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GREED VERSUS FEAR: OPTIMAL TIME-CONSISTENT TAXATION WITH DEFAULT

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Greed versus Fear: Optimal Time-Consistent Taxation with Default^{*}

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Abstract

This paper studies optimal time-consistent distortionary taxation with default. The government taxes labor income or issues non-contingent debt to finance an exogenous stochastic stream of fiscal shocks. Debt can be repudiated subject to default costs. Optimal policy is characterized by two opposing incentives that capture the double lack of commitment to old tax policies and to repaying debt, respectively: an incentive to postpone taxes by issuing more debt for the future (“Greed”), and an incentive to tax more currently to avoid punishing default premia (“Fear”). A Generalized Euler Equation captures these forces and determines the optimal back-loading or front-loading of tax distortions. Even with small default risk, tax-smoothing and debt issuance are substantially limited when short-term debt is available. The same mechanisms operate in environments with long-term debt, but debt serves more as a fiscal absorber.

Keywords: Taxation, default, Markov-perfect equilibrium, time-consistency, generalized Euler equation, commitment, long-term debt.

JEL classification: D52; E43; E62; H21; H63.

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1 Introduction

This paper studies the optimal time-consistent allocation of tax distortions and the optimal issuance of debt in an environment where government debt can be defaulted on. We consider a government that has to finance an exogenous stream of stochastic government expenditures and maximizes the utility of the representative household. The government uses distortionary labor taxes or issues non-contingent debt. The government can default on its debt subject to a default cost. Our setup is fully time-consistent; the government commits to neither old tax promises nor debt repayment.

Our analysis builds on the concept of Markov-perfect equilibrium of [Klein et al. \(2008\)](#). Optimal policy is time-consistent in the payoff-relevant state variables, which for our case are government debt and government expenditures. Furthermore, we model default as in the work of [Arellano \(2008\)](#) and [Aguiar and Gopinath \(2006\)](#), that builds on the debt repudiation setup of [Eaton and Gersovitz \(1981\)](#). This setup allows us to observe default in equilibrium.¹

In most of the sovereign default literature, government debt is assumed to be held only by foreigners, whereas domestic households are hand-to-mouth consumers. However, [Reinhart and Rogoff \(2011\)](#) find that, on average, domestic debt accounts for nearly two-thirds of total public debt for a large group of countries. We consider a closed economy in which domestic households hold government debt. Thus, our model takes into account that default events often involve default on debt held by domestic households. This assumption is supported by the empirical literature. While domestic default events are more difficult to identify than external default events, [Reinhart and Rogoff \(2011\)](#) document 68 cases of overt default on domestic debt since 1800. Moreover, often even when default is only on external debt (which we do not model), a significant portion of external debt is held by domestic investors ([Sturzenegger and Zettelmeyer 2006](#)). For these reasons, it is of interest to understand the tradeoffs governments face when considering whether to default on domestic households.

Our purpose is to analyze optimal tax-smoothing and debt issuance in such an environment. The lack of state-contingent insurance markets hinders the ability of the government to smooth taxes. Default can in principle make debt partially state-contingent. This state-contingency does not come for free: default risk is reflected in equilibrium prices, altering the tradeoffs that the government is facing in smoothing taxes over states and dates.

The government has an incentive to default when either debt or expenditures are high. By defaulting, the government can avoid high distortionary taxation. However, default entails a temporary collapse of debt markets and utility losses. We analyze optimal time-consistent policy through the lens of a Generalized Euler Equation (GEE henceforth), which summarizes the dynamic costs and benefits that the government faces when it issues debt. An increase in debt issuance induces a marginal welfare loss (higher debt is repaid in the future with higher distortionary taxes), which is balanced with the benefits of additional revenue from debt issuance. Additional revenue relaxes the government budget and allows lower taxes today.

Under commitment, the planner makes average tax distortions constant over time. In

¹See the surveys of [Aguiar and Amador \(2014\)](#) and [Aguiar et al. \(2016\)](#) for further references.

contrast, our GEE captures two opposite forces that reflect the double lack of commitment to old tax promises and to debt repayment, respectively. Facing a fiscal shock, the policymaker can either raise taxes or issue more debt that is due next period. The current policymaker realizes that higher debt issuance makes the future policymaker tax more, reducing future consumption, conditional on repaying. This raises future marginal utility and increases the current price of debt. Consequently, debt becomes cheaper, creating the incentive to back-load tax distortions. We dub this force “Greed,” to capture the desire of the current government to reduce interest rates by bequeathing more debt to the future government.

However, this desire to postpone taxes and issue more debt is counteracted by the “Fear” of default. Higher debt issuance increases the probability of default, decreasing the price of debt and raising default premia. Debt becomes more expensive, forcing the policymaker to use higher current taxes to absorb fiscal shocks. Consequently, default risk creates an incentive to front-load tax distortions and issue less debt. Ultimately, the dynamics of taxes and debt are determined by the relative strength of these opposite forces, which depend on the curvature of the pricing kernel and the size of the region in which government debt is subject to default risk.

We also provide a full-blown analysis of the time-consistent and the commitment problem, when the government issues long-term debt as in [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#). Long-term debt is of particular interest to our optimal taxation exercise, since future long-term debt prices provide an additional fiscal hedging tool. Moreover, it has been shown in the sovereign default literature that long-term debt specifications can sustain higher debt in equilibrium and capture well the mean of spreads without generating counterfactually high spread volatility.²

Under commitment, the planner still wants to keep average tax distortions constant over time when there is long-term debt. The long duration of debt, however, makes the planner take into account the effects of his actions on all past prices, introducing additional history dependence to the commitment plan. In contrast, when there is double lack of commitment with long-term debt, the actions of all future policymakers matter for the determination of taxes and debt. We show that a richer version of our GEE goes through, reflecting the *present value* of the two opposing forces of “Greed” and “Fear” on debt prices.

We conduct a series of numerical exercises to highlight the implications of the double lack of commitment for the typical optimal taxation problem. We use a small domestic default probability as in the study of [Reinhart and Rogoff \(2011\)](#). We contrast time-consistent policy with default (“Greed versus Fear”) to time-consistent policy without default (“Greed”), and to optimal policy under commitment. We perform our numerical exercises for both the short- and the long-term debt specification.

In the short-term debt case, we find that the [Barro \(1979\)](#) and [Aiyagari et al. \(2002\)](#) random walk results about the tax rate are substantially altered. When there is no commitment to old tax policies (“Greed”) but no default option, tax-smoothing is substantially limited. Tax rates are very volatile and exhibit low autocorrelation. When we turn on default risk and activate

²See, for example, [Chatterjee and Eyigungor \(2012\)](#).

the “Fear” force, tax rates remain volatile, but governments issue limited amounts of debt, even if default is rare. Hence, governments rely mostly on taxes to absorb fiscal shocks in such an environment. In the case of long-term debt, governments issue, on average, more debt and spreads are relatively less volatile. We also find that future prices of long-term debt are particularly important in the time-consistent problem with default, since they react the most to adverse fiscal shocks.

Related literature. This work is related to two strands of literature: the literature that drops the commitment assumption in optimal policy and the literature that follows [Eaton and Gersovitz \(1981\)](#) in modelling the default decision of the government. The commitment benchmark for short-term debt is the work of [Aiyagari et al. \(2002\)](#), who endogenize the tax-smoothing results of [Barro \(1979\)](#) and analyze optimal taxation under incomplete markets when there is commitment to both tax policies and debt repayment.

Several papers have dropped the commitment assumption of [Lucas and Stokey \(1983\)](#) and [Aiyagari et al. \(2002\)](#), and considered other timing protocols. For example, [Chari and Kehoe \(1993\)](#) and [Aguiar and Amador \(2016\)](#) study environments with sustainability constraints using the concept of sustainable plans of [Chari and Kehoe \(1990\)](#). [Dovis \(2019\)](#) analyzes efficient sovereign default under information frictions. Another relevant study is [Sleet and Yeltekin \(2006\)](#), which endogenizes the incompleteness of government debt markets through limited commitment and private information, and considers the optimal design of fiscal policy.

Following a different route in exploring optimal policy beyond commitment, several papers have considered environments with Markov-perfect policy, which is the relevant concept for this work. More specifically, [Krusell et al. \(2004\)](#), [Debortoli and Nunes \(2013\)](#) and [Occhino \(2012\)](#) analyze time-consistent taxation and debt issuance in deterministic setups without default with either exogenous government spending or utility-providing government spending. [Krusell et al. \(2004\)](#), in particular, discovered that optimal policy may involve discontinuous policy functions when the government issues debt in a time-consistent fashion. Their deterministic setup is at the heart of our stochastic setup with default. [Martin \(2009, 2011\)](#) has made extensive progress in analyzing the joint determination of time-consistent fiscal and monetary policy.

Turning to the default literature, [Cuadra et al. \(2010\)](#) have considered distortionary taxation in a small open economy sovereign default model, where debt is only external. A paper of interest is [Pouzo and Presno \(2022\)](#), which has been the first to consider the possibility of debt repudiation à la Arellano in the closed economy optimal taxation problem. [Pouzo and Presno](#) alter the [Aiyagari et al. \(2002\)](#) setup only in one dimension; they allow the government to default but they retain a notion of commitment. In their setup, the government cannot commit to repay debt, but, as long as the government decides to repay, it honors the marginal utility promises of the plan devised in previous periods and commits therefore to the sequence of taxes and interest rates. Thus, only the “Fear” force shows up in their work and not the “Greed” element. In contrast, we treat debt and taxes symmetrically and derive the fully time-consistent policy. Furthermore, we extend the analysis to environments with long-term debt.

Another paper of interest is [Sosa-Padilla \(2018\)](#), who builds a quantitative model of banking

crises and sovereign default in closed economies. He considers optimal time-consistent debt, tax and default policies and focuses on the endogenous costs of default through the collapse of credit. In another strand of the literature, [Espino et al. \(2025\)](#) study the jointly optimal fiscal and monetary policy in an open economy with default risk, and analyze the tradeoffs between distortionary taxation, outright default and “default” through inflation.³ In both of these contributions our “Greed” force is absent, since in the former one, domestic debt is priced by risk-neutral banks, whereas in the latter one, debt is external.

A dimension of the problem that we do not examine is the redistributive effect of domestic default. A prominent example is [D’Erasmus and Mendoza \(2021\)](#), who study optimal domestic sovereign default in a quantitative model with heterogeneous agents, bringing to the forefront the “forgotten” history of domestic debt of [Reinhart and Rogoff \(2011\)](#). They focus on the redistributive effects of default and the liquidity properties of government debt, but abstain from the optimal distortionary taxation problem.

Organization. Section 2 lays out the economy. Section 3 sets up the time-consistent taxation problem with default. The forces of “Greed” and “Fear” are analyzed through the lens of a GEE in section 4 and are contrasted to tax-smoothing under commitment. Section 5 extends the analysis to environments with long-term debt. Section 6 provides various numerical exercises and section 7 concludes. Appendices A and B provide the proofs of the time-consistent problems with short- and long-term debt respectively, and Appendix C provides additional numerical results. In addition, Online Appendix D analyzes a battery of relevant commitment problems with short- and long-term debt.

Time-consistent problems present their own numerical challenges. Online Appendix E illustrates the relevant numerical issues that involve potential discontinuities in policy functions. Online Appendix F reformulates our optimal taxation problem with discrete choice methods, which deliver smooth decision rules. Online Appendix G provides details on the computational methods employed.

2 Economy

We start the analysis of optimal time-consistent policy considering first defaultable short-term debt. Our environment is similar to the environment of [Lucas and Stokey \(1983\)](#) and [Aiyagari et al. \(2002\)](#). Time is discrete and the horizon is infinite. The uncertainty is coming from exogenous government expenditure shocks g_t , that take values in the set $\mathcal{G}$, and are assumed to be Markov. Let $g^t \equiv (g_0, g_1, \dots, g_t)$ denote the partial history of shocks up to t and assume that the initial shock g_0 is given.

The economy is populated by a representative household with preferences over consumption and leisure and an endowment of one unit of time every period. Let $c_t(g^t)$ denote private consumption and let $h_t(g^t)$ denote labor, which is used for the production of output. Leisure

³Other setups that analyze inflation and default include [Bassetto and Galli \(2019\)](#) and [Roettger \(2019\)](#).

is $l_t(g^t) = 1 - h_t(g^t)$. Let $d_t(g^t)$ denote an indicator variable that captures the decision of the government to default, so $d_t(g^t) = 1$ if there is default, and $d_t(g^t) = 0$ if the government repays.

The resource constraint in the economy reads

$$c_t(g^t) + g_t = h_t(g^t). \quad (1)$$

Household. The household's preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t),$$

where u is increasing and concave, and $\mathbb{E}_0$ the expectation operator with respect to the physical measure of uncertainty, conditional on the initial shock g_0 . The household trades with the government a discount bond that gives one unit of consumption next period at any state of the world in which the government repays, and zero in the event of default. The price of the bond is $q_t(g^t)$. The household pays a linear tax $\tau_t(g^t)$ on labor income $w_t(g^t)h_t(g^t)$. The household's budget constraint when the government is not defaulting at t reads

$$c_t(g^t) + q_t(g^t)b_{t+1}(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + b_t(g^{t-1}). \quad (2)$$

Note that the bond position b_{t+1} is function of the partial history g^t . The household has also initial debt b_0 and is also subject to borrowing limits that are assumed to be large enough so that they do not bind.

Default. We follow [Bianchi et al. \(2023\)](#) and [Fourakis and Karabarbounis \(2025\)](#) and model the cost of default as a utility loss for the policymaker. When the government defaults, it incurs a utility cost that depends on the government spending shock g , $\psi(g)$. We turn to the functional form of $\psi(g)$ in the numerical section. At the period of default, current debt is wiped out and the government debt market collapses. The government cannot issue any new debt and runs a balanced budget. The government reenters the market in the next period with zero initial debt.⁴

Consequently, the household's budget constraint in the event of default is

$$c_t(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t). \quad (3)$$

⁴It is common to add stochastic exclusion after default in the open economy sovereign default literature to capture the observed limited access to international markets after default; see for example [Arellano \(2008\)](#). We abstain from this feature in our closed economy. See also [D'Erasmus and Mendoza \(2021\)](#) for a similar assumption.

Firms. Firms are price-takers and hire labor in competitive factor markets. They operate a linear technology that converts one unit of labor to one unit of output.

Government. The budget constraint of the government in the event of repayment is

$$B_t(g^{t-1}) = \tau_t(g^t)w_t(g^t)h_t(g^t) - g_t + q_t(g^t)B_{t+1}(g^t). \quad (4)$$

If $B_t(g^{t-1}) > 0$, the government borrows from the household, whereas if $B_t(g^{t-1}) < 0$, the government lends to the household.

If there is default, the government runs a balanced budget,

$$\tau_t(g^t)w_t(g^t)h_t(g^t) = g_t. \quad (5)$$

Equilibrium. A competitive equilibrium with taxes and default is a price-tuple $\{q, w\}$, a government policy $\{\tau, B, d\}$, and a household's allocation and bond holdings $\{c, h, b\}$ such that: 1) Given prices and government policies, the household maximizes its utility subject to the budget constraint, 2) Given wages, firms maximize profits, 3) Prices and government policies are such that the resource constraint and the government budget constraint hold. Furthermore, the bond market clears, $b_t = B_t$.⁵

2.1 Optimality conditions

Competitive factor markets lead to an equilibrium wage rate that is unity, $w_t(g^t) = 1$. The household's labor supply condition equates the marginal rate of substitution between consumption and leisure to the after-tax wage,

$$\frac{u_{lt}}{u_{ct}} = (1 - \tau_t)w_t. \quad (6)$$

The Euler equation for government bonds is

$$q_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{ct}} (1 - d_{t+1}). \quad (7)$$

Equation (7) is the standard pricing equation that prices an asset with risky payoff $1 - d_{t+1}$. The equilibrium price of debt is zero, if the government defaults with certainty. If the government repays with certainty, then (7) reduces to the standard Euler equation without default.

⁵Given $b_t = B_t$ and the rest of the equilibrium conditions, the government budget constraint is redundant.

Default premium. We use (7) to contrast the return of government debt to the return of a risk-free asset. Assume that there is a private risk-free asset that is in *zero* net supply, so that the equilibrium allocation is not affected. This asset has a price $q_t^F = \beta \mathbb{E}_t u_{c,t+1}/u_{ct}$. We express the default *premium* as⁶

$$\frac{q_t^F}{q_t} - 1 = \frac{r_t - r_t^F}{1 + r_t^F} = \frac{\mathbb{E}_t x_{t+1} d_{t+1}}{\mathbb{E}_t x_{t+1} (1 - d_{t+1})} = \frac{\text{Prob}_t^x(\text{default at } t+1)}{\text{Prob}_t^x(\text{repayment at } t+1)}, \quad (8)$$

where $x_{t+1} \equiv u_{c,t+1}/\mathbb{E}_t u_{c,t+1} \geq 0$, with $\mathbb{E}_t x_{t+1} = 1$, the *risk-adjusted* change of measure, and Prob_t^x the respective conditional risk-adjusted probability. The variables r_t and r_t^F refer to the net returns of government debt and the risk-free asset, $r_t \equiv 1/q_t - 1$, $r_t^F \equiv 1/q_t^F - 1$, respectively. Therefore, the default premium at t is equal to the ratio of the conditional risk-adjusted probability of default next period over the respective risk-adjusted probability of repayment.

3 Markov-perfect policy

The policymaker chooses tax, debt and default policies to maximize the utility of the representative household. His choice of (τ, B, d) is subject to the constraints of the competitive equilibrium, that is, optimality conditions, budget and resource constraints. We use the primal approach of Lucas and Stokey (1983) and eliminate tax rates and prices (τ, q) from the budget constraints by using the respective optimality conditions (6) and (7).

We assume a Markov-perfect timing protocol as in Klein et al. (2008), so the solution to the policy problem is time-consistent in the payoff-relevant state variables.⁷ The Markov-perfect equilibrium has two “natural” state variables: government debt B , and the exogenous shock g . At the beginning of each period, the government decides to repay or default. Let $V^r(B, g)$ denote the value function if the government decides to repay, and $V^d(g)$ the value function if the government defaults. The value function of the government is

$$V(B, g) = \max\{V^r(B, g), V^d(g)\}. \quad (9)$$

Value of default. When the government defaults, the consumption and labor allocation is determined by the resource constraint, the labor supply condition and the balanced budget requirement. Using (6) to eliminate the after-tax wage rate from the household’s budget (3), the balanced budget requirement becomes

⁶In small open economy models the default premium is measured against the exogenous return of an international risk-free asset.

⁷See Bassetto (2005) for a careful analysis of the timing protocols underlying optimal policy design.

$$\Omega(c, h) = 0, \quad (10)$$

where

$$\Omega(c, h) \equiv u_c(c, 1 - h)c - u_l(c, 1 - h)h, \quad (11)$$

that is, consumption net of after-tax labor income, in marginal utility units. Ω is exactly equal to the primary surplus in marginal utility units. Let $(c^d(g), h^d(g))$ denote the default consumption-labor allocation that satisfies the resource constraint (1) and the balanced budget requirement (10) at g .⁸ The value of default is

$$V^d(g) = u(c^d(g), 1 - h^d(g)) - \psi(g) + \beta \mathbb{E}_{g'|g} V(0, g'). \quad (12)$$

Moreover, the default allocation induces the default tax rate, $\tau^d(g) = 1 - \frac{u_l(c^d(g), 1 - h^d(g))}{u_c(c^d(g), 1 - h^d(g))}$.

Default decision. Define the default set and its complement repayment set as

$$D(B) \equiv \{g \in \mathcal{G} | V^d(g) > V^r(B, g)\} \quad \text{and} \quad A(B) \equiv D(B)^c = \{g \in \mathcal{G} | V^d(g) \leq V^r(B, g)\}.$$

Given an amount of debt B at the beginning of the period, the default set denotes the set of values of g for which the government decides to default, so if $g \in D(B)$, $d(B, g) = 1$ and $V(B, g) = V^d(g)$. When the government is indifferent between repaying or defaulting, $V^r(B, g) = V^d(g)$, we assume that it repays. Thus, if $g \in A(B)$, we have $d(B, g) = 0$ and $V(B, g) = V^r(B, g)$. Define also the (endogenous) upper and lower debt limits as

$$\bar{B} \equiv \inf\{B | D(B) = \mathcal{G}\} \quad \text{and} \quad \underline{B} \equiv \sup\{B | D(B) = \emptyset\}.$$

$\bar{B}$ is the lowest amount of debt above which the government defaults with certainty, whereas $\underline{B}$ is the highest amount of debt below which the government repays with certainty.

Value of repayment. In a Markov-perfect equilibrium optimal policy has *two* features: a) the policymaker is not bound to past promises, a fact which is embedded in the state variables (B, g) , and b) the policymaker at t takes into account that an optimal policy will be followed by the *future* policymaker, given the value of the natural state variables next period, (B', g') .

⁸Note that if there are multiple feasible allocations that satisfy the balanced budget requirement (10), we should pick the one with the highest consumption. This allocation should correspond to a tax rate that is at the increasing portion of the tax Laffer curve.

The choices of the future policymaker are the future default decision, captured in the future repayment set $A(B')$, and the future consumption and labor decisions, that affect future marginal utility, and therefore, current prices of debt. Let $\mathcal{C}(B', g')$ and $\mathcal{H}(B', g')$ denote the repayment consumption and labor policy functions next period when the state is (B', g') . We obviously have $\mathcal{C}(B', g') + g' = \mathcal{H}(B', g'), \forall (B', g')$. The current policymaker takes into account that by choosing debt B' , he affects the consumption-labor choice of the future policy maker through $(\mathcal{C}, \mathcal{H})$, as well as the future repayment decision through $A(B')$. The value of repayment is

$$V^r(B, g) = \max_{c \geq 0, h \in [0, 1], B'} u(c, 1 - h) + \beta \mathbb{E}_{g'|g} V(B', g') \quad (13)$$

subject to

$$u_c(c, 1 - h)B \leq \underbrace{\Omega(c, h)}_{\text{surplus in } u_c \text{ units}} + \underbrace{\beta \mathbb{E}_{g' \in A(B')|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))}_{Q(B', g)} \cdot B' \quad (14)$$

$$c + g = h \quad (15)$$

The policymaker chooses a consumption-labor allocation and issues new debt subject to the implementability constraint (14) and the resource constraint (15). The implementability constraint is just the government's (or household's) budget constraint in current marginal utility units, after (6) and (7) are used to eliminate τ and q . In particular, let $Q_t \equiv u_{ct} \cdot q_t$ denote the price of government debt in current *marginal utility* units. From (7) we know that Q_t is equal to average future marginal utility, conditional on repaying, $Q(B', g) \equiv \beta \mathbb{E}_{g' \in A(B')|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))$. This is exactly the object that shows up in the right-hand side of (14).

Markov-perfect requirement. Let $c(B, g)$, $h(B, g)$ and $B'(B, g)$ be the policy functions that solve (13). The Markov-perfect requirement is that $c(B, g) = \mathcal{C}(B, g)$ and $h(B, g) = \mathcal{H}(B, g)$ for all (B, g) .⁹

Default threshold. It is easy to see that the repayment value function of the government is decreasing in B . As a result, default sets are increasing in the amount of debt issued, that is, if $B_1 < B_2$, then $D(B_1) \subseteq D(B_2)$. Moreover, let $\omega(g)$ denote the amount of debt such that the government is indifferent between repaying or defaulting at g , $V^d(g) = V^r(\omega(g), g)$. The government defaults if debt is above the threshold, $B > \omega(g)$, and repays if $B \leq \omega(g)$.

⁹A more precise MPE requirement would be that $\mathcal{C}(B, g)$ and $\mathcal{H}(B, g)$ are maximizers of the stated problem in order to account for the existence of multiple solutions. This is for example what Klein et al. (2008) do.

4 Greed versus Fear and tax-smoothing

We proceed with some simplifying assumptions, to understand the dynamic tradeoffs that the policymaker is facing.

4.1 Assumptions

Assume a continuous distribution of shocks with bounded support $[g, \bar{g}]$ and a conditional density $f(g'|g)$. Furthermore, assume differentiability of the value functions, the policy functions of the future planner $(\mathcal{C}, \mathcal{H})$ and the default threshold ω . We conjecture that the government defaults more in “bad” times of high government expenditures, that is, if $g' > g$ and $g \in D(B)$, then $g' \in D(B)$. This statement is equivalent to a monotonically decreasing threshold, $\omega(g') \leq \omega(g)$ for $g' > g$. If ω is strictly decreasing, then we can invert it and interpret $\omega^{-1}(B)$ as the value of government spending that makes the government indifferent between repaying or defaulting for $B \in [\underline{B}, \bar{B}]$. In that case, the repayment and default sets become intervals, $A(B) = [g, \omega^{-1}(B)]$ and $D(B) = (\omega^{-1}(B), \bar{g}]$.¹⁰

These assumptions are *not* innocuous, since any differentiability holds at most locally. A smooth (or even a continuous) Markov-perfect equilibrium may not even exist. We make these assumptions only to gain intuition and to develop insights about the (local) dynamic tradeoffs. In the numerical treatment of the problem we abstain from these assumptions and work directly with the Bellman equations of the default and repayment problem.¹¹

4.2 Optimal debt issuance

The main object of the analysis throughout the paper is $\Phi \geq 0$, the multiplier on the implementability constraint (14). We call Φ the *excess burden of taxation*, since it denotes the welfare cost stemming from the absence of lump-sum taxes, and captures essentially tax distortions.

Consider the first-order necessary condition with respect to B' in problem (13) at a locally smooth interior optimum,

$$-\beta \underbrace{\frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g')}_{\text{MC: future taxes}} = \Phi \cdot \underbrace{[Q(B', g) + Q_B(B', g) \cdot B']}_{\text{MR from debt issuance}}. \quad (16)$$

¹⁰To see what is necessary for $\frac{d\omega}{dg} < 0$ in a smooth equilibrium, apply the implicit function theorem to get $\frac{d\omega}{dg} = \frac{\frac{dV^d}{dg} - \frac{\partial V^r}{\partial g}}{\frac{\partial V^r}{\partial B}}$. The repayment value function is decreasing in B , $\frac{\partial V^r}{\partial B} < 0$. Thus, we have $\frac{d\omega}{dg} < 0$ when $\frac{dV^d}{dg} > \frac{\partial V^r}{\partial g}$. We typically expect the value of repayment to be decreasing in g , $\frac{\partial V^r}{\partial g} < 0$. The value of default could potentially be increasing in g , depending on the specification of the utility cost $\psi(g)$. If this is the case, then the condition for monotonicity for ω is satisfied. In the more likely scenario of $\frac{\partial V^d}{\partial g} < 0$, we have $\frac{d\omega}{dg} < 0$ when the value of default decreases in *absolute* terms less than the value of repayment, when g increases. In the numerical section we show that the threshold is indeed decreasing in g for our calibration.

¹¹We discuss the various issues about non-differentiabilities and discontinuities in section 6 and provide further details in Online Appendices E, F and G.

Optimality condition (16) captures the dynamic tradeoffs of issuing debt, and consequently, the tradeoffs between taxing the current period versus issuing debt and postponing taxes to the future. The left-hand side denotes the marginal cost of issuing debt. Debt is costly in this second-best world because the government has to raise revenues through distortionary taxation in order to repay. Thus, the left-hand side captures effectively future taxes. The right-hand side of (16) denotes the marginal benefit of issuing debt, which is equal to the marginal revenue from debt issuance, multiplied by the shadow value of relaxing the government budget, Φ . Having more revenue from debt issuance allows the government to tax less in the current period.

The crucial element in (16) is the manipulation of interest rates embedded in the price derivative Q_B . We expand Q_B and rewrite (16) in sequential notation as a *Generalized Euler Equation* (GEE henceforth) in the following proposition.¹²

Proposition 1. (“GEE with short-term debt”)

The GEE with short-term debt at a locally smooth equilibrium takes the form

$$\underbrace{\mathbb{E}_t m_{t+1} \Phi_{t+1}}_{\text{Average future taxes}} = \Phi_t \cdot \left\{ 1 + \overbrace{\left[\underbrace{\mathbb{E}_t m_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} C_B(B_{t+1}, g_{t+1})}_{\text{“Greed” (+): } Q \uparrow \text{ due to an increase of } u'_c} + \underbrace{f^m(\omega^{-1}(B_{t+1}), B_{t+1}|g_t) \frac{d\omega^{-1}}{dB'}}_{\text{“Fear” (-): } Q \downarrow \text{ due to default risk}} \right]}^{\eta \equiv Q_B/Q: \text{ price semi-elasticity}} \cdot B_{t+1} \right\}, \quad (17)$$

where $m_{t+1} \geq 0$ is the default-and-risk-adjusted change of measure ($\mathbb{E}_t m_{t+1} = 1$),

$$m_{t+1} = m(B_{t+1}, g_{t+1}|g_t) \equiv \frac{(1 - d_{t+1})u_{c,t+1}}{\mathbb{E}_t(1 - d_{t+1})u_{c,t+1}}, \quad (18)$$

and $f^m(g_{t+1}, B_{t+1}|g_t) \equiv m(B_{t+1}, g_{t+1}|g_t) \cdot f(g_{t+1}|g_t)$, the induced conditional density.

Proof. See Appendix A. □

The left-hand side of (17) captures *average* future tax distortions with respect to the default-and-risk-adjusted measure. It corresponds to the left-hand side of (16), after the respective envelope condition is used, and expresses the marginal cost of issuing debt in terms of the future excess burden of taxation, $\Phi_{t+1} = \Phi(B_{t+1}, g_{t+1})$. The right-hand side of (17) captures how the reactions of the future policymaker affect prices. In particular, the terms in the square brackets constitute the price *semi-elasticity* with respect to debt, $\eta_t \equiv Q_B(B_{t+1}, g_t)/Q_t$. The price semi-elasticity η_t can be decomposed into two parts that reflect both dimensions of lack of commitment: a) no commitment to tax policies designed in the past, b) no commitment to repaying debt.

Greed. Consider first the lack of commitment to tax policies designed in the past and assume that $u_{cl} \geq 0$. The current policymaker has to take into account how the future policymaker

¹²Recall that a GEE is an optimality condition that involves derivatives of future policy functions. See Klein et al. (2008).

chooses consumption and labor, since these choices affect future marginal utility and, therefore, the price of government debt. In particular, we expect that an increase in debt B' leads to a decrease in the consumption of the future policymaker ($\mathcal{C}_B < 0$), since, conditional on repaying, the future policymaker has to use higher distortionary taxes. This fall in future consumption raises marginal utility, $(u'_{cc} - u'_{cl})\mathcal{C}_B > 0$, and, therefore, *increases* the price of debt, $Q_B > 0$. In other words, by bequeathing higher debt to the future government, the current policymaker reduces the cost of new debt. So higher debt issuance leads to *lower* interest rates due to the manipulation of the actions of the future policymaker. We dub this effect “Greed”.

