penalizes the zero interest rate in an asymmetric fashion is $P = P(a_t)$ where the occasionally binding constraint is $a_t \equiv R_{n,t} - 1 \geqslant 0$ with

$$P = P(a_t) = \frac{\exp(-w_r a_t)}{w_r} \tag{25}$$

asymmetric forms of a ZLB mandate result in a probability of hitting the ZLB

$$p = p(\Pi, \rho^*(\Pi, w_r)) \tag{26}$$

where $\rho^*(\Pi, w_r)$ is the optimized form of the rule given the steady state target Π and the weight on the interest rate volatility, w_r.

Given a target low probability $\bar{p}$ and given w_r, $\Pi = \Pi^*$ is chosen so satisfy

$$p(R_{n,t} \leqslant 1) \equiv p(\Pi^*, \rho^*(\Pi^*, w_r)) \leqslant \bar{p} \tag{27}$$

This then achieves the ZLB constraint

$$R_{n,t} \geqslant 1 \text{ with high probability } 1 - \bar{p} \tag{28}$$

given both w_r and $\Pi = \Pi^*(\bar{p}, w_r)$ where $R_{n,t}$ is the nominal interest rate.

6.2 Stage 1: Design of the Mandate

The policymaker first chooses a per period probability $\bar{p}$ of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the actual household intertemporal welfare

$$\Omega_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau U_{t+\tau} \right] = U_t + \beta(1+g)^{1-\sigma} \mathbb{E}_t \left[\Omega_{t+1} \right] \tag{29}$$

with respect to w_r.

This two-stage delegation game defines an equilibrium in choice variables w_r^*, ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (28).

6.3 The Liquidity Rule

For the liquidity rule there is an upper bound constraint $\chi_{m,t} \equiv \frac{M_t}{L_t} \leqslant 1$ where at the upper bound the central bank provides all the credit required by entrepreneurs. Since $M_t < 0$ is possible for the case where banks go short and lend to the central bank there is no ZLB for this instrument. In order to impose this constraint we extend the penalty function to

and chooses a large w_r. $P(a_t)$ then has the property

$$\begin{aligned} \lim_{w_r \to \infty} P(a_t) &= \infty \text{ for } a_t < 0 \\ &= 0 \text{ for } a_t > 0 \end{aligned}$$

Thus $P(a_t)$ enforces the ZLB approximately but with more accuracy as w_r becomes large. Stages 2–1 then proceed as before, but now confined to a large w_r which will enable Π to be close to unity. This alternative mandate leads to similar results and conclusions.

$P = w_r\left(R_{n,t} - R_n\right)^2 + w_\chi\left(\chi_{m,t} - \chi_m\right)^2$ with a modified welfare at stage 2 becoming

$$\Omega_t^{mod} \equiv \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^\tau\left(U_{t+\tau} - w_r\left(R_{n,t+\tau} - R_n\right)^2 - w_\chi\left(\chi_{m,t+\tau} - \chi_m\right)^2\right)\right] \quad (30)$$

There are now *two* probabilities to consider: one is as before, the probability of $R_{n,t}$ hitting its lower bound

$$p_{ZLB} = p_{ZLB}(\Pi, \chi_m, \rho^*(\Pi, \chi_m, w_r, w_\chi)) \quad (31)$$

where $\rho^*(\Pi, \chi_m, w_r, w_\chi)$ is the optimized form of the rule given the steady state target (Π, χ_m) and the weights on the interest rate and liquidity volatility, (w_r, w_χ). The other probability is the probability of $\chi_{m,t}$ hitting its upper bound

$$p_{UB} = p_{UB}(\Pi, \chi_m, \rho^*(\Pi, \chi_m, w_r, w_\chi)) \quad (32)$$

Given low target probabilities $(\bar{p}_{ZLB}, \bar{p}_{UB})$ and given (w_r, w_χ), $\Pi = \Pi^*$ and $\chi = \chi^*$ are chosen so satisfy

$$p(R_{n,t} \leqslant 1) \equiv p_{ZLB}(\Pi^*, \rho^*(\Pi^*, w_r, w_\chi)) \leqslant \bar{p}_{ZLB} \quad (33)$$

$$p(\chi_{m,t} \geqslant 1) \equiv p_{UB}(\Pi^*, \rho^*(\Pi^*, w_r, w_\chi)) \leqslant \bar{p}_{UB} \quad (34)$$

This then achieves both the ZLB constraint for $R_{n,t}$ and the upper bound for $\chi_{m,t}$

At Stage 1 of the game, the policymaker first chooses a per period probability $\bar{p}_{ZLB}, \bar{p}_{UB}$ which define the tightness of the both constraints. Then it maximizes the actual household intertemporal welfare (29) as before but now with respect to $(w_r, w_{chi}$. This two-stage delegation game defines an equilibrium in choice variables (w_r^*, w_χ^*), ρ^* and (Π^*, χ_m^*) that maximizes the true household welfare subject to the both constraints.

7 Optimized Liquidity and Interest Rate Rules

This section presents the main results of our work regarding the optimized liquidity rules. We provide our computations of the optimized liquidity rule alongside an optimized monetary rule subject to the ZLB and upper bound constraints that maximizes welfare. Our main finding is that the liquidity rule is welfare increasing relative to the no-rule case, but this is conditional to the high level of monitoring by the central bank. We show that when this is below a threshold, liquidity injections are then welfare reducing. Additionally, we provide the determinacy properties of the two rules and lastly we show the impulse responses of our economy to a risk shock when the monetary and liquidity rules are set to their optimized value.

7.1 Optimized Welfare Results

In order to assess the differences between the two economies with and without an optimized liquidity rule we proceed as follows. Given a particular equilibrium for C_t and H_t and single-period utility, $U_t = U(C_t, C_{t-1}, H_t)$ we then compute CE_t, the increase in the given by a 1% increase in consumption, by defining the consumption equivalence (CE) variable:

$$\begin{aligned} CE_t &\equiv U_t(1.01\,C_t, 1.01\,C_{t-1}/(1+g), H_t) - U_t \\ &+ \mathbb{E}_t\left[(1+g_{t+1})\beta_{g,t+1}CE_{t+1}\right] \end{aligned}$$

Then we use the deterministic steady state of CE_t, CE, to compare the welfare outcome compared with a baseline. If these two values are Ω_2 and Ω_1 respectively, the consumption equivalent variation is then given by $CEV = \frac{\Omega_2 - \Omega_1}{CE}$. Table 4 has the results of this exercise.

	The Optimized Simple Monetary and Liquidity Rules															
	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	5	0.00002	1.715	0.721	-0.388	-1.115	-5	0.00000007	1.004	-472.203	-0.005	0.01	15	0.1	0.1
(b)	1	5	0.000002	1.687	0	0	0	0	0	1.004	-472.243	-0.016	0.01	25	0	0.1
(c)	0.806	5.000	0.011	1.894	0	0	0	0	0	1.000	-472.193	-0.003	0.073	0	0	0.1
(d)	0.791	5.000	0.014	1.881	0	0	0	0	0	1.000	-472.183	0.000	0.074	0	0	0
	Estimated model															
	ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	ρ_r^{l*}	$\frac{\alpha_\pi^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_y^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{dy}^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{sp}^{l*}}{1-\rho_r^{l*}}$	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(e)	0.713	2.610	0.053	0.204	0	0	0	0	0	1.0073	-472.337	-0.041	0.007	0	0	0
(f)	0.713	2.610	0.053	0.204	0	0	0	0	0	1.000	-472.200	-0.005	0.101	0	0	0
	Original Taylor Rule															
	ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	ρ_r^{l*}	$\frac{\alpha_\pi^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_y^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{dy}^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{sp}^{l*}}{1-\rho_r^{l*}}$	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(g)	0	1.5	0.5	0	0	0	0	0	0	1.0073	-490.807	-4.952	0.2408	0	0	0
(h)	0	1.5	0.5	0	0	0	0	0	0	1.000	-484.015	-3.146	0.3218	0	0	0

Table 4: Welfare Analysis: $\omega = 0.9$

As explained previously in Section 6, the policymaker chooses a per period probability of hitting the ZLB and designs the optimal loss function in the delegation game. In these results, in rows (a) and (b), we only consider a probability $p_{zlb} = 0.01$ per period (a quarter) thus imposing an increasing severe constraint of a zero lower bound episode of once every 20 quarters (5 years) to 100 quarters (25 years). But a looser constraint could be assumed.

So far we have only discussed the lower bound on the nominal interest rate and that leaves the question of the upper bound on $\chi_{m,t} = \frac{M_t}{L_t} \leqslant 1$ which we impose by allowing $w_\chi > 0$ in (30). However this upper bound is different from the ZLB constraint in that by construction $M_t \leqslant L_t$ unlike $R_{n,t} \geqslant 1$ which can be (and has been for some central banks) violated. We therefore choose $\bar{p}_{UB} = 10^{-6} \ll 0.01$ as a soft constraint for the liquidity ratio, alongside our calibrated choice $\chi_m = 0.1$ to fit Euro area data during the LTROs period (see Section 3.1).[15]

We set as the benchmark model the one without a liquidity rule, nor a ZLB constraint, but with an optimized monetary rule which is column (d) in table 4. Therefore, for this case (in red) we have a consumption equivalent variation (CEV) of 0. The remaining combinations of the rules we consider are compared to this one. For the optimization exercise we impose a lower bound $\alpha_y^l \geqslant -5$ which turns out to be binding. The main result comes from comparing the case with no liquidity rule but with a ZLB constraint, with the case when we allow for an optimized liquidity rule together with a optimized monetary rule and a ZLB constraint. These are the columns (b) and (a) respectively. Shifting from (b) to (a) then results to a welfare gain of 0.015% in consumption equivalent terms.

We also compare our estimated model (columns (e) and (f)) which run with an estimated but not welfare-optimized monetary rule with our welfare optimized liquidity rule models. The estimated models perform worse than the benchmark model (d) and are welfare reducing. Row (e) sets inflation at its the empirical trend $\Pi^* = 1.0073$. Then compared with row (a) we see a welfare loss of 0.037 % in CEV terms for the empirical estimated rule.

So far, welfare differences in CEV terms are small which reflects the low welfare costs of business cycles pointed out by Lucas (1987). In the context of our estimated NK model this is the consequence of efficient stabilization policy conducted using welfare-optimized rules. To see this we examine the performance of the *original (inefficient) Taylor rule* with its parameters taken from Taylor (1993) and *not* optimized. From rows (g) and (h) we see that there is a large welfare cost associated with the original Taylor rule of approximately 3.15 to 4.95 CEV% for the same target inflation rates and a very high probability of the nominal interest rate hitting the ZLB. This indicates that the welfare costs can be high and furthermore the inertia term on nominal interest rate, which is absent in the original Taylor rule, plays a crucial role in stabilizing the economy, raising welfare and lowering the possibility of the ZLB episode.

The results in (a) show that the optimized liquidity provision attaches a negative weight on prices and output changes but a positive, although small, weight to changes in the credit spread. Given that both rules are activated, the liquidity rule provides determinacy.

The monitoring parameter of the central bank, ω is crucial for our welfare results. Specifically, although with a high monitoring parameter value of $\omega = 0.9$ our welfare-optimized monetary and liquidity rules are welfare enhancing as shown, this is not the case for a low monitoring parameter. The intuition is the following. Liquidity loosens the GK banks' friction and stimulates the macro-economy after a negative supply or demand-side shock. At the same time, although the probability of default increases, the

[15] Appendix G explores higher values of χ_m which slightly increase welfare but at the expense of raising the $\bar{p}_{UB}$.

lower credit spread brought about by the liquidity injection increases aggregate investment albeit for a smller proportion or remaining firms. But a high monitoring parameter leads to a stronger mitigating effect increased liquidity on the the GK friction by loosening the banks' constraint enough so the gains outweigh the implementation costs resulting in a welfare improvement.