The “Greed” effect is present as long as there is curvature in the utility function. If utility were linear in consumption, there would be no room for interest rate manipulation through the consumption and labor choices of the future policymaker. Note also that this effect is not present in the typical small open economy models of sovereign default as in [Arellano \(2008\)](#), since external investors are either risk-neutral, or their pricing kernel is outside the control of the domestic government.

Fear. The positive “Greed” effect through the pricing kernel is naturally accompanied with an opposite effect of *negative* sign, that is related to the lack of commitment to repaying. Higher debt issuance B' increases the default region and raises the probability of default of the future policymaker. This effect is reflected in the price semi-elasticity in (17) as the default-and-risk-adjusted probability mass at the threshold value of government expenditures $\omega^{-1}(B_{t+1})$, times the change in the threshold, $\frac{d\omega^{-1}}{dB'} < 0$. Thus, the price of debt *falls*, $Q_B < 0$, raising interest rates. In other words, default risk reduces the marginal revenue from debt issuance. We call this force “Fear,” referring to the fear of no repayment, which leads to rising default premia.

4.3 Optimal tax rate

Consider now the optimal tax rate of the repayment problem (13) and its relationship to the excess burden of taxation $\Phi_t = \Phi(B_t, g_t)$.

Proposition 2. 1. (“Optimal tax rate”) The optimal repayment tax rate $\tau_t = \tau(B_t, g_t)$ takes the form

$$\tau_t = \frac{\Phi_t \left[(\varepsilon_{cc,t} + \varepsilon_{hc,t}) \left(1 - \frac{B_t}{c_t}\right) + \varepsilon_{ch,t} + \varepsilon_{hh,t} \right]}{1 + \Phi_t \left(1 + \varepsilon_{hh,t} + \varepsilon_{hc,t} \left(1 - \frac{B_t}{c_t}\right)\right)}, \quad (19)$$

where $\varepsilon_{cc,t} \equiv -u_{cc,t}c_t/u_{ct}$, $\varepsilon_{ch,t} \equiv u_{cl,t}h_t/u_{ct}$ the own and cross elasticity of the marginal utility of consumption and $\varepsilon_{hh,t} \equiv -u_{ll,t}h_t/u_{lt}$, $\varepsilon_{hc,t} \equiv u_{cl,t}c_t/u_{lt}$, the own and cross elasticity of the marginal disutility of labor.

2. (“Constant Frisch elasticity”) Assume the utility function $U = \frac{c^{1-\rho}-1}{1-\rho} - a_h \frac{h^{1+\phi_h}}{1+\phi_h}$. Then τ_t becomes

$$\tau_t = \frac{\Phi_t(\rho(1 - \frac{B_t}{c_t}) + \phi_h)}{1 + \Phi_t(1 + \phi_h)}. \quad (20)$$

Proof. See Appendix A. □

Discussion. The optimal tax rate (19) in the event of repayment depends on the excess burden of taxation Φ_t , the elasticities of the period utility function $\varepsilon_{i,j}, i, j = c, h$, and the amount of outstanding debt, B_t . Debt enters the determination of τ_t through the lack of commitment of the planner to past promises. To see that, consider the tax rate for the constant Frisch case, where the elasticity channel is held constant. An increase B_t leads to a decrease in the tax rate, keeping the excess burden of taxation constant. This is the typical time-inconsistency issue that shows up in Lucas and Stokey (1983), and it refers to the incentive of the planner to *devalue* initial debt.¹³ In the time-consistent solution though, the excess burden of taxation is increasing when debt increases, leading typically to an optimal tax rate that is an increasing function of debt.

4.4 Tax-smoothing

To understand deeper the implications of the GEE in proposition 1 for tax-smoothing, it is instructive to consider how the excess burden of taxation Φ_t , and, therefore, taxes, varies across states and dates in environments with commitment to old tax policies and commitment to repayment.¹⁴

Complete markets with commitment. Consider first the environment of Lucas and Stokey (1983), that is, an economy with a full set of state-contingent Arrow securities, and a policy-maker that is committed both to the plan designed at $t = 0$, and to debt repayment. The optimal issuance of state-contingent debt satisfies

$$\Phi_{t+1} = \Phi_t, \quad (21)$$

for all t, g^t . Consequently, the government makes the excess burden of taxation *constant* over states and dates, $\Phi_t = \Phi, \forall t, g^t$, which constitutes effectively the main source of tax-smoothing.

Incomplete markets with commitment. A similar tax-smoothing condition for an interior solution emerges also in the case of *non-contingent* debt as in Aiyagari et al. (2002) and Bhandari et al. (2017),

¹³If we set $t = 0$ and make Φ constant in formula (19), we get the initial period optimal tax rate of Lucas and Stokey (1983), when there are no past promises to uphold.

¹⁴All relevant problems in this section are fully stated and analyzed in Appendix A and Online Appendix D.

$$\mathbb{E}_t x_{t+1} \Phi_{t+1} = \Phi_t, \quad (22)$$

where x_{t+1} stands, as previously, for the risk-adjusted change of measure, $x_{t+1} \equiv u_{c,t+1}/\mathbb{E}_t u_{c,t+1}$. Therefore, when there are not sufficient securities to hedge fiscal shocks, the policymaker makes the excess burden of taxation on *average* constant, with respect to the risk-adjusted measure. With either complete or incomplete markets, the policymaker under commitment does not take into account the reaction of the future policymaker. As a result, the pricing effects Q_B that we saw in proposition 1 are *absent* in (21) and (22).

Markov-perfect policy with no default option: Greed. Consider now retaining the assumption of non-contingent debt and commitment to debt repayment, but dropping the assumption of commitment to old tax policies. The optimal debt issuance of the time-consistent policymaker satisfies

$$\mathbb{E}_t x_{t+1} \Phi_{t+1} = \Phi_t \cdot \underbrace{\left[1 + \mathbb{E}_t x_{t+1} \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \mathcal{C}_B(B_{t+1}, g_{t+1}) \cdot B_{t+1} \right]}_{\text{“Greed”: } Q_B > 0}, \quad (23)$$

where $\mathcal{C}$ is the consumption policy function of the future policymaker. As in the Aiyagari et al. economy, the left-hand side of (23) captures average tax-distortions with respect to the risk-adjusted measure. Considering the right-hand side of (23), the lack of commitment to old tax policies, combined with full enforceability of debt contracts, activates *only* the “Greed” effect in the price semi-elasticity η of proposition 1. If debt is positive, $B_{t+1} > 0$, the reduction in interest rates due to the reduction of consumption of the future policymaker ($\mathcal{C}_B < 0$) leads to higher average tax distortions in the future relative to today, $\mathbb{E}_t x_{t+1} \Phi_{t+1} \geq \Phi_t$. In other words, the “Greed” effect creates an incentive to the policymaker to *back-load* tax distortions, since the interest cost of debt is low.¹⁵

Greed versus fear: back-loading versus front-loading. Finally, when we drop also the commitment to repayment assumption, we get the full version of the GEE (17) in proposition 1. Average tax distortions are not constant due to the two dimensions of lack of commitment.¹⁶

¹⁵Krusell et al. (2004) analyze optimal-time consistent taxation in the Lucas and Stokey (1983) setup without uncertainty and default. Their respective GEE is the deterministic version of (23). We provide further comments on their setup in Online Appendix E. Occhino (2012) and Debortoli and Nunes (2013) consider the same setup as Krusell et al. (2004), but allow government expenditures to provide utility. Utility-providing government consumption alters non-trivially the reaction of private consumption to debt, making private consumption increase when debt increases, $\mathcal{C}_B > 0$, instead of $\mathcal{C}_B < 0$ in the current paper. This feature creates the *opposite* incentives for interest rate manipulation to Krusell et al. (2004) and to the current paper, generating incentives to *front-load* tax distortions.

¹⁶In Pouzo and Presno (2022), the “Greed” effect is not present, due to the commitment of the policymaker to previously designed tax policies. Consequently, their respective optimality condition exhibits only the “Fear” term.

The negative “Fear” effect due to default risk opposes the positive “Greed” effect, mitigating in a sense the time-inconsistency problem. If the reduction in interest rates due to the “Greed” effect is larger than the increase in interest rates due to the “Fear” effect, then it is optimal for the planner to tax less today, issue more debt and back-load on average tax distortions. This translates to a positive drift in the excess burden with respect to the default-and-risk-adjusted measure, $\mathbb{E}_t m_{t+1} \Phi_{t+1} \geq \Phi_t$, when $B_{t+1} > 0$. The opposite happens if the “Fear” force is stronger, i.e. if default premia increase so much that they dominate the decrease in interest rates from the “Greed” effect. In that case, the policymaker wants to decrease on average the excess burden over time and to *front-load* tax distortions, $\mathbb{E}_t m_{t+1} \Phi_{t+1} \leq \Phi_t$, when $B_{t+1} > 0$, imparting a negative drift (with respect to m_{t+1}) to the excess burden of taxation.¹⁷

4.5 Interest rate manipulation under commitment

A large part of the action in the GEE (17) is coming from manipulating the consumption choices of the future policymaker, in order to make future marginal utility higher and debt cheaper. The incentives to manipulate marginal utility to lower interest rates should be also present under commitment, although they did not emerge in the tax-smoothing conditions (21) and (22).¹⁸ To see how these incentives materialize in the commitment case, it is instructive to take a step back and consider the environment of Lucas and Stokey (1983). The first-order condition with respect to consumption for $t \geq 1$ under commitment takes the form¹⁹

$$u_{ct} + \underbrace{\Phi_t \Omega_{ct}}_{\text{effect on surplus}} - \underbrace{\Phi_t u_{cc,t} B_t}_{R_{t,t+1} \downarrow: \text{relax IC at } t} + \underbrace{\Phi_{t-1} u_{cc,t} B_t}_{R_{t-1,t} \uparrow: \text{tighten IC at } t-1} = \lambda_t. \quad (24)$$

The variable λ_t in the right-hand side of (24) denotes the shadow value of output. Turning to the left-hand side, an increase in consumption increases utility and affects the surplus in u_c units, Ω . These are the first two terms in the left-hand side of (24). Moreover, an increase in consumption reduces marginal utility. As a result, the value of debt in marginal utility units, $u_{ct} B_t$, goes down when $B_t > 0$, relaxing the current implementability constraint, with shadow value Φ_t . This initial debt devaluation incentive is the third term in the left-hand side of (24). To see clearly the price implications, let $\pi_{t+1|t}$ denote the conditional probability of a state. The equilibrium price of the state-contingent claim, $p_t = \beta \pi_{t+1|t} \frac{u_{c,t+1}}{u_{ct}}$, increases when u_{ct} falls. In even simpler terms, if we had no uncertainty, the interest rate from t to $t+1$, $R_{t,t+1} \equiv \frac{u_{ct}}{\beta u_{c,t+1}}$, falls.

The debt devaluation incentive is the one we identified in the discussion of the optimal tax rate in proposition 2.²⁰ Under commitment though, the policymaker has also to take into

¹⁷The discussion is silent about drifts with respect to the physical measure. It is still possible to have a positive drift or no drift with respect to m_{t+1} (or x_{t+1} in the case without default) and still have a negative drift with respect to the physical measure.

¹⁸We are thankful to Marco Bassetto and Javier Bianchi for raising this question.

¹⁹See Online Appendix D for the derivation.

²⁰See the first-order condition with respect to consumption in the proof of proposition 2.

account the marginal utility promises given in the past. A fall in u_{ct} reduces the price of state-contingent claims issued at $t - 1$, $p_{t-1} = \beta \pi_{t|t-1} \frac{u_{ct}}{u_{c,t-1}}$, reducing the revenue from debt issuance in the previous period. Thus, the implementability constraint at $t - 1$ is tightened, an effect that has shadow value Φ_{t-1} . In a deterministic environment, the fall in p_{t-1} raises the interest rate from $t - 1$ to t , $R_{t-1,t} \equiv \frac{u_{c,t-1}}{\beta u_{ct}}$. This increase in interest rates of the previous period contributes a fourth term in the left-hand side of (24) that is of *opposite* sign to the third term of debt devaluation.

However, the excess burden of taxation is constant in the [Lucas and Stokey](#) economy, $\Phi_t = \Phi$, as we saw in (21). So the benefit of reducing current interest rates in the third term is exactly equal to the cost of raising previous interest rates, and the third and fourth terms in the left-hand side of (24) *cancel* each other for each $t \geq 1$. Only at the initial period $t = 0$, the third term survives, since there is no promise to uphold.²¹

To summarize, the incentives to manipulate interest rates through marginal utility are muted in the commitment case, due to the upholding of price promises designed at $t = 0$.

5 Long-term debt

Consider now the possibility of long-term debt as in the work of [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#) in the sovereign default literature. In particular, we adopt the probabilistic bond structure of [Chatterjee and Eyigungor \(2012\)](#). The government issues long-term debt B_t . With probability $\delta \in [0, 1]$, the bond matures and provides one unit of consumption. With probability $1 - \delta$, the bond does not mature and provides a coupon payment z . The short-term debt specification we considered previously is captured by setting $\delta = 1$.²²

Assume that the government has the option to default on its entire debt obligations B_t , subject to the same costs as in the case of short-term debt ($\delta = 1$). Under this assumption, the definitions of a) the value function in (9), b) the default value function in (12), c) the default and repayment sets, remain the same as previously.

The household's budget constraint when the government repays its debt reads²³

$$c_t(g^t) + q_t(g^t) \underbrace{[B_{t+1}(g^t) - (1 - \delta)B_t(g^{t-1})]}_{\text{new debt issuance}} = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + \chi B_t(g^{t-1}), \quad (25)$$

where q_t is the price of the long-term bond, and $\chi \equiv \delta + (1 - \delta)z$, a composite parameter

²¹Same comments apply also to the non-contingent debt economy of [Aiyagari et al. \(2002\)](#). The first-order condition with respect to consumption, provided in Online Appendix D, is the same as (24), with debt being non-contingent, $B_t = B_t(g^{t-1})$. The third and fourth term do *not* cancel out though, since the excess burden is not constant anymore, as we emphasized in (22). The sum of the third and fourth term is proportional to $\Delta\Phi_t = \Phi_t - \Phi_{t-1}$, which is equal to the *innovation* in Φ_t , $\Phi_t - E_{t-1}x_t\Phi_t$, from (22). The innovation is typically small, muting again the incentives for marginal utility manipulation, except for the initial period.

²²If $\delta = 0$, we have a consol with coupon payments z .

²³To save on notation, we do not differentiate here between the household's asset position b_t , and the government's debt position B_t .

which collects the principal payments of maturing government bonds and the coupon payments of non-maturing government bonds.²⁴

The labor supply condition of the household is given by (6). The Euler equation with defaultable long-term debt becomes

$$q_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{ct}} (1 - d_{t+1}) [\chi + (1 - \delta) q_{t+1}]. \quad (26)$$

The current price of the bond q_t reflects the discounted principal payments δ of maturing bonds, together with the coupon payments and the future price q_{t+1} of non-maturing bonds. It is useful for later purposes to rewrite the Euler equation as

$$Q_t = \beta \mathbb{E}_t (1 - d_{t+1}) [\chi u_{c,t+1} + (1 - \delta) Q_{t+1}], \quad (27)$$

where, as in the short-term debt case, Q_t is defined as the price of debt in marginal utility units, $Q_t \equiv u_{ct} \cdot q_t$.

5.1 Value of repayment

We follow again the primal approach and use optimality conditions (6) and (26) to eliminate taxes and prices from the household's budget constraint. Let $\mathcal{C}(B', g')$ and $\mathcal{H}(B', g')$ stand for the policy function for consumption and labor next period in the event of repayment. With long-term debt ($\delta < 1$), the current policymaker has also to take into account the debt policy function for next period in the event of repayment, $\mathcal{K}(B', g')$, since it affects, through future prices Q_{t+1} , the current price Q_t , as we can see from (27).

Given $\mathcal{C}(B', g')$, $\mathcal{H}(B', g')$ and $\mathcal{K}(B', g')$, the value of repayment is

$$V^r(B, g) = \max_{c \geq 0, h \in [0, 1], B'} u(c, 1 - h) + \beta \mathbb{E}_{g'|g} V(B', g') \quad (28)$$

subject to

$$\chi u_c(c, 1 - h) B \leq \Omega(c, h) + Q(B', g)[B' - (1 - \delta)B] \quad (29)$$

$$c + g = h, \quad (30)$$

where the price of long-term debt satisfies the recursion,

$$Q(B', g) = \beta \mathbb{E}_{g' \in A(B')|g} [\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta) Q(\mathcal{K}(B', g'), g')]. \quad (31)$$

²⁴Note that if we set $z = 1$, we get $\chi = 1$ and recover the specification of Hatchondo and Martinez (2009), who consider a long-dated security with an exponentially decaying coupon structure. See also Woodford (2001) for the use of these bonds in the analysis of non-Ricardian fiscal regimes.

The Markov-perfect requirement is $c(B, g) = \mathcal{C}(B, g)$, $h(B, g) = \mathcal{H}(B, g)$ and $B'(B, g) = \mathcal{K}(B, g)$ for all (B, g) .

5.2 Greed versus fear with long-term debt

To build intuition, we make again the same assumptions as in section 4.1 and consider a locally smooth equilibrium. The marginal benefits and costs of issuing debt are captured by the optimality condition

$$-\beta \frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g') = \Phi \cdot [Q(B', g) + Q_B(B', g) \cdot \underbrace{(B' - (1 - \delta)B)}_{\text{new debt issuance}}] \quad (32)$$

As in the analysis of short-term debt in (16), the left-hand side denotes the marginal cost of issuing more debt, whereas the right-hand side denotes the marginal benefit of the additional revenue from debt issuance, with Φ standing for the excess burden of taxation, that is, the multiplier on (29). Note that a change in price affects only the price of *newly* issued debt, $B' - (1 - \delta)B$, and not the entire stock of debt.

By using the envelope condition and recursion (31) to expand the price derivative Q_B , we can express (32) as a Generalized Euler Equation for the case of long-term debt.

Proposition 3. (“GEE with long-term debt”)

1. The GEE with long-term debt at a locally smooth equilibrium takes the form

$$\mathbb{E}_t n_{t+1}^d \Phi_{t+1} = \Phi_t \cdot [1 + \eta_t \cdot (B_{t+1} - (1 - \delta)B_t)] \quad (33)$$

where η_t is the price semi-elasticity with respect to debt, $\eta_t \equiv Q_B(B_{t+1}, g_t)/Q_t$, and $n_{t+1}^d \geq 0$ the default-and-long-term-debt-adjusted change of measure ($\mathbb{E}_t n_{t+1}^d = 1$),

$$n_{t+1}^d = n^d(B_{t+1}, g_{t+1}|g_t) \equiv \frac{(1 - d_{t+1})[\chi u_{c,t+1} + (1 - \delta)Q_{t+1}]}{\mathbb{E}_t(1 - d_{t+1})[\chi u_{c,t+1} + (1 - \delta)Q_{t+1}]} \quad (34)$$

2. The price semi-elasticity η_t follows the forward recursion

$$\begin{aligned} \eta_t = & \underbrace{\left[\mathbb{E}_t n_{t+1}^d \frac{\chi(u_{cc,t+1} - u_{cl,t+1})}{\chi u_{c,t+1} + (1 - \delta)Q_{t+1}} \mathcal{C}_B(B_{t+1}, g_{t+1}) \right]}_{\text{“Greed” (+)}} + \underbrace{f^n(\omega^{-1}(B_{t+1}), B_{t+1}|g_t) \frac{d\omega^{-1}}{dB_{t+1}}}_{\text{“Fear” (-)}} \\ & + (1 - \delta) \mathbb{E}_t n_{t+1}^d \frac{Q_{t+1}}{\chi u_{c,t+1} + (1 - \delta)Q_{t+1}} \mathcal{K}_B(B_{t+1}, g_{t+1}) \underbrace{\cdot \eta_{t+1}}_{\text{future price semi-elasticity}} \end{aligned} \quad (35)$$

where $f^n(g_{t+1}, B_{t+1}|g_t) \equiv n^d(B_{t+1}, g_{t+1}|g_t) \cdot f(g_{t+1}|g_t)$, the default-and-long-term-debt-adjusted conditional density.

Proof. See Appendix B. □

Proposition 4. (“Optimal tax rate with long-term debt”)

The optimal tax rate $\tau_t = \tau(B_t, g_t)$ with long-term debt in the event of repayment is

$$\tau_t = \frac{\Phi_t[(\varepsilon_{cc,t} + \varepsilon_{hc,t})(1 - \chi_{c_t}^{B_t}) + \varepsilon_{ch,t} + \varepsilon_{hh,t}]}{1 + \Phi_t(1 + \varepsilon_{hh,t} + \varepsilon_{hc,t}(1 - \chi_{c_t}^{B_t}))}. \quad (36)$$

Proof. See Appendix B. □

Discussion. As in the case of short-term debt, the GEE in (33) expresses the pricing effects of the reactions of the future policymakers, and therefore the effects on marginal revenue, through the price semi-elasticity $\eta \equiv Q_B/Q$.²⁵ Whenever there is positive net debt issuance ($B_{t+1} > (1 - \delta)B_t$), average tax distortions (with respect to n_{t+1}^d) increase (decrease) when the price semi-elasticity is being positive (negative), $\eta_t > (<)0$. The price semi-elasticity in (35) exhibits the two forces that we previously identified: the increase in prices due to increased marginal utility (“Greed”), and the decrease in prices due to an increased default probability (“Fear”). Debt though is long-lived ($\delta < 1$), therefore changes in future prices Q_{t+1} affect current prices. Thus, besides next period’s “Greed” and “Fear” effects, the current semi-elasticity is determined also by the future semi-elasticity η_{t+1} , taking into account how B_{t+1} affects the debt decisions of the future policymaker, $\mathcal{K}_B$. We typically expect $\mathcal{K}_B$ to be positive. If we solve (35) forward, we get

$$\begin{aligned} \eta_t = & \mathbb{E}_t^{n^d} \sum_{i=1}^{\infty} (1 - \delta)^{i-1} \left(\prod_{j=1}^{i-1} \frac{Q_{t+j}}{\chi^{u_{c,t+j}} + (1 - \delta)Q_{t+j}} \cdot \mathcal{K}_{B,t+j} \right) \cdot \left[\frac{\chi(u_{cc,t+i} - u_{cl,t+i})}{\chi^{u_{c,t+i}} + (1 - \delta)Q_{t+i}} \mathcal{C}_{B,t+i} \right. \\ & \left. + f^n(\omega^{-1}(B_{t+i}), B_{t+i}|g_{t+i-1}) \frac{d\omega_{t+i}^{-1}}{dB_{t+i}} \right], \end{aligned} \quad (37)$$

where $\mathbb{E}_t^{n^d}$ denotes the conditional expectation operator according to the default-and-long-term-debt-adjusted measure (34). Consequently, with long-term debt ($\delta < 1$), the present discounted value of *all* future “Greed” and “Fear” effects determines the sign and the size of the price semi-elasticity. If the present discounted value of the “Greed” effects is larger (smaller) than the respective present discounted value of the “Fear” effects, then $\eta_t > (<)0$. Obviously, when $\delta = 1$, the formulas of proposition 3 reduce to the formulas of proposition 1.

²⁵It is clear from the long-term debt case that we based the analysis (even for $\delta = 1$) on the price semi-elasticity η_t and *not* on the price elasticity $\epsilon_t^Q \equiv \eta_t \cdot B_{t+1}$, because the change in prices Q_B affects only new debt issuance.

Tax rate. Turning to the optimal tax rate in proposition 4, we note that expression (36) for the optimal tax rate remains the same as in proposition 2 (with an evolution of Φ_t that satisfies now (33)), except for the fact that the debt devaluation incentives are determined now by the size of *debt service* (principal and coupon payments), χB_t , and not by the entire stock of debt B_t .

No default option. Lastly, as in the case of short term debt ($\delta = 1$) in (23), if we removed the default option from a time-consistent policymaker, only the “Greed” effect would survive, making the respective price semi-elasticity positive, $\eta_t^{\text{no def.}} > 0$.²⁶ In that case, the price semi-elasticity $\eta_t^{\text{no def.}}$ satisfies the forward recursion

$$\begin{aligned} \eta_t^{\text{no def.}} = & \underbrace{\left[\mathbb{E}_t n_{t+1} \frac{\chi(u_{cc,t+1} - u_{cl,t+1})}{\chi u_{c,t+1} + (1-\delta)Q_{t+1}} \mathcal{C}_B(B_{t+1}, g_{t+1}) \right]}_{\text{“Greed” (+)}} \\ & + (1-\delta) \mathbb{E}_t n_{t+1} \frac{Q_{t+1}}{\chi u_{c,t+1} + (1-\delta)Q_{t+1}} \mathcal{K}_B(B_{t+1}, g_{t+1}) \cdot \eta_{t+1}^{\text{no def.}}, \end{aligned} \quad (38)$$

where $n_{t+1} \geq 0$ the long-term-debt-adjusted change of measure, $n_{t+1} \equiv \frac{\chi u_{c,t+1} + (1-\delta)Q_{t+1}}{\mathbb{E}_t(\chi u_{c,t+1} + (1-\delta)Q_{t+1})}$, and $(\mathcal{C}, \mathcal{H}, \mathcal{K})$ the future consumption, labor and debt policy functions for the time-consistent problem without default. The price semi-elasticity (38) enters a GEE that is the same as (33), with an averaging of the excess burden of taxation in the left-hand side with respect to n_{t+1} (and *not* the n_{t+1}^d of (34)).

5.3 Long-term debt and commitment

We contrast here our results to the respective results of the commitment problem with long-term debt. The optimality condition for debt in the commitment case satisfies²⁷

$$\Phi_t = \mathbb{E}_t n_{t+1} \Phi_{t+1}, \quad (39)$$

where, as previously, n_{t+1} stands for the long-term-debt-adjusted change of measure. So, the policymaker is making the average (with respect to n_{t+1}) excess burden of taxation *constant*. Condition (39) reduces to the typical tax-smoothing condition (22) of Aiyagari et al. (2002), when $\delta = 1$. The “Greed” and “Fear” effects that we identified in (33) of proposition 3 are, as expected, absent.

Old promises versus future policymakers. The nature of long-term debt introduces a strong dependence on the *past* in the commitment solution. To see that, consider the optimality condition with respect to consumption for $t \geq 0$,

²⁶We analyze the respective problem in Appendix B.

²⁷See Online Appendix D for the full analysis of the commitment problem with long-term debt.

$$u_{ct} + \Phi_t \Omega_{ct} - \Phi_t \chi u_{cc,t} B_t + \underbrace{\chi u_{cc,t} \xi_{t-1}}_{\text{previous price promises}} = \lambda_t, \quad (40)$$

where λ_t is the shadow value of output, and ξ_t is the shadow value of long-term debt prices. The first three terms in the left-hand side of (40) have the same interpretation as the respective three terms in the short-term debt case, (24). Besides the debt devaluation incentive (third term), the decrease in marginal utility resulting from an increase in consumption, affects the price of the previous period Q_{t-1} , with shadow value ξ_{t-1} , which is the fourth term in the left-hand side of (40).

The variable ξ_t follows the law of motion

$$\xi_t = \Phi_t b_{t+1}^L + (1 - \delta) \xi_{t-1}, \quad (41)$$

where $b_{t+1}^L \equiv B_{t+1} - (1 - \delta) B_t$, the new debt issuance, and $\xi_{-1} \equiv 0$, since there are no price promises at $t = 0$. Note that for short-term debt ($\delta = 1$), we have $\chi = 1$ and $b_{t+1}^L = B_{t+1}$. Thus, ξ_t in (41) becomes $\xi_t = \Phi_t B_{t+1}$, and (40) reduces to (24).

To understand (41) for $\delta < 1$, solve it backwards to get

$$\xi_t = \sum_{i=0}^t (1 - \delta)^i \Phi_{t-i} b_{t+1-i}^L. \quad (42)$$

When we have long-term debt, a change in Q_t , affects the previous prices, Q_{t-i} , $i = 1, \dots, t$, as we can see from (27). This change in *all* past prices is multiplied with the respective (net) debt issuances at each period, b_{t+1-i}^L , $i = 1, \dots, t$, and influences all past implementability constraints, with respective excess burden Φ_{t-i} , $i = 1, \dots, t$. Consequently, the entire sequence of the past Φ_{t-i} , $i = 1, \dots, t$ enters the solution, introducing an additional dependence on the past. This is exactly what (42) reveals.

To summarize, there is a stark contrast between the commitment and the time-consistent solution with long-term debt. With commitment, the planner has to take into account the effect of its actions on all *past* prices, as we see in (42). Instead, without commitment, the policymaker has to take into account the reactions of all *future* policymakers, as we saw in (37).

6 Numerical exercises

Our purpose in this section is to highlight the quantitative implications of time-consistent optimal taxation when debt repayment is not enforceable. We use a standard calibration, typical of optimal taxation exercises, and consider a low probability of default, as documented in Rein-

hart and Rogoff (2011). We contrast time-consistent policy with default (“Greed versus Fear”) to time-consistent policy without default (“Greed”), and to optimal policy under commitment. We perform our quantitative exercises for both the short- and the long-term debt specification.

Calibration. We use the constant Frisch elasticity utility function that delivers the tax rate (20) in proposition 2 and employ standard parameter values. We use an annual frequency and set $(\beta, \phi_h, \rho) = (0.96, 1, 1)$, so the utility function is logarithmic in consumption with a unitary Frisch elasticity. Government spending is $g_t = G \exp(x_t^g)$, where

$$x_t^g = \rho_x x_{t-1}^g + \sigma_\varepsilon \varepsilon_t, \quad (43)$$

and $\varepsilon_t \sim N(0, 1)$.²⁸ We use annual US data on government spending for the period 1947 – 2022 and set $\rho_x = 0.8443$ and $\sigma_\varepsilon = 0.0914$.²⁹ We set $G = 0.0593$, so that the economy generates an average government spending-to-output ratio of 15.39% at the first-best.³⁰ We set the labor disutility parameter to $a_h = 7.3378$, so that the household works 40% of its time at the first-best when spending takes the value G .³¹

Default costs. We allow asymmetric utility costs of default, that is, it is less costly to default when government expenditures are high. In particular, we use the quadratic function

$$\psi(g) = \max\{0, \psi_0 - \psi_1 \cdot (g - \underline{g})^2\}, \quad (44)$$

where $\psi_0, \psi_1 > 0$, and $\underline{g}$ the lowest value of the spending shock. The utility costs, whenever positive (i.e. when $g \leq \underline{g} + \sqrt{\psi_0/\psi_1}$), are decreasing and concave in g . Thus, a reduction in the “bad” shock g increases the utility cost from default at a decreasing rate.³²

Numerical challenges and computational method. We use value function iteration and work directly with the Bellman equations, without any of the assumptions of section 4.1. There are several issues with the numerical computation of the problem. These issues have to do with

²⁸The mean and standard deviation of g_t are respectively $\mathbb{E}(g_t) = G \exp(\frac{1}{2}\sigma_x^2)$, and $\sigma_g = G \exp(\frac{1}{2}\sigma_x^2)(\exp(\sigma_x^2) - 1)^{\frac{1}{2}}$, where $\sigma_x = \frac{\sigma_\varepsilon}{\sqrt{1-\rho_x^2}}$.

²⁹We measure g_t as the sum of the US federal government consumption and investment (NIPA table 3.9.5, lines 10 and 11) for the years 1947 – 2022. We deflate using the consumer price index, take logarithms, de-trend with a linear trend, and estimate (43) on the residuals.

³⁰This is the mean ratio of government spending to output in the US data for the period 1947 – 2022. To calculate that, we divide our measure of government spending over the model-consistent measure of output, that is, our measure of spending plus private consumption. To measure private consumption, we use nondurable goods and services expenditure (NIPA table 1.1.5 lines 5 and 6).

³¹We refer to the implied value of output, 0.4, as “first-best” output for the rest of the paper.

³²Pouzo and Presno (2022) use a linear specification for the utility costs of default. Our quadratic specification follows the logic of Bianchi et al. (2023) and Fourakis and Karabarbounis (2025), who consider utility costs of default that are an increasing and concave function of the “good” shocks in their economies, that is, tradeable income or productivity shocks, respectively.

Table 1: Utility costs of default.

Short-term debt	Long-term debt
$\psi_0 = 0.2653$	$\psi_0 = 0.6710$
$\psi_1 = 99.0341$	$\psi_1 = 259.4705$

Calibrated values of (ψ_0, ψ_1) for the short- and long-term debt specification.

non-convexities which lead to discontinuous, non-monotonic policy functions and touch upon the Markov-perfect nature of optimal policy, echoing the results of [Krusell et al. \(2004\)](#) in deterministic economies. Stochastic government expenditures can smooth these discontinuities and eliminate non-monotonicities, if we constrain ourselves to a small state space for debt and i.i.d. shocks, as we show in the Online Appendix. However, for larger state spaces and persistent shocks the issues of jumps and non-monotonicities re-emerge. We deal with these issues by using discrete choice taste shocks, effectively “convexifying” the problem.³³ We adapt this methodology to our optimal taxation problem and provide an exhaustive analysis, as well as robustness exercises, in Online Appendices [E](#), [F](#) and [G](#).

For the commitment problems, we solve the short-term debt case with value function iteration, using a recursive formulation in the spirit of [Kydland and Prescott \(1980\)](#). For the long-term debt case, we use the Parameterized Expectations Algorithm of [den Haan and Marcet \(1990\)](#), implemented through the artificial neural networks machinery of [Valaitis and Villa \(2024\)](#).³⁴

Debt limits. The government is not allowed to lend so the lower debt limit is zero, as in [Pouzo and Presno \(2022\)](#). In the short-term debt case, we are cautious about the size of the upper debt limit and set it to 40% of first-best output.³⁵ We use a larger state space in the long-term debt case and set the upper debt limit to 100% of first-best output. Given these exogenous limits, the model with default generates an endogenous debt limit that is smaller.

6.1 Analysis with short-term debt

For the short-term debt case, we pick the parameters (ψ_0, ψ_1) of the default cost function [\(44\)](#), reported in [table 1](#), to match two targets: a) a domestic default probability of 1.1% from

³³See [Dvorkin et al. \(2021\)](#) for a successful application of these methods in sovereign default models. [Train \(2009\)](#) provides a textbook treatment of discrete choice methods. In a similar vein, [Chatterjee and Eyigungor \(2012\)](#) introduce an additional shock in the resource constraint to “convexify” the constraint set. A formal introduction of lotteries in debt as in [Chatterjee and Eyigungor \(2016\)](#) could, in principle, perform the same role.

³⁴See Online Appendices [D](#) and [G](#), for the recursive formulation of the commitment problem and for all the details pertaining to computation, respectively.

³⁵Computational issues for the time-consistent problem are aggravated for large state spaces for debt, especially in the short-term debt case. Even if these issues can be alleviated with carefully chosen taste shocks, we are prudent in our choices for the size of the state space, since we do not want the taste shocks to be the main driver of the dynamics. See Online Appendix [E](#) for further discussion.

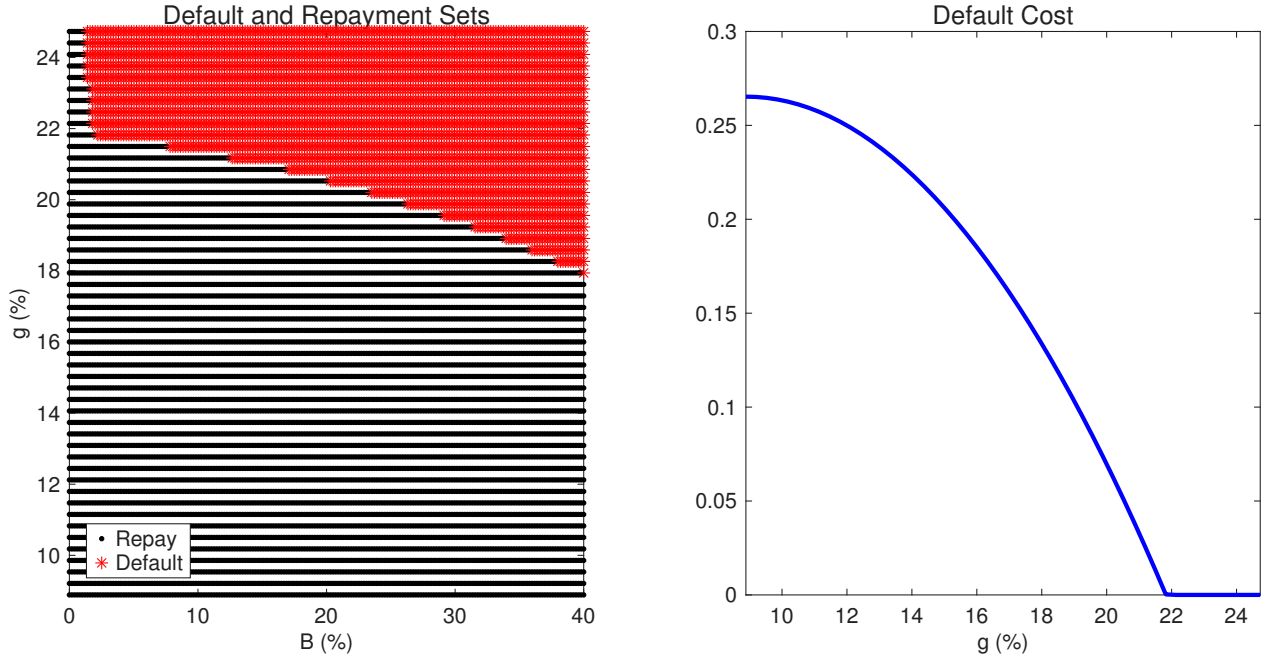


Figure 1: The left panel depicts the default and repayment sets for short-term debt. Given the probabilistic nature of the default decision due to the taste shocks, we label a state (B, g) as “default”, whenever the probability of default is above 99.9%. The right panel displays the default cost function $\psi(g)$ in (44) for the baseline calibration (first column in table 1). To facilitate interpretation, both B and g are expressed in terms of first-best output.

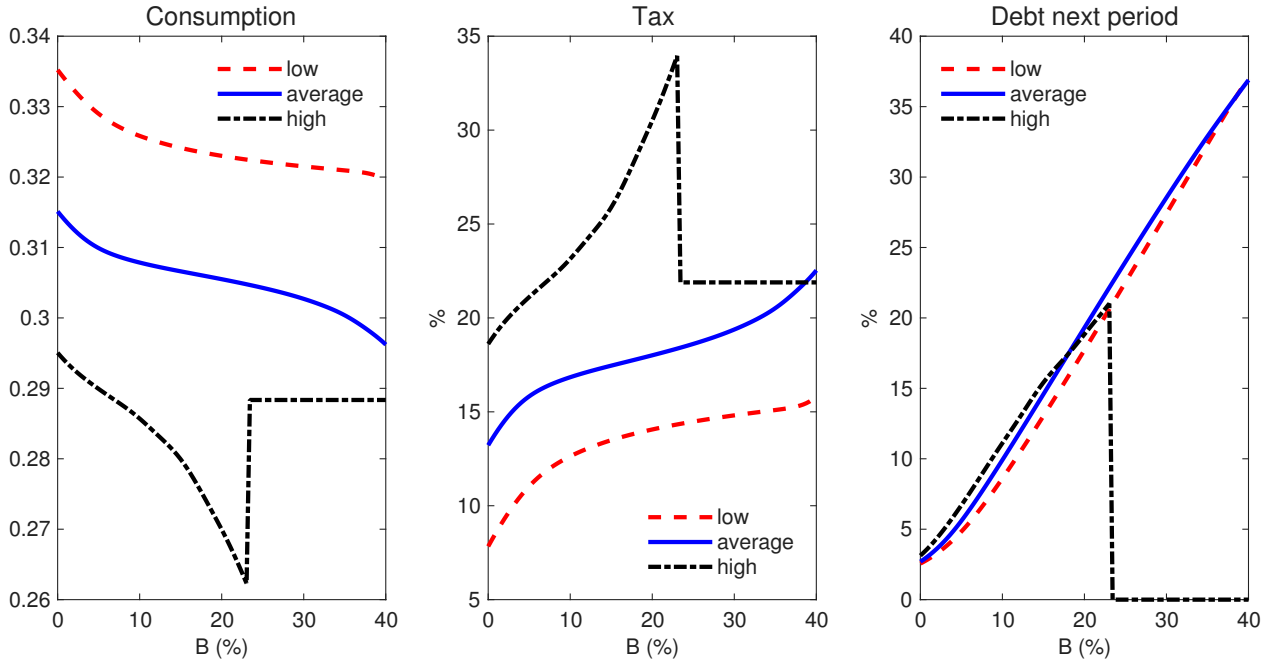


Figure 2: Policy functions for consumption, taxes and debt in the full-blown model with default. The policy functions are plotted for the mean g (“average”), as well as for the level of g two standard deviations below (“low”), or above (“high”) the mean. The taste shocks in the debt and default decision are averaged out. Debt is expressed in terms of first-best output.

the [Reinhart and Rogoff \(2011\)](#) data, as reported by [D’Erasmus and Mendoza \(2021\)](#) b) the maturity-adjusted debt ratio in the U.S. The average debt held privately as share of output is

40.88% for the period 1947-2022.³⁶ The average maturity is 3.85 years.³⁷ Thus, the maturity-adjusted value of debt is 10.62%.³⁸

Optimal policy with default. Before we proceed to the analysis of the moments of interest, we delve here deeper into the mechanics of the model. Consider first the default or repayment decision in the beginning of the period. The left panel of figure 1 plots the default and repayment sets, whereas the right panel plots the utility costs of default. As we expect, default sets are increasing in debt. Moreover, the incentives for default are higher in “bad” times, that is, when government expenditures are high. Thus, the default threshold $\omega(g)$ is monotonically decreasing.

Similarly, figure 2 splices together the policy functions for consumption, taxes and debt in the events of repayment and default. These panels are instructive about how the government uses taxes or debt issuance to finance a fiscal shock, depending on the current debt position. The policy functions are plotted for the mean g (“average”), as well as for a level of g that is two standard deviations below (“low”) or above (“high”) the mean.

For a low or average spending shock, an increase in current debt leads to a reduction in consumption, and an increase in the tax rate and new debt, capturing the tradeoff between taxing today versus postponing taxes to the future. When the spending shock is high and debt is low, the government is still absorbing an increase in debt by increasing partially the tax rate and issuing new debt, reducing consumption. Note though that taxes can become very high due to default risk: revenue from debt issuance is becoming relatively small, because the public is expecting a high probability of default next period, forcing the government to tax too much today. For example, at a level of debt of 20%, the optimal tax rate is about 27%, whereas the respective time-consistent tax rate with no default would be about 21%.³⁹ When current debt surpasses a level (about 23% in the figure) though, it is not beneficial anymore to repay debt, since distortionary taxes become too high. The government defaults on its current debt, gets excluded from government debt markets, and runs a balanced budget that allows a lower tax rate and higher consumption than in the case of repayment.

Price Q and debt revenue. To understand better the mechanism, consider the price schedule $Q(B', g)$ and the respective debt revenue in figure 3. We see in the left panel that Q is increasing in the amount of new debt issued, B' , for either a low spending shock that is one standard deviation below the mean, or for the average spending shock. This reflects the positive “Greed” force we identified in our analysis, since new debt reduces consumption next period and increases marginal utility ($Q_B > 0$). However, for a high spending shock that is one stan-

³⁶This is the par value. The respective market value is 41.38%. Our source is <https://www.dallasfed.org/research/econdata/govdebt>

³⁷We use the reported maturity of Hall and Sargent (2011) for the years 1947-2009, and the reported maturity of De Graeve and Mazzolini (2023) for the years 2010-2020.

³⁸Similarly, D’Erasmus and Mendoza (2021) target a maturity-adjusted debt ratio of 7.45% for the countries of the Eurozone, and Pouzo and Presno (2022) target a maturity-adjusted debt ratio of 5.5% for Spain.

³⁹We provide the policy functions for the time-consistent problem without default in figure C.1 in Appendix C.

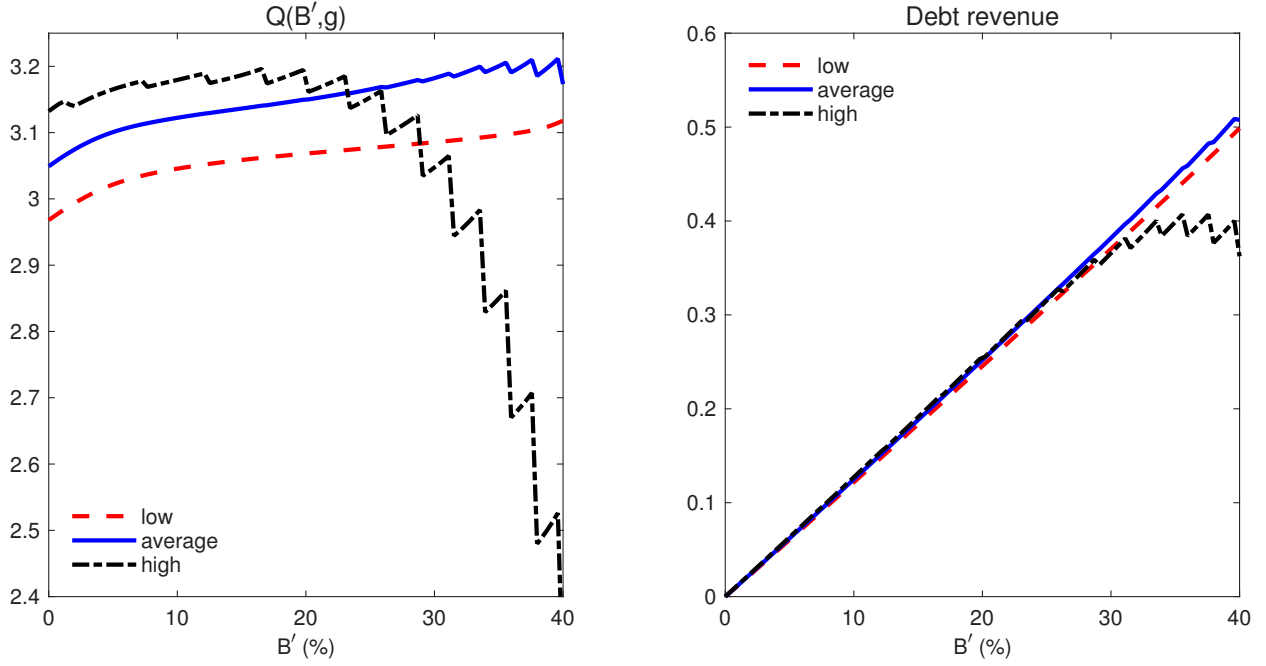


Figure 3: The left panel plots the price schedule in marginal utility units Q , as a function of B' , for the mean level of g (“average”), and for a g that is one standard deviation above (“high”) or below (“low”) the mean. The right panel plots the revenue from debt issuance, $Q(B',g) \cdot B'$. Debt is expressed in terms of first-best output.

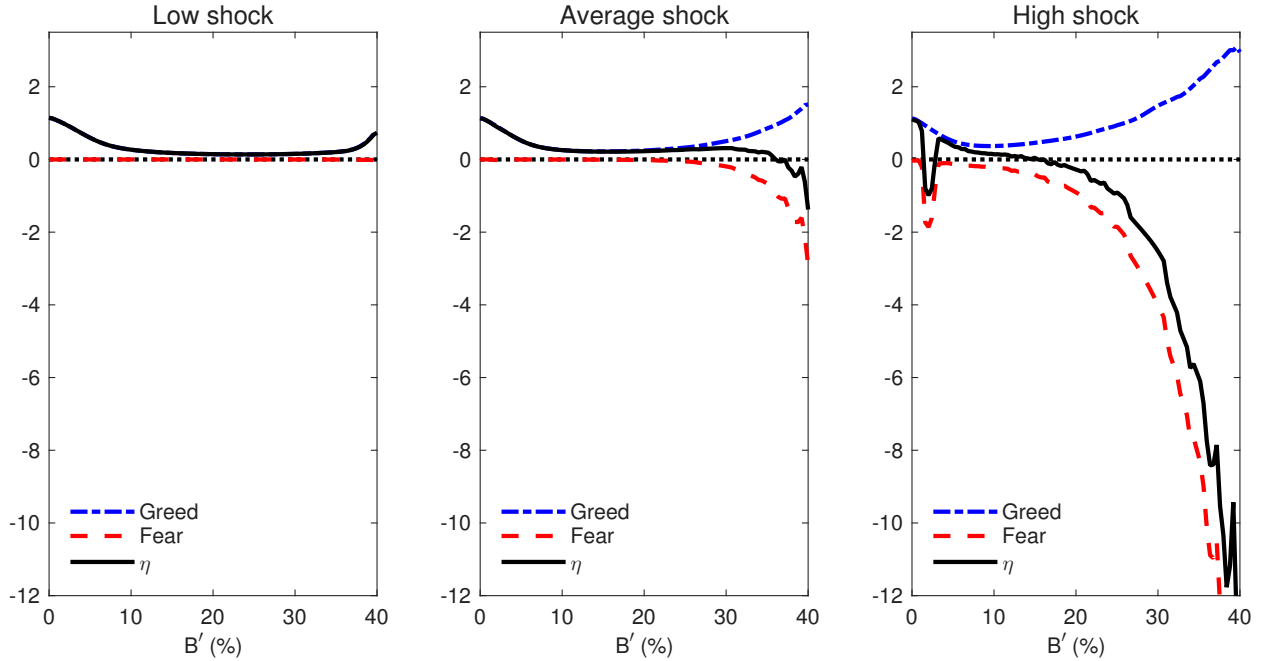


Figure 4: Decomposition of η in the GEE (17) of Proposition 1 into the “Greed” and “Fear” parts. The solid black line denotes the sum of the two parts, η . Debt is expressed in terms of first-best output.

standard deviation above the mean, the situation is *reversed*, since default risk gets activated with higher debt issuance. For low levels of debt issuance, the positive effect of “Greed” dominates the negative effect of the “Fear” force ($Q_B < 0$), and Q increases. At some point the “Fear” force dominates and the price starts falling with new debt issuance. Consequently, debt revenue (right panel) at a high spending shock increases for low levels of debt issuance, but at some point the fall in prices due to default risk is so large, that the revenue falls.

Greed versus Fear decomposition. We dig deeper into the *strength* of the forces behind the change in the price of debt and decompose the price semi-elasticity of proposition 1, $\eta(B', g) \equiv Q_B/Q$, into the respective positive “Greed” and negative “Fear” part for different values of g .⁴⁰ Figure 4 plots the decomposition of η . Consider first the left panel, where the current shock is one standard deviation below the mean. In that case, default risk is minimal, even for high levels of debt issuance. The “Fear” part is zero for each level of B' , and the price semi-elasticity is determined only by the positive “Greed” part. In that case, the incentive of the government is to back-load tax distortions. In the middle panel we consider the same decomposition for the mean spending shock. For low levels of debt issuance, the “Fear” part is again zero, and therefore the positive “Greed” part determines η . When debt issuance exceeds 30%, then the negative “Fear” part gets activated. The sum of the two parts remains positive for most levels of new debt issuance, till debt reaches the upper bound, where the “Fear” part makes η slightly negative. A different picture emerges in the right panel, where we consider a high spending shock that is one standard deviation above the mean. Both the “Greed” and the “Fear” part are active even for low levels of debts. The “Greed” part mostly dominates for low levels of debt, making η positive. However, for B' above 20%, the “Fear” part is negative enough so that it dominates the overall behavior of the price semi-elasticity, making it strongly negative.

6.2 Long-run analysis

We start with the calculation of stationary moments. For that purpose, we conduct a long simulation of 500,000 periods where we discard the first 50,000 periods. The first column of Table 2 reports moments from the time-consistent plan with default (“Default”), the second column the respective moments from the time-consistent plan without default (“MPE”), and the third column (“Commitment”) the moments from the problem of Aiyagari et al. (2002), where there is commitment to both old tax policies and to debt repayment. We are interested in a meaningful comparison between the three environments, especially between the time-consistent problem without default and the commitment problem. For that reason, the first three columns use the same upper debt limit, set to 40% of first-best output.⁴¹

We begin with the moments from the commitment problem in the third column of table 2. The mean debt ratio is 15.94% and the mean tax rate is 16.92% with a standard deviation of 2.2 percentage points. Note that the tax rate has high persistence, as we may expect from the tax-smoothing condition (22) and the analysis of Aiyagari et al. (2002). Moving to the second column (“MPE”), we notice that the mean tax rate remains similar, but tax volatility increases to 5.46 percentage points, and the tax rate exhibits almost zero autocorrelation. There is a noticeable change in the level of debt. When the “Greed” force of the GEE (23) comes into

⁴⁰See Online Appendix G on how we approximate these objects with our numerical solution.

⁴¹Long-run analysis of optimal taxation problems with incomplete markets depend crucially on the exogenously set debt limits, see for example Aiyagari et al. (2002) or Bhandari et al. (2017). Given our prudent choice of the upper debt limit for the time-consistent case with short-term debt (see footnote 35), we can have a fair comparison between the three environments only if we use the same debt limit for the commitment case. As mentioned earlier, given the exogenous upper debt limit, the default model will generate a smaller endogenous borrowing limit.

Table 2: Stationary moments with short-term debt.

	Default	MPE	Commitment	Commitment <i>high debt limit</i>
Tax rate in %				
Mean	16.64	17.21	16.92	20.03
St. Dev	4.15	5.46	2.19	3.54
Autocorrelation	0.2461	0.0797	0.9199	0.9330
Corr with g	0.6153	0.3930	0.8099	0.3386
St. Dev of $\Delta\tau$	4.95	7.27	0.70	0.62
Debt-to-output ratio in %				
Mean	10.63	23.72	15.94	84.36
St. Dev	5.73	11.58	13.23	64.65
Autocorrelation	0.8316	0.8835	0.9863	0.9882
Default probability in %	1.0938	0	0	0
Spread in %				
Mean	1.24			
St. Dev	5.96			
Autocorrelation	0.4238			

Stationary moments on repayment periods from a long simulation of 500,000 periods, where we discarded the first 50,000 periods. The moments for the MPE and the commitment case are calculated on the same realizations of shocks as in the default model. The debt-to-output ratio refers to B_t/h_t . The first three columns use an upper debt limit that is set to 40% of first-best output. The last column uses an upper debt limit that is set to 200% of first-best output.

play, the policymaker is issuing more debt on average, at the level of 23.72%. Turning to the full-blown model with default in the first column of the table, the “Fear” force gets activated in the GEE (17). The default probability of 1.09% and the mean debt ratio match the targeted Reinhart and Rogoff default probability and the maturity-adjusted debt ratio, respectively. Mean tax rates are close to the case without the default option or the commitment case in the second and third column respectively, with very low autocorrelation though, and high volatility, either in terms of the level of the tax rate or the change in the tax rate. Equilibrium average debt is much smaller, although the government defaults approximately once every 100 years, capturing high “debt intolerance”. The mean spread, $r_t - r_t^F$, is at 1.24% with a standard deviation close to 6 percentage points.

We solve also the commitment problem for a high upper debt limit that is 200% of first-best output, and report the relevant moments in the fourth column of table 2. Higher upper debt limits under commitment allow the government to sustain more debt in equilibrium. Furthermore, higher debt limits allow the government to make the tax rate even more persistent and less correlated with spending, confirming the ad hoc analysis of Barro (1979), with a standard deviation of the *change* in tax rates that is very small. Naturally, with higher upper

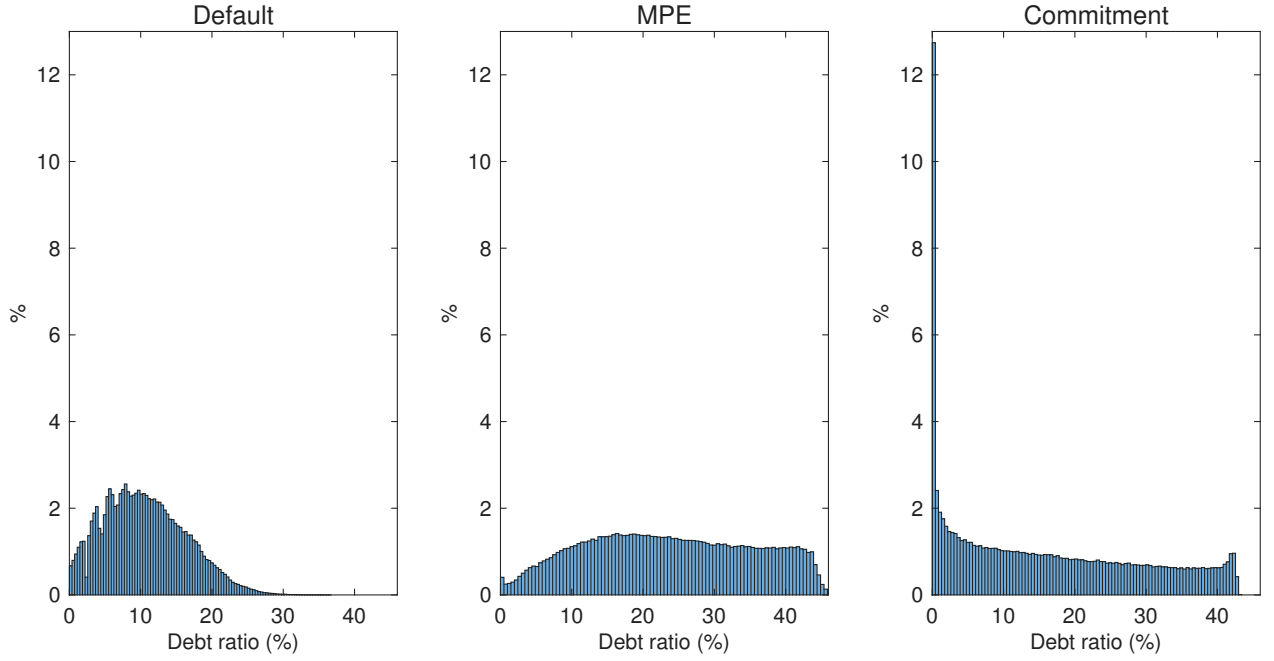


Figure 5: Long-run distribution of the debt-to-output ratio (B_t/h_t) in repayment periods for all three environments.

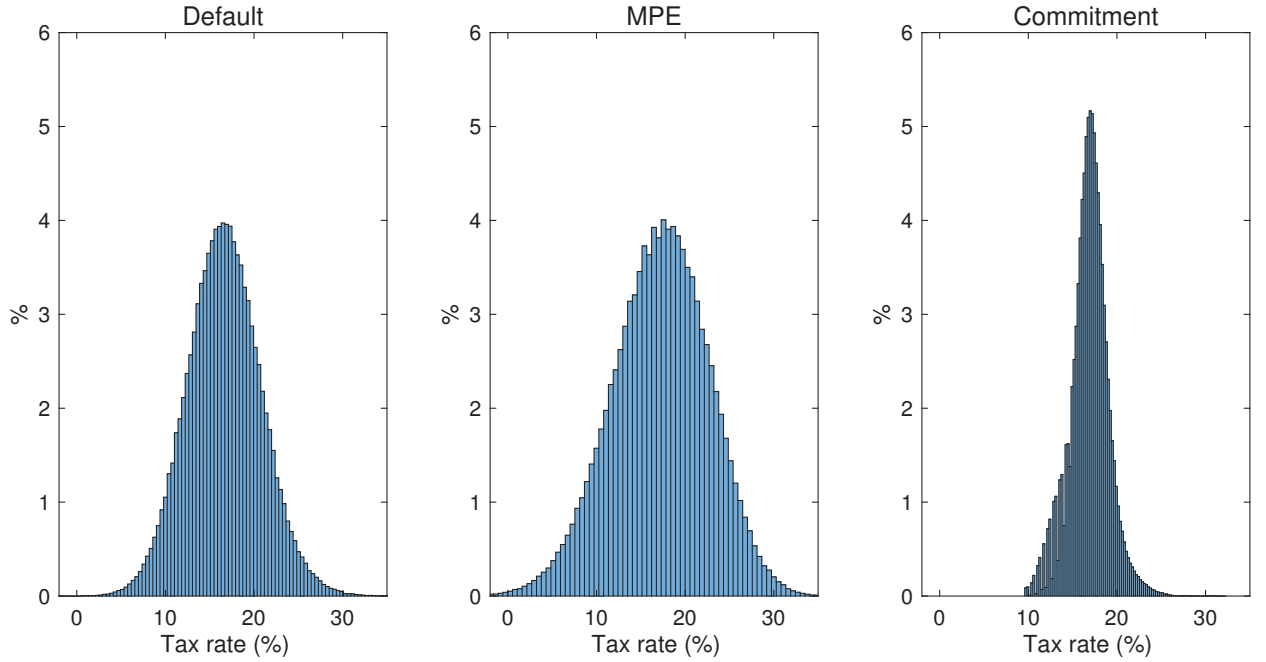


Figure 6: Long-run distribution of the tax rate in repayment periods for all three environments.

debt limits, the contrast between commitment and default is even starker.