We quantify this difference by performing the same exercise as above but with a different, lower, level of central bank monitoring. Specifically now we change ω from 0.9 to 0.5. The results are shown in Table 5 where for the optimization exercise we now impose an upper bound $\alpha_\pi \leqslant 5$ which turns out to be binding. Here, we have not repeated the estimated model results since they are identical to Table 4. The model with the welfare optimized liquidity injections (column (a)) are, under the new parametrization, are no longer welfare improving. Specifically it produces a welfare *loss* of 0.016% in consumption equivalent terms compared to the benchmark model specification (column (d)) which is better off without liquidity.

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	1.9	0.0000003	0.675	0.700	-2	-0.757	-2	0.0000001	1.004	-472.301	-0.0315	0.01	70	0.1	0.1
(b)	1	5	0.00001	1.686	0	0	0	0	0	1.004	-472.275	-0.025	0.01	20	0	0.1
(c)	0.800	5	0.013	1.890	0	0	0	0	0	1.000	-472.226	-0.011	0.073	0	0	0.1
(d)	0.794	5	0.014	1.881	0	0	0	0	0	1.000	-472.183	0.000	0.074	0	0	0

Table 5: Welfare Analysis: $\omega = 0.5$

Three important results stand out from tables 4 and 5. First, to summarize our comments above, when the monitoring activity is low ($\omega = 0.5$), the economy is better off without the rule owing to the fact that the BGG friction dominates. By contrast when the monitoring activity is high ($\omega = 0.9$), the BGG friction no longer dominates. These results imply that the liquidity rule can be welfare enhancing *but only if the central bank monitoring ability is high.*

Second, both the monetary and liquidity optimized rules are inertial (that is have a have a high persistence parameter) and involve a strong response to output growth and the inflation coefficient relative to the other components of the rule. For the case of liquidity rule in almost all cases the persistence parameter is very high and close to one.

Third, the inclusion of the response to spread deviations in the interest rate monetary rule does not have a significant effect as shown by the very low weight it gets in our computations.

In Appendix H, for completeness, we present the results for values of ω equal to in the range [0.6(0.1)0.8] which lies in the middle of the results shown here. The model with the optimized liquidity rule remains welfare enhancing compared to the benchmark, but much less so than in the case of a high monitoring parameter.

7.2 Determinacy and Stability Analysis

In addition to computing welfare-optimal parameters in the policy rules it is useful for policy advice to explore the available entire policy space that yields determinacy and stability. Figure 5 depicts the these regions for different pairs of all the liquidity and monetary rule parameters. Whereas up to now we have imposed $\alpha_{sp} = 0$ in the monetary rule, we now allow for a response to the spread for that rule as well as in the liquidity rule. On the x-axis are the monetary rule parameters, while in the y-axis the liquidity rule ones. We choose the same grid for each parameter of [-10,10] apart from the autoregressive coefficient of the rules and we set the monitoring value to its high value, $\omega = 0.9$. At each iteration of a parameter the remaining parameters are set at their optimized values as in Table 4. The blue are depicts the determinacy region where the light the regions were no determinacy occurs.

Part a. of the figure 5 shows the couple α_{sp} and α^l_{sp}; the spread coefficients in the two rules. There is an inverse relationship between the two parameters. Determinacy occurs when the monetary rule coefficient is positive and the liquidity rule negative and also when the liquidity rule coefficient is positive and the monetary rule negative. Part b. shows the inflation coefficients of the two rules. Determinacy is achieved for any combination that includes a positive response of the monetary rule to inflation. Part c. of the figure shows the determinacy regions for the couples α_y and α^l_y, the output coefficients for both rules. A higher response of the monetary rule to output is more likely to provide stability in the system. A same inference comes from part d. regarding the coefficient on the output changes. Lastly, part e. shows the determinacy region for the two autoregressive parameters. Given that the rest of the parameters are in their optimized values, any couple of ρ and ρ^l provides determinacy.

The main conclusion from this section is that it confirms that the combination of monetary and liquidity rules that include a response to the spread in both rules are robust in the sense by that they allow for a large space of determinate and stable choice of response parameters in the region of those that are welfare-optimal.

7.3 Impulse Responses

We complete our analysis by showing the impulse responses of the model to a risk shock. A risk shock is an unanticipated increase in the idiosyncratic dispersion of an entrepreneur similarly to Christiano et al. (2014). We choose this shock since it directly affects the entrepreneurs' probability of default and harms their networth. Therefore more liquidity and more loans would lead to higher default and more economic contraction if the BGG friction is stronger. We test this by considering two model versions. The first is our estimated

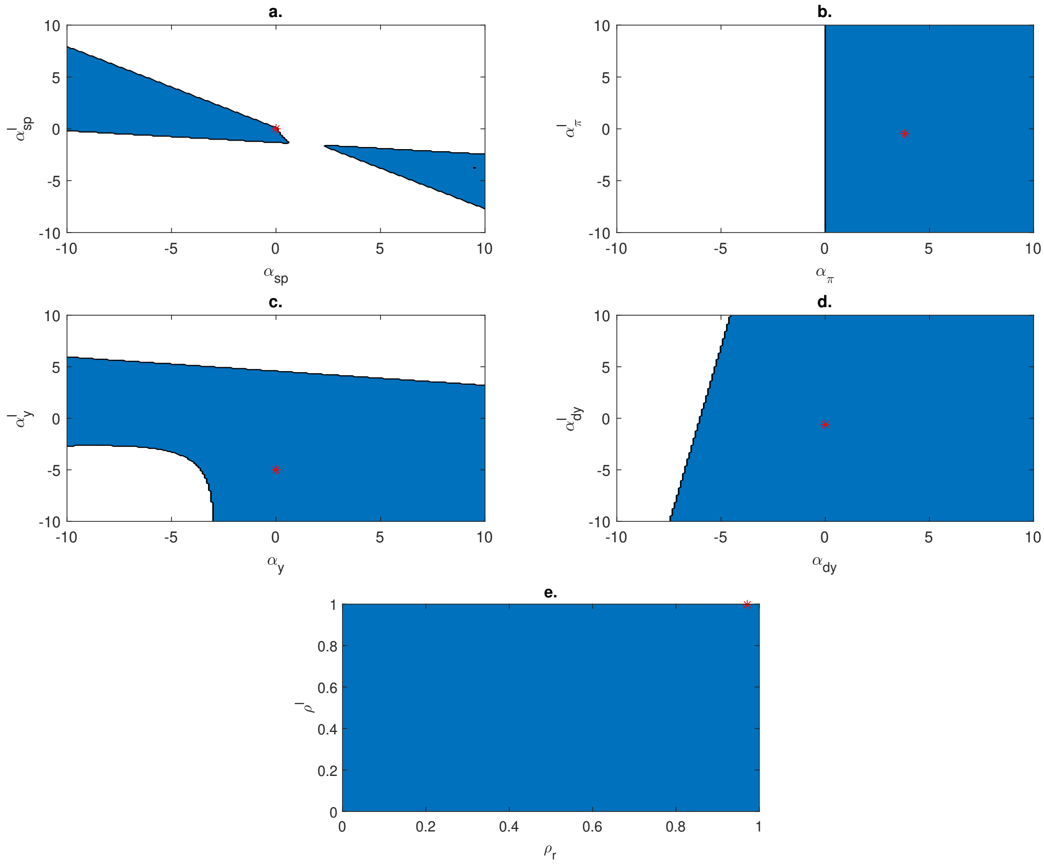

Figure 5: Determinacy and Stability Analysis for the monetary and liquidity rule feedback parameters. Notes: Remaining parameters at their optimized values as in Table 4. The blue area corresponds to the determinacy region while the light are in the indeterminacy region. The red star shows the optimized value for each parameter.

model with the estimated monetary rule and no liquidity rule. The second specification is the same but under welfare optimized liquidity and a monetary rules as shown in the previous subsection. Figure 6 displays the dynamic responses of various variables to the shock. The blue thick line shows the responses under no liquidity policy intervention, labelled as "Estimated Model" while the yellow dotted line shows the responses of the model when we consider our optimized policy rules labelled as "Optimized Policy".

The impulse responses produce a direct sharp increase in the default probability of the entrepreneurs in both modelling specifications. Owing to this, the credit spread, the difference between the loan and the deposit rate, increases and banks provide less credit leading to a reduction in investment and output and economic contraction. To counteract the economic downturn, the central bank adapts a policy rate reduction that according to

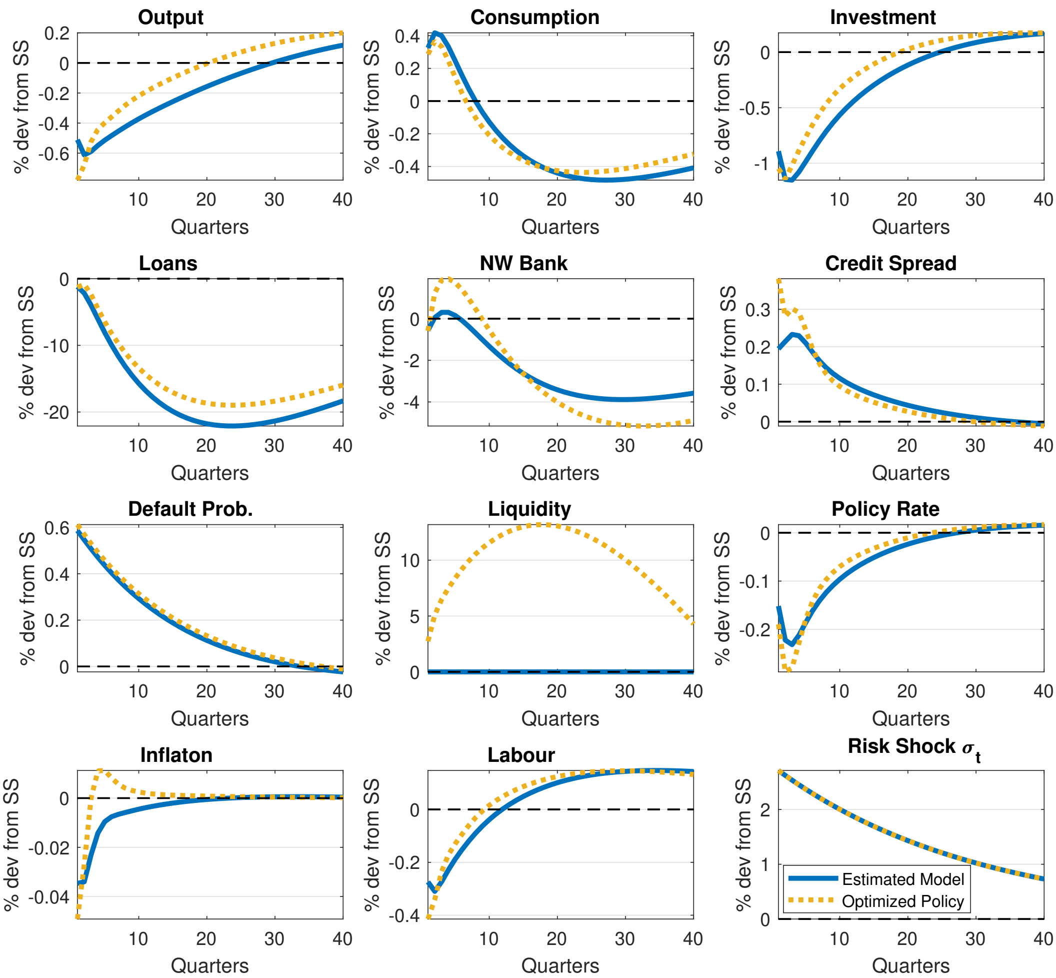

Figure 6: Impulse Responses to a Risk Shock.

the Taylor rule accommodates the fall in output and inflation. This makes them gradually return back to their steady state levels.