To visualize better the behavior of the debt ratio and the tax rate in the three environments of the first three columns of table 2, we plot in figures 5 and 6 the long-run distributions of the debt ratio and the tax rate. Notice the differences especially in the distribution of debt in figure 5. When there is no commitment to old tax policies and repayment (left panel), the distribution of debt is concentrated at low levels of debt. Instead, when there is no commitment to old tax policies but there is commitment to repayment (middle panel), the distribution of debt is more dispersed, with much higher probability mass at high levels of debt. Lastly, under commitment

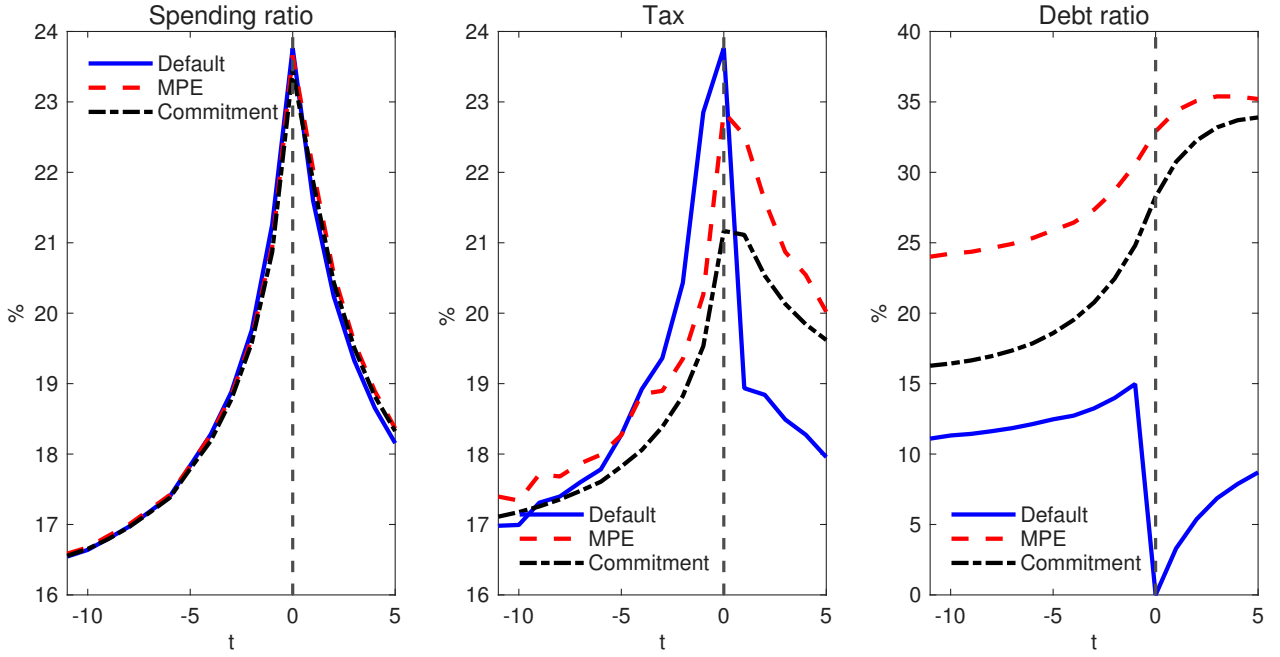


Figure 7: Ensemble means of g_t/h_t , τ_t and B_{t+1}/h_t around default episodes across the three environments.

(right panel) the distribution of debt is again dispersed, but with higher probability mass at low levels of debt, and a sizable probability mass at zero debt, capturing the precautionary savings motive of the government in the [Aiyagari et al. \(2002\)](#) economy.

6.3 Default episodes

In this section we focus on the dynamics around default episodes. We pick 2000 default episodes from our simulation, in which the government is in repayment for at least 15 periods before default, and at least 5 periods after default. We also calculate several variables of interest on the same realizations of shocks for the time-consistent case with no default option (“MPE”) and the commitment case.

We contrast first the ensemble means of government expenditures, issued debt and taxes around default episodes across the three environments.⁴² Default episodes are triggered by a steady increase in government spending, as we can see in the left panel of figure 7, which is naturally accompanied by an increase in taxes and debt in the middle and right panel of figure 7 respectively. In the commitment case, the policymaker is absorbing the increase in spending with a large increase in debt and a relatively more moderate increase in the tax rate. This increase in debt is even larger for the “MPE” case due to the “Greed” force. Instead, when there is default risk, the increase in debt is small due to punishing default premia, and the government resorts to higher distortionary taxes, till it eventually finds it optimal to default.

Figure 8 displays the median dynamics (together with the 25th and 75th percentile) of spending, taxes, debt and the spread around default episodes, and table 3 reports the respective ensemble moments at the last period before default. Overall, the dynamics are as we expect

⁴²As it is typically done in the sovereign default literature, we also report the mean statistics in pre-default periods across the three environments in table C.1 in Appendix C.

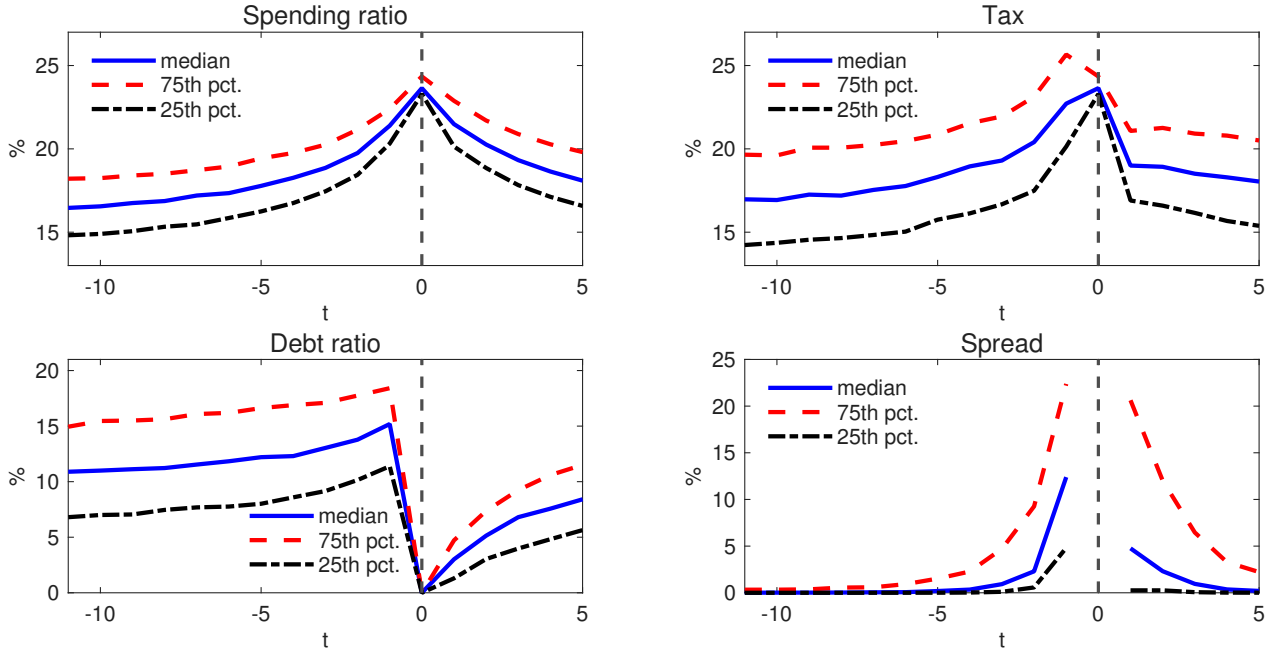


Figure 8: Dynamics of government spending (g_t/h_t), taxes (τ_t), debt (B_{t+1}/h_t), and spread ($r_t - r_t^F$) around default episodes. The spread is not defined at the period of default.

Table 3: Pre-default ensemble moments with short-term debt.

	Mean	St. Dev
Spending ratio in %	21.26	1.45
Tax rate in %	22.85	3.95
Debt ratio in %	14.97	5.13
Spread in %	15.05	12.33

Ensemble means and standard deviations at the last period before default. The reported debt ratio refers to the debt issued at the last period before default, divided over the respective output before default. This is the amount of debt that is defaulted on.

them. The median spending starts rising about 10 periods before default. Debt rises slowly whereas taxes rise faster. At about 5 periods before default spreads start to rise. At the period before default spending reaches on average about 21% and the tax rate 22.85%, with a standard deviation of almost 4 percentage points (see table 3). New debt issuance becomes prohibitively expensive with spreads reaching on average 15% before default with a standard deviation of similar magnitude. The average amount of debt defaulted upon is about 15%. After default, taxes take some periods to reach their median value, since government expenditures are persistent and remain relatively high. This explains also why spreads, although much lower than before default, remain positive. Debt ratios take longer to recover.

Table 4: Stationary moments with long-term debt.

	Default	MPE	Commitment	Commitment <i>high debt limit</i>
Tax rate in %				
Mean	17.12	19.48	18.04	19.88
St. Dev	4.08	4.10	2.39	3.29
Autocorrelation	0.2052	0.2276	0.9397	0.9597
Corr with g	0.4691	0.4422	0.6000	0.3826
St. Dev of $\Delta\tau$	4.97	4.66	0.54	0.42
Debt-to-output ratio in %				
Mean	22.52	78.24	42.47	84.90
St. Dev	15.53	33.53	31.59	60.99
Autocorrelation	0.9368	0.9044	0.9920	0.9920
Macauley duration				
Mean	3.4984	3.5752	3.5727	3.5721
St. Dev	0.1631	0.0614	0.0175	0.0159
Default probability in %				
	1.0944	0	0	0
Spread in %				
Mean	1.06			
St. Dev	2.78			
Autocorrelation	0.5596			

Stationary moments on repayment periods from a long simulation of 500,000 periods, where we discarded the first 50,000 periods, for the long-term debt case. The moments for the MPE and the commitment case are calculated on the same realizations of shocks as in the default model. The first three columns use an upper debt limit that is set to 100% of first-best output. The last column uses an upper debt limit that is set to 200% of first-best output.

6.4 Long-term debt

Turning to the analysis of the long-term debt model, we set $\delta = 0.25$, which implies an expected maturity of 4 years, consistent with the average maturity of 3.85 years for our postwar sample. We set $z = 0.025$, thus the composite parameter χ becomes $\chi = \delta + (1 - \delta)z = 0.26875$.⁴³ We pick the default cost parameters in the second column of table 1 to match the Reinhart and Rogoff default probability of 1.1%, as in our short-term debt exercise, and to maximize the debt-to-output ratio.

⁴³To understand what the value of z implies, note that at a deterministic steady state with long-bonds, the debt-to-output ratio is given by $\frac{B}{y} = \frac{1}{\chi - \delta q} \cdot \frac{S}{y}$, where y is output and S the primary surplus. The price of the bond at the steady state satisfies $q = \beta[\chi + (1 - \delta)q]$, so $q = \frac{\beta\chi}{1 - \beta(1 - \delta)}$. Using that, we get $\frac{B}{y} = \frac{1}{\chi} \cdot \frac{1 - \beta(1 - \delta)}{1 - \beta} \cdot \frac{S}{y}$. If $z = 0.025$ and the primary surplus ratio is 1.5%, the resulting debt ratio is 39.06%. Note that the primary surplus ratio has to be equal to the interest payments ratio at the steady state, which was on average 1.75% in the period 1947-2022 (series FYOIGDA188S from FRED). If we set a primary surplus ratio to 1.75%, the resulting debt ratio is 45.58%. Thus, our value for z is broadly consistent with the observed debt ratios.

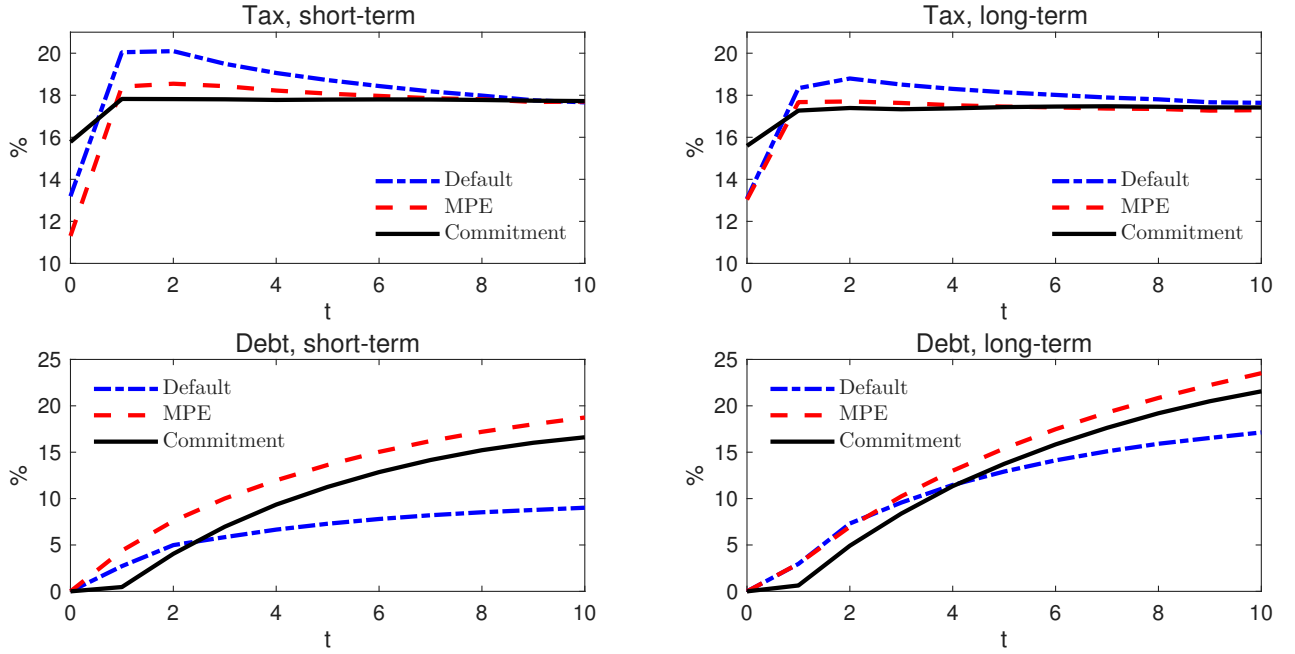


Figure 9: Average response of τ_t and B_t to a two standard deviation shock above mean at $t = 1$ for all three environments in the short- (left panels) and long-term debt case (right panels). All economies start at $t = 0$ with zero initial debt and average government expenditures. Debt is expressed in terms of first-best output.

Table 4 reports the moments from a long simulation for the long-term debt case, where we contrast again our full blown model with default (first column), to the time-consistent case without default (second column), and the commitment case (third column) for an upper debt limit that is 100% of first-best output. For completeness, the fourth column reports moments from the commitment case for an upper debt limit that is 200% of first-best output. The debt-to-output ratio, when default happens approximately once every one hundred years, reaches on average 22.52%, twice as large as in the short-term debt case of table 2.⁴⁴ Overall, debt-to-output ratios are substantially higher than in the short-term debt case for all environments. We draw similar conclusions contrasting the three environments as in the short-term debt case: debt is higher on average for the time-consistent case without default, and tax rates for the default case or the time-consistent case are very volatile in comparison to the commitment case, with minimal autocorrelation. Note that in the commitment case tax rates exhibit even higher autocorrelation than in the short term debt case (see either the third or the fourth column of the table), reflecting the higher dependence on the past that we highlighted in the analysis of (42).

To report the spread for long-term debt, $r_t - r_t^F$, we calculate the internal rate of return of the long bond r_t and the respective return of a risk-free long bond r_t^F that is in zero net supply.⁴⁵ The mean spread is small and of similar size as in the short-term debt case in table

⁴⁴There are choices of (ψ_0, ψ_1) that can generate an even higher debt-to-output ratio, with the drawback that the ω threshold may become non-monotonic for small parts of the state space that are associated with high government expenditures. We refrained from using these calibrations to obtain a monotonic threshold everywhere.

⁴⁵The internal rate of return equalizes the present value of future payments to the price q_t . The payments of the long bond at period i ahead are $\chi(1-\delta)^{i-1}$, $i \geq 1$. Thus, $q_t = \frac{\chi}{1+r_t} \sum_{i=1}^{\infty} \left(\frac{1-\delta}{1+r_t} \right)^{i-1} = \frac{\chi}{r_t+\delta}$. Consequently,

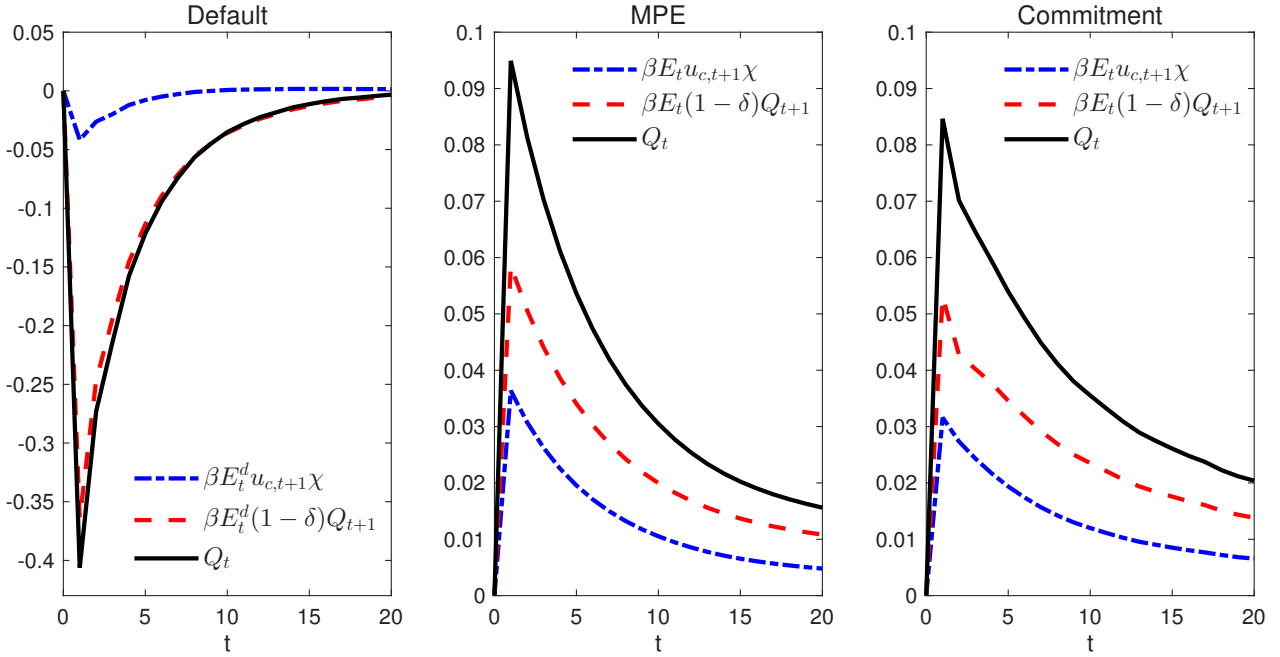


Figure 10: Impulse responses to a two standard deviation shock above mean at $t = 1$ of the price in marginal utility units Q_t and its respective parts for all three environments. E_t^d in the left panel is shorthand for $E_t(1 - d_{t+1})$. All economies start at $t = 0$ with zero initial debt and average government expenditures.

2, but with a standard deviation that is 2.78 percentage points, about *half* the respective standard deviation in the short term debt case. This is similar to the findings of Chatterjee and Eyigungor (2012), who emphasized this property of long-term debt spreads in small open economies without optimal distortionary taxation.⁴⁶

We conduct also a default episode exercise for long-term debt that we report fully in Appendix C. Average debt is about 27% in the 15 periods before default. The average debt that is defaulted upon is about 39% (versus 15% in the short-term debt case in table 3), and the average spread at the last period before default is about 8% with a standard deviation about 5 percentage points, levels which are about half the respective magnitudes in table 3.⁴⁷

Responses of taxes and debt. In figure 9 we plot the average response of taxes and debt to a two-standard deviation shock above the mean at $t = 1$. The left panels consider the short-term debt case and the right panels the long-term debt case. In the commitment case, the shock at $t = 1$ is absorbed by an almost permanent increase in the tax rate and a slow buildup of debt. In the time-consistent case without default (“MPE”), the policymaker increases debt more aggressively. In contrast, the presence of default risk reduces the ability of the government to absorb fiscal shocks with new debt issuance, since debt becomes more expensive. As a result,

$r_t = \frac{\chi}{q_t} - \delta$. For the respective risk-free bond with price q_t^F , we have $r_t^F = \frac{\chi}{q_t^F} - \delta$. See Online Appendix G for the calculation of q_t and q_t^F .

⁴⁶We report also the Macaulay duration of the long bond in table 4. The Macaulay duration is defined as the *weighted* duration of the long bond, $D_t^M \equiv \sum_{i=1}^{\infty} w_{t,i} \cdot i$, where $w_{t,i} \geq 0$, $\sum_{i=1}^{\infty} w_{t,i} = 1$. The weights are given by the present value of the respective bond payments at period i , as a fraction of the price q_t , $w_{t,i} \equiv \frac{\chi(1-\delta)^{i-1}}{(1+r_t)^i} \cdot \frac{1}{q_t}$, $i \geq 1$. Consequently, $D_t^M = \frac{\chi}{q_t(1+r_t)} \sum_{i=1}^{\infty} \left(\frac{1-\delta}{1+r_t}\right)^{i-1} \cdot i = \frac{\chi}{q_t(1+r_t)} \left(\frac{r_t+\delta}{1+r_t}\right)^{-2}$, which simplifies to $D_t^M = \frac{1+r_t}{\delta+r_t} < \frac{1}{\delta}$, after using $q_t = \frac{\chi}{r_t+\delta}$.

⁴⁷See tables C.2, C.3 and figure C.2 in Appendix C.

the government absorbs the shock with a much larger increase of the tax rate at $t = 1$ and a smaller increase in debt. Similar comments apply to long-term debt case, with the difference that the levels of debt are larger for all three environments, since more debt can be sustained in equilibrium, as we saw in table 4.

Price of long-term debt and the absorption of shocks. We conclude our analysis with figure 10, which plots the impulse response of the price in marginal utility units Q_t to a two-standard deviation shock above the mean at $t = 1$. Our purpose is to understand how the government is using future prices to absorb fiscal shocks in the case of long-term debt. Using as a guide the pricing equation (27), we decompose the response of Q_t in terms of the response of future marginal utility and future prices Q_{t+1} . We note that both in the time-consistent case without default (middle panel) and in the commitment case (right panel), Q_t increases, capturing the increase in debt over time, that increases both future marginal utility and prices. The policymaker is using *both* channels to absorb the fiscal shock as can be seen in the plotted decomposition, with about one third of the adjustment through future marginal utility and two thirds of the adjustment through future prices. This holds both in the time-consistent case without default and in the commitment case. When there is a default option (left panel) though, an adverse fiscal shock increases the probability of default, reducing therefore Q_t . Interestingly, almost the *entire* adjustment is done by the fall in future prices Q_{t+1} , with barely any change in future marginal utilities. Thus, the “Fear” effect dominates and operates mainly through long-term prices.

7 Concluding remarks

In this paper, we analyze optimal distortionary taxation in a setup where policymakers can neither commit to tax policies devised in the past nor to debt repayment. We want to understand how this double absence of commitment alters the basic tax-smoothing and debt issuance prescriptions. We are motivated by the fact that domestic sovereign default is an empirically relevant phenomenon, as Reinhart and Rogoff (2011) showed.

Two opposite forces shape optimal policy in both short- and long-term debt environments. They manifest themselves as a desire to back-load or front-load tax distortions and alter the basic tax-smoothing prescriptions under commitment. We show that lack of commitment to old tax policies increases debt issuance and tax volatility. However, when we introduce even a small risk of default, the fiscal hedging opportunities of the government are curbed, and debt issuance becomes limited. Higher debt is sustained when the government can issue long-term debt.

A Lack of commitment and short-term debt

A.1 Proof of proposition 1

Proof. The assumptions in section 4.1 allow us to use Leibniz's rule to calculate the various derivatives. The expected value function next period and the price Q are

$$\mathbb{E}_{g'|g}V(B', g') = \int_{\underline{g}}^{\omega^{-1}(B')} V^r(B', g')f(g'|g)dg' + \int_{\omega^{-1}(B')}^{\bar{g}} V^d(g')f(g'|g)dg' \quad (\text{A.1})$$

$$\begin{aligned} Q(B', g) &= \beta \mathbb{E}_{g'|g}(1 - d(B', g'))u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) \\ &= \beta \int_{\underline{g}}^{\omega^{-1}(B')} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))f(g'|g)dg'. \end{aligned} \quad (\text{A.2})$$

Use Leibniz's rule and differentiate (A.1) to get

$$\begin{aligned} \frac{\partial}{\partial B'} \mathbb{E}_{g'|g}V(B', g') &= f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \left[\overbrace{V^r(B', \omega^{-1}(B')) - V^d(\omega^{-1}(B'))}^{=0, \text{ since } V^d(\omega^{-1}(B')) = V^r(B', \omega^{-1}(B'))} \right] \\ &\quad + \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B', g')}{\partial B'} f(g'|g)dg' \\ &= \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B', g')}{\partial B'} f(g'|g)dg' \\ &= \mathbb{E}_{g'|g}(1 - d(B', g')) \frac{\partial V^r(B', g')}{\partial B'}. \end{aligned} \quad (\text{A.3})$$

Similarly, differentiate (A.2) and recall that $\mathcal{C}(B', g') + g' = \mathcal{H}(B', g')$, so $\mathcal{C}_B(B', g') = \mathcal{H}_B(B', g')$. We get

$$\begin{aligned} Q_B(B', g) &= \beta \left[\int_{\underline{g}}^{\omega^{-1}(B')} (u'_{cc} - u'_{cl}) \mathcal{C}_B(B', g') f(g'|g)dg' \right. \\ &\quad \left. + u_c(\mathcal{C}(B', \omega^{-1}(B')), 1 - \mathcal{H}(B', \omega^{-1}(B'))) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \right] \\ &= \beta \left[\mathbb{E}_{g'|g}(1 - d(B', g'))(u'_{cc} - u'_{cl}) \mathcal{C}_B(B', g') \right. \\ &\quad \left. + u_c(B', \omega^{-1}(B')) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \right], \end{aligned} \quad (\text{A.4})$$

where the shorthand $u_c(B', \omega^{-1}(B')) \equiv u_c(\mathcal{C}(B', \omega^{-1}(B')), 1 - \mathcal{H}(B', \omega^{-1}(B')))$ is used. Using (A.3) and (A.4) in (16) we get

$$\begin{aligned} -\beta \mathbb{E}_{g'|g}(1 - d(B', g')) \frac{\partial V^r(B', g')}{\partial B'} &= \Phi \cdot \left\{ Q(B', g) + \beta \left[\mathbb{E}_{g'|g}(1 - d(B', g'))(u'_{cc} - u'_{cl}) \mathcal{C}_B(B', g') \right. \right. \\ &\quad \left. \left. + u_c(B', \omega^{-1}(B')) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \right] \cdot B' \right\} \end{aligned} \quad (\text{A.5})$$

The envelope condition is $\frac{\partial V^r(B,g)}{\partial B} = -\Phi u_c < 0$. Update one period to get

$$\frac{\partial V^r(B',g')}{\partial B'} = -\Phi(B',g')u_c(\mathcal{C}(B',g'), 1 - \mathcal{H}(B',g')). \quad (\text{A.6})$$

Use now (A.6) in (A.5) to replace the derivative of the repayment value function in the left-hand side, divide the resulting expression over Q (see A.2), simplify, note that at the indifference threshold we have $d(B', \omega^{-1}(B')) = 0$, and rewrite (A.5) as

$$\mathbb{E}_{g'|g} m(B', g'|g) \Phi(B', g') = \Phi \cdot [1 + \eta(B', g) \cdot B'] \quad (\text{A.7})$$

where $\eta(B', g) \equiv \frac{Q_B(B',g)}{Q(B',g)}$, the price semi-elasticity with respect to B' , with

$$\eta(B', g) = \mathbb{E}_{g'|g} m(B', g'|g) \frac{u'_{cc} - u'_{cl}}{u'_c} \mathcal{C}_B(B', g') + m(B', \omega^{-1}(B')|g) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \quad (\text{A.8})$$

and $m(B', g'|g)$ the default-and-risk-adjusted change of measure,

$$m(B', g'|g) \equiv \frac{(1 - d(B', g'))u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))}{\mathbb{E}_{g'|g}(1 - d(B', g'))u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))}. \quad (\text{A.9})$$

Rewrite (A.7), (A.8) and (A.9) in sequential notation to get (17) and (18) in the text. $\square$

A.2 Proof of proposition 2

Proof. Let λ denote the multiplier on the resource constraint (15). First-order necessary conditions for an interior solution are

$$c : \quad u_c + \Phi[\Omega_c - u_{cc}B] = \lambda \quad (\text{A.10})$$

$$h : \quad u_l - \Phi[\Omega_h + u_{cl}B] = \lambda \quad (\text{A.11})$$

Combining (A.10-A.11) and eliminating λ gives

$$1 = \frac{u_l}{u_c} \cdot \frac{1 - \Phi[\frac{\Omega_h}{u_l} + \frac{u_{cl}}{u_l}B]}{1 + \Phi[\frac{\Omega_c}{u_c} - \frac{u_{cc}}{u_c}B]} = \frac{u_l}{u_c} \cdot \frac{1 + \Phi[1 + \varepsilon_{hh} + \varepsilon_{hc}(1 - \frac{B}{c})]}{1 + \Phi[1 - \varepsilon_{cc}(1 - \frac{B}{c}) - \varepsilon_{ch}]},$$

where the second line follows by expressing the derivatives of $\Omega_i, i = c, h$ in terms of the elasticities defined in proposition 2, $\Omega_c/u_c = 1 - \varepsilon_{cc} - \varepsilon_{ch}$, and $\Omega_h/u_l = -1 - \varepsilon_{hh} - \varepsilon_{hc}$. Use now $u_l/u_c = 1 - \tau$ and rewrite the above expression in terms of τ . Note that the denominator of

(19) is positive, since condition (A.11) and $\lambda > 0$ imply that $1 + \Phi(1 + \varepsilon_{hh} + \varepsilon_{hc}(1 - B/c)) > 0$. $\square$

A.3 Time-consistent policy with no default option

Consider a situation where the policymaker cannot commit to tax policies designed in the past, but where there is *no* option to default. Let $\mathcal{C}(B', g')$ and $\mathcal{H}(B', g')$ denote the consumption and labor policy functions next period when the state is (B', g') . We obviously have $\mathcal{C}(B', g') + g' = \mathcal{H}(B', g'), \forall (B', g')$. Let $V(B, g)$ denote the value function of the Markov-perfect policymaker. Given $\mathcal{C}$ and $\mathcal{H}$, V solves the following problem:

$$V(B, g) = \max_{c \geq 0, h \in [0, 1], B'} u(c, 1 - h) + \beta \mathbb{E}_{g'|g} V(B', g') \quad (\text{A.12})$$

subject to

$$u_c(c, 1 - h)B \leq \Omega(c, h) + \beta \mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) \cdot B' \quad (\text{A.13})$$

$$c + g = h \quad (\text{A.14})$$

$$\underline{M} \leq B' \leq \bar{M}, \quad (\text{A.15})$$

where $(\underline{M}, \bar{M})$ are some exogenous bounds on debt. Let $c(B, g)$ and $h(B, g)$ denote the policy functions that solve (A.12). The Markov-perfect requirement is that $c(B, g) = \mathcal{C}(B, g)$ and $h(B, g) = \mathcal{H}(B, g), \forall (B, g)$.

Assign now multipliers Φ on (A.13) and λ on the resource constraint (A.14), and focus on an interior solution of the problem. Consider again an optimum that is locally differentiable. The first-order conditions with respect to (c, h) are exactly the same as in (A.10) and (A.11), leading to exactly the same formula (19) for the optimal tax rate as in proposition 2. The formula for the tax rate holds in this case for every period, and not only when the government decides to repay. Turning to the optimality condition with respect to B' , we have

$$\begin{aligned} -\mathbb{E}_{g'|g} V_B(B', g') &= \Phi \left[\mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) \right. \\ &\quad \left. + \mathbb{E}_{g'|g} (u_{cc}(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) - u_{cl}(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))) \mathcal{C}_B(B', g') \cdot B' \right], \quad (\text{A.16}) \end{aligned}$$

where we already used the fact that $\mathcal{C}_B = \mathcal{H}_B$. The envelope condition is $V_B(B, g) = -\Phi u_c$. Update one period, and eliminate the derivative of the value function in the left hand-side in (A.16), to get

$$\begin{aligned} \mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) \Phi'(B', g') &= \Phi \left[\mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) \right. \\ &\quad \left. + \mathbb{E}_{g'|g} (u_{cc}(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) - u_{cl}(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))) \mathcal{C}_B(B', g') \cdot B' \right]. \end{aligned}$$

Divide now over average marginal utility, $\mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g'))$, turn into sequential notation, and define the risk-adjusted change of measure

$$x_{t+1} \equiv \frac{u_c(\mathcal{C}(B_{t+1}, g_{t+1}), 1 - \mathcal{H}(B_{t+1}, g_{t+1}))}{E_t u_c(\mathcal{C}(B_{t+1}, g_{t+1}), 1 - \mathcal{H}(B_{t+1}, g_{t+1}))}, \quad (\text{A.17})$$

to finally get the respective expression in terms of the excess burden of taxation (23) in the text.

B Lack of commitment and long-term debt

B.1 Proof of proposition 3

Proof. To derive (33) and (35) we work with (32).

Derivative of expected value function. As in the proof of proposition 1, the derivative of the expected value function takes the form

$$\frac{\partial}{\partial B'} \mathbb{E}_{g'|g} V(B', g') = \int_{\underline{g}}^{\omega^{-1}(B')} \frac{\partial V^r(B', g')}{\partial B'} f(g'|g) dg' = \mathbb{E}_{g'|g} (1 - d(B', g')) \frac{\partial V^r(B', g')}{\partial B'}. \quad (\text{B.1})$$

Price derivative. Turning to the price $Q(B', g)$, recall that

$$\begin{aligned} Q(B', g) &= \beta \mathbb{E}_{g'|g} (1 - d(B', g')) \left[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta) Q(\mathcal{K}(B', g'), g') \right] \\ &= \beta \int_{\underline{g}}^{\omega^{-1}(B')} \left[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta) Q(\mathcal{K}(B', g'), g') \right] f(g'|g) dg'. \end{aligned} \quad (\text{B.2})$$

Differentiating using Leibniz's rule, and recalling that $\mathcal{C}_B = \mathcal{H}_B$, we get

$$\begin{aligned} Q_B(B', g) &= \beta \left[\chi u_c(B', \omega^{-1}(B')) + (1 - \delta) Q(\mathcal{K}(B', \omega^{-1}(B')), \omega^{-1}(B')) \right] f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \\ &\quad + \beta \mathbb{E}_{g'|g} (1 - d(B', g')) \chi (u'_{cc} - u'_{cl}) \mathcal{C}_B(B', g') \\ &\quad + \beta (1 - \delta) \mathbb{E}_{g'|g} (1 - d(B', g')) \mathcal{K}_B(B', g') Q_B(\mathcal{K}(B', g'), g') \end{aligned} \quad (\text{B.3})$$

where $u_c(B', \omega^{-1}(B'))$ is the same shorthand as in the proof of proposition 1.

Envelope condition. The envelope condition with long-term debt is

$$\frac{\partial V^r(B, g)}{\partial B} = -\Phi[\chi u_c + (1 - \delta) Q]. \quad (\text{B.4})$$

Updating one period delivers

$$\frac{\partial V^r(B', g')}{\partial B'} = -\Phi(B', g')[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta)Q(\mathcal{K}(B', g'), g')]. \quad (\text{B.5})$$

GEE. Use now the derivative (B.1) and the envelope condition (B.5) in (32), divide by Q and recall that Q satisfies (B.2), to rewrite (32) as

$$\mathbb{E}_{g'|g} n^d(B', g'|g) \Phi(B', g') = \Phi \cdot [1 + \eta(B', g) \cdot (B' - (1 - \delta)B)] \quad (\text{B.6})$$

where

$$n^d(B', g'|g) \equiv \frac{(1 - d(B', g')) \left[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta)Q(\mathcal{K}(B', g'), g') \right]}{\mathbb{E}_{g'|g}(1 - d(B', g')) \left[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta)Q(\mathcal{K}(B', g'), g') \right]}, \quad (\text{B.7})$$

the default-and-long-term-debt-adjusted change of measure, and η the price semi-elasticity, $\eta(B', g) \equiv \frac{Q_B(B', g)}{Q(B', g)}$. Write (B.6) and (B.7) in sequential notation to get respectively (33) and (34) in the text.

Price semi-elasticity. For the recursion for η , divide (B.3) over Q , recall that Q satisfies (B.2), and use the change of measure (B.7), to get

$$\begin{aligned} \underbrace{\frac{Q_B(B', g)}{Q(B', g)}}_{\eta} &= n^d(B', \omega^{-1}(B')|g) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'} \\ &\quad + \mathbb{E}_{g'|g} n^d(B', g'|g) \frac{\chi(u'_{cc} - u'_{cl})}{\chi u'_c + (1 - \delta)Q'} \mathcal{C}_B(B', g') \\ &\quad + (1 - \delta) \mathbb{E}_{g'|g} n^d(B', g'|g) \frac{Q'}{\chi u'_c + (1 - \delta)Q'} \mathcal{K}_B(B', g') \underbrace{\frac{Q_B(\mathcal{K}(B', g'), g')}{Q'}}_{\eta'} \end{aligned} \quad (\text{B.8})$$

where $Q' \equiv Q(\mathcal{K}(B', g'), g')$. Use the definition of the default-and-long-term-debt-adjusted conditional density f^n in the proposition and write (B.8) in sequential notation to get (35). $\square$

B.2 Proof of proposition 4

Proof. First-order necessary conditions for an interior solution are

$$c : \quad u_c + \Phi[\Omega_c - u_{cc}\chi B] = \lambda \quad (\text{B.9})$$

$$h : \quad u_l - \Phi[\Omega_h + u_{cl}\chi B] = \lambda, \quad (\text{B.10})$$

where λ is the multiplier on the resource constraint (30). Combine (B.9) and (B.10) to

eliminate λ , and follow the same steps as in the proof of proposition 2, to get the optimal tax rate in the text. □

B.3 Time-consistent policy with long-term debt and no default option

For completeness, we state here the time-consistent problem with no default option in the case of long-term debt. Let $\mathcal{C}(B', g')$, $\mathcal{H}(B', g')$ and $\mathcal{K}(B', g')$ denote the consumption, labor, and debt policy functions of the future policymaker. The value function for a time-consistent policymaker with no default option is

$$V(B, g) = \max_{c \geq 0, h \in [0, 1], B'} u(c, 1 - h) + \beta \mathbb{E}_{g'|g} V(B', g') \quad (\text{B.11})$$

subject to

$$\chi u_c(c, 1 - h)B \leq \Omega(c, h) + Q(B', g)[B' - (1 - \delta)B] \quad (\text{B.12})$$

$$c + g = h \quad (\text{B.13})$$

$$\underline{M} \leq B' \leq \bar{M}, \quad (\text{B.14})$$

where the price of long-term debt satisfies the recursion,

$$Q(B', g) = \beta \mathbb{E}_{g'|g} \left[\chi u_c(\mathcal{C}(B', g'), 1 - \mathcal{H}(B', g')) + (1 - \delta)Q(\mathcal{K}(B', g'), g') \right] \quad (\text{B.15})$$

The Markov-perfect requirement is $c(B, g) = \mathcal{C}(B, g)$, $h(B, g) = \mathcal{H}(B, g)$ and $B'(B, g) = \mathcal{K}(B, g)$ for all (B, g) .

Use the same steps as in the proof of proposition 3 or in the analysis of the time-consistent problem with no default in section A.3, to get the recursion for $\eta_t^{\text{no def.}}$ in (38). The respective GEE in sequential notation is

$$\mathbb{E}_t n_{t+1} \Phi_{t+1} = \Phi_t \cdot \left[1 + \eta_t^{\text{no def.}} \cdot (B_{t+1} - (1 - \delta)B_t) \right], \quad (\text{B.16})$$

where the long-term-debt-adjusted change of measure $n_{t+1} = n(B_{t+1}, g_{t+1}|g_t)$ is

$$n(B_{t+1}, g_{t+1}|g_t) \equiv \frac{\chi u_c(\mathcal{C}(B_{t+1}, g_{t+1}), 1 - \mathcal{H}(B_{t+1}, g_{t+1})) + (1 - \delta)Q(\mathcal{K}(B_{t+1}, g_{t+1}), g_{t+1})}{\mathbb{E}_t \left[\chi u_c(\mathcal{C}(B_{t+1}, g_{t+1}), 1 - \mathcal{H}(B_{t+1}, g_{t+1})) + (1 - \delta)Q(\mathcal{K}(B_{t+1}, g_{t+1}), g_{t+1}) \right]}. \quad (\text{B.17})$$

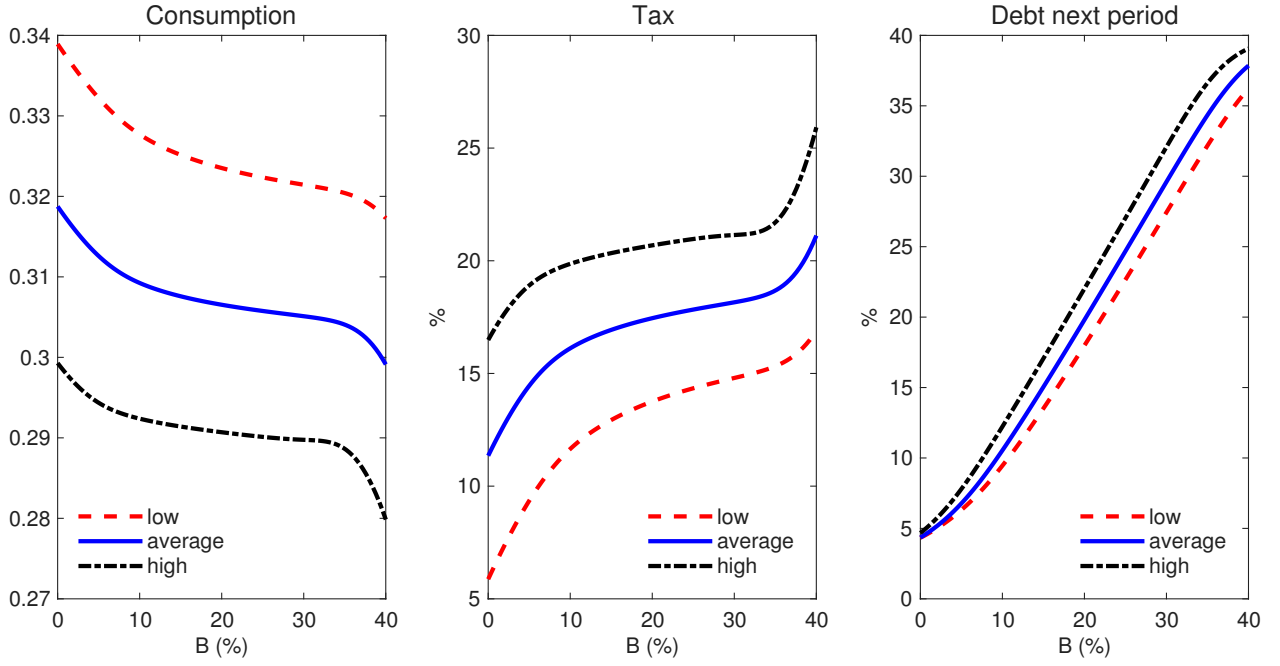


Figure C.1: Policy functions for consumption, taxes and debt next period in the time-consistent model without the option to default. The policy functions are plotted for the mean g (“average”), as well as for the level of g two standard deviations below (“low”), or above (“high”) the mean. The taste shocks that perturb the debt decision are averaged out. To facilitate the interpretation of debt, we express it in terms of first-best output in all panels.

C Additional results

C.1 Short-term debt

For completeness, figure C.1 plots the policy functions for consumption, taxes and debt coming from the time-consistent problem without default. In this case, only the “Greed” force emerges in the GEE, as we saw in equation (23). The policy functions are plotted for the same shocks as the policy functions in figure 2, which correspond to the full blown model with default. Due to the absence of default risk, we can see in the left panel that consumption is at a higher level for each (B, g) than the respective repayment consumption in figure 2 in the text. Similarly, the tax rates in the middle panel are at a lower level than the respective repayment tax rates.

Table C.1 reports the mean statistics of the default episode exercise. More specifically, we calculate each respective statistic for the 15 periods before default, and report the mean statistic across the 2000 sub-samples. The average spread in pre-default periods is about 3%, whereas in the long-run analysis of table 2, which was not conditioning on default episodes, is naturally lower at 1.24%.

C.2 Long-term debt

Table C.2 reports the mean statistics for 2000 default episodes in the long-term debt case. As in the case of short-term debt in table C.1, we calculate each statistic for the 15 periods before default and report the mean statistic across samples.

Table C.1: Mean pre-default statistics with short-term debt.

	Default	MPE	Commitment	Commitment <i>high debt limit</i>
Tax rate in %				
Mean	18.07	18.06	17.65	20.44
St. Dev	4.10	5.08	1.09	0.71
Autocorrelation	0.1382	-0.0386	0.7594	0.7049
Corr with g	0.5847	0.2743	0.9121	0.6488
St. Dev of $\Delta\tau$	5.11	7.23	0.64	0.48
Debt-to-output ratio in %				
Mean	11.76	25.07	17.54	85.86
St. Dev	3.71	5.69	3.76	5.80
Autocorrelation	0.5776	0.5582	0.8924	0.8602
Spread in %				
Mean	2.92			
St. Dev	5.70			
Autocorrelation	0.4460			

Mean statistics across 2000 default episodes for the 15 periods before default. The upper debt limit is 40% of first-best output for the first three columns and 200% for the last column. The same realizations of shocks have been used for the MPE and the commitment case.

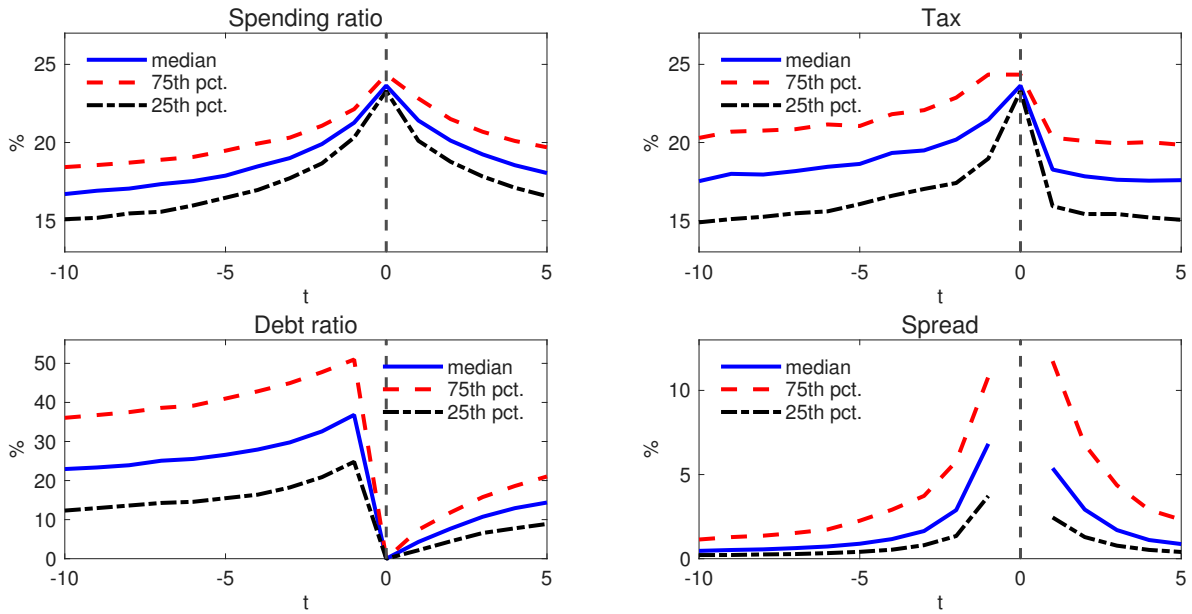


Figure C.2: Dynamics of government spending (g_t/h_t), taxes (τ_t), debt (B_{t+1}/h_t), and spread ($r_t - r_t^F$) around default episodes for long-term debt. The spread is not defined at the period of default.

Similarly to table 3 in the text, table C.3 reports the ensemble moments at the period before default for the long-term debt case. Figure C.2 performs the same exercise as figure 8 for the long-term debt case.

Table C.2: Mean pre-default statistics with long-term debt.

	Default	MPE	Commitment	Commitment <i>high debt limit</i>
Tax rate in %				
Mean	18.41	20.44	18.71	20.45
St. Dev	3.74	3.45	0.82	0.69
Autocorrelation	0.0568	0.0484	0.7494	0.7789
Corr with g	0.4027	0.3825	0.8846	0.8464
St. Dev of $\Delta\tau$	4.98	4.63	0.49	0.38
Debt-to-output ratio in %				
Mean	27.15	81.71	45.76	88.60
St. Dev	6.75	6.02	5.30	6.20
Autocorrelation	0.6913	0.4888	0.9046	0.8952
Macauley duration				
Mean	3.4135	3.5653	3.5716	3.5712
St. Dev	0.0982	0.0193	0.0137	0.0133
Spread in %				
Mean	2.13			
St. Dev	2.78			
Autocorrelation	0.5290			

Mean statistics across 2000 default episodes for the 15 periods before default, in the long-term debt case. The same realizations of shocks have been used for the MPE and the commitment case. The first three columns use an upper debt limit that is set to 100% of first-best output. The last column uses an upper debt limit that is set to 200% of first-best output.

Table C.3: Pre-default ensemble moments with long-term debt.

	Mean	St. Dev
Spending ratio in %	21.14	1.29
Tax rate in %	21.61	3.92
Debt ratio in %	38.85	17.79
Spread in %	7.74	4.88

Ensemble means and standard deviations across sub-samples at the last period before default, for the long-term debt case. The reported debt ratio refers to the debt issued at the last period before default, divided over the respective output before default. This is the amount of debt that is defaulted on.

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Greed versus Fear: Optimal Time-Consistent Taxation with Default

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Online Appendix
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JEL classification: D52; E43; E62; H21; H63.

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D Commitment problems

In this section we consider three policy problems under commitment: the optimal fiscal policy problem with complete markets, and the optimal fiscal policy problem with incomplete markets with either short- or long-term debt. In all three environments, the labor supply condition of the household is given by (6) and the equilibrium wage is unity, $w_t(g^t) = 1, \forall t, g^t$ and will not be repeated. Recall also the definition of the surplus in marginal utility units Ω in (11).

D.1 Complete markets

Consider first the complete markets economy of Lucas and Stokey (1983). There is a full set of Arrow securities, with price $p_t(g_{t+1}, g^t)$. The household's budget constraint reads¹

$$c_t(g^t) + \sum_{g_{t+1}} p_t(g_{t+1}, g^t) B_{t+1}(g^{t+1}) = (1 - \tau_t(g^t)) w_t(g^t) h_t(g^t) + B_t(g^t), \quad (\text{D.1})$$

where $B_t(g^t)$ is the state-contingent debt position. The household's optimality condition with respect to an Arrow security that pays one unit of consumption at g_{t+1} and zero otherwise is given by $p_t(g_{t+1}, g^t) = \beta \pi_{t+1}(g_{t+1} | g^t) \frac{u_c(g^{t+1})}{u_c(g^t)}$.

Using the primal approach to eliminate after-tax wages and prices, the Ramsey planner chooses $\{c_t(g^t), h_t(g^t), B_{t+1}(g^{t+1})\}_{t \geq 0, g^t}$ at $t = 0$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi_t(g^t) u(c_t(g^t), 1 - h_t(g^t)) \quad (\text{D.2})$$

subject to

$$u_c(c_t(g^t), 1 - h_t(g^t)) B_t(g^t) = \Omega(c_t(g^t), h_t(g^t)) + \beta \sum_{g_{t+1}} \pi_{t+1}(g_{t+1} | g^t) u_c(c_{t+1}(g^{t+1}), 1 - h_{t+1}(g^{t+1})) B_{t+1}(g^{t+1}) \quad (\text{D.3})$$

$$c_t(g^t) + g_t = h_t(g^t), \quad (\text{D.4})$$

with $c_t(g^t) \geq 0$, $h_t(g^t) \in [0, 1]$ and (g_0, B_0) given. Assign scaled multipliers $\{\beta^t \pi_t(g^t) \Phi_t(g^t)\}$, and $\{\beta^t \pi_t(g^t) \lambda_t(g^t)\}$ on (D.3) and (D.4) respectively. The first-order necessary conditions for an interior solution are

¹To save on notation, we will not differentiate here between the bond holdings of the consumer b_t , and the government debt liability, B_t .

$$c_t, t \geq 1 : \quad u_{ct} + \Phi_t \Omega_{ct} - u_{cc,t}[\Phi_t - \Phi_{t-1}]B_t = \lambda_t \quad (\text{D.5})$$

$$h_t, t \geq 1 : \quad u_{lt} - \Phi_t \Omega_{ht} - u_{cl,t}[\Phi_t - \Phi_{t-1}]B_t = \lambda_t \quad (\text{D.6})$$

$$c_0 : \quad u_{c0} + \Phi_0[\Omega_{c0} - u_{cc,0}B_0] = \lambda_0 \quad (\text{D.7})$$

$$h_0 : \quad u_{l0} - \Phi_0[\Omega_{h0} + u_{cl,0}B_0] = \lambda_0 \quad (\text{D.8})$$

$$B_{t+1}, t \geq 0 : \quad \Phi_t = \Phi_{t+1} \quad (\text{D.9})$$

Equation (D.5) is equation (24) in the text. The optimality condition with respect to state-contingent debt (D.9) implies that the excess burden is *constant* across states and dates, $\Phi_t(g^t) = \Phi \forall t, g^t$, as we emphasized in (21).

D.2 Incomplete markets and short-term debt

Consider now the case where government debt is *non-contingent*, as in Aiyagari et al. (2002) and Bhandari et al. (2017). The household's budget constraint reads

$$c_t(g^t) + q_t(g^t)B_{t+1}(g^t) = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + B_t(g^{t-1}). \quad (\text{D.10})$$

The households's optimality condition with respect to non-contingent debt is $q_t = \beta E_t \frac{u_{c,t+1}}{u_{ct}}$.

Using the primal approach, we have the following policy problem: the Ramsey planner chooses $\{c_t(g^t), h_t(g^t), B_{t+1}(g^t)\}_{t \geq 0, g^t}$ at $t = 0$ to maximize (D.2) subject to

$$u_c(c_t(g^t), 1 - h_t(g^t))B_t(g^{t-1}) = \Omega(c_t(g^t), h_t(g^t)) + \beta \sum_{g^{t+1}} \pi_{t+1}(g_{t+1}|g^t) u_c(c_{t+1}(g^{t+1}), 1 - h_{t+1}(g^{t+1}))B_{t+1}(g^t) \quad (\text{D.11})$$

$$c_t(g^t) + g_t = h_t(g^t) \quad (\text{D.12})$$

$$\underline{M} \leq B_{t+1}(g^t) \leq \bar{M}, \quad (\text{D.13})$$

with $c_t(g^t) \geq 0$, $h_t(g^t) \in [0, 1]$ and (g_0, B_0) given. Let $\{\beta^t \pi_t(g^t) \Phi_t(g^t)\}$ and $\{\beta^t \pi_t(g^t) \lambda_t(g^t)\}$ be the scaled multipliers on (D.11) and (D.12) respectively and focus on an interior solution. The first-order necessary conditions are

$$c_t, t \geq 1 : \quad u_{ct} + \Phi_t \Omega_{ct} - u_{cc,t}(\Phi_t - \Phi_{t-1})B_t = \lambda_t \quad (\text{D.14})$$

$$h_t, t \geq 1 : \quad u_{ht} - \Phi_t \Omega_{ht} - u_{cl,t}(\Phi_t - \Phi_{t-1})B_t = \lambda_t \quad (\text{D.15})$$

$$c_0 : \quad u_{c0} + \Phi_0(\Omega_{c0} - u_{cc,0}B_0) = \lambda_0 \quad (\text{D.16})$$

$$h_0 : \quad u_{h0} - \Phi_0(\Omega_{h0} + u_{cl,0}B_0) = \lambda_0 \quad (\text{D.17})$$

$$B_{t+1} : \quad \Phi_t = \mathbb{E}_t x_{t+1} \Phi_{t+1}, \quad (\text{D.18})$$

where $x_{t+1} \equiv \frac{u_{c,t+1}}{\mathbb{E}_t u_{c,t+1}}$, the risk-adjusted change of measure. Footnote 21 refers to (D.14), which takes the same form as (24), except that debt is non-contingent. First-order condition (D.18) is the tax-smoothing condition (22) in the text.

D.2.1 Recursive formulation

Consider now a recursive formulation of the commitment problem, in the spirit of Kydland and Prescott (1980). Shocks follow a time-invariant Markov chain, with transition probability $\pi(g'|g)$. To represent the problem recursively, we need to capture the marginal utility promises (which translate to interest rate promises) that the planner has to keep.

Define $\tilde{B}_t \equiv \mathbb{E}_t u_{c,t+1} \cdot B_{t+1}$, that is, average marginal utility times debt, which is a function of information at t . This variable serves as a *pseudo-state* variable for the problem from period one onward, with an initial value that is chosen optimally at $t = 0$. The implementability constraint (D.11) can be expressed in terms of $\tilde{B}_t$ as

$$\frac{u_{ct}}{\mathbb{E}_{t-1} u_{ct}} \tilde{B}_{t-1} = \Omega(c_t, h_t) + \beta \tilde{B}_t.$$

We express the commitment problem before the realization of uncertainty at time t , to capture promises *across* states. Let W_t denote average (or *ex-ante*) utility, $W_t \equiv \mathbb{E}_t V_{t+1}$, with respective recursion $W_{t-1} = \mathbb{E}_{t-1}(u(c_t, 1 - h_t) + \beta W_t)$.

Bellman equation. Let underscore ‘ $-$ ’ denote previous period and let $W(\tilde{B}_-, g_-)$ denote the optimal ex-ante value of the commitment problem from period one onward at $(\tilde{B}_-, g_-)$. The Bellman equation is given by

$$W(\tilde{B}_-, g_-) = \max_{c_g, h_g, \tilde{B}_g} \sum_g \pi(g|g_-) [u(c_g, 1 - h_g) + \beta W(\tilde{B}_g, g)] \quad (\text{D.19})$$

subject to

$$\frac{u_c(c_g, 1 - h_g)}{\sum_g \pi(g|g_-) u_c(c_g, 1 - h_g)} \tilde{B}_- = \Omega(c_g, h_g) + \beta \tilde{B}_g, \forall g \quad (\text{D.20})$$

$$c_g + g = h_g, \forall g \quad (\text{D.21})$$

$$c_g \geq 0, h_g \in [0, 1], \forall g \quad (\text{D.22})$$

$$\underline{B}_g \leq \tilde{B}_g \leq \bar{B}_g \forall g \quad (\text{D.23})$$

Note that the debt limits in (D.23) are formulated directly in terms of the average marginal-utility adjusted state variable $\tilde{B}_t$.

Initial period problem. The planner chooses $c_0 \geq 0, h_0 \in [0, 1]$ and $\tilde{B}_0$ within the debt limits to maximize

$$u(c_0, 1 - h_0) + \beta W(\tilde{B}_0, g_0) \quad (\text{D.24})$$

subject to

$$u_c(c_0, 1 - h_0) B_0 = \Omega(c_0, h_0) + \beta \tilde{B}_0 \quad (\text{D.25})$$

$$c_0 + g_0 = h_0, \quad (\text{D.26})$$

where (g_0, B_0) given. The solution of the $t = 0$ problem furnishes the optimal value $\tilde{B}_0^*$ as function of the initial conditions (g_0, B_0) . This value is used to initialize the state variable in (D.19).