The responses of the estimated model and the model with the optimized rules are similar. Nevertheless, the drop in the values of the macro variables is much smaller for the optimized policy rules model. A liquidity rule that responds to all three variables' deviations, namely output, inflation and spread changes, manages to keep the economy closer to the steady state level and alleviate some of the negative consequences of the shock. Liquidity increases at about 10% higher than its steady state value. Note than in the estimated model there is no liquidity response since we do not include a liquidity rule. Although there is not much of a change in the default probabilities compared to the estimated model, investment and thus output decrease less due to the stabilizing forces of the two rules. The credit spread increases but falls faster than the estimated model. The policy rate decreases even more than the estimated model and this makes inflation to return quickly to its steady state value.

8 Conclusions

This paper has employed a medium-sized NK model estimated by Bayesian methods to study a combination of interest rate and liquidity rules. The novel feature of the model is the combination of two financial frictions, one for the bank-household side and one for the bank-firm side. These two financial frictions are modelled using the frameworks of Gertler and Kiyotaki (2010) and Bernanke et al. (1999a) respectively. The motivation for including both these features is that the implementation of the liquidity rule is welfare-enhancing for the first of these frictions but welfare-reducing for the second. The reason for this is that on the household side liquidity injections by the central bank bypasses the financial friction, but on the firm side increases the probability of default by firms.

Our main results are first, we find a welfare-optimized combination of rules where the welfare benefits of the liquidity rule outweigh the implementation costs, *but only if the monitoring ability of the policymaker is high.* Second, both the monetary and liquidity rules are highly inertial and involve a strong response to output growth, but a very small response to the interest rate spread. Third, including a response to the spread in both rules allows for a large policy space of determinate and stable rules.

The focus of our paper is the interaction of conventional monetary policy and liquidity injections, but the general framework and methodology is well-suited for other dimensions of policy. In particular, it would be interesting to compare our liquidity policy with Quantitative Easing (QE) in the form of asset purchase policies, such as the Asset Purchase Programme of the ECB, which have also been extensively employed to alleviate market frictions. In the context of our model for a QE scenario, as for LTROs, the central bank's asset purchases loosens the banks' leverage constraint stemming from the GK friction, thereby increasing credit provision. The ultimate outcome of QE would then depend on the interplay between the GK and BGG frictions, which could lead to either a welfare-enhancing or welfare-reducing impact. A critical aspect in the QE context, analogous to the monitoring parameter in our present framework, would be the risk weight parameter assigned to the bonds held by banks. A higher risk weight on government bonds would result in a more effective QE policy, thereby mitigating any negative outcome associated with the higher default probability from increased liquidity.

References

An, S. and F. Schorfheide (2007): "Bayesian analysis of DSGE models," *Econometric Reviews*, 26, 113–172.

Angelini, P., S. Neri, and F. Panetta (2014): "The interaction between capital requirements and monetary policy," *Journal of money, credit and Banking*, 46, 1073–1112.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

Bernanke, B., M. Gertler, and S. Gilchrist (1999a): "The Financial Accelerator in Quantitative Business Cycles," in *Handbook of Macroeconomics, vol. 1C*, ed. by M. Woodford and J. B. Taylor, Amsterdam: Elsevier Science.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999b): "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, 1, 1341–1393.

Bocola, L. (2016): "The Pass-Through of Sovereign Risk," *Journal of Political Economy*, 124, 879–926.

Boehl, G., G. Goy, and F. Strobel (2021): "A structural investigation of quantitative easing," *The Review of Economics and Statistics*, 1–45.

Cahn, C., J. Matheron, and J.-G. Sahuc (2017): "Assessing the Macroeconomic Effects of LTROs during the Great Recession," *Journal of Money, Credit and Banking*, 49, 1443–1482.

Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Carlstrom, C. T. and T. S. Fuerst (1997): "Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis," *The American Economic Review*, 893–910.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk shocks," *The American Economic Review*, 104, 27–65.

Christoffel, K. P., G. Coenen, and A. Warne (2008): "The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis," .

Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, and A. Vardoulakis (2015): "Capital regulation in a macroeconomic model with three layers of default," 40.

Coenen, G., P. Karadi, S. Schmidt, and A. Warne (2018): "The New Area-Wide Model II: an extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector," *ECB Working Paper, No 2200.*

Curdia, V. and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetarypolicy," *Journal of Monetary Economics*, 58, 54–79.

Deak, S., P. Levine, and S. T. Pham (2023): "Simple Mandates, Monetary Rules and Trend Inflation," *Macroeconomic Dynamics*, Published Online, 7 July, 2023, DOI: https://doi.org/10.1017/S136510052300024X.

Den Haan, W. and J. D. Wind (2012): "Nonlinear and Stable Perturbation Based Approximation," *Journal of Economic Dynanamics and Control*, 36, 1477–1479.

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic Competition and Optimal Product Diversity," *American Economic Review*, 67, 297–308.

Eggertsson, G. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, No. 1, 139–211.

Fernandez-Villaverde, J., J. Rubio-Ramirez, T. Sargent, and M. W. Watson (2007): "ABC (and Ds) of Understanding VARs," *American Economic Review*, 97, 1021–1026.

Ferrero, A., R. Harrison, and B. Nelson (2022): "House price dynamics, optimal LTV limits and the liquidity trap," Bank of England Working Paper No 969, CEPR Discussion Paper No. DP13400 and forthcoming Review of Economic Studies.

Gertler, M. and S. Gilchrist (2018): "What Happened: Financial Factors in the Great Recession," Tech. rep., NBER Working Paper.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58, 17–34.

——— (2013): "Qe 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool," *International Journal of Central Banking.*

Gertler, M. and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 3, 547–599.

Gertler, M., N. Kiyotaki, and A. Queralto (2012a): "Financial Crises, Bank Risk Exposure and Government Financial Policy," *Journal of Monetary Economics*, 59, S17–S34.

——— (2012b): "Financial crises, bank risk exposure and government financial policy," *Journal of Monetary Economics*, 59, S17–S34.

King, R., C. Plosser, and S. Rebelo (1988): "Production, Growth and Business Cycles I: The basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195–231.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211 – 248.

Levine, P. and D. Currie (1987): "The Design of Feedback Rules in Linear Stochastic Rational Expectations Models," *Journal of Economic Dynamics and Control*, 11, 1–28.

Levine, P., P. McAdam, and J. Pearlman (2008): "Quantifying and Sustaining Welfare Gains from Monetary Commitment," *Journal of Monetary Economics*, 55, 1253–1276.

Levine, P., J. Pearlman, S. Wright, and B. Yang (2023): "Imperfect Information and Hidden Dynamics," School of Economics Discussion Papers 1223, School of Economics, University of Surrey.

Lucas, R. E. (1987): *Models of Business Cycles*, Oxford: Basil Blackwell.

Rannenberg, A. (2016): "Bank Leverage Cycles and the External Finance Premium," *Journal of Money Credit and Banking*, 48, 1477–1479.

Schmitt-Grohe, S. and M. Uribe (2007): "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54, 1702–1725.

Sims, E. and J. C. Wu (2021): "Evaluating central banks' tool kit: Past, present, and future," *Journal of Monetary Economics*, 118, 135–160.

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US business cycles: A Bayesian DSGE approach," *American Economic Review*, 97, 586–606.

Taylor, J. B. (1993): "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Tsiaras, S. (2023): "Asset purchases, limited asset markets participation and inequality," *Journal of Economic Dynamics and Control*, 154, 104721.

Woodford, M. (2003): *Interest and Prices. Foundations of a Theory of Monetary Policy*, Princeton University Press.

Wu, J. C. and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money Credit and Banking*, 48, 253–291.

Online Appendix

The structure of the Appendix is as follows. In Section A, we lay out the standard NK model without financial frictions. In the end of the section we provide the full standard NK model listing. In Sections B and C, we present details of the BGG and GK frictions respectively. Our full model combines the standard NK presented here with these two frictions and is presented in Section D. Section E analyzes the steady state. Section F provides a robustness assessment of the assumptions associated with the costs of liquidity policy. Sections G and H provide more results for optimized rules for different values of the parameters χ_m and ω respectively.

A The Core NK Model without a Banking Sector

We now develop an NK model with a stationarized RBC model at its core. Now we add sticky prices and nominal wages. The household sector and its supply of homogeneous is as in the RBC core. The only difference with the textbook NK model is that households invest in bank deposits instead of bonds which is usually the investment vehicle in the NK model. We therefore focus on the supply side and the modelling of price and wage stickiness.

A.1 Households

We choose preferences compatible with balanced growth (see King, Plosser, and Rebelo (1988)). With external habit in consumption, household j has a single-period utility

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}) \exp\left(\frac{(\sigma_c - 1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1) \quad \sigma_l > 0$$

$$\to \log(C_t^j - \chi C_{t-1}) - \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

where C_{t-1} is aggregate per capita consumption whereas with internal habit we have

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}^j) \exp\left(\frac{(\sigma_c - 1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1) \quad \sigma_l > 0$$

$$\to \log(C_t^j - \chi C_{t-1}^j) - \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

Defining an instantaneous marginal utility by

$$U_{C,t} = (C_t - \chi C_{t-1}) \exp\left(\frac{(\sigma_c - 1)H_t^{1+\sigma_l}}{1+\sigma_l}\right)$$

Then in a symmetric equilibrium he household first-order conditions for external habit and internal habit respectively are

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right] \\
\Lambda_{t,t+1} &= \beta\frac{\lambda_{t+1}}{\lambda_t} \\
U_{H,t} &= -H_t^{\sigma_l}(C_t - \chi C_{t-1})\exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right) \\
\frac{U_{H,t}}{\lambda_t} &= -W_t
\end{aligned}
$$

where for external habit and internal habit respectively we have

$$
\begin{aligned}
\lambda_t &= U_{C,t} \\
\lambda_t &= U_{C,t} - \beta\chi\mathbb{E}_t[U_{C,t+1}]
\end{aligned}
$$

Parameter σ_l is referred to by Smets and Wouters (2007) as the labour supply elasticity.

A.2 Sticky Prices

First we introduce a retail sector producing differentiated goods under monopolistic competition. This sector converts homogeneous output from a competitive wholesale sector. The aggregate prices in the two sectors are given by P_t and P_t^W respectively and $P_t > P_t^W$ from the *mark-up* possible under monopolistic competition. The *real marginal cost* of producing each differentiated good $MC_t \equiv \frac{P_t^W}{P_t}$. In the RBC model $P_t = P_t^W$ so $MC_t = 1$ and the *marginal cost is constant*. In the NK model retailers are locked into price-contracts and cannot their prices every period. Their marginal costs therefore vary. In periods of high demand they simply increase output until they are able to change prices.

The retail sectors then uses a homogeneous wholesale good to produce a basket of differentiated goods for consumption

$$
C_t = \left(\int_0^1 C_t(m)^{(\zeta-1)/\zeta}dm\right)^{\zeta/(\zeta-1)} \tag{A.1}
$$

where $\zeta > 1$ is the elasticity of substitution. For each m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize (A.1) given total expenditure $\int_0^1 P_t(m)C_t(m)dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_t(m)$ of the form

$$
C_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} C_t
$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta}dm\right]^{\frac{1}{1-\zeta}}$. P_t is the aggregate price index. Note that C_t and P_t

are Dixit-Stiglitz aggregators – see Dixit and Stiglitz (1977). Demand for investment and government services takes the same form, so in aggregate

$$Y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} Y_t$$

Following Calvo (1983), we now assume that there is a probability of $1-\xi_p$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then it is held fixed.[16] For each retail producer m, given its real marginal cost $MC_t = \frac{P_t^W}{P_t}$, the objective is at time t to choose $\{P_t^0(m)\}$ to maximize discounted real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - P_{t+k} MC_{t+k}\right]$$

subject to

$$Y_{t+k}(m) = \left(\frac{P_t^O(m)}{P_{t+k}}\right)^{-\zeta} Y_{t+k} \tag{A.2}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the (non-stationarized) stochastic discount factor[17] over the interval $[t, t+k]$. The solution to this optimization problem is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k}\right] = 0$$

Using (A.2) and rearranging this leads to

$$P_t^O = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k}} \tag{A.3}$$

where the m index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history.