D.3 Incomplete markets and long-term debt

Consider now the long-term bond of Chatterjee and Eyigungor (2012). The budget constraint of the consumer reads

$$c_t(g^t) + q_t(g^t)[B_{t+1}(g^t) - (1 - \delta)B_t(g^{t-1})] = (1 - \tau_t(g^t))w_t(g^t)h_t(g^t) + \chi B_t(g^{t-1}), \quad (\text{D.27})$$

where $\chi \equiv \delta + (1 - \delta)z$. The optimality condition of the household with respect to the bond is

$$q_t(g^t) = \beta \sum_{g^{t+1}} \pi_{t+1}(g^{t+1}|g^t) \frac{u_c(g^{t+1})}{u_c(g^t)} [\chi + (1 - \delta)q_{t+1}(g^{t+1})]. \quad (\text{D.28})$$

To derive the proper implementability constraints, eliminate first the after-tax wage from the household's budget constraint to get

$$u_c(c_t, 1 - h_t)[\chi + (1 - \delta)q_t]B_t = \Omega(c_t, h_t) + u_c(c_t, 1 - h_t)q_t B_{t+1}, t \geq 0. \quad (\text{D.29})$$

D.3.1 Ramsey problem

The Ramsey planner chooses $\{c_t(g^t), h_t(g^t), B_{t+1}(g^t), q_t(g^t)\}_{t \geq 0, g^t}$ at $t = 0$ to maximize (D.2) subject to

$$u_c(c_t(g^t), 1 - h_t(g^t))[\chi + (1 - \delta)q_t(g^t)]B_t(g^{t-1}) = \Omega(c_t(g^t), h_t(g^t)) + u_c(c_t(g^t), 1 - h_t(g^t))q_t(g^t)B_{t+1}(g^t) \quad (\text{D.30})$$

$$u_c(c_t(g^t), 1 - h_t(g^t))q_t(g^t) = \beta \sum_{g^{t+1}} \pi_{t+1}(g_{t+1}|g^t) u_c(c_{t+1}(g^{t+1}), 1 - h_{t+1}(g^{t+1})) \cdot [\chi + (1 - \delta)q_{t+1}(g^{t+1})] \quad (\text{D.31})$$

$$c_t(g^t) + g_t = h_t(g^t) \quad (\text{D.32})$$

$$\underline{M} \leq B_{t+1}(g^t) \leq \bar{M} \quad (\text{D.33})$$

with $c_t(g^t) \geq 0$, $h_t(g^t) \in [0, 1]$ and (g_0, B_0) given. Note that the planner chooses also long-term debt prices q_t , so he has to take into account how current prices depend on future marginal utility and future prices q_{t+1} through (D.31).

Assign scaled multipliers $\{\beta^t \pi_t(g^t) \Phi_t(g^t)\}$, $\{\beta^t \pi_t(g^t) \xi_t(g^t)\}$ and $\{\beta^t \pi_t(g^t) \lambda_t(g^t)\}$ on (D.30), (D.31) and (D.32) respectively, and focus again on an interior solution. First-order necessary conditions for $t \geq 0$ are as follows.

$$c_t : \quad \begin{aligned} & u_{ct} + \Phi_t [\Omega_{ct} + u_{cc,t} [q_t B_{t+1} - (\chi + (1 - \delta)q_t) B_t]] \\ & - u_{cc,t} [\xi_t q_t - \xi_{t-1} (\chi + (1 - \delta)q_t)] = \lambda_t \end{aligned} \quad (\text{D.34})$$

$$h_t : \quad \begin{aligned} & u_{ht} - \Phi_t [\Omega_{ht} - u_{cl,t} [q_t B_{t+1} - (\chi + (1 - \delta)q_t) B_t]] \\ & - u_{cl,t} [\xi_t q_t - \xi_{t-1} (\chi + (1 - \delta)q_t)] = \lambda_t \end{aligned} \quad (\text{D.35})$$

$$B_{t+1} : \quad \Phi_t u_{ct} q_t = \beta \mathbb{E}_t u_{c,t+1} (\chi + (1 - \delta)q_{t+1}) \Phi_{t+1} \quad (\text{D.36})$$

$$q_t : \quad \xi_t = \Phi_t [B_{t+1} - (1 - \delta)B_t] + (1 - \delta)\xi_{t-1}, \xi_{-1} \equiv 0 \quad (\text{D.37})$$

Note that at $t = 0$ there are no price promises to uphold, which makes the shadow value of the initial price zero, $\xi_{-1} \equiv 0$.

Simplification. We can simplify further the first-order conditions with respect to (c, h) . Consider first (D.34) and collect the terms that multiply $u_{cc,t} q_t$:

$$u_{ct} + \Phi_t \left[\Omega_{ct} - \chi u_{cc,t} B_t \right] - \underbrace{u_{cc,t} q_t \left[\xi_t - \Phi_t (B_{t+1} - (1 - \delta) B_t) - (1 - \delta) \xi_{t-1} \right]}_{=0, \text{ from (D.37)}} + \chi u_{cc,t} \xi_{t-1} = \lambda_t.$$

The third term above is identically equal to zero, due to the law of motion of ξ_t in (D.37). Thus, (D.34) simplifies to

$$u_{ct} + \Phi_t \left[\Omega_{ct} - \chi u_{cc,t} B_t \right] + \chi u_{cc,t} \xi_{t-1} = \lambda_t \quad (\text{D.38})$$

which corresponds to (40) in the text.

Following similar steps with the first-order condition with respect to labor (D.35), we get

$$u_{lt} - \Phi_t \left[\Omega_{ht} + \chi u_{cl,t} B_t \right] + \chi u_{cl,t} \xi_{t-1} = \lambda_t. \quad (\text{D.39})$$

Excess burden of taxation. Using the optimality condition with respect to debt B_{t+1} , we get again an “averaging” result with respect to the proper measure. Rewrite (D.36) as

$$\Phi_t = \beta \frac{\mathbb{E}_t u_{c,t+1} (\chi + (1 - \delta) q_{t+1}) \Phi_{t+1}}{u_{ct} q_t}$$

But from (D.28) we have $u_{ct} q_t = \beta \mathbb{E}_t u_{c,t+1} (\chi + (1 - \delta) q_{t+1})$. Use this fact to eliminate the current price q_t to finally get condition (39) in the text, $\Phi_t = \mathbb{E}_t n_{t+1} \Phi_{t+1}, t \geq 0$, where $n_{t+1} \equiv \frac{u_{c,t+1} (\chi + (1 - \delta) q_{t+1})}{\mathbb{E}_t u_{c,t+1} (\chi + (1 - \delta) q_{t+1})} \geq 0$, that is, the long-term-debt adjusted change of measure, with $\mathbb{E}_t n_{t+1} = 1$.

D.3.2 Recursive formulation

As in the case of short-term debt, consider a recursive formulation of the commitment problem with long-term debt. Let $Q_t \equiv u_{ct} q_t$, denote the price of the asset in marginal utility units, which follows recursion

$$Q_t = \beta \mathbb{E}_t (\chi u_{c,t+1} + (1 - \delta) Q_{t+1}), \quad (\text{D.40})$$

from (D.28). Our state variables are the exogenous shock g_t , debt B_t , and the *pseudo-state* variable Q_t , that captures promises to the past and follows (D.40).²

²Note that in the case of short-term debt ($\delta = 1$) in the previous section, $\tilde{B}_t$ was effectively capturing *both* debt *and* marginal utility promises, reducing the endogenous state variables to one. With long-term debt instead, we have to keep track of both B_t and Q_t , which is the sum of the entire stream of future marginal utilities.

Let $W(B, Q_-, g_-)$ denote the respective optimal ex-ante value of the problem from period one onward, when the state is (B, Q_-, g_-) , that is, before the realization of uncertainty (recall that B is predetermined). The Bellman equation is

$$W(B, Q_-, g_-) = \max_{c_g, h_g, B'_g, Q_g} \sum_g \pi(g|g_-) [u(c_g, 1 - h_g) + \beta W(B'_g, Q_g, g)] \quad (\text{D.41})$$

subject to

$$[\chi u_c(c_g, 1 - h_g) + (1 - \delta)Q_g]B = \Omega(c_g, h_g) + Q_g B'_g, \forall g \quad (\text{D.42})$$

$$Q_- = \beta \sum_g \pi(g|g_-) [\chi u_c(c_g, 1 - h_g) + (1 - \delta)Q_g] \quad (\text{D.43})$$

$$c_g + g = h_g, \forall g \quad (\text{D.44})$$

$$\underline{M} \leq B'_g \leq \bar{M}, \forall g \quad (\text{D.45})$$

where $c_g \geq 0$, $h_g \in [0, 1]$, and Q_g belong to the proper sets of marginal-utility adjusted prices, that can be generated by the competitive equilibrium.

Initial period problem. The planner's problem at $t = 0$ is to choose $\{c_0, h_0, B_1, Q_0\}$ to maximize

$$u(c_0, 1 - h_0) + \beta W(B_1, Q_0, g_0) \quad (\text{D.46})$$

subject to

$$[\chi u_c(c_0, 1 - h_0) + (1 - \delta)Q_0]B_0 = \Omega(c_0, h_0) + Q_0 B_1 \quad (\text{D.47})$$

$$c_0 + g_0 = h_0, \quad (\text{D.48})$$

where (g_0, B_0) given. Note that there are no price “promises” from the past (Q_{-1}), a fact which makes equation (D.43) redundant at the initial period. The *optimal* Q_0^* that solves the initial period problem is used to initialize (D.41), and is ultimately a function of the initial conditions (g_0, B_0) .

E Jumps and non-monotonicities without taste shocks

There are several issues with the numerical computation of the problem. These issues stem from the Markov-perfect nature of optimal policy, that generates non-convexities in the implementability constraint, and leads to discontinuous and non-monotonic policy functions. As we

mention in the main text, we deal with these issues using the discrete choice taste shock machinery. Before we proceed here to the re-formulation of the policy problem with taste shocks, we illustrate first the discontinuities without taste shocks.

E.1 Deterministic problem

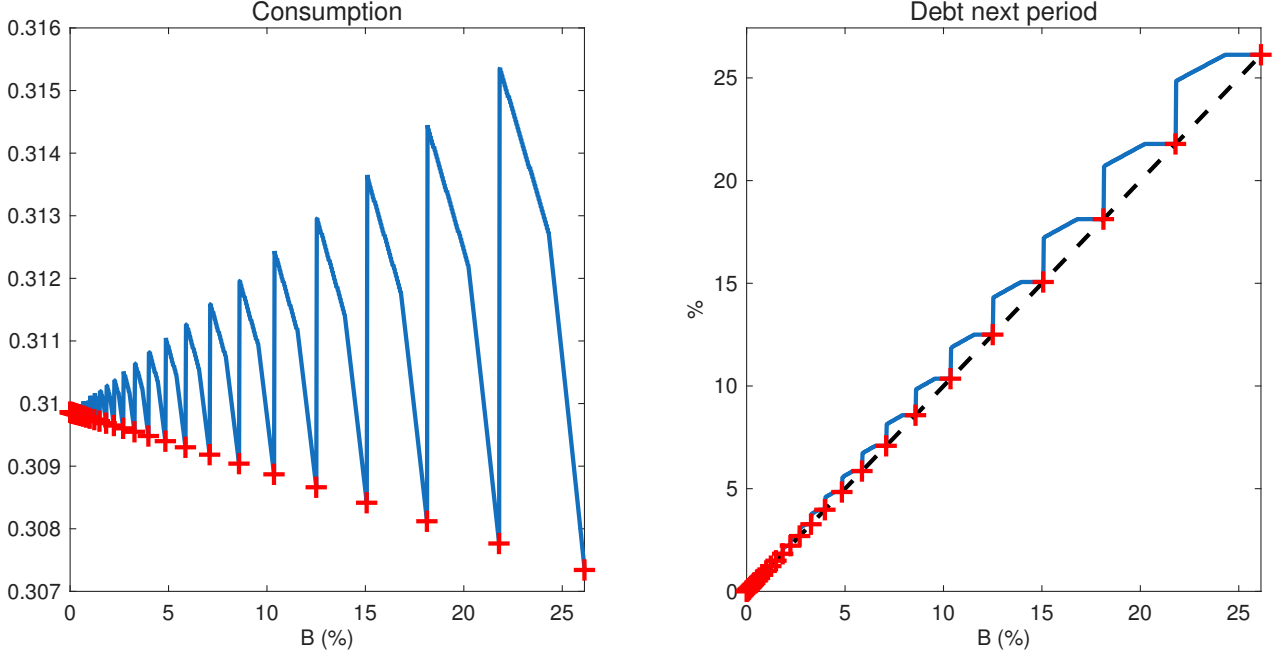


Figure E.1: Consumption and debt policy function in the deterministic [Krusell et al. \(2004\)](#) economy. The preference parameters are set to the baseline values in the text. Government expenditures are set to G . The upper debt limit is set to the maximum debt that can be repaid in one period. The crosses ‘+’ correspond to steady state values of consumption (left panel) and capital (right panel). At the right-hand side of each of these points, there is a jump upwards in consumption and debt. Debt is expressed in terms of first-best output.

To understand better the issues involved, we shut off first uncertainty and the option to default in the short-term debt model. The setup reduces to a deterministic [Lucas and Stokey \(1983\)](#) economy, with the crucial difference that policy is set in a time-consistent manner. [Krusell et al. \(2004\)](#) have analyzed this problem. The GEE for an interior solution at a locally smooth equilibrium is

$$\Phi_{t+1} = \Phi_t \cdot \left[1 + \frac{u_{cc,t+1} - u_{cl,t+1}}{u_{c,t+1}} \mathcal{C}_B(B_{t+1}) \cdot B_{t+1} \right]. \quad (\text{E.1})$$

As we mentioned in the text, equation (E.1) is the deterministic version of the GEE (23). [Krusell et al. \(2004\)](#) have shown that the non-convexities associated with the Markov-perfect assumption in this economy introduce serious discontinuities in the policy functions for consumption and debt. In figure E.1 we illustrate the severity of these discontinuities. The debt policies are continuous from the left. At each jump, if current debt increases marginally, the current planner has an incentive to issue a large amount of debt for next period, reducing a lot the interest rate (since the consumption of the *future* policymaker will jump downwards and

therefore marginal utility will jump upwards) and allowing therefore to tax less currently– which is why also current consumption jumps upwards as well. As a result, we get *non-monotonic* policy functions for consumption. The size of jumps increases for higher values of debt and the GEE is valid only locally, at the increasing parts of the debt policy function.

E.2 The smoothing effect of uncertainty

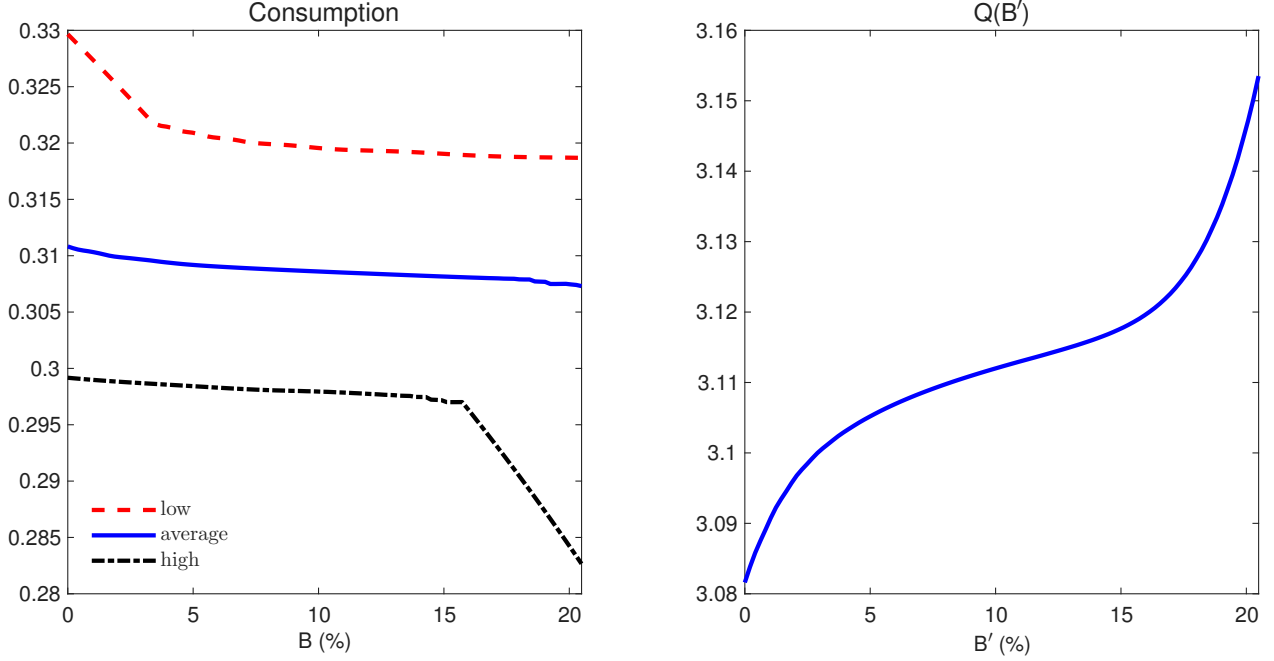


Figure E.2: The left panel plots consumption policy functions for the time-consistent policy problem without default for the short-term debt case. The policy functions are plotted for the mean g (“average”), as well as for a level of g that is two standard deviations below (“low”), or above (“high”) the mean. Government expenditures are i.i.d. with the same unconditional variance as in the baseline calibration. The upper debt limit is set to the maximum debt that can be repaid in one period. The right panel plots the price in marginal utility $Q(B')$. Debt is expressed in terms of first-best output.

When we have stochastic government expenditures, it may be possible to eliminate the jumps in the policy functions, due to the *smoothing* effect of uncertainty. To see that in a heuristic way, drop for simplicity the option to default, assume that $u_{cl} = 0$, and consider the average marginal utility over the entire support of government expenditures, $\mathbb{E}_{g'|g} u_c(\mathcal{C}(B', g'))$. This object captures equilibrium interest rates and is the source of discontinuity in the implementability constraint, through the actions of the future policymaker $\mathcal{C}$. Assume for the sake of the argument that each policy function of consumption for next period (indexed by g'), is discontinuous in B' . As long as the points where the policy functions have jumps are countable and not the same across shocks g' , average marginal utility will smooth out the jumps, leading to a continuous price function of debt. At a more fundamental level, uncertainty partially “convexifies” the constraint set.

This intuition is partially confirmed, under *restrictive* assumptions for the size of the state space and the persistence of shocks. To see that, we solve first the time-consistent problem without default, using the baseline calibration, with the difference that government expendi-

tures are i.i.d., with a variance that is equal to the unconditional variance of the persistent process used in the text. We set the upper debt limit to the maximum static primary surplus that can be generated in the worst case of the highest government expenditure shock. This amounts to a value a little bit above 20% of first-best output. Figure E.2 displays the policy functions for consumption (left panel) and the respective price in marginal utility units $Q(B')$ (right panel), which does not depend on the current shock, due to the i.i.d. assumption. We see that uncertainty smooths out the jumps in the policy functions that we saw in figure E.1. Policy functions for consumption are monotonically decreasing, leading to a continuous and increasing pricing schedule that captures the “Greed” force.

However, jumps and the resulting non-monotonicities may still show up in the stochastic model for less restrictive assumptions. Their severity depends on a) the size of the state space b) the persistence of the exogenous process c) the presence of default risk d) the duration of debt. We illustrate these points in a series of numerical experiments.

E.3 Experiments

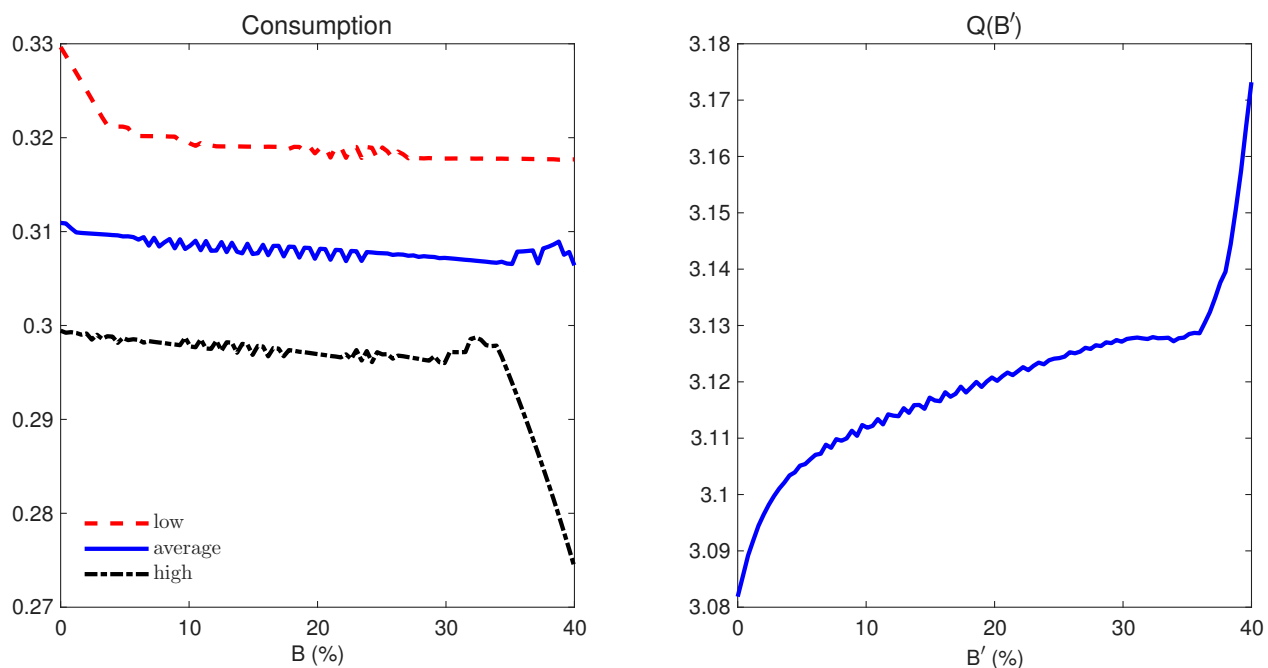


Figure E.3: Consumption policy functions and price schedule $Q(B')$ for the time-consistent model without default in the short-term debt case. Shocks are i.i.d. as in figure E.2. The upper debt limit is set to 40% of first-best output.

Size of state space. Consider first an increase of the upper debt limit to 40% of first-best output, our baseline value, keeping government expenditures i.i.d. Figure E.3 plots the respective policy functions for consumption and the price schedule Q . Small jumps and non-monotonicities in consumption begin to appear at higher levels of debt, and this also starts to affect the price function.

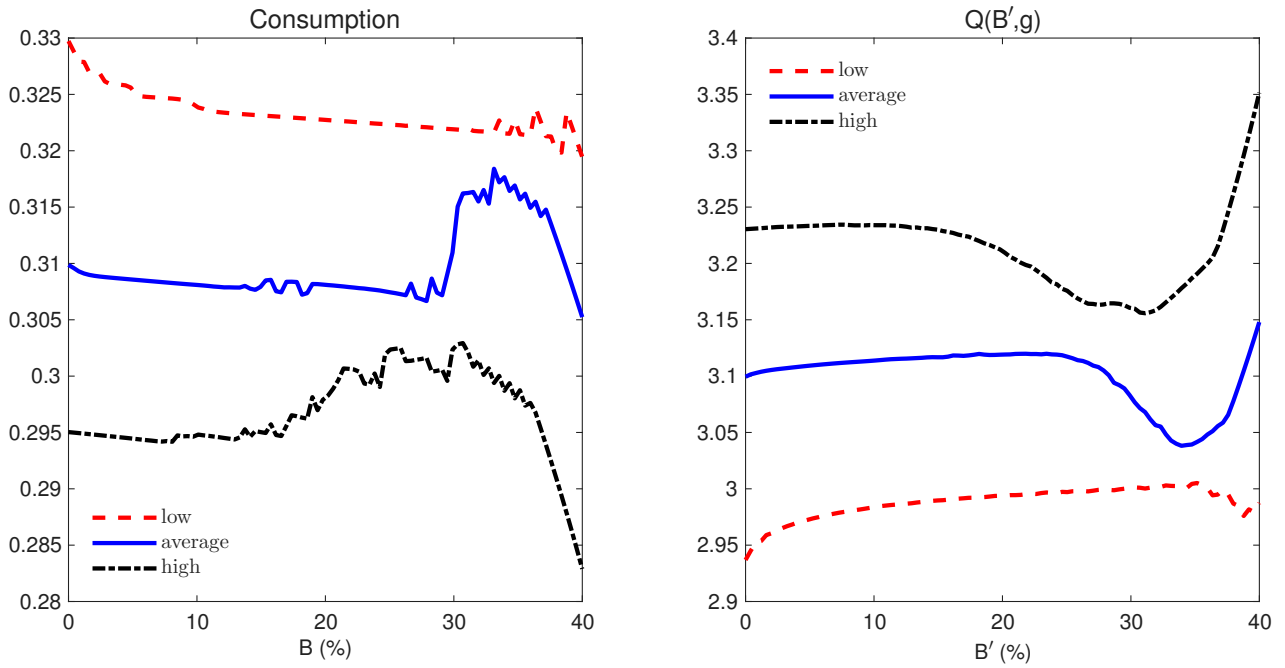


Figure E.4: Consumption policy functions and price schedule $Q(B', g)$ for the time-consistent model without default in the short-term debt case. “Average” g corresponds to the mean shock, and “low” (“high”) to a shock two standard deviations below (above) the mean. The process for g_t and the upper debt limit are as in the baseline calibration in the text.

Persistence of shocks. Consider now using the persistent process for g_t from our baseline calibration and solve for the time-consistent policy without default. Figure E.4 plots the respective policy functions for consumption and the price schedule $Q(B', g)$ for various levels of shocks. The figure reveals that with a persistent shock process, which in a sense makes g_t stay close to its realization in the previous period, resembling the deterministic case, jumps and non-monotonicities in consumption become severe, leading also to decreasing parts of the price schedule for high levels of debt.

Default option. Finally, consider the full model with a default option in the short-term debt case, using the baseline calibration in the text. Figure E.5 plots the respective policy functions and the price schedule Q . With the default option, high values of debt are less likely to occur in equilibrium. As this is where the discontinuities typically occur, the default option effectively makes the size of the jumps less severe, as we can see in the left panel of the figure. Intuitively, the “Fear” force, that acts opposite to the “Greed” force, mitigates the desire to issue more debt, alleviating partially the discontinuities, at least in the short-term debt case.

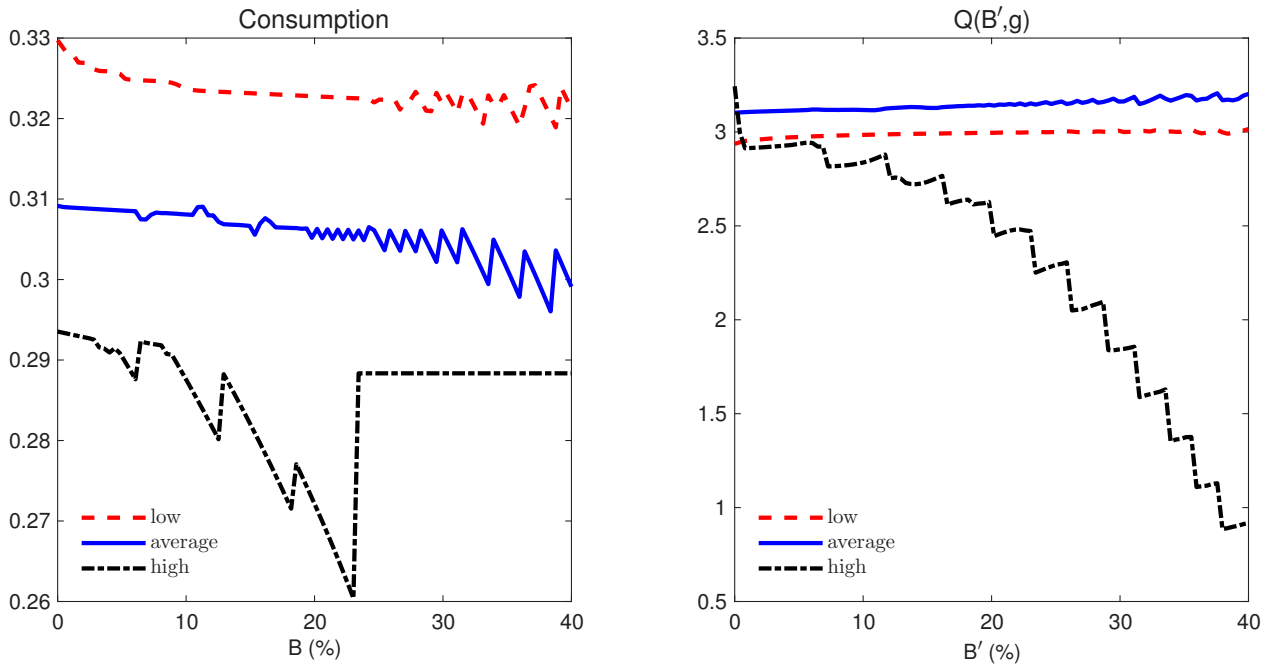


Figure E.5: Consumption policy functions and price schedule $Q(B', g)$ for the time-consistent model with default in the short-term debt case. “Average” g corresponds to the mean shock, and “low” (“high”) to a shock two standard deviations below (above) the mean. The process for g_t and the upper debt limit are as in the baseline calibration in the text.