By the law of large numbers the evolution of the price index is given by

$$P_t^{1-\zeta} = \xi_p P_{t-1}^{1-\zeta} + (1-\xi_p)(P_t^0)^{1-\zeta} \tag{A.4}$$

Now define k periods ahead inflation as

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$$

To ease the notation in what follows we denote $\Pi_t = \Pi_{t-1,t}$ and $\Pi_{t+1} = \Pi_{t,t+1}$.

[16] Thus we can interpret $\frac{1}{1-\xi_p}$ as the average duration for which prices are left unchanged.

[17] We stationarize the model later.

We can now write the fraction (A.3)

$$\frac{P_t^O}{P_t} = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta-1} Y_{t+k}}$$

and (A.4) as

$$1 = \xi_p (\Pi_t)^{\zeta-1} + (1-\xi_p) \left(\frac{P_t^O}{P_t}\right)^{1-\zeta}$$

A.3 Price Dynamics

In order to set up the model in non-linear form as a set of difference equations, required for software packages such a Dynare, we need to represent the price dynamics as *difference equations*. .

First we assume a zero-growth steady state so that we do not yet need to stationarize any variables. Then using the Lemma in that section , price dynamics are given by

$$\begin{aligned}
\frac{P_t^0}{P_t} &= \frac{J_t^p}{JJ_t^p} \\
JJ_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1} \Pi_{t+1}^{\zeta-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1} \Pi_{t+1}^{\zeta} J_{t+1}^p] &= \left(\frac{1}{1-\frac{1}{\zeta}}\right) Y_t MC_t MCS_t \\
1 &= \xi_p \Pi_t^{\zeta-1} + (1-\xi_p) \left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta} \\
MC_t &= \frac{P_t^W}{P_t} = \frac{W_t}{F_{H,t}} \qquad (A.5)
\end{aligned}$$

where (A.5) allows for $P_t \neq P_t^W$. We have also introduced a mark-up shock MCS_t to MC_t. Notice that the real marginal cost, MC_t, is no longer fixed as it was in the RBC model.

A.4 Indexing

Prices are now are indexed to last period's aggregate inflation, with a price indexation parameter γ_p. Then the price trajectory with no re-optimization is given by $P_t^O(j)$, $P_t^O(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}$, $P_t^O(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_p}$, $\cdots$. where $Y_{t+k}(m)$ is given by (A.2) with indexing so that

$$Y_{t+k}(m) = \left(\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p}\right)^{-\zeta} Y_{t+k}$$

With indexing by an amount $\gamma_p \in [0,1]$ and an exogenous mark-up shock MS_t as before, the optimal price-setting first-order condition for a firm j setting a new optimized price $P^0_t(j)$ is now given by

$$P^0_t = \frac{\frac{\zeta}{\zeta-1}\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} MC_{t+k}MS_{p,t+k}Y_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma}\right]}.$$

Price dynamics are now given by

$$\begin{aligned}
\frac{P^0_t}{P_t} &= \frac{J^p_t}{JJ^p_t} \\
JJ^p_t - \xi_p\mathbb{E}_t[\Lambda_{t,t+1}\tilde{\Pi}^{\zeta-1}_{t+1}JJ^p_{t+1}] &= Y_t \\
J^p_t - \xi_p\mathbb{E}_t[\Lambda_{t,t+1}\tilde{\Pi}^{\zeta}_{t+1}J^p_{t+1}] &= \frac{\zeta}{\zeta-1}MC_tMS_{p,t}Y_t \\
\tilde{\Pi}_t &\equiv \frac{\Pi_t}{\Pi^{\gamma_p}_{t-1}} \\
1 &= \xi_p\tilde{\Pi}^{\zeta-1}_t + (1-\xi_p)\left(\frac{J^p_t}{JJ^p_t}\right)^{1-\zeta}
\end{aligned}$$

A.5 Price Dynamics in a Non-Zero-Growth Steady State

Stationarizing J^p_t and JJ^p_t as in the RBC model, price dynamics with indexing become

$$\begin{aligned}
\frac{P^0_t}{P_t} &= \frac{J^p_t}{JJ^p_t} \\
JJ^p_t - \xi_p\mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}^{\zeta-1}_{t+1}JJ^p_{t+1}] &= Y_t \\
J^p_t - \xi_p\mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}^{\zeta}_{t+1}J^p_{t+1}] &= \frac{\zeta}{\zeta-1}MC_tMS_{p,t}Y_t \\
\tilde{\Pi}_t &\equiv \frac{\Pi_t}{\Pi^{\gamma_p}_{t-1}} \\
1 &= \xi_p\tilde{\Pi}^{\zeta-1}_t + (1-\xi_p)\left(\frac{J^p_t}{JJ^p_t}\right)^{1-\zeta}
\end{aligned}$$

A.6 Sticky Wages

To introduce wage stickiness we now assume that each household supplies homogeneous labour at a nominal wage rate $W_{h,t}$ to a monopolistic trade-union who differentiates the labour and sells type $H_t(j)$ at a nominal wage $W_{n,t}(j) > W_{h,t}$ to a labour packer in a sequence of Calvo staggered nominal wage contracts. The real wage is then defined as $W_t \equiv \frac{W_{n,t}}{P_t}$. We now have to distinguish between *price inflation* which now uses the notation $\Pi^p_t \equiv \frac{P_t}{P_{t-1}}$ and *wage inflation*, $\Pi^w_t \equiv \frac{W_{n,t}}{W_{n,t-1}} = \frac{W_t\Pi^p_t}{W_{t-1}}$.

As with price contracts we employ Dixit-Stiglitz quantity and price aggregators. Calvo probabilities are now ξ_p and ξ_w for price and wage contracts respectively. The competitive labour packer forms a composite labour service according to $H_t = \left(\int_0^1 H_t(j)^{(\mu-1)/\mu} dj\right)^{\mu/(\mu-1)}$ and sells onto the intermediate firm. where $\mu > 1$ is the elasticity of substitution. For each j, the labour packer chooses $H_t(j)$ at a wage $W_{n,t}(j)$ to maximize H_t given total expenditure $\int_0^1 W_{n,t}(j)H_t(j)dj$. This results in a set of labour demand equations for each differentiated labour type j with wage $W_{n,t}(j)$ of the form

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} H_t \tag{A.6}$$

where $W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\mu} dj\right]^{\frac{1}{1-\mu}}$ is the aggregate nominal wage index. H_t and $W_{n,t}$ are Dixit-Stigliz aggregators for the labour market.

Wage setting by the trade-union again follows the standard Calvo framework supplemented with indexation. At each period there is a probability $1 - \xi_w$ that the wage is set optimally. The optimal wage derives from maximizing discounted profits. For those trade-unions unable to reset, wages are indexed to last period's aggregate inflation, with wage indexation parameter γ_w. Then as for price contracts the wage rate trajectory with no re-optimization is given by $W^O_{n,t}(j)$, $W^O_{n,t}(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}$, $W^O_{n,t}(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_w}$, $\cdots$. The trade union then buys homogeneous labour at a nominal price $W_{h,t}$ and converts it into a differentiated labour service of type j. The trade union time t then chooses $W^O_{n,t}(j)$ to maximize real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \frac{\Lambda_{t,t+k}}{P_{t+k}} H_{t+k}(j) \left[W^O_{n,t}(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - W_{h,t+k}\right]$$

where using (A.6) with indexing $H_{t+k}(j)$ is given by

$$H_{t+k}(j) = \left(\frac{W^O_{n,t}(j)}{W_{n,t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}\right)^{-\mu} H_{t+k}$$

and μ is the elasticity of substitution across labour varieties.

This leads to the following first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \frac{\Lambda_{t,t+k}}{P_{t+k}} H_{t+k}(j)\Big[W^0_t(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - \frac{\mu}{\mu-1} W_{h,t+k}\Big] = 0$$

and hence by analogy with price-setting, this leads to the optimal real wage

$$\frac{W^O_{n,t}}{P_t} = \frac{\mu}{\mu-1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k}\left(\Pi^w_{t,t+k}\right)^{\zeta} H_{t+k}\frac{W_{h,t+k}}{P_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k}\left(\Pi^w_{t,t+k}\right)^{\zeta}\left(\Pi^p_{t,t+k}\right)^{-1} H_{t+k}} = \frac{J^w_t}{JJ^w_t}$$

Then by the law of large numbers the evolution of the wage index is given by

$$W_{n,t}^{1-\mu} = \xi_w \left(W_{n,t-1} \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_w} \right)^{1-\mu} + (1-\xi_w)(W_{n,t}^0(j))^{1-\mu}$$

A.7 Price and Wage Dynamics

We now apply the analysis of A.3-A.5 to wage dynamics and bring the two forms together. The model is now stationarized.

$$\begin{aligned}
\Pi_t^p &\equiv \frac{P_t}{P_{t-1}} \\
\tilde{\Pi}_t^p(\gamma) &\equiv \frac{\Pi_t^p}{\Pi_{p,t-1}^\gamma} \\
JJ_t^p - \xi_p \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^p(\gamma_p)^{\zeta-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^p(\gamma_p)^{\zeta} J_{t+1}^p] &= \frac{\zeta}{\zeta-1} Y_t MC_t MS_{p,t} \\
1 &= \xi_p \tilde{\Pi}_t^p(\gamma_p)^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta} \\
\frac{P_t^O}{P_t} &= \frac{J_t^p}{JJ_t^p}
\end{aligned}$$

$$\Pi_t^w \equiv \frac{W_{n,t}}{W_{n,t-1}} = (1+g_t)\frac{\Pi_t W_t}{W_{t-1}} \tag{A.7}$$

$$\tilde{\Pi}_t^w \equiv \frac{\Pi_t^w}{(\Pi_{t-1}^p)^{\gamma_w}} \tag{A.8}$$

$$MRS_t = -\frac{U_{H,t}}{U_{C,t}} = \frac{W_{h,t}}{P_t} \tag{A.9}$$

$$JJ_t^w - \xi_w \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{(\tilde{\Pi}_{t,t+1}^w)^\mu}{\tilde{\Pi}_{t,t+1}^p(\gamma_w)} JJ_{t+1}^w\right] = H_{d,t} \tag{A.10}$$

$$J_t^w - \xi_w \mathbb{E}_t\left[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{w,t+1}^\mu J_{t+1}^w\right] = -\frac{\mu}{\mu-1} MRS_t MS_{w,t} H_{d,t} \tag{A.11}$$

$$(W_{n,t})^{1-\mu} = \xi_w \left((W_{n,t-1}) \frac{1}{\tilde{\Pi}_t^p(\gamma_w)} \right)^{1-\mu} + (1-\xi_w)\left(W_{n,t}^O\right)^{1-\mu} \Rightarrow$$

$$1 = \xi_w \left(\frac{\Pi_t^w \tilde{\Pi}_{p,t}(\gamma_w)}{\Pi_t^p} \right)^{\mu-1} + (1-\xi_w)\left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{1-\mu} \tag{A.12}$$

$$W_t^O \equiv \frac{W_{n,t}^O}{W_{n,t}} = \frac{W_{n,t}^O/P_t}{W_{n,t}/P_t} = \frac{J_t^w}{W_t JJ_t^w} \tag{A.13}$$

$$\Pi_t^w = (1+g_t)\frac{\Pi_t W_t}{W_{t-1}} \tag{A.14}$$

A.8 Capacity Utilization and Fixed Costs of Production

We now add two remaining features to the model. As in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) we assume that using the stock of capital with intensity u_t produces a cost of $a(u_t)K_t$ units of the composite final good. The functional form is chosen consistent with the literature:

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 \tag{A.15}$$

and satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. Then we must add a term $(r_t^K - a(u_t)K_t$ to the household budget constraint on the income side where r_t^K is the rental rate leading to the following first-order condition determines capacity utilization:

$$r_t^K = a'(u_t) \tag{A.16}$$

Capital now enters the production function as $u_t K_{t-1}$.