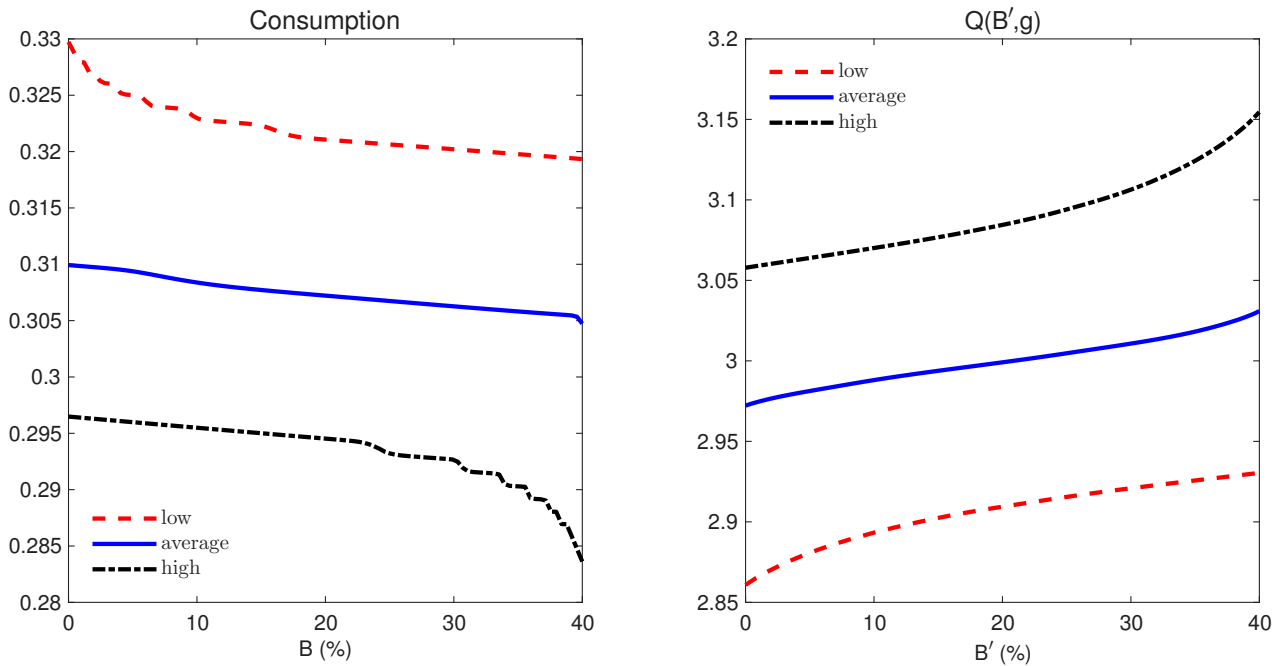


Figure E.6: Consumption policy functions and price schedule $Q(B', g)$ for the time-consistent model without default in the long-term debt case. “Average” g corresponds to the mean shock, and “low” (“high”) to a shock two standard deviations below (above) the mean. Government expenditures are persistent as in the baseline calibration, but the upper debt limit is set to only 40% of first-best output.

Long-term debt. Consider now the long-term debt specification. The same issue of jumps and non-monotonicities appears, but it is less pronounced than in the short-term debt case, and tends to appear at higher levels of debt, allowing us to increase the size of the state space. The reason can be seen in equation (27) and (31) in the text. Any discontinuities that may show

up in the time-consistent problem may stem from the future consumption policy function $\mathcal{C}$, which affects marginal utility next period, and the future debt policy function $\mathcal{K}$, which affects future prices. The composite parameter χ , which multiplies future marginal utility and captures coupon payments and maturing debt, is small for our baseline calibration ($\chi = 0.26875$), which at least tempers issues coming from $\mathcal{C}$.³

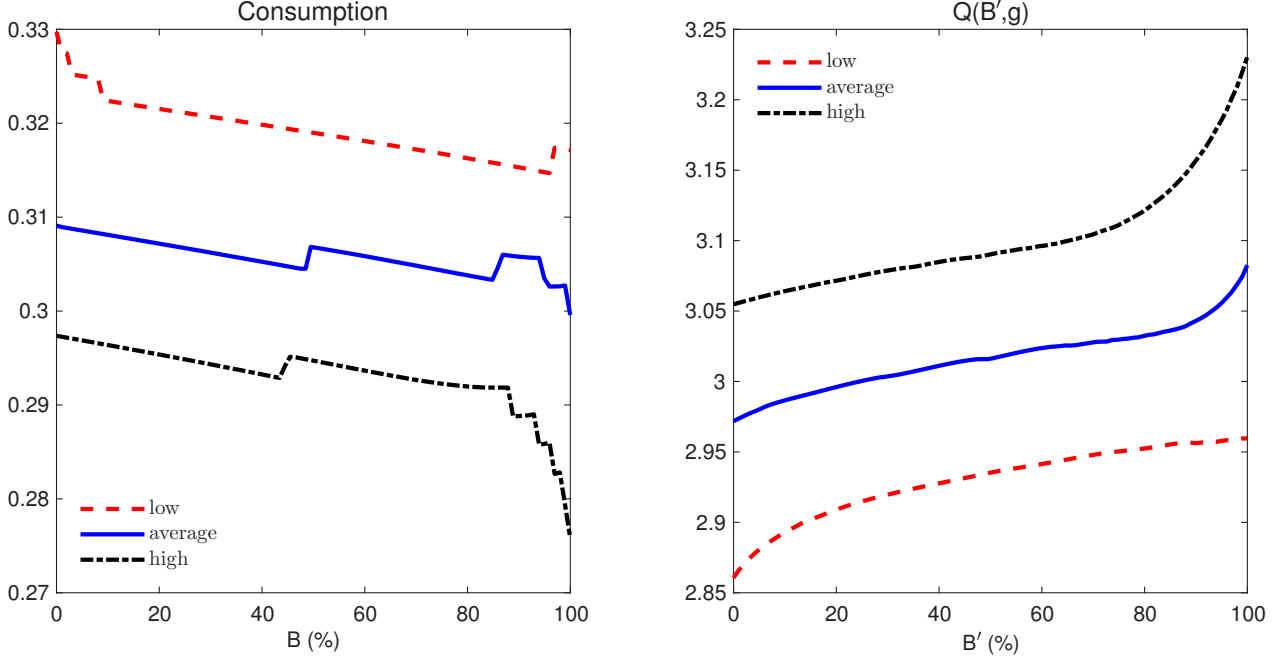


Figure E.7: Consumption policy functions and price schedule $Q(B', g)$ for the time-consistent model without default in the long-term debt case. “Average” g corresponds to the mean shock, and “low” (“high”) to a shock two standard deviations below (above) the mean. Government expenditures are persistent as in the baseline calibration, and the upper debt limit is set to the baseline value of 100% of first-best output.

We solve first the time-consistent model without default in the long-term debt case. We start with figure E.6, where we plot the consumption policy functions and Q , using the baseline persistent process for g_t , but setting the upper debt limit to only 40% of first-best output, which is our baseline value for the short-term debt specification. For this size of the state space, jumps are not present. Consumption policy functions in the left panel of the figure are smooth and monotonic. Instead, when we increase the upper debt limit to 100%, our baseline value for the long-term debt case, jumps and non-monotonicities re-appear, as we can see in the left panel of figure E.7. Finally, we solve for the full version of our model with default and long-term debt for the baseline calibration. Figure E.8 plots the respective policy functions and price schedule. Jumps and non-monotonicities become again prevalent.

³It is well known in the small open economy sovereign default literature that jumps and discontinuities show up with long-term debt- see for example the detailed discussion in [Chatterjee and Eyigungor \(2012\)](#). Note that there are typically no such issues in the sovereign default literature with short-term debt, since the “Greed” force is absent, given the fact that debt is priced by risk-neutral foreign investors. This would also be true if external investors are risk-averse, since the domestic government is “small” and takes the pricing kernel of foreign investors as given. Thus, the typical source of jumps in the sovereign default literature is the future debt decision of the government ($\mathcal{K}$ in our notation) in the long-term debt case. Instead, in the closed economy time-consistent optimal taxation problem that we analyze, both $\mathcal{C}$ and $\mathcal{K}$ matter.

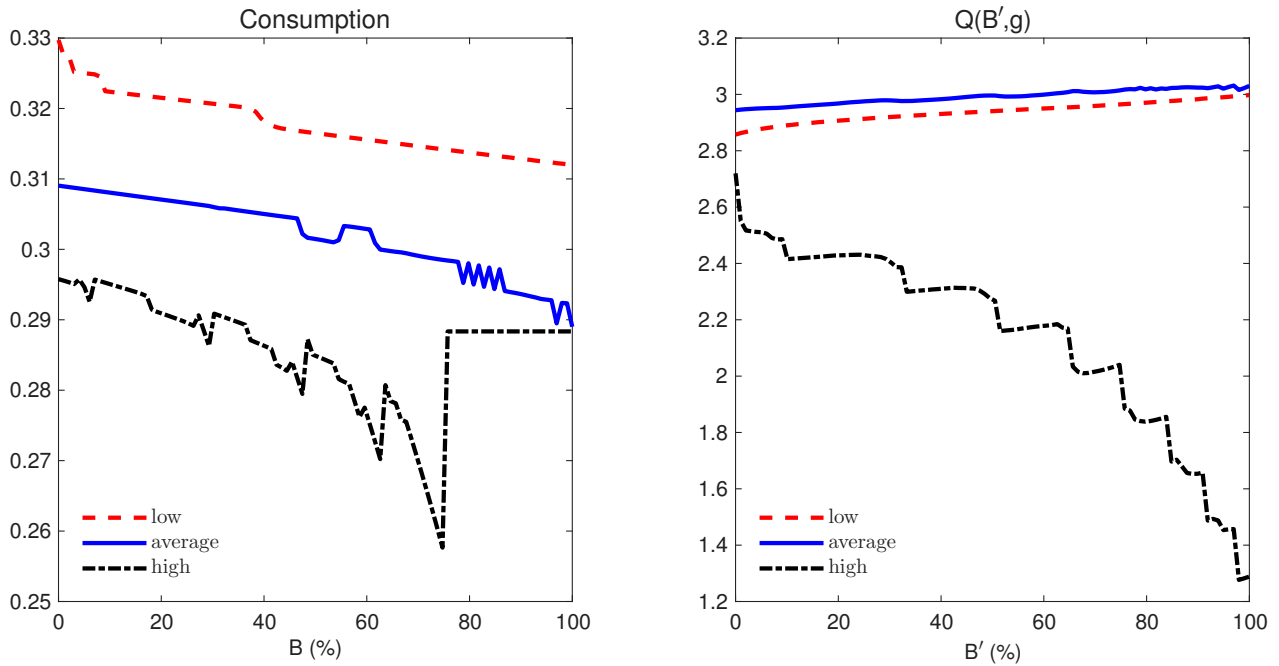


Figure E.8: Consumption policy functions and price schedule $Q(B', g)$ for the time-consistent model with default in the long-term debt case. “Average” g corresponds to the mean shock, and “low” (“high”) to a shock two standard deviations below (above) the mean. Government expenditures and the upper debt limit are as in the baseline calibration.

F Taste shocks formulation

Our desire is to have a more relevant calibration with a relatively large state space and persistent shocks, and to avoid the complications with the jumps and non-monotonicities that we illustrated in the previous section. Moreover, we want a reliable solution algorithm that is convergent. For that purpose, we adopt the discrete choice taste shock machinery, pioneered by [Dvorkin et al. \(2021\)](#) in the sovereign default literature, and adapt it to our time-consistent optimal taxation problem.

In particular, we assume that the policymaker has a discrete choice set for debt, and that he faces additive extreme value type I (Gumbel) taste shocks to both the debt and the default decision. Taste shocks make the choice of debt and default probabilistic. The induced uncertainty helps ‘smooth’ out the constraints of the policy problem, reducing jumps and discontinuities, and facilitating numerical computation and convergence.

For notational simplicity, we assume that utility is separable in consumption and leisure, $u_{cl} = 0$. Assume a finite grid for government debt, $B_i, i = 1, \dots, N_B$. Assume also a finite grid for government spending shocks, $g_k, k = 1, \dots, N_g$, with transition probability $\pi(j|i) \equiv \text{Prob}(g_{t+1} = g_j | g_t = g_i)$.

F.1 Short-term debt

F.1.1 Time-consistent policy without default

Consider first the case when there is no commitment to the tax plans designed in the previous periods, but no default option. The policymaker faces a *vector* of additive taste shocks $\epsilon = (\epsilon_1, \dots, \epsilon_{N_B})$. The j th component of ϵ , ϵ_j , is associated with the choice of debt B_j , $j = 1, \dots, N_B$. The taste shocks are identically and independently (from each other and from spending shocks) distributed with an extreme value type I (Gumbel) distribution, with location parameter $-\gamma\sigma_\epsilon$ and scale parameter σ_ϵ , where γ is the Euler-Mascheroni constant.⁴

Competitive equilibrium. The policymaker chooses from a finite grid for debt, but the household's choice of b is *continuous*. This allows us to use the household's Euler equation to replace prices in the implementability constraint of the policy problem. In equilibrium, we obviously have $b = B$, where B belongs to the finite grid. The aggregate state in the economy includes government debt B , the exogenous shock g , and the vector of additive shocks ϵ .⁵ The household has to make forecasts for the future aggregate state, so the household's Euler equation reflects the uncertainty coming from the taste shocks, $q = \beta E_{g'|g} E_{\epsilon'} u_c(c')/u_c(c)$.

Policy problem. The policymaker faces a vector of taste shocks ϵ at (B_i, g_k) . Given (B_i, g_k) and policy functions of the future policymaker (to be specified later), a choice of debt B_j allows us to solve for consumption from the implementability constraint (IC), and the resource constraint (RC). Let this mapping be denoted by $\mathbf{f}(i, k, j)$, that is, the feasible consumption that satisfies the IC and the RC at (B_i, g_k) , when the policymaker chooses B_j for next period.⁶ Let

$$U(i, k, j) \equiv u(\mathbf{f}(i, k, j), 1 - \mathbf{f}(i, k, j) - g_k) \quad (\text{F.1})$$

denote the induced period utility when action j is chosen at (B_i, g_k) . Let $V(i, k, \epsilon)$ denote the *value* function of the policymaker who faces the vector of taste shocks ϵ at (B_i, g_k) . Averaging the taste shocks delivers the *ex-ante* value function, $\bar{V}(i, k) \equiv E_\epsilon V(i, k, \epsilon)$.⁷ Let $v(i, k, j)$ denote the *conditional* (on action B_j) value function, which is *net* of the taste shock,

⁴If $Y \sim \text{Gumbel}(A, B)$, where A is the location parameter and B is the scale parameter, then $E(Y) = A + B\gamma$, and $V(Y) = B^2 \frac{\pi^2}{6}$. Consequently, we have $E(\epsilon_j) = 0$ and $\text{Var}(\epsilon_j) = \sigma_\epsilon^2 \frac{\pi^2}{6}$.

⁵When we allow for default, we also add to the aggregate state the vector of shocks u that perturb the default decision.

⁶If there is more than one value for consumption, we pick the highest one. We leave the dependence of the mapping $\mathbf{f}$ on the policy function of the future policymaker implicit.

⁷For a useful review of dynamic discrete choice models and the associated terminology, see [Arcidiacono and Ellickson \(2011\)](#).

$$\begin{aligned}
v(i, k, j) &= U(i, k, j) + \beta \sum_{k'} \pi(k'|k) E_{\epsilon'} V(j, k', \epsilon') \\
&= U(i, k, j) + \beta \sum_{k'} \pi(k'|k) \bar{V}(j, k').
\end{aligned} \tag{F.2}$$

The policymaker solves the problem

$$V(i, k, \epsilon) = \max_j \{v(i, k, j) + \epsilon_j\}, \tag{F.3}$$

which delivers a decision rule for debt,

$$\vartheta(i, k, \epsilon) \equiv \operatorname{argmax}_j \{v(i, k, j) + \epsilon_j\}. \tag{F.4}$$

Optimal consumption is given by

$$c(i, k, \epsilon) \equiv \mathbf{f}(i, k, \vartheta(i, k, \epsilon)) = \sum_j 1_{\{\vartheta(i, k, \epsilon)=j\}} \mathbf{f}(i, k, j), \tag{F.5}$$

where 1 is the indicator function. The Gumbel assumption allows us to get the conditional choice probabilities of action j and the ex-ante value function $\bar{V}$ at state (i, k) in closed form.⁸

$$p(j|i, k) = E_{\epsilon} 1_{\{\vartheta(i, k, \epsilon)=j\}} = \frac{\exp\left(\frac{v(i, k, j)}{\sigma_{\epsilon}}\right)}{\sum_j \exp\left(\frac{v(i, k, j)}{\sigma_{\epsilon}}\right)} \tag{F.6}$$

$$\bar{V}(i, k) = \sigma_{\epsilon} \ln \sum_j \exp(v(i, k, j)/\sigma_{\epsilon}) \tag{F.7}$$

Implementability constraint. Let's dig now deeper in the implementability constraint. Let $\Theta(j, k', \epsilon')$ and $\mathcal{F}(j, k', l)$ denote respectively the decision rule for debt and the consumption mapping for the *future* policymaker, when the state next period is (j, k', ϵ') and the choice of debt next period B_l . Θ and $\mathcal{F}$ induce a policy function for consumption next period, $\mathcal{C}(j, k', \epsilon') \equiv \mathcal{F}(j, k', \Theta(j, k', \epsilon'))$, and conditional choice probabilities for the future policymaker, $\mathcal{P}(l|j, k') \equiv E_{\epsilon'} 1_{\{\Theta(j, k', \epsilon')=l\}}$. Given ϑ and $\mathcal{F}$, the implementability constraint at (i, k) is

⁸In practice, whenever we calculate conditional choice probabilities and ex-ante value functions in (F.6) and (F.7) at state (i, k) , we subtract a state-specific constant K_{ik} to avoid numerical overflow. More precisely, $p(j|i, k) = \frac{\exp\left(\frac{v(i, k, j)}{\sigma_{\epsilon}} - K_{ik}\right)}{\sum_j \exp\left(\frac{v(i, k, j)}{\sigma_{\epsilon}} - K_{ik}\right)}$, and $\bar{V}(i, k) = \sigma_{\epsilon} (K_{ik} + \ln \sum_j \exp(v(i, k, j)/\sigma_{\epsilon} - K_{ik}))$, where $K_{ik} \equiv \max_j (v(i, k, j)/\sigma_{\epsilon})$.

$$u_c(c)B_i = \Omega(c, c + g_k) + Q(j, k) \cdot B_j, \quad (\text{F.8})$$

where $Q(j, k)$ stands for the price in marginal utility units, when the current shock is g_k and the debt chosen is B_j . Q is calculated as

$$\begin{aligned} Q(j, k) &\equiv \beta \sum_{k'} \pi(k'|k) E_{\epsilon'} u_c(\mathcal{C}(j, k', \epsilon')) = \beta \sum_{k'} \pi(k'|k) E_{\epsilon'} \sum_l 1_{\{\Theta(j, k', \epsilon')=l\}} u_c(\mathcal{F}(j, k', l)) \\ &= \beta \sum_{k'} \pi(k'|k) \sum_l E_{\epsilon'} (1_{\{\Theta(j, k', \epsilon')=l\}}) u_c(\mathcal{F}(j, k', l)) \\ &= \beta \sum_{k'} \pi(k'|k) \sum_l \mathcal{P}(l|j, k') u_c(\mathcal{F}(j, k', l)). \end{aligned} \quad (\text{F.9})$$

Given state (i, k) , choice of debt j , and the policy functions for the future policymaker, we get the mapping $\mathbf{f}(i, k, j)$ for consumption c from (F.8). Analogously to the case without taste shocks, the Markov-perfect requirement for time-consistency is $\vartheta(i, k, \epsilon) = \Theta(i, k, \epsilon)$ and $\mathbf{f}(i, k, j) = \mathcal{F}(i, k, j) \forall i, k, j$. These two requirements imply that $c(i, k, \epsilon) = \mathcal{C}(i, k, \epsilon)$.

F.1.2 Time-consistent policy with default

Turn now to the full model, which includes the option to default. We introduce *two* types of additive taste shocks: a) analogously to the previous section, the vector ϵ , which perturbs the debt decision in the event of *repayment*, with each component of ϵ following a *Gumbel* $(-\gamma\sigma_\epsilon, \sigma_\epsilon)$, b) the vector $u = (u_r, u_d)$, which perturbs the repayment and default decision. Shocks $u_i, i = r, d$, are identically distributed, and follow a *Gumbel* $(-\gamma\sigma_u, \sigma_u)$. Shocks u_i and ϵ_j are independent from each other and from the spending shocks g , for all $i = r, d$ and $j = 1, \dots, N_B$.

The timing is as follows: given state (B_i, g_k) the policymaker faces first shocks u and decides if she will default or repay. If the policymaker decides to repay, the vector of taste shocks ϵ is realized, which perturbs the debt issuance decision, B_j . Given the timing, the policymaker has to form expectations of ϵ at the repayment/default stage.

Decision to default or repay. Let $V(i, k, u)$ denote the value function at the default/repayment stage at (i, k) . We have

$$V(i, k, u) = \max\{\bar{V}^r(i, k) + u_r, V^d(k) + u_d\}, \quad (\text{F.10})$$

where $\bar{V}^r(i, k) \equiv E_\epsilon V^r(i, k, \epsilon)$, the *ex-ante* repayment value function (to be specified later), and $V^d(k)$ the default value function at g_k . The indicator variable that captures the default decision is a function of both (B_i, g_k) , and the shocks u , $d = d(i, k, u)$. Given the Gumbel assumption, the probabilities of default and repayment at (i, k) are

$$p^d(i, k) \equiv E_u d(i, k, u) = \frac{\exp\left(\frac{V^d(k)}{\sigma_u}\right)}{\exp\left(\frac{\bar{V}^r(i, k)}{\sigma_u}\right) + \exp\left(\frac{V^d(k)}{\sigma_u}\right)} \quad (\text{F.11})$$

$$p^r(i, k) = 1 - p^d(i, k). \quad (\text{F.12})$$

The ex-ante value function $\bar{V}(i, k)$ is

$$\bar{V}(i, k) \equiv E_u V(i, k, u) = \sigma_u \ln \left(\exp\left(\frac{\bar{V}^r(i, k)}{\sigma_u}\right) + \exp\left(\frac{V^d(k)}{\sigma_u}\right) \right). \quad (\text{F.13})$$

Default value function. The default value function at g_k that corresponds to (12) in the text is given by

$$V^d(k) = u(c^d(k), 1 - h^d(k)) - \psi(g_k) + \beta \sum_{k'} \pi(k'|k) \bar{V}(j_0, k'), \quad (\text{F.14})$$

where $(c^d(k), h^d(k))$ is the default consumption-labor allocation at g_k , and j_0 is the index in the debt grid that corresponds to zero debt.

Repayment value function. Given the policy functions of the future policymaker, let $\mathbf{f}(i, k, j)$ denote now the consumption in the event of *repayment*, when the state is (i, k) and debt B_j is issued. Let $U(i, k, j)$ denote the respective period utility as in (F.1). The conditional value function of repayment at (i, k) and debt issuance j is given by

$$v^r(i, k, j) = U(i, k, j) + \beta \sum_{k'} \pi(k'|k) \bar{V}(j, k'), \quad (\text{F.15})$$

where $\bar{V}(j, k')$ is given by (F.13). The value function of repayment at (i, k, ϵ) is given by

$$V^r(i, k, \epsilon) = \max_j \{v^r(i, k, j) + \epsilon_j\}, \quad (\text{F.16})$$

which generates a decision rule $\vartheta(i, k, \epsilon)$ as in (F.4). The conditional choice probabilities of debt and the respective ex-ante value of repayment are given by

$$p(j|i, k) = \frac{\exp\left(\frac{v^r(i, k, j)}{\sigma_\epsilon}\right)}{\sum_j \exp\left(\frac{v^r(i, k, j)}{\sigma_\epsilon}\right)} \quad (\text{F.17})$$

$$\bar{V}^r(i, k) \equiv E_\epsilon V^r(i, k, \epsilon) = \sigma_\epsilon \ln \sum_j \exp(v^r(i, k, j)/\sigma_\epsilon) \quad (\text{F.18})$$

Implementability constraint. To finish the description of the problem, we need to see how the implementability constraint is modified when there are also taste shocks in the default decision. Analogously to the previous section, let $\Theta(j, k', \epsilon')$ and $\mathcal{F}(j, k', l)$ denote respectively the decision rule for debt issuance and the consumption mapping implied by the future policymaker's choice of B_l at state (j, k', ϵ') in the event of *repayment*. The induced policy function for repayment consumption is $\mathcal{C}(j, k', \epsilon') \equiv \mathcal{F}(j, k', \Theta(j, k', \epsilon'))$, and the conditional choice probabilities for debt issuance are $\mathcal{P}(l|j, k') \equiv E_{\epsilon'} 1_{\{\Theta(j, k', \epsilon')=l\}}$. Recall also that the decision rule for default is given by $d(j, k', u')$, with probabilities of default and repayment given by (F.11) and (F.12).

The implementability constraint at (i, k) and debt issuance j is given, as previously, by (F.8), where $Q(j, k)$ incorporates now both the default option, and the uncertainty with respect to shocks u' and ϵ' ,

$$\begin{aligned} Q(j, k) &\equiv \beta \sum_{k'} \pi(k'|k) E_{u'} (1 - d(j, k', u')) E_{\epsilon'} u_c(\mathcal{C}(j, k', \epsilon')) \\ &= \beta \sum_{k'} \pi(k'|k) p^r(j, k') \sum_l \mathcal{P}(l|j, k') u_c(\mathcal{F}(j, k', l)). \end{aligned} \quad (\text{F.19})$$

As previously, the requirement for time-consistency is $\vartheta(i, k, \epsilon) = \Theta(i, k, \epsilon)$ and $\mathbf{f}(i, k, j) = \mathcal{F}(i, k, j) \forall i, k, j$, so that the policy functions for repayment consumption for the current and the future policymaker agree, $c(i, k, \epsilon) = \mathcal{C}(i, k, \epsilon)$.

F.2 Long-term debt

Consider now the problem with long-term debt. We focus directly on the time-consistent problem with default. The definitions for the value functions, the optimal decision rules, and the respective derivations of the conditional choice probabilities for default and debt issuance of the previous section go through and will not be repeated. We only need to specify the proper implementability constraint for the case of long-term debt. Let $\Theta(j, k', \epsilon')$ and $\mathcal{F}(j, k', l)$ be defined as in the previous section, with the corresponding policy function for repayment consumption next period $\mathcal{C}(j, k', \epsilon')$, conditional choice probabilities for debt issuance next period $\mathcal{P}(l|j, k')$, and default choice $d(j, k', u')$. The implementability constraint at state (i, k) and choice B_j , that corresponds to (29), after eliminating labor by using the resource constraint (30), reads

$$[\chi u_c(c) + (1 - \delta)Q(j, k)]B_i = \Omega(c, c + g_k) + Q(j, k) \cdot B_j. \quad (\text{F.20})$$

Equation (F.20) delivers the mapping for consumption $\mathbf{f}(i, k, j)$ at state (i, k) and choice B_j . The price $Q(j, k)$ solves the functional equation

$$\begin{aligned}
Q(j, k) &= \beta \sum_{k'} \pi(k'|k) E_{u'}(1 - d(j, k', u')) E_{\epsilon'} [\chi u_c(\mathcal{C}(j, k', \epsilon')) + (1 - \delta) Q(\Theta(j, k', \epsilon'), k')] \\
&= \beta \sum_{k'} \pi(k'|k) p^r(j, k') \sum_l \mathcal{P}(l|j, k') [\chi u_c(\mathcal{F}(j, k', l)) + (1 - \delta) Q(l, k')]. \quad (\text{F.21})
\end{aligned}$$

The Markov-perfect requirement is $\vartheta(i, k, \epsilon) = \Theta(i, k, \epsilon)$ and $\mathbf{f}(i, k, j) = \mathcal{F}(i, k, j) \forall i, k, j$.

G Computational details

G.1 Time-consistent problems

In this section, we describe the solution algorithm to the problem without commitment. We describe the general case of default and long-term debt as it nests every other specification.

Implementation. We use $N_g = 50$ grid points for g , and $N_B = 100$ grid points for debt. We discretize g with Tauchen's method using 3 standard deviations. We keep these features constant for all model specifications. The lower bound for debt is $\underline{M} = 0$. The upper bound is $\bar{M} = 0.16$ for the short-term debt specification, and $\bar{M} = 0.4$ for the case of long-term debt. These numbers correspond to 40% and 100% of first-best output respectively.

Pre-solving. We speed up our algorithm by pre-solving for consumption. Given state (B_i, g_k) , and a choice B_j , we back out consumption from the implementability constraint (F.20). The implementability constraint provides a nonlinear relation between the variable Υ , defined in (G.1), the state (B_i, g_k) , and c ,

$$\chi u_c(c) B_i - \Omega(c, c + g_k) = \underbrace{Q(j, k)(B_j - (1 - \delta) B_i)}_{\equiv \Upsilon} \quad (\text{G.1})$$

We pre-solve for the mapping $c(g, B, \Upsilon)$ outside of the value function iteration loop. When evaluating $c(g, B, \Upsilon)$, the algorithm only needs to evaluate on the gridpoints for g and B . We interpolate in the Υ dimension using a cubic spline.

Value function iteration. The maximization step in the value function iteration algorithm is redundant given the Gumbel distributional assumption for the taste shocks, and the resulting expressions for the conditional choice probabilities.

- Let $Q^\kappa(j, k)$ denote the price function, $V^{d, \kappa}(k)$ the default value function, and $\bar{V}^\kappa(i, k)$ the ex-ante value function at iteration κ , when the state is (i, k) .
 - Given $Q^\kappa(j, k)$, solve from the implementability constraint for the consumption mapping $\mathbf{f}^\kappa(i, k, j)$ in the event of repayment, and calculate the respective period utility $U^\kappa(i, k, j)$ in (F.1).

- Given $U^\kappa(i, k, j)$ and the ex-ante value function $\bar{V}^\kappa(i, k)$, calculate the conditional value function of repayment $v^{r, \kappa+1}(i, k, j)$ in (F.15). Use $v^{r, \kappa+1}(i, k, j)$ to calculate the conditional choice probabilities of debt $p^\kappa(j|i, k)$ in (F.17), and to update the ex-ante value of repayment to $\bar{V}^{r, \kappa+1}(i, k)$ in (F.18).
- Given $V^{d, \kappa}(k)$ and $\bar{V}^\kappa(i, k)$, use (F.14) to update the default value function to $V^{d, \kappa+1}(k)$. Use $V^{d, \kappa+1}(k)$ and $\bar{V}^{r, \kappa+1}(i, k)$ to calculate the default and repayment probabilities $p^{d, \kappa}(i, k)$ and $p^{r, \kappa}(i, k)$ in (F.11) and (F.12). Update finally the ex-ante value function to $\bar{V}^{\kappa+1}(i, k)$ through (F.13).

- Define $\hat{Q}(j, k)$ as

$$\hat{Q}(j, k) = \beta \sum_{k'} \pi(k'|k) p^{r, \kappa}(j, k') \sum_l p^\kappa(l|j, k') [u_c(\mathbf{f}^\kappa(j, k', l))\chi + (1 - \delta)Q^\kappa(l, k')]. \quad (\text{G.2})$$

- We use dampening in the updating of Q and set $Q^{\kappa+1}(j, k) = \alpha_Q \hat{Q}(j, k) + (1 - \alpha_Q)Q^\kappa(j, k)$, where $\alpha_Q \in [0, 1]$. We use $\alpha_Q = 0.1$.
- To initialize $Q^0(j, k)$, we calculate the allocation that corresponds to a debt-rollover policy, and derive the induced price Q .
- We initialize the value functions at zero, $\bar{V}^0(i, k) = V^{d, 0}(k) = 0$.⁹

G.2 Policy functions and spreads

Policy functions. We average the preference shocks when we plot policy functions in the text. For instance, we plot the policy functions for consumption and debt as

$$c(i, k) = \sum_j p(j|i, k) \mathbf{f}(i, k, j)$$

$$B'(i, k) = \sum_j p(j|i, k) B_j$$

We perform similar calculations when we plot the tax rate.

Spread with short-term debt. To calculate default spreads, we need to price an asset that is in zero net supply and provides one unit of consumption next period at every contingency. For the case of short-term debt, the price of the risk-free asset is given by $q_t^F = \beta \mathbb{E}_t \frac{u_{c, t+1}}{u_{ct}}$, with a respective price in marginal utility units $Q_t^F = \beta \mathbb{E}_t u_{c, t+1}$.

Let $Q^F(j, k)$ denote the price of the default-free asset in a repayment state, when the current shock is k and the government issues B_j debt for next period. This is given by

⁹We experimented also with different initial values for the value functions and got the same results.

$$Q^F(j, k) = \beta \sum_{k'} \pi(k'|k) \left[p^r(j, k') \sum_l \mathcal{P}(l|j, k') u_c(\mathcal{F}(j, k', l)) + p^d(j, k') u_c(c^d(k')) \right]. \quad (\text{G.3})$$

Expression (G.3) takes into account that the risk free asset provides a unit of consumption independent of government default, and contrasts to the price $Q(j, k)$ of the risky government asset that we derived in (F.19). The respective prices q and q^F are derived by dividing (F.19) and (G.3) with current marginal utility,

$$q(j|i, k) = \frac{Q(j, k)}{u_c(\mathbf{f}(i, k, j))} \Rightarrow r(j|i, k) \equiv \frac{1}{q(j|i, k)} - 1 \quad (\text{G.4})$$

$$q^F(j|i, k) = \frac{Q^F(j, k)}{u_c(\mathbf{f}(i, k, j))} \Rightarrow r^F(j|i, k) \equiv \frac{1}{q^F(j|i, k)} - 1, \quad (\text{G.5})$$

with a spread that is given by $r(j|i, k) - r^F(j|i, k)$.

Spread with long-term debt. In long-term debt case, the price of risk-free debt q_t^F and the price in marginal utility units $Q_t^F \equiv u_{ct} \cdot q_t^F$ are given respectively by

$$q_t^F = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{ct}} [\chi + (1 - \delta) q_{t+1}^F] \quad (\text{G.6})$$

$$Q_t^F = \beta \mathbb{E}_t [\chi u_{c,t+1} + (1 - \delta) Q_{t+1}^F]. \quad (\text{G.7})$$

We are interested in the spread in a repayment state, since the price of the risky asset is not defined in a default state. However, with long-term debt, the price of the risk-free asset in the default state affects prices in the repayment state, and therefore, the respective spread. Let $Q^{F,r}(j, k)$ denote the price (in marginal utility units) of the risk-free asset in a repayment state, when the shock is k and the government issues B_j debt for next period. Let $Q^{F,d}(k)$ denote the price of the risk-free asset in a default state, when the shock is k . We have the following recursion for $Q^{F,r}$:

$$\begin{aligned} Q^{F,r}(j, k) = & \beta \sum_{k'} \pi(k'|k) \left[p^r(j, k') \sum_l \mathcal{P}(l|j, k') [\chi u_c(\mathcal{F}(j, k', l)) + (1 - \delta) Q^{F,r}(l, k')] \right. \\ & \left. + p^d(j, k') [\chi u_c(c^d(k')) + (1 - \delta) Q^{F,d}(k')] \right] \end{aligned} \quad (\text{G.8})$$

Recursion (G.8) collapses to (G.3) when $\delta = 1$. The recursion for the price of long bonds $Q^{F,d}$ in the default state is

$$\begin{aligned}
Q^{F,d}(k) &= \beta \sum_{k'} \pi(k'|k) \left[p^r(j_0, k') \sum_l \mathcal{P}(l|j_0, k') [\chi u_c(\mathcal{F}(j_0, k', l)) + (1 - \delta) Q^{F,r}(l, k')] \right. \\
&\quad \left. + p^d(j_0, k') [\chi u_c(c^d(k')) + (1 - \delta) Q^{F,d}(k')] \right], \tag{G.9}
\end{aligned}$$

where j_0 is the index in the debt grid that corresponds to zero debt, as in (F.14). The price of the risk-free asset in the default state takes into account the probability of repayment or default after re-entry in the market. In particular, the first line in (G.9) captures the event of repayment, after re-entry in the market. The second line in (G.9) captures the event when there is default after re-entry.

Recursions (G.8) and (G.9) contrast to recursion (F.21) that determines the price of the risky asset. To get the price of long-bonds and the respective spread, divide (F.21) and (G.8) with current marginal utility to get

$$q(j|i, k) = \frac{Q(j, k)}{u_c(\mathbf{f}(i, k, j))} \Rightarrow r(j|i, k) \equiv \frac{\chi}{q(j|i, k)} - \delta \tag{G.10}$$

$$q^{F,r}(j|i, k) = \frac{Q^{F,r}(j, k)}{u_c(\mathbf{f}(i, k, j))} \Rightarrow r^{F,r}(j|i, k) \equiv \frac{\chi}{q^{F,r}(j|i, k)} - \delta. \tag{G.11}$$

The default spread at state (i, k) , when the government issues B_j units of long bonds, is $r(j|i, k) - r^{F,r}(j|i, k)$.

G.3 Greed and Fear decomposition

Figure 4 in the text plots the respective “Greed” and “Fear” forces in the Generalized Euler Equation (17) of proposition 1. However, the derivation of the “Greed” and “Fear” objects in the text assumes that debt is a continuous choice variable, and that default is a deterministic choice given (B, g) . We show here how we use our numerical solution with the taste shocks to approximate the “Greed” and “Fear” objects.

Recall that the derivative of the price in marginal utility units Q_B is given by (A.4), which is repeated here for convenience:

$$\begin{aligned}
Q_B(B', g) &= \beta \left[\underbrace{\mathbb{E}_{g'|g}(1 - d(B', g')) \frac{\partial}{\partial B'} u_c(\mathcal{C}(B', g'))}_{\text{(I): “Greed”}} \right. \\
&\quad \left. + \underbrace{u_c(\mathcal{C}(B', \omega^{-1}(B'))) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}}{dB'}}_{\text{(II): “Fear”}} \right], \tag{G.12}
\end{aligned}$$

where, to ease notation, we assumed that $u_{cl} = 0$, and where we have avoided using the chain rule for $\frac{\partial}{\partial B'} u_c(\mathcal{C}(B', g'))$. Terms (I) and (II) are *proportional* to the “Greed” and “Fear”

Table G.1: Taste shocks calibration.

	Short-term debt	Long-term debt
MPE	$\sigma_\epsilon = 2.5 \times 10^{-3}$	$\sigma_\epsilon = 10^{-3}$
Default	$\sigma_\epsilon = 10^{-3}$ $\sigma_u = 10^{-6}$	$\sigma_\epsilon = 10^{-3}$ $\sigma_u = 10^{-6}$

Scale parameters of the taste shocks that perturb the debt (σ_ϵ) and default decisions (σ_u). “MPE” refers to the time-consistent policy problem without default.

components; to determine the price semi-elasticity η , we divide Q_B with Q and get the respective terms in (17).

We utilize the decision rules with taste shocks, $d(B', g', u')$ and $\mathcal{C}(B', g', \epsilon')$, to approximate (I) and (II) in (G.12), and use formula (F.19) for the division with the price Q to get our final decomposition. In particular, we set

$$I \simeq \sum_{g'} \pi(g'|g) E_{u'} (1 - d(B', g', u')) \frac{\partial E_{\epsilon'} u_c(\mathcal{C}(B', g', \epsilon'))}{\partial B'} \quad (\text{G.13})$$

$$II \simeq E_{\epsilon'} u_c(\mathcal{C}(B', \omega^{-1}(B'), \epsilon')) f(\omega^{-1}(B')|g) \frac{d\omega^{-1}(B')}{dB'} \quad (\text{G.14})$$

- In (G.13), we average marginal utility of consumption over the preference shocks ϵ' and then calculate $\frac{\partial E_{\epsilon'} u_c(\mathcal{C}(B', g', \epsilon'))}{\partial B'}$. We evaluate the derivative numerically by interpolating $E_{\epsilon'} u_c(\mathcal{C}(B', g', \epsilon'))$ and using central differences. In this way, we do not need to explicitly apply the chain rule.
- The taste shock u makes the default decision probabilistic, whereas the default threshold ω^{-1} in (G.14) assumes a deterministic choice. We circumvent this issue, by assuming that there is “default” if the probability of default is at least 99.9% at state (B, g) . To get $\omega^{-1}(B)$, we fix B and find the highest value of g for which there is repayment. We calculate $\frac{d\omega^{-1}(B)}{dB}$ using finite differences by interpolating $\omega^{-1}(B)$ linearly.
- We use the Matlab function “lognpdf”, which returns the lognormal probability density function, to evaluate the conditional density $f(\omega^{-1}(B)|g)$.
- To get a smooth approximation for the decomposition, we re-solve our model using a finer grid for g and set $N_g = 300$.

G.4 Volatility of taste shocks

Table G.1 lists our choices for the scale parameters ($\sigma_\epsilon, \sigma_u$) in the baseline calibration. These parameters are proportional to the respective standard deviation of the taste shocks by the

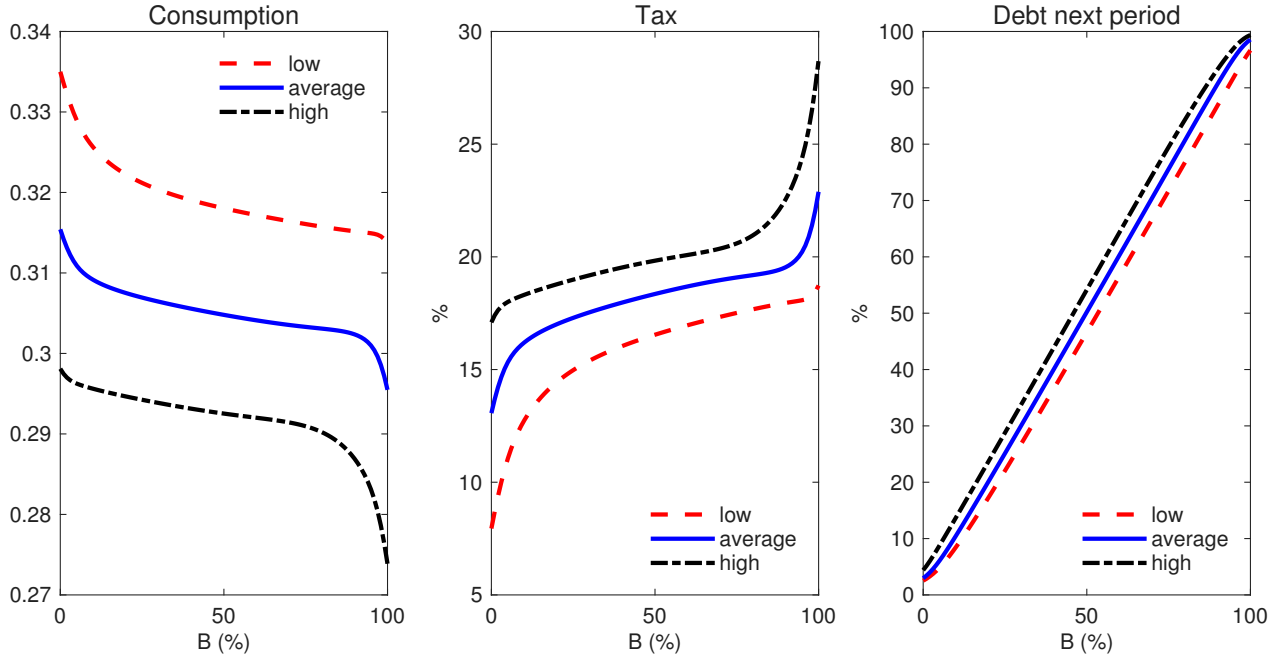


Figure G.1: Policy functions for consumption, taxes and debt next period in the time-consistent model without default for the long-term debt case. The policy functions are plotted for the mean g (“average”), as well as for a level of g two standard deviations below (“low”), or above (“high”) the mean. The taste shocks that perturb the debt decision are averaged out. Debt is expressed in terms of first-best output.

factor $\pi/\sqrt{6}$. The scale parameters are set to a different number in each specification, balancing the tradeoff between obtaining monotonic policy functions for consumption and minimizing the influence of these shocks on economic dynamics.

First of all, the parameter σ_u , which controls the volatility of the taste shock that perturbs the default decision, is very small, for both the short-term and long-term debt case. A more substantial level of σ_ϵ is needed for the taste shock that perturbs debt choices. When we have short-term debt, we need a smaller value of σ_ϵ in the default model than in the time-consistent model without default, confirming our analysis about the mitigation of jumps when there is default risk in Online Appendix E. Our baseline smooth and monotonic consumption policy functions in figures C.1 and 2 in the text, calculated with the additional taste shocks, contrast to the policy functions without taste shocks in the left panels of figures E.4 and E.5 respectively.

For the interested reader, we provide also the policy functions for the long-term debt case, without or with the default option. The smooth consumption policy functions with long-term debt in figure G.1 contrast to the respective policy functions in figure E.7 without taste shocks. In the default case with long-term debt, we retain the same σ_ϵ , and get the (essentially) smooth and monotonic repayment consumption policy functions in figure G.2, which contrast to the respective policy functions without taste shocks in figure E.8.

Robustness. To check the robustness of our results with respect to the choice of σ_ϵ , we repeat the long simulation exercise of tables 2 and 4 in the text by setting each σ_ϵ to *half* its respective value in table G.1. Table G.2 reports the stationary moments for the short-term debt case and contrasts them to the moments coming from the baseline calibration. The moments of the

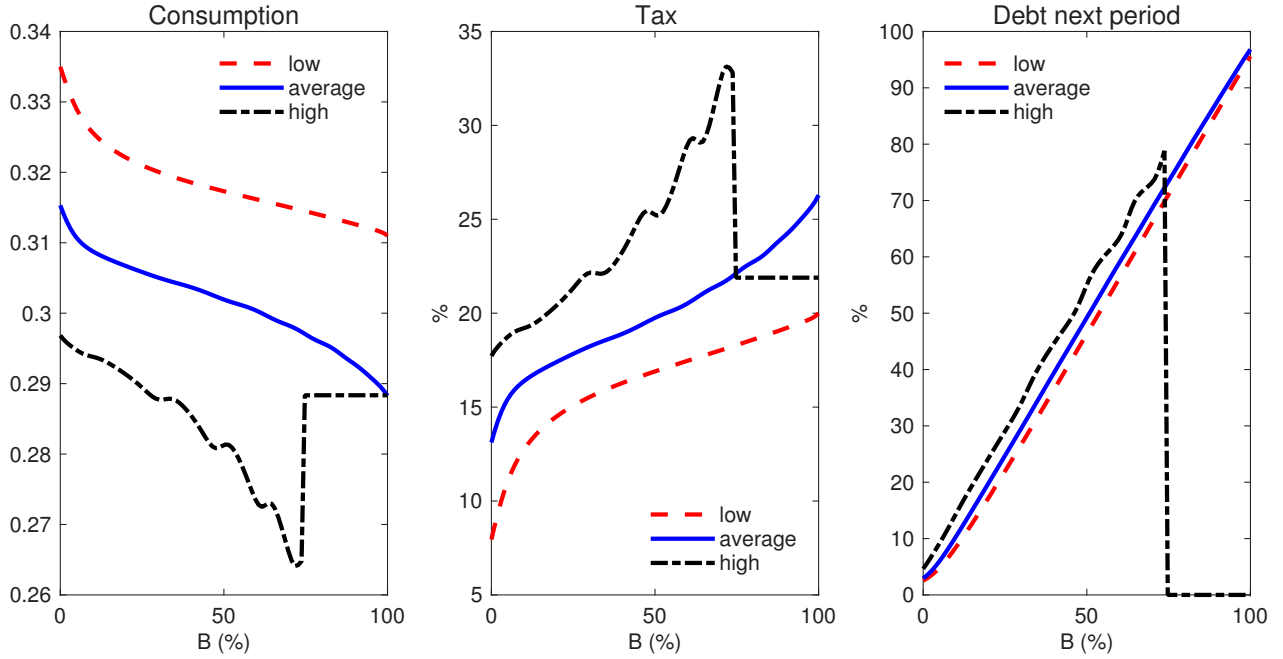


Figure G.2: Policy functions for consumption, taxes and debt next period in the time-consistent model with default for the long-term debt case. The policy functions are plotted for the mean g (“average”), as well as for a level of g two standard deviations below (“low”), or above (“high”) the mean. The taste shocks are averaged out. Debt is expressed in terms of first-best output.

spreads are generally robust to the value of σ_ϵ . More differences can be seen in the moments of debt and tax rates. Mean debt is reduced by two percentage points, and the volatility of the tax rate is reduced by 0.7 percentage points in the default problem. For the time-consistent problem without default, we have a small increase in average debt (since taste shocks mitigate the “Greed” force by adding noise to debt decisions), and the tax rate volatility is reduced by 1.3 percentage points. We observe similar patterns in the respective exercise with long-term debt in table G.3. The moments of spreads display small changes. Tax volatility is reduced for both the default model and the time-consistent model without default. Mean debt falls for the default model, but increases relatively more when there is no default option.

G.5 Commitment problems

G.5.1 Short-term debt

We solve the commitment problem with short-term debt using the recursive formulation in the spirit of Kydland and Prescott (1980), that we provided in Online Appendix D.2. We use the same grid for g as in the time-consistent problems, so $N_g = 50$. We can eliminate labor h_g using the resource constraint (D.21), so there are $2 \times N_g = 100$ choice variables ($c_g, \tilde{B}_g$) in the optimization problem. We use 100 linearly spaced grid points for debt in average marginal utility units, $\tilde{B}$. For the lower bound of the state variable in (D.23), we set $\underline{B}_g = 0$. For the upper bound $\bar{B}_g$, which is state-contingent, we proceed as follows: we find a constant tax policy that supports a maximum level of debt $\bar{M}$ and derive the respective consumption allocation, $c^{init}(\bar{M}, g)$. We set then $\bar{B}_g = \bar{M} \cdot \sum_{g'} \pi(g'|g) u_c(c^{init}(\bar{M}, g'))$. We set the upper bound for debt

Table G.2: Stationary moments for different values of σ_ϵ in the short-term debt case.

	Default $0.5 \times \sigma_\epsilon$	Default (baseline)	MPE $0.5 \times \sigma_\epsilon$	MPE (baseline)
Tax rate in %				
Mean	16.5497	16.6378	17.2737	17.2105
St. Dev	3.4156	4.1505	4.1541	5.4579
Autocorrelation	0.3834	0.2461	0.1750	0.0797
Corr with g	0.7205	0.6153	0.4920	0.3930
St. Dev of $\Delta\tau$	3.6265	4.9494	5.1371	7.2689
Debt-to-output ratio in %				
Mean	8.6510	10.6336	25.1007	23.7239
St. Dev	5.0448	5.7296	12.9265	11.5793
Autocorrelation	0.8780	0.8316	0.9265	0.8835
Default probability in %				
	1.0820	1.0938	0	0
Spread in %				
Mean	1.1640	1.2381		
St. Dev	5.3949	5.9627		
Autocorrelation	0.4358	0.4238		

Stationary moments from a long simulation for the short-term debt case, with σ_ϵ half its respective baseline value. The baseline (unrounded for more precision) moments from table 2 are reported here for comparison.

to $\bar{M} = 0.16$, corresponding to 40% of first-best output, for the third column of table 2. We set $\bar{M} = 0.8$, corresponding to 200% of first-best output, for the last column of table 2.

We use value function iteration, augmented with Howard's policy improvement algorithm to speed up convergence. We generate a first guess for the value function $W(\tilde{B}_-, g_-)$ using again a constant tax policy. Since grids for debt in average utility units differ for each g , we approximate W separately for each g using a cubic spline in the $\tilde{B}_-$ dimension. We use the interior-point algorithm to perform the maximization step.

G.5.2 Long-term debt

Turning to the long-term debt case, we follow a different computational strategy. The recursive formulation provided in Online Appendix D.3 in the spirit of Kydland and Prescott (1980) has 3 state variables (two of which are endogenous), $3 \times N_g = 150$ choice variables (c_g, B'_g, Q_g) (eliminating labor using the resource constraint), and $N_g + 1 = 51$ constraints. Moreover, the state space is endogenous. To handle the size of the problem, and to avoid issues with potential infeasibilities if we use a Cartesian grid for the state space, we work directly with the first-order conditions of the commitment problem. In particular, we use as state variables (B_t, g_t) , and ξ_{t-1} , that is, the multiplier on the price of long-term debt, q_t , in the spirit of Marcet and

Table G.3: Stationary moments with different values of σ_ϵ in the long-term debt case.

	Default $0.5 \times \sigma_\epsilon$	Default (baseline)	MPE $0.5 \times \sigma_\epsilon$	MPE (baseline)
Tax rate in %				
Mean	17.0287	17.1208	19.9401	19.4814
St. Dev	3.2842	4.0811	3.4321	4.1017
Autocorrelation	0.3311	0.2052	0.4040	0.2276
Corr with g	0.5650	0.4691	0.5919	0.4422
St. Dev of $\Delta\tau$	3.5740	4.9699	3.3044	4.6607
Debt-to-output ratio in %				
Mean	20.4514	22.5191	89.6470	78.2408
St. Dev	15.3928	15.5254	29.6792	33.5305
Autocorrelation	0.9591	0.9368	0.8838	0.9044
Default probability in %				
	1.1149	1.0944	0	0
Spread in %				
Mean	1.0249	1.0590		
St. Dev	2.3822	2.7770		
Autocorrelation	0.5609	0.5596		

Stationary moments from a long simulation for the long-term debt case, with σ_ϵ that is half its respective value. The baseline (unrounded for more precision) moments from table 4 are reported here for comparison.

Marimon (2019). We follow the methods of Valaitis and Villa (2024), who use artificial neural networks to generalize the Parameterized Expectations Algorithm of den Haan and Marcet (1990).

For convenience, we provide here the first-order necessary conditions and the constraints of the commitment problem that we analyzed in Online Appendix D.3 for $t \geq 0$, taking now into account the upper and lower bounds on debt (D.33):

$$u_{ct} - u_{lt} + \Phi_t \left[\Omega_{ct} + \Omega_{ht} - \chi(u_{cc,t} - u_{cl,t})B_t \right] + \chi(u_{cc,t} - u_{cl,t})\xi_{t-1} = 0 \quad (\text{G.15})$$

$$\Phi_t u_{ct} q_t = \beta \mathbb{E}_t u_{c,t+1} (\chi + (1 - \delta)q_{t+1}) \Phi_{t+1} + \xi_t^U - \xi_t^L \quad (\text{G.16})$$

$$\xi_t = \Phi_t (B_{t+1} - (1 - \delta)B_t) + (1 - \delta)\xi_{t-1} \quad (\text{G.17})$$

$$u_{c,t} [\chi + (1 - \delta)q_t] B_t = \Omega(c_t, h_t) + u_{c,t} q_t B_{t+1} \quad (\text{G.18})$$

$$u_{ct} q_t = \beta \mathbb{E}_t u_{c,t+1} [\chi + (1 - \delta)q_{t+1}] \quad (\text{G.19})$$

$$c_t + g_t = h_t \quad (\text{G.20})$$

Moreover, B_{t+1} satisfies the upper and lower bounds (D.33), with respective multipliers $\xi_t^U, \xi_t^L \geq 0$, and the complementary slackness conditions hold. Note that we eliminated the

multiplier λ_t from the first-order conditions (D.38) and (D.39) and combined them to get (G.15).

The conditional expectations of interest are

$$\Psi^1(\mathcal{I}_t) \equiv \mathbb{E}_t u_c(c_{t+1}) [\chi + (1 - \delta)q_{t+1}] \quad (\text{G.21})$$

$$\Psi^2(\mathcal{I}_t) \equiv \mathbb{E}_t u_c(c_{t+1}) (\chi + (1 - \delta)q_{t+1}) \Phi_{t+1} \quad (\text{G.22})$$

where $\mathcal{I}_t$ stands for the information set used to train the network, to be specified below. We parameterize the conditional expectations $\Psi^1(\mathcal{I}_t)$ and $\Psi^2(\mathcal{I}_t)$ with a single artificial neural network, denoted as $\mathcal{ANN}(\mathcal{I}_t)$.

We provide the gist of the method here and refer the interested reader for details to Valaitis and Villa (2024): Simulate a sequence of the exogenous shocks g_t of length T and fix it. Given the initial values of the variables in the information set, $\mathcal{I}_0$, and weights for the $\mathcal{ANN}(\mathcal{I}_t)$ at iteration κ , generate a sequence of $\{c_t, \Phi_t, \xi_t, B_{t+1}, q_t, \xi_t^U, \xi_t^L\}_{t=0}^T$ that satisfies (G.15)-(G.20). Given this sequence, calculate the implied output of the model (the terms inside the conditional expectations in (G.21)-(G.22)) and train the $\mathcal{ANN}(\mathcal{I}_t)$, to get a new set of weights for the next iteration $\kappa + 1$. Iterate until the network prediction errors converge below a small threshold and the simulated sequences for $\{c_t, \Phi_t, \xi_t, B_{t+1}, q_t, \xi_t^U, \xi_t^L\}_{t=0}^T$ are stable across iterations.

Implementation. To help reach convergence, we follow Maliar and Maliar (2003) and impose moving bounds on debt at each iteration that are gradually relaxed.¹⁰ T is set to 20000. We use a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 30 neurons in the hidden layer. We use the hyperbolic tangent sigmoid activation function and train the network using Levenberg-Marquardt back-propagation.

Lagrange multipliers. In the practical implementation of the algorithm, instead of checking the debt constraints and the complementary slackness conditions appended to the system (G.15)-(G.20), we follow Valaitis and Villa (2024) and allow the debt constraints to be violated, subject to a utility penalty. We replace the multipliers ξ_t^U and ξ_t^L in the system (G.15)-(G.20) with smooth functions of B_{t+1} that correspond to the derivatives of the penalty function.¹¹

In particular, we set

$$\xi_t^U(B_{t+1}) = \begin{cases} \phi(B_{t+1} - \bar{M}) + \ln(1 + \phi(B_{t+1} - \bar{M})) & \text{if } B_{t+1} \geq \bar{M} \\ 0 & \text{if } B_{t+1} < \bar{M}. \end{cases} \quad (\text{G.23})$$

¹⁰See Valaitis and Villa (2024) for details.

¹¹We use the utility penalty function $\Xi(B_{t+1}) = \frac{\phi}{2}(B_{t+1} - \bar{M})^2 + \phi^{-1} \left\{ (1 + \phi(B_{t+1} - \bar{M})) [\ln(1 + \phi(B_{t+1} - \bar{M})) - 1] + 1 \right\}$ if $B_{t+1} \geq \bar{M}$, $\Xi(B_{t+1}) = 0$ if $B_{t+1} \in (\underline{M}, \bar{M})$, and $\Xi(B_{t+1}) = \frac{\phi}{2}(B_{t+1} - \underline{M})^2 + \phi^{-1} \left\{ (1 + \phi(\underline{M} - B_{t+1})) [\ln(1 + \phi(\underline{M} - B_{t+1})) - 1] + 1 \right\}$ if $B_{t+1} \leq \underline{M}$, with $\phi > 0$. Using L'Hospital's rule, we can show that $\Xi(B_{t+1}) = 0, \forall B_{t+1}$, when $\phi = 0$. The penalty function is increasing and convex for $B_{t+1} \geq \bar{M}$, and decreasing and convex for $B_{t+1} \leq \underline{M}$, with $\Xi(\bar{M}) = \Xi(\underline{M}) = \Xi'(\bar{M}) = \Xi'(\underline{M}) = 0$, and $\Xi(B_{t+1}) > 0$ for $B_{t+1} > \bar{M}$ or $B_{t+1} < \underline{M}$. Set $\xi_t^U(B_{t+1}) \equiv \Xi'(B_{t+1})$ for $B_{t+1} \geq \bar{M}$, and $\xi_t^L(B_{t+1}) \equiv -\Xi'(B_{t+1})$ for $B_{t+1} \leq \underline{M}$.

Similarly, we set

$$\xi_t^L(B_{t+1}) = \begin{cases} \phi(\underline{M} - B_{t+1}) + \ln(1 + \phi(\underline{M} - B_{t+1})) & \text{if } B_{t+1} \leq \underline{M} \\ 0 & \text{if } B_{t+1} > \underline{M}. \end{cases} \quad (\text{G.24})$$

We set the penalty parameter $\phi = 10$. The lower bound for debt is zero, $\underline{M} = 0$, and the upper bound is either $\bar{M} = 0.4$ (100% of first-best output), or $\bar{M} = 0.8$ (200% of first-best output).

Information set. The information set includes the state variables of the problem, (g_t, B_t, ξ_{t-1}) . We also include in $\mathcal{I}_t$ the lagged values of the multipliers on the borrowing constraints, so $\mathcal{I}_t = (g_t, B_t, \xi_{t-1}, \xi_{t-1}^U, \xi_{t-1}^L)$. We adopt this approach for two reasons. First, it is computationally less demanding and, second, it allows for a smooth approximation of the model dynamics close to the constraint, which helps the artificial neural network deliver more accurate forecasts. We use as initial values for $\mathcal{I}_0 = (g_0, B_0, \xi_{-1}, \xi_{-1}^U, \xi_{-1}^L) = (G, 0, 0, 0, 0)$.