The final change is to add fixed costs F, necessary to transform homogeneous wholesale goods into differentiated retail goods. To pin down F we make the assumption that entry occurs until retail profits are eliminated in the steady state , i.e., $P^W Y^W = PY$. It follows that

$$\frac{P^W}{P} = MC = \frac{Y}{Y^W} = \frac{\left(1 - \frac{F}{Y^W}\right)}{\Delta_p} \tag{A.17}$$

It follow that

$$\frac{F}{Y^W} = 1 - \Delta_p MC \tag{A.18}$$

For the zero inflation, $MC = 1 - \frac{1}{\zeta}$ and $\Delta_p = \Delta_w = 1$ and therefore $\frac{F}{Y^W} = \frac{1}{\zeta}$.

A.9 Price and Wage Dispersion

The output and labour market clearing conditions must take into account relative price dispersion across varieties and wage dispersion across firms. Integrating across all firms, taking into account that the capital-labour ratio is common across firms and that the wholesale sector is separated from the retail sector we obtain aggregate demand for intermediate (wholesale) goods necessary to produce final retail goods as

$$Y_t^W - F = \int_0^1 \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} dm(C_t + I_t + G_t) = \Delta_t^p Y_t$$

where labour market clearing gives total demand for labour, H_t^d, as

$$H_t = \int_0^1 H_t(j)dj = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} dj\, H_t^d = \Delta_t^w H_t^d$$

where the price dispersion is given by $\Delta_t^p = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} df$ and wage dispersion is given by $\Delta_t^w = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} dj$. We have:

$$\Delta_t^p = \xi_p + \tilde{\Pi}_t^{\zeta}\Delta_{t-1}^p + (1-\xi_p)\left(\frac{P_t^O}{P_t}\right)^{-\zeta}$$

$$\Delta_t^w = \xi_w \tilde{\Pi}_{w,t}^{\mu}\Delta_{t-1}^w + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\mu}$$

A.10 Summary of Supply Side

Wholesale, Retail and capital producer firm behaviour is given by

$$\begin{aligned}
\textbf{Wholesale Production} &: Y_t^W = (A_t H_t^d)^{\alpha} K_{t-1}^{1-\alpha} \\
\textbf{Retail Aggregate Production} &: Y_t = \frac{Y_t^W - F}{\Delta_t^p} \\
\textbf{Aggregate Employed Labour} &: H_t^d = \frac{H_t}{\Delta_t^w} \\
\textbf{Labour Demand} &: W_t = \frac{P_t^W}{P_t} F_{H,t} = \frac{P_t^W}{P_t}\frac{\alpha Y_t^W}{H_t^d} \\
\textbf{Capital Demand} &: r_t^K = \frac{P_t^W}{P_t} F_{K,t} = \frac{P_t^W}{P_t}\frac{(1-\alpha)Y_t^W}{K_{t-1}}
\end{aligned}$$

where K_t is *end-of-period* $[t, t+1]$ capital, W_t is the wage rate of the composite differentiated labour orovided by the labour packer (trade-union) and Δ_t^p and Δ_t^w are price dispersion and wage dispersion (defined below), r_t^K is the rental net rate for capital and we have imposed labour demand equal to labour supply in a labour market equilibrium. Production is assumed to be Cobb-Douglas.

Capital accumulation with investment adjustment costs carried out by capital goods producers is given by

$$\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t IS_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X (X_t - 1 - g)^2 \\
S'(X_t) &= 2\phi_X (X_t - 1)
\end{aligned}$$

$$Q_t IS_t(1 - S(X_t) - X_t S'(X_t)) \quad + \quad \mathbb{E}_t\left[\Lambda_{t,t+1} Q_{t+1} IS_{t+1} S'(X_{t+1}) X_{t+1}^2\right] = 1$$

where I_t, and Q_t are investment and the real price of capital respectively. IS_t is a capital specific shock process. $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state with output, consumption, capital, investment and the real wage growing at a rate g.

Then this completes the supply side with price and wage dynamics and dispersion as given in sections A.7 and A.9.

A.11 Capital Return and Expected Spread

The gross return on capital by

$$R_t^K = \left[\frac{r_t^K + (1-\delta) Q_t}{Q_{t-1}}\right]$$

Then in the *absence of financial frictions* including the risk-premium shock RPS_t we have *arbitrage* between discounted returns on capital and deposits given by

$$\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}^K] = \mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = 1 \tag{A.19}$$

In the main model, where we include BGG financial frictions in (B.8), we have

$$\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}^K] \neq \mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = 1$$

A.12 The Monetary Rule and Output Equilibrium

The nominal interest rate is given by the following Taylor-type rule

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\Big[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \\ &+ \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\Big] + \epsilon_{MPS,t}\,; \quad \rho_r \in [0,1) \end{aligned}$$

where $\epsilon_{M,t}$ is a monetary policy shock process. θ_π and θ_y are the long-run elasticities of the inflation and output respectively with respect to the interest rate. The "Taylor principle" requires $\theta_\pi > 1$. The conventional Taylor rule stabilizes output about its flexi-price level which is that found by solving the RBC core of this model or simply allowing the contract parameter ξ_p to tend to zero. Unlike the implementable form, this requires observations of the output gap $\frac{Y_t}{Y_t^F}$ to implement and monitor.[18] The output equilibrium is given by

$$Y_t = C_t + G_t + I_t$$

A.13 The Stationary Equilibrium

To stationarize the model labour-augmenting technical progress parameter is decomposed into a cyclical component, stationary A_t, and a deterministic trend $\bar{A}_t$:

$$\begin{aligned} A_t &= \bar{A}_t A_t^c \\ \bar{A}_t &= (1+g)\bar{A}_{t-1} \end{aligned}$$

Then we can define stationarized variables by

$$\begin{aligned}
\frac{\Omega_t}{\bar{A}_t^{1-\sigma}} &= \frac{U_t}{\bar{A}_t^{1-\sigma}} + \beta E_t \frac{\Omega_{t+1}}{\bar{A}_{t+1}^{1-\sigma}} \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{1-\sigma} \\
\frac{U_t}{\bar{A}_t^{1-\sigma}} &= \frac{\left[\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \\
\Lambda_{t,t+1} &= \beta \frac{U_{C,t+1}}{U_{C,t}} = \beta(1+g)^{(1-\varrho)(1-\sigma)-1} \frac{U_{C,t+1}^c}{U_{C,t}^c} \equiv \beta_g \frac{U_{C,t+1}^c}{U_{C,t}^c}
\end{aligned}$$

where the growth-adjusted discount rate is defined as

$$\beta_g \equiv \beta(1+g)^{1-\sigma},$$

the Euler equation is still

$$E_t\left[\Lambda_{t,t+1} R_{t+1}\right]$$

Now stationarize remaining variables by defining cyclical components:

$$\begin{aligned}
\frac{U_{C,t}}{\bar{A}_t^{-\sigma}} &= \frac{(1-\sigma)\frac{U_t}{\bar{A}_t^{1-\sigma}}}{\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}} - \beta\chi \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma} \frac{(1-\sigma)\frac{U_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{A}_{t+1}} - \chi \frac{C_t}{\bar{A}_t} \frac{\bar{A}_t}{\bar{A}_{t+1}}} \\
Y_t^c &\equiv \frac{Y_t}{\bar{A}_t} = \frac{(A_t H_t^d)^\alpha \left(\frac{K_{t-1}}{\bar{A}_t}\right)^{1-\alpha} - \frac{F_t}{\bar{A}_t}}{\Delta_t^p} = \frac{(A_t H_t^d)^\alpha \left(\frac{K_{t-1}^c}{(1+g_t)}\right)^{1-\alpha} - F}{\Delta_t^p} \\
K_t^c &\equiv \frac{K_t}{\bar{A}_t} \\
K_t^c &= (1-\delta)\frac{K_{t-1}^c}{1+g_t} + (1 - S(X_t^c))I_t^c
\end{aligned}$$

[18] Technically this should pose no problems in a perfect information rational expectations equilibrium, but the rationale for 'simple rules' is to have policies that are easy to observe without relying on the perfect information solution.

$$
\begin{aligned}
X_t^c &= (1+g_t)\frac{I_t^c}{I_{t-1}^c} \\
S(X_t^c) &= \phi_X(X_t^c - 1 - g_t)^2 \\
S'(X_t^c) &= 2\phi_X(X_t^c - 1 - g_t) \\
C_t^c &\equiv \frac{C_t}{\bar{A}_t} \\
I_t^c &\equiv \frac{I_t}{\bar{A}_t} \\
W_t^c &\equiv \frac{W_t}{\bar{A}_t}
\end{aligned}
$$

In what follows all variables are in stationary form and we drop the superscript ^{c} for variables

A.14 Full Core NK Model Listing

The full model in stationarized form is given by:

A.15 Dynamic Model

$$
\begin{aligned}
\beta_g &\equiv \beta\ (1+g)^{-\sigma_c} \\
U_t &= \frac{(C_t - \chi C_{t-1}/(1+g))^{1-\sigma_c}\exp\left(\frac{(\sigma_c-1)(H_t)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c} \\
CE_t &= \frac{(1.01(C_t - \chi C_{t-1}/(1+g)))^{1-\sigma_c}\exp\left(\frac{(\sigma_c-1)(H_t)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c} - U_t \\
&+ \mathbb{E}_t[(1+g)\ \beta_{g,t+1}\, CE_{t+1}] \\
\Omega_t &= U_t + \beta_g\, \mathbb{E}_t[\Omega_{t+1}] \\
U_{C_t} &= (C_t - \chi C_{t-1}/(1+g))^{-\sigma_c}\exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right) \\
U_{H_t} &= -H_t^{\sigma_l}(C_t - \chi C_{t-1}/(1+g))^{-\sigma_c}\exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right) \\
\lambda_t &= \mathbb{E}_t[\beta_g\, R_{t+1}]\, RPS_t\, \lambda_{t+1} \\
\lambda_t &= U_{C,t} - \chi\, \mathbb{E}_t[\beta_g U_{C,t+1}] \\
\frac{-U_{H_t}}{\lambda_t} &= W_{h,t} \\
R_t &= \frac{R_{n,t-1}}{\Pi_t} \\
Y_t &= \frac{Y_t^W - F}{\Delta_t^p}
\end{aligned}
$$

$$
\begin{aligned}
H_{d,t} &= \frac{H_t}{\Delta_t^w} \\
Y_t^W &= (H_{d,t}\, A_t)^\alpha \left(\frac{K_{t-1}}{1+g_t}\right)^{1-\alpha} \\
R_t^K &= \frac{\left(\frac{Y_t^W\,(1-\alpha)\,MC_t}{\frac{K_{t-1}}{1+g_t}} + (1-\delta)\; Q_t\right)}{Q_{t-1}} \\
\Lambda_{t-1,t} &= \frac{\beta_{g,t}\,\lambda_t}{\lambda_{t-1}} \\
1 &= Q_t\,\left(1 - S_t - X_t\, S_t'\right) + \mathbb{E}_t[\Lambda_{t,t+1}\, Q_{t+1}\, S_{t+1}'\,(X_{t+1})^2] \\
\frac{\alpha\, MC_t\, Y_t^W}{H_t} &= W_t \\
MC_t &= \frac{P_t^W}{P_t} \\
K_t &= \left((1-S_t)\; I_t + \frac{K_{t-1}\;(1-\delta)}{1+g_t}\right) \\
X_t &= \frac{(1+g_t)\; I_t}{I_{t-1}} \\
S_t &= \phi_X\;(X_t - 1 - g)^2 \\
S_t' &= 2\,\phi_X\,(X_t - 1 - g) \\
1 &= \Lambda_{t,t+1}\, R_{t+1}^K = 1 \\
Y_t &= C_t + I_t + G_t \\
Y_t &= JJ_t^p - \mathbb{E}_t[(1+g)\;\Lambda_{t,t+1}\,\tilde{\Pi}_{t+1}^{\zeta-1}\, JJ_{t+1}^p] \\
\frac{\zeta}{\zeta-1}\, Y_t\, MC_t\, MCS_t &= J_t^p - \mathbb{E}_t\left[(1+g)\;\Lambda_{t,t+1}\,\tilde{\Pi}_{t+1}^{\zeta}\, J_{t+1}^p\right] \\
\Lambda_{t,t+1} &= \frac{\beta_{g,t+1} U_{C,t+1}}{U_{C,t}} \\
\tilde{\Pi}_t &= \frac{\Pi_t}{\Pi_{t-1}^{\gamma_p}} \\
P_t^O &= \frac{J_t^p}{JJ_t^p} \\
1 &= \tilde{\Pi}_t^{\zeta-1} + (1-\xi_p)\;(P_t^O)^{1-\zeta} \\
\Delta_t^p &= \xi_p\,\tilde{\Pi}_t^{\zeta}\,\Delta_{t-1}^p + (1-\xi_p)\;(P_t^O)^{(-\zeta)} \\
\Pi_t^w &= \Pi_t \frac{W_t(1+g)}{W_{t-1}} \\
\tilde{\Pi}_t^w &= \frac{\Pi_t}{\Pi_{t-1}^{\gamma_w}} \\
H_t &= JJ_t^w - \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}\,\xi_w\,(\tilde{\Pi}_{t+1}^w)^{\mu_w}}{\tilde{\Pi}_{t+1}(\gamma_w)}\, JJ_{t+1}^w\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mu_w}{\mu_w - 1} W_{h,t}\, H_t\, MRSS_t &= J_t^w - \mathbb{E}_t[(1+g)\Lambda_{t,t+1}\, \xi_w\, (\tilde{\Pi}_{t+1}^w)^{\mu_w}\, J_{t+1}^w] \\
W_t^O &= \frac{J_t^w}{W_t\, JJ_t^w} \\
1 &= \xi_w \left(\frac{\Pi_t^w\, \tilde{\Pi}_t(\gamma_w)}{\Pi_t} \right)^{\mu_w - 1} + (1-\xi_w)\ (W_t^O)^{1-\mu_w} \\
\Delta_t^w &= \xi_w\, (\tilde{\Pi}_t^w)^{\mu_w}\, \Delta_{t-1}^w + (1-\xi_w)\ (W_t^O)^{-\mu_w} \\
\text{Invmarkup}_t &= \frac{W_{h,t}}{W_t} \\
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right)\right. \\
&+ \left.\theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \log(MPS_t) \\
Y_t &= C_t + C_{E,t} + G_t + I_t + \alpha(u_t)K_{t-1}
\end{aligned}
$$

with AR(1) processes for A_t, G_t, MC_t, $MRSS_t$, IS_t, MPS_t and RPS_t.

A.16 Balanced Growth Steady State

With non-zero steady state growth, the steady state for the rest of the system is the same as the zero-growth RBC model except for the following relationships: for particular steady state inflation rate $\Pi_p = \Pi_w > 1$ the NK features of the blanced growth steady state become:

$$
\begin{aligned}
R_n &= \Pi R \\
\tilde{\Pi}_p(\gamma) &\equiv \Pi^{1-\gamma} \\
\frac{P^O}{P} = \frac{J^p}{JJ^p} &= \left(\frac{1-\xi_p \tilde{\Pi}_p(\gamma_p)^{\zeta-1}}{1-\xi_p} \right)^{\frac{1}{1-\zeta}} \\
MC = \frac{P^W}{P} &= \left(1 - \frac{1}{\zeta}\right) \frac{J^p(1-\beta(1+g)\xi_p \tilde{\Pi}_p(\gamma_p)^{\zeta})}{H_p(1-\beta(1+g)\xi_p \tilde{\Pi}_p(\gamma_p)^{\zeta-1})} \\
&= \text{Inverse of price mark-up} \\
\Delta_p &= \frac{1-\xi_p}{1-\xi_p \tilde{\Pi}_p(\gamma_p)^{\zeta}} \left(\frac{J^p}{JJ^p}\right)^{-\zeta}
\end{aligned}
$$

and for wage dynamics

$$
\begin{aligned}
\frac{W^O}{W} = \frac{\frac{J^w}{JJ^w}}{\frac{W}{P}} &= \left(\frac{1-\xi_w \tilde{\Pi}_p(\gamma_w)^{\mu-1}}{1-\xi_w} \right)^{\frac{1}{1-\mu}} \\
\frac{J^w}{JJ^w} &= MS_w \frac{W_h}{P} \frac{(1-\beta\xi_w(1+g)\tilde{\Pi}_p(\gamma_w)^{\mu-1}}{(1-\beta\xi_w \tilde{\Pi}_p(\gamma_w)^{\mu})}
\end{aligned}
$$

$$
\begin{aligned}
\text{i.e., } \frac{\frac{W_h}{P}}{\frac{W}{P}} &= \left(1-\frac{1}{\mu}\right)\frac{\frac{J^w}{JJ^w}}{\frac{W}{P}}\frac{(1-\beta\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu}}{(1-\beta\xi_w(1+g)\tilde{\Pi}_p(\gamma_w)^{\mu-1})} \\
&= \text{Inverse of wage mark-up} \\
\Delta_w &= \frac{1-\xi_w}{1-\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu}}\left(\frac{\frac{J^w}{JJ^w}}{\frac{W}{P}}\right)^{-\mu}
\end{aligned}
$$

B Details of the BGG Model Component

Following on from Section 2.1 in the main text, we now describe the incentive compatibility constraint, the optimal contract for the risk-neutral entrepreneur (the firm), aggregation over old and new entrepreneurs and banks and the choice of density function for ψ.

B.1 Incentive Compatibility Constraint

The bank's incentive compatibility constraint is

$$
\mathbb{E}_t\left[(1-\mu)R^K_{t+1}Q_tK_{e,t}\int_{\psi_{min}}^{\bar{\psi}_{t+1}}\psi f(\psi)d\psi+(1-p(\bar{\psi}_{t+1}))\frac{R^L_t}{\Pi_{t+1}}L_{e,t}=R^B_{t+1}L_{e,t}\geqslant R_{t+1}L_{e,t}\right] \tag{B.1}
$$

The left hand side part of (B.1) is the expected return to the bank from the contract averaged over all realizations of the shock. From (B.1) R^B_t is defined as

$$
(1-\mu)R^K_tQ_{t-1}K_{e,t-1}\int_{\psi_{min}}^{\bar{\psi}_t}\psi f(\psi)d\psi+(1-p(\bar{\psi}_t))\frac{R^L_{t-1}}{\Pi_t}L_{e,t-1}=R^B_tL_{e,t-1} \tag{B.2}
$$

In the pure BGG case we have $R^B_t = R_t = \frac{R_{n,t-1}}{\Pi_t}$, where $R_{n,t-1}$ is the nominal interest rate. In the pure GK case ψ_{min} is sufficiently high to give $p(\bar{\psi}_t) = \int_{\psi_{min}}^{\bar{\psi}_t} f(\psi)d\psi = \int_{\psi_{min}}^{\bar{\psi}_t}\psi f(\psi)d\psi = 0$. Then $R^L_t = R_{n,t}$.

Eliminating the real loan rate from (3), this becomes

$$
\mathbb{E}_t\left[R^K_{t+1}Q_tK_{e,t}\left((1-\mu)\int_{\psi_{min}}^{\bar{\psi}_{t+1}}\psi f(\psi)d\psi+\bar{\psi}_{t+1}(1-p(\bar{\psi}_{t+1}))\right)=R^B_{t+1}L_{e,t}\right] \tag{B.3}
$$

Defining

$$
\Gamma(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}}\psi f(\psi)d\psi+\bar{\psi}_{t+1}(1-p(\bar{\psi}_{t+1})) \tag{B.4}
$$

$$
G(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}}\psi f(\psi)d\psi \tag{B.5}
$$

(B.3) becomes

$$\mathbb{E}_t\Big[R^K_{t+1}Q_tK_{e,t}\left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})\right] = R^B_{t+1}L_{e,t}\Big] \tag{B.6}$$

B.2 The Optimal Contract

The optimal contract for the risk neutral entrepreneur maximizes the average return to capital over the distribution of ψ_t taking into account the possibility of default and the cost of loans in its absence. She chooses $K_{e,t}$ and the loan rate R^L_t, which from (3) is equivalent to choosing the threshold shock ψ_{t+1}, and solves

$$\max_{\bar{\psi}_{t+1},K_{e,t}} \mathbb{E}_t\left[\left(1-\Gamma(\bar{\psi}_{t+1})\right)R^K_{t+1}Q_tK_{e,t}\right]$$

given initial net worth $n_{E,e,t}$, subject to (B.6) which, using (1) can be rewritten as

$$\mathbb{E}_t\Big[R^K_{t+1}Q_tk_{e,t}\left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})\right] = R^B_{t+1}(Q_tK_{e,t}-N_{E,e,t})\Big] \tag{B.7}$$

Let λ_t be the Lagrange multiplier associated with the constraint. Then the first order conditions are

$$\begin{aligned}
k_t &: \mathbb{E}_t\left[(1-\Gamma(\bar{\psi}_{t+1})R^K_{t+1} + \lambda_t\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))R^K_{t+1} - R^B_{t+1}\right]\right] = 0 \\
\bar{\psi}_{t+1} &: \mathbb{E}_t\left[-\Gamma'(\bar{\psi}_{t+1}) + \lambda_t(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right] = 0 \\
u_t &: r^K_t = \alpha'(u_t)
\end{aligned}$$

Combining the two first order conditions, we arrive at

$$\mathbb{E}_t[R^K_{t+1}] = \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R^B_{t+1}] \tag{B.8}$$

where the *premium on external finance*, $\rho(\bar{\psi}_{t+1})$ is given by

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1-\Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]}. \tag{B.9}$$

B.3 Aggregation

We now aggregate assuming that entrepreneurs exit with fixed probability $1-\sigma_E$. To allow new entrants start up we assume exiting entrepreneurs transfer a proportion ξ_E of their wealth to new entrants. Aggregating, the net worth of the entrepreneur, $K_{e,t-1}$, becomes K_{t-1}, $N_{E,e,t}$ becomes $N_{E,t}$ which then accumulates according to

$$N_{E,t} = (\sigma_E + \xi_E)(1-\Gamma(\bar{\psi}_t))R^K_tQ_{t-1}K_{t-1}$$

and on exiting the entrepreneur consumes

$$C_{E,t} = (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t))R_t^K Q_{t-1}K_{t-1}.$$

The equilibrium is completed with the aggregate incentive compatibility constraint, assumed to be always binding and be independent from each entrepreneur type e.[19]

$$\mathbb{E}_t\left[R_{t+1}^K Q_t k_t\left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})\right]\right] = \mathbb{E}_t\left[R_{t+1}^B (Q_t k_t - n_{E,t})\right].$$

B.4 Choice of Density Function

We choose a *log-normal distribution* for ψ, $\log(\psi) \sim \mathcal{N}\left(-\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right)$. With the mean set to $-\frac{\sigma_\psi^2}{2}$, $\mathbb{E}[\psi] = 1$. This which has the benefit of being mean preserving if extending to consider volatility in σ_ψ. We then have

$$p(\bar{\psi}_t) = \int_0^{\bar{\psi}_t} f\left(\psi; -\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right) d\psi$$

$$G(\bar{\psi}_t) \equiv \int_0^{\bar{\psi}_t} \psi\ f\left(\psi; -\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right) d\psi$$

$$\Gamma(\bar{\psi}_{t+1}) \equiv G(\bar{\psi}_t) + \bar{\psi}_t(1 - p(\bar{\psi}_t))$$

Then it can be shown that

$$G'(\bar{\psi}_t) = \frac{1}{\sigma_\psi\sqrt{2\pi}} \exp\left[-\frac{\left(\log(\bar{\psi}_t) + \frac{1}{2}\sigma_\psi^2\right)^2}{2\sigma_\psi^2}\right]$$

$$\Gamma'(\bar{\psi}_{t+1}) = 1 - p(\bar{\psi}_t)$$

C The Banker's Problem Solution in the GK Model Component

The aggregate solution is assumed to take the form

$$V_{B,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}N_{B,t+1}] \tag{C.1}$$

We write the Bellman equation as

$$V_{B,t-1} = \max_{l_t, m_t} \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)N_{B,t} + \sigma_B V_{B,t}]$$

[19] This follows from (B.1).

$$= \max_{l_t,m_t} \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)N_{B,t} + \sigma_B \mathbb{E}_t(\Lambda_{t,t+1}N_{B,t+1})] \tag{C.2}$$

where corresponding to (9)

$$\mathbb{E}_t(\Lambda_{t,t+1}N_{B,t+1}) = \mathbb{E}_t[\Lambda_{t,t+1}(R_{t+1}N_{B,t} + (R^B_{t+1} - R_{t+1})L_t - (R^M_{t+1} - R_{t+1})M_t)]$$

This is subject to the condition that $V_{B,t} \geqslant \theta[L_t - \omega M_t]$, which implies the constraint

$$\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}[R_{t+1}N_{B,t} + (R^B_{t+1} - R_{t+1})L_t - (R^M_{t+1} - R_{t+1})M_t)] \geqslant \theta(L_t - \omega M_t) \tag{C.3}$$

If $\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}[(R^B_{t+1} - R_{t+1})l_t + (R^M_{t+1} - R_{t+1})M_t)] < \theta(L_t - \omega M_t)$, then maximization takes place if and only if the constraint binds, so that the solution is:

$$L_t = \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]} n_{B,t} + m_t \frac{(\theta\omega - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^M_{t+1} - R_{t+1})]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]}. \tag{C.4}$$

The arbitrage condition between the interest rates implies the following relation:

$$\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^M_{t+1} - R_{t+1})] = \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]. \tag{C.5}$$

Substituting this to C.4 it simplifies to:

$$L_t = \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]} N_{B,t} + \omega M_t. \tag{C.6}$$

Substituting C.6 to the terminal wealth:

$$V_{B,t} = \mathbb{E}_t N_{B,t}[(\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})(\frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]})) + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_t)]$$

and the Bellman equation becomes

$$\begin{aligned} V_{B,t-1} &= \max_{L_t,M_t} \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)N_{B,t} + \sigma_B V_{B,t}] \\ &= (1-\sigma_B)n_{B,t} + \sigma_B N_{B,t}\{\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})\left(\frac{\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}}{\theta - \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})}\right) \\ &+ \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1}R_t)\} \end{aligned}$$

It follows that

$$\Omega_t = (1-\sigma_B)+\sigma_B[(\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1}-R_{t+1})\left(\frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]})\right)+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_t)] \tag{C.7}$$

Equivalently, defining for banks the maximum adjusted leverage

$$\phi_t^B = (L_t - \omega M_t)/N_{B,t} \tag{C.8}$$

we can rewrite this last equation as

$$\Omega_t = 1 - \sigma_B + \sigma_B \theta \phi_t^B. \tag{C.9}$$

D The Full NK Financial Frictions Model Listing

In this section we describe the system of equations of the financial frictions part of the model. The stationarized form can be summarized as:

$$\begin{aligned}
\mathbb{E}_t[R_{t+1}^K] &= \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R_{t+1}^B] \\
(1+g)N_{E,t} &= (\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}_t))R_t^K Q_{t-1}K_{t-1} \\
\phi_t &= \frac{(\phi_t - 1)\mathbb{E}_t\left[R_{t+1}^B\right]}{\mathbb{E}_t\left[R_{t+1}^K\left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})\right]\right]} \\
\phi_t &= \frac{Q_t K_t}{N_{E,t}} \\
\rho(\bar{\psi}_{t+1}) &= \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]} \\
L_t &= Q_t K_t - N_{E,t} \\
\bar{\psi}_t &= \frac{R_{l,t-1}L_{t-1}}{R_t^K Q_{t-1}K_{t-1}}\frac{1}{\Pi_t} \\
R_t^K &= \frac{r_t^K u_t - \alpha(u_t) + (1-\delta)Q_t}{Q_{t-1}} \\
r_t^K &= \frac{(1-\alpha)P_t^W Y_t^W}{u_{t-1}K_{t-1}/(1+g)} \\
L_t &= \phi_t^B N_{B,t} + \omega M_t \\
\phi_t^B &= \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \Omega_{t+1}\mathbb{E}_t\Lambda_{t,t+1}[R_{t+1}^B - R_{t+1}]} \\
\Omega_t &= 1 - \sigma_B + \sigma_B\theta\phi_t^B \\
N_{B,t}(1+g) &= (\sigma_B + \xi_B)R_t^B L_{t-1} - \sigma_B(R_t D_{t-1} + R_t^M M_{t-1}) \\
D_t &= L_t - N_{B,t} - M_t \\
\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^M - R_{t+1})] &= \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^B - R_{t+1})] \\
lever_t &= \frac{L_t}{N_{B,t} + M_t} \\
T_t &= G_t + \Psi_t(M_t) - R_{m,t}M_{t-1}
\end{aligned}$$

$$M_t = \chi_{m,t} L_t$$
$$Y_t = C_t + C_{E,t} + G_t + \Psi_t(M_t) + I_t + \mu G(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1}/(1+g) + a(u_t) K_{t-}$$
$$(1+g) C_{E,t} = (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t)) R_t^K Q_{t-1} K_{t-1}$$

E Steady State Derivations

E.1 Steady State of the Bankers Problem

We begin by finding the steady state of the financial sector variables and then proceed with the real sector variables. A method that simplifies the calculations is to divide all variables in the bankers' problem over the loans (L) and in the entrepreneurs' problem over the capital K. Firstly, we show the steady state values for Q, R, Λ. From the capital producers problem we have that $Q = 1$ and from the Euler equation, we have that $R = \frac{1}{\beta^g}$ and $\Lambda = \beta^g$.

The goal here is to have two equations with unknowns the bank leverage (ϕ^B) and the interest rate on loans (R^B). The incentive constraint of the bank in steady state is

$$L = \phi^B N^B + \omega M, \tag{E.1}$$

where $M = \chi_m L \rightarrow \frac{M}{L} = \chi_m$ By dividing (E.1) over loans we have $\frac{L}{L} = \phi^B \frac{(N^B)}{L} + \omega \frac{M}{L}$. Rearranging terms :

$$\frac{N^B}{L} = \frac{1}{\phi^B}(1 - \omega\chi_m).$$

From the bank's balance sheet constraint we have $D = L - N - M$. Dividing over L:

$$\frac{N^B}{L} = 1 - \frac{D}{L} - \chi_m. \tag{E.2}$$

The bank's net worth is $N^B(1+g) = (\sigma^B + \xi^B)(R^B L) - \sigma^B(RD + R^M M)$. Again dividing over L and rearranging terms, yields:

$$\frac{N^B}{L} = \frac{1}{1+g}[(\sigma^B + \xi^B)R^B - \sigma^B(R\frac{D}{L} + R^M \chi_m]. \tag{E.3}$$

Substituting (E.2) in (E.3) and using $R = 1/\beta^g$ we have

$$\frac{N^B}{L} = (\sigma^B + \xi^B)(R^B) - \sigma^B(\frac{1}{\beta^g}(1 - \frac{N}{L} - \chi_m) + R^M M)$$

Rearranging terms and substituting $R^M = \omega R^B + (1-\omega)R$

$$\frac{N^B}{L} = \frac{(\sigma^B + \xi^B)R^B - \sigma^B/\beta^g + \omega\sigma^B\chi_m(R - R^B)}{1 + g - \sigma^B/\beta^g} = \frac{1}{\phi^B}(1 - \omega\chi_m).$$

So we get the **first equation** for the steady state leverage,

$$\boxed{\phi^B = \frac{(1+g-\sigma^B/\beta)(1-\omega\chi_m)}{(\sigma^B+\xi)R_B-\sigma^B/\beta+\omega\sigma^B\chi_m(R-R^B)}} \tag{E.4}$$

Now I turn in finding the steady state value of the leverage using the definition of leverage. We know that

$$\phi^B = \frac{\nu_{d,j}}{\theta - spread}$$

We also know that $\nu_d = \Lambda\Omega R = \beta\Omega\frac{1}{\beta} = \Omega$ After substituting ν_d, the leverage (ϕ^B) becomes

$$\phi^B = \frac{\Omega}{\theta - \Lambda\Omega(R^B - R)}$$

. Rearranging terms and substituting Ω given by

$$\Omega = (1-\sigma^B) + \sigma^B\phi^B\theta$$

the leverage yields:

$$\boxed{\phi^B = \frac{(1-\sigma^B)+\sigma^B\phi^B\theta}{\theta - ((1-\sigma^B)+\sigma^B\phi^B\theta)(\beta R^B - 1)}} \tag{E.5}$$

being **the second equation** in the system. Hence, we have 2 equations (E.4, E.5) and 2 unknowns (ϕ^B, R^B). After solving this system it is straightforward to find ($\frac{N}{L}, \frac{D}{L}$).

E.2 Steady State of the Entrepreneurs' Problem

Here the solution strategy is the same with the bankers' problem. We find two equations with only unknowns the entrepreneurial leverage (ϕ^E) and the return on capital (R^K).

$$\phi^E = \frac{QK}{N^E}$$

Rearranging,

$$\frac{N^E}{K} = \frac{1}{\phi^E}$$

From, the entrepreneur's net worth equation, $N^E(1+g) = (\sigma^E + \xi^E)(1-\Gamma(\bar{\psi}))QKR^k$ we get the entrepreneurial net worth in steady state. Dividing with capital and substituting $Q = 1$ we have:

$$\frac{N_E}{K} = \frac{1}{(1+g)}(\sigma^E+\xi^E)(1-\Gamma(\bar{\psi}))R^k.$$

From entrepreneurs balance sheet constraint $L = QK - N^E$, dividing with $K^s s$ we have

$$\frac{N_E}{K} = 1 - \frac{L}{K} = \frac{1}{\phi^E}$$

Hence and using the fact that $R^K = \rho(\psi)R^B$,

$$\boxed{\phi_E = \frac{1+g}{(\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))\rho(\psi)R^B}} \tag{E.6}$$

which is **the first equation** for the system.

From the Zero Profit Condition , solving for ϕ_E we get

$$\phi_E = -\frac{R^B}{R^K(\Gamma(\bar{\psi}) - \mu G(\bar{\psi})) - R^B}.$$

Substituting again:

$$\boxed{\phi_E = -\frac{R^B}{\frac{\rho(\bar{\psi})}{\beta}(\Gamma(\bar{\psi}) - \mu G(\bar{\psi})) - R^B}} \tag{E.7}$$

yields **the second equation** for the system.

We have 2 equations (E.6, E.7) and 2 unknowns $(\bar{\psi}, \phi_E)$. Since we know the distribution of ψ we can solve the system. After solving the system and have a value for ψ, ϕ^E, we find R_k from $(R^K = \rho R^B)$ since we know $\bar{\psi}$ and R. We have four equations, two for the bank's problem and two for the entrepreneur with all the distribution equations for ρ,ψ,$G(\psi)$ etc. These are solved by the function `SS_formal_fct.m` To find R^L we go to the only equation that has it:

$$R^L = \bar{\psi}R^K Q\frac{K}{L}.$$

We then know

$$\frac{K}{L} = \frac{1}{1 - \frac{1}{\phi^E}} = \frac{\phi^E}{\phi^E - 1}$$

E.3 Steady State of the Real Sector

Having solved for the financial sector, it's straightforward to find the steady values for the real economy. The interest rate on capital yields $R^K = r_K + 1 - \delta$. We can calculate rK since we know all the other variables. Solve for L/K:

$$\frac{H^d}{K} = \left(\frac{1}{1+g}\right)\left(\frac{u * r_K}{(1-\alpha)PWP}\right)^{\frac{1}{\alpha}}$$

From the law of motion for capital, we have

$$\frac{I}{K} = \frac{\delta + g}{1 + g} \tag{E.8}$$

From the entrepreneur's consumption equation we get

$$\frac{C^E}{K} = (1 - \sigma^E)(1 - \xi^E)(1 - \Gamma(\bar{\psi})) R_{k,ss} Q/(1 + g) \tag{E.9}$$

since we know everything. The resource constraint of the economy in steady state yields $Y = C + I + G + \mu G(\bar{\psi_t}) R_k Q K$. Let's name g_y the steady state fraction of government spending relative to output ($\eta = G/Y$). Using the production function,

$$C = L^{1-\alpha} K^{\alpha} - \delta K - g_y L^{1-\alpha} K^{\alpha} - C^E - \mu G(\bar{\psi_t}) R_k Q K$$

$$\frac{Y}{K} = \frac{(H/K(1+g))^{\alpha}}{(1+F)\Delta^P}$$

Rearranging terms we get :

$$\frac{C}{K} = (1 - g_y)(\frac{Y}{K})^{1-\alpha} - \frac{I}{K} - \frac{C^E}{K} - \mu G(\bar{\psi_t}) R_k Q \tag{E.10}$$

$$Y/C = \frac{Y/K}{C/K}.$$

Finally we have:

$$W = \frac{P^W}{P} \alpha \left(\frac{H^d}{K(1+g)} \right)^{(\alpha-1)}$$

and

$$W^h = W * Invmarkup$$

To find C and H^d we need the labour FOC and the equations therein. We put altogether in a function `SS_formal_realsector_fct.m` with inputs W and Y/C. The system is

- U_C
- U_H
- labour FOC
- $C = \frac{Y^W/\Delta_P}{Y/C}$
- lam
- W equation solved wrt H^d

Finally, knowing L and $\frac{Y}{K}$ from the production function we find the capital. Hence, having capital, by reverse engineering we can find the values for (I, C, C^E, Y) from the equations (E.8, E.9, E.10) respectively. We know from the entrepreneur problem that

$$\frac{L}{K} = 1 - \frac{N^E}{K} = 1 - \frac{1}{\phi^E}$$

Since we know ϕ^E and the capital in steady state we now can find the value for L_t and by the same method we can find (D, N_b, N_e).

F Robustness: Steady State Welfare and Penalty Costs

Figure 7 shows the welfare, Ω_t, path as a function of the liquidity ratio χ_m. Together, we plot the liquidity costs from transaction costs, Ψ_t. The figure is similar to Figure 2 in the main text but for different parametrizatio of the penalty function. Specifically, we set $\tau_1 = 0.000755$ and $\tau_2 = 0.0025$ which yield a 25 basis points cost. While the figure in th emain text shows the path for a 10 bps cost. The figure again is plotted for three different values of ω: (0.1,0.5,0.9), the monitoring ability of the central bank to the liquidity funds. Results remain fairly similar with the figure in the main text with a lower penalty cost. The main difference is that a liquidity rule under high monitoring does not increase welfare as much as in the case of a lower penatly.

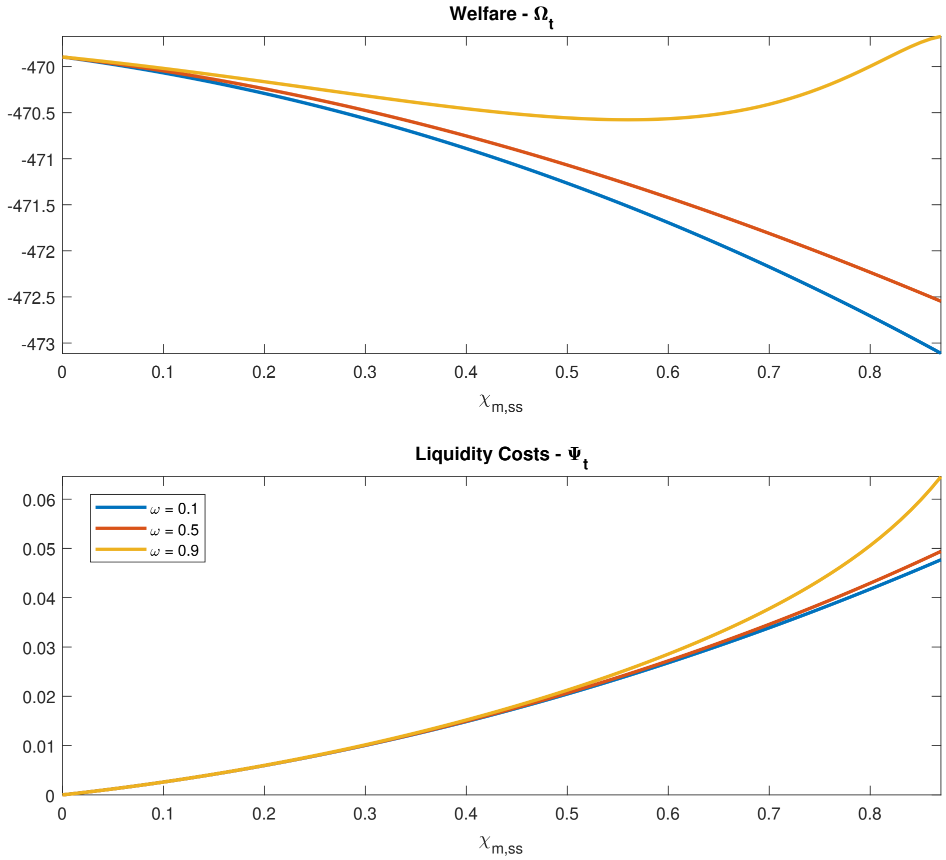

Figure 7: Welfare conditional on central bank liquidity for a 25 bps liquidity cost

G Welfare Changes to Different Steady State Liquidity Values

The Optimized simple rule (with ZLB) - χ_m=0.5															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.839	4.106	4.312	5.091	22.804	0.932	13.652	0.016	0.252	37.708	34.860	1.0084	-472.145	-0.047	0.010	27
The Optimized simple rule (without ZLB) - χ_m=0.5															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.927	2.554	0.505	5.796	23.390	0.036	5.204	0.003	0.001	31.752	22.022	1.0000	-471.911	0.015	0.158	0
The Optimized simple rule (with ZLB) - χ_m=0.4															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.7516	4.0799	3.7832	4.3652	22.45002	0.9492	13.1109	0.01676	0.3317	34.94229	34.5395	1.00792	-472.1473	-0.0529	0.010	36
The Optimized simple rule (without ZLB) - χ_m=0.4															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.9837	10.1087	0.5544	4.5871	20.9133	0.7038	8.0492	0.0015	0.0004	43.2372	15.7983	1.0000	-471.922	0.0071	0.14816	0
The Optimized simple rule (with ZLB) - χ_m=0.3															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.8379	3.7763	3.3186	4.6987	22.3175	0.9778	13.2211	0.0102	0.3094	34.8012	34.5717	1.00808	-472.1461	-0.0525	0.010	36
The Optimized simple rule (without ZLB) - χ_m=0.3															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.8570	5.4204	0.5823	8.7590	23.319	0.8001	7.2222	0.0045	0.0017	27.792	20.847	1.0000	-471.9213	0.0073	0.14269	0
The Optimized simple rule (with ZLB) - χ_m=0.1															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.8202	2.7032	2.8754	4.3372	23.0275	0.9968	12.8465	0.0072	0.3773	35.0029	34.5753	1.0076	-472.1508	-0.0511	0.010	41
The Optimized simple rule (without ZLB) - χ_m=0.1															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.8233	10.8666	5.2209	6.3496	70.6351	0.8888	12.5412	0.0263	0.0135	35.8040	70.4397	1.0000	-471.934	0.0039	0.15434	0
The Optimized simple rule (with ZLB) - χ_m=0.0001															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.857	3.322	2.881	4.158	22.723	0.999	13.175	0.011	0.237	34.741	34.406	1.0082	-472.154	-0.050	0.010	27
0.980	3.415	3.751	5.048	22.899	0.999	13.529	0.024	0.009	34.862	34.777	1.0078	-472.088	-0.032	0.025	5
0.821	4.175	3.049	4.053	22.724	0.999	13.307	0.011	0.003	34.867	34.465	1.0051	-472.007	-0.010	0.050	3
The Optimized simple rule (without ZLB) - χ_m=0.0001															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.990	3.586	1.567	0.892	19.614	0.997	13.829	0.006	0.004	25.159	35.567	1.0000	-471.968	0.000	0.156	0
0.736	19.857	0.109	6.225	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.0000	-472.065	-0.026	0.090	0
Estimated model															
ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	$\frac{\alpha_{sp}^*}{1-\rho_r^*}$	ρ_r^{l*}	$\frac{\alpha_\pi^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_y^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{dy}^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{sp}^{l*}}{1-\rho_r^{l*}}$	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.0073	-472.337	-0.103	0.007	-
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.0000	-472.200	-0.066	0.101	-
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.0000	-472.194	-0.065	0.100	-

Table 6: Welfare Analysis

H Optimized Rules and Monitoring

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	5	0.00002	1.715	0.721	-0.388	-1.115	-5	0.00000007	1.004	-472.203	-0.005	0.01	15	0.1	0.1
(b)	1	5	0.000002	1.687	0	0	0	0	0	1.004	-472.243	-0.016	0.01	25	0	0.1
(c)	0.806	5.000	0.011	1.894	0	0	0	0	0	1.000	-472.193	-0.003	0.073	0	0	0.1
(d)	0.791	5.000	0.014	1.881	0	0	0	0	0	1.000	-472.183	0.000	0.074	0	0	0

Table 7: Welfare Analysis: $\omega = 0.9$

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	5	0.0002	1.846	0.843	-1.410	-5	-0.88	0.000007	1.004	-472.222	-0.0103	0.01	40	0.1	0.1
(b)	1	5	0.000004	1.742	0	0	0	0	0	1.004	-472.252	-0.0182	0.01	15	0	0.1
(c)	0.822	5	0.015	1.887	0	0	0	0	0	1.000	-472.202	-0.0049	0.073	0	0	0.1
(d)	0.795	5	0.0133	1.873	0	0	0	0	0	1.000	-472.183	0.0000	0.074	0	0	0

Table 8: Welfare Analysis: $\omega = 0.8$

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	5	0.0001	1.740	0.701	-4.908	-0.734	-5	0.0000007	1.004	-472.237	-0.0144	0.01	5	0.1	0.1
(b)	1	5	0.00005	1.812	0	0	0	0	0	1.004	-472.260	-0.0205	0.01	10	0	0.1
(c)	0.747	5	0.012	1.887	0	0	0	0	0	1.000	-472.210	-0.0071	0.076	0	0	0.1
(d)	0.769	5	0.013	1.879	0	0	0	0	0	1.000	-472.183	0.0000	0.075	0	0	0

Table 9: Welfare Analysis: $\omega = 0.7$

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	4.996	0.000007	1.608	0.700	-4.994	-0.740	-2	0.0000007	1.004	-472.254	-0.0187	0.01	50	0.1	0.1
(b)	1	5	0.00003	1.773	0	0	0	0	0	1.004	-472.268	-0.0225	0.01	15	0	0.1
(c)	0.867	5	0.015	1.900	0	0	0	0	0	1.000	-472.218	-0.0092	0.071	0	0	0.1
(d)	0.828	5	0.014	1.900	0	0	0	0	0	1.000	-472.183	0.0000	0.072	0	0	0

Table 10: Welfare Analysis: $\omega = 0.6$

The Optimized Simple Monetary and Liquidity Rules

	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*	$w_{\chi_m}^*$	χ_m
(a)	1	1.9	0.0000003	0.675	0.700	-2	-0.757	-2	0.0000001	1.004	-472.301	-0.0315	0.01	70	0.1	0.1
(b)	1	5	0.00001	1.686	0	0	0	0	0	1.004	-472.275	-0.025	0.01	20	0	0.1
(c)	0.800	5	0.013	1.890	0	0	0	0	0	1.000	-472.226	-0.011	0.073	0	0	0.1
(d)	0.794	5	0.014	1.881	0	0	0	0	0	1.000	-472.183	0.000	0.074	0	0	0

Table 11: Welfare Analysis: $\omega = 0.5$