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OPTIMAL CLIMATE POLICY IN A GLOBAL ECONOMY

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Optimal climate policy in a global economy^{*}

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Abstract

This paper studies the optimal climate policy in a global economy. Emissions impose a dynamic, global, negative externality, raising natural questions about international policy coordination. To understand the issues involved, I first build a simple dynamic multi-country model and study the optimal cooperative climate policy that corrects the global externality. Moreover, I move beyond cooperation and study the optimal climate policy for a large country that faces a passive rest of the world. In this setup, corrective taxation motives are intertwined with interest rate manipulation through capital controls. Incentives for carbon taxation are amplified for net importers and mitigated for net exporters.

Keywords: climate change, carbon taxation, international cooperation, non-cooperative policy, capital controls.

JEL classification: E61; H21; H23; Q43; Q54.

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1 Introduction

One of the fundamental challenges for humankind is the risk of climate change and its repercussions for growth and welfare. A voluminous literature in climate science has developed over the last decades about global warming. Similarly, economists have started to think hard about aggregative models that incorporate climate issues, an effort that has led to the environmental macroeconomics literature.¹ The stakes are high, necessitating the need for intergovernmental agreements and for a concerted effort among policymakers, industry leaders and international organizations.²

Integrated Assessment models (IAMs) have been built to study the interactions between climate and the economy. One prominent model, especially for economists, is the DICE (Dynamic Integrated Climate Economy) model of Nordhaus (1994) and Nordhaus (2008). Climate-economic models typically integrate a climate module, which is usually a simplified summary of the climate system coming from the natural sciences, into a dynamic, aggregative model of the economy. Schematically, economic activity generates greenhouse gas emissions, which contribute to the rise of atmospheric temperature, leading to damages to the environment and output, maybe of catastrophic proportions.

New generations of models like Golosov et al. (2014) and Cai and Lontzek (2019) have introduced climate and growth risks in dynamic macroeconomic models and have analyzed the implications for the social cost of carbon (SCC), that is, the marginal cost of increasing carbon emissions, a cost which is not internalized by private agents in the economy. The *negative* externality of carbon emissions can be corrected with a Pigouvian carbon tax that is exactly equal to the social cost of carbon.

There is a fundamental *international* dimension in the nature of the climate externality. Emissions of one country, increase the *global*, slow to depreciate, stock of emissions, which increases global temperature. A rise in global temperature leads to country-specific damages not only to the country that generated the emissions, but to all other countries. Thus, emissions act effectively as a durable, public “bad”. This feature necessitates international cooperation, a fact long acknowledged by policymakers. International agreements as the Paris Agreement in 2015 have been put in place. However, agreements are non-binding and compliance to targets by large emitters as China (30% of global emissions in 2020) or the United States (14% of emissions in 2020) is crucial.

In this paper I want to understand deeper the global nature of the emission externality and its implications for international policy and coordination. In particular, I study environments that go beyond cooperation policy and consider setups that may involve a large country that

¹See for example Hassler and Krusell (2018) for a detailed survey.

²The implications of climate change for the economy are topical also for monetary policy makers. See for example Hale et al. (2019) and Del Negro et al. (2023).

acts in a unilateral way.³

To understand the tradeoffs that emerge in a multiple-country model, I build a model based on a simplified version of [Goloso et al. \(2014\)](#). Consider first a simple deterministic closed economy, without physical capital. There is a representative household that consumes a final good, net of climate-induced damages. The final good is produced with labor and ‘dirty’ energy. Energy is produced with labor in an energy sector. Production of energy contributes to the stock of emissions over time, reducing net (of damages) output that is available for consumption. A social planner that maximizes the utility of the representative household chooses the optimal intersectoral allocation of labor, and the optimal production of energy, taking into account the output damages induced by energy. Instead, competitive firms in the market economy do not internalize the negative production externality of energy. The planner’s allocation can be implemented by using a *corrective* carbon tax on energy producers that is rebated lump-sum. Such a tax reduces the socially inefficient production of energy and shifts labor to the production of the final good.

I extend this simple setup in a multiple-country model. Country-specific production of energy contributes to global emissions, which induce country-specific damages. I consider two distinct policy environments: a) a *cooperative* environment where a global social planner maximizes the weighted average of the representative household’s utility in each country, subject to the global resource constraint, production technologies and the law of motion of the stock of emissions, b) a setup beyond cooperation, where a *large* country, in terms of both emissions and participation in international capital markets, designs a unilateral policy as a *dynamic monopolist* who faces a passive rest of the world, in the spirit of [Costinot et al. \(2014\)](#).

The cooperative equilibrium is characterized by three conditions: a) *consumption efficiency*, which posits that the ratio of marginal utilities across countries should be *constant*, b) *intersectoral labor efficiency in each country*, which determines the efficient allocation of labor between the two sectors in each country, taking into account the damages that an increase in the production of energy creates, c) *intertemporal* climate efficiency, which posits that the shadow cost of the stock of emissions should reflect the *global* current and future marginal damages. A global carbon tax can implement the Pareto-efficient allocation.

Turning to optimal *unilateral* policies, a large country (Home) with market power in international capital markets has incentives to manipulate global interest rates to improve its trading position with the rest of the world (Foreign). These incentives, which are manifested as *capital controls* that are implemented with a tax on borrowing or saving from abroad, are intertwined with incentives for carbon taxation. As in the no-externality, endowment economy of [Costinot et al. \(2014\)](#), Home has incentives to tax capital inflows/subsidize capital outflows, when it grows

³See [Hassler et al. \(2021\)](#) for an exercise that goes beyond the standard Pigouvian taxation problem by treating suboptimal policies.

faster than the rest of the world. Instead, if Home grows slower than the rest of the world, it implements a subsidy on capital inflows/tax on capital outflows.

Turning to the domestic unilateral carbon tax, the domestic policymaker takes into account foreign output damages, even if she maximizes only domestic welfare, because they affect the trading position with the rest of the world, and, therefore, interest rates. Foreign damages are harmful for Home, making it impose a domestic carbon tax that is *above* its own country-specific damages, if Home is a net *buyer* of goods, so a net importer. Instead, if Home is a net *seller* of goods, foreign damages are *beneficial* to Home. This leads to an incentive to impose a carbon tax that is *smaller* than Home's damages. If this incentive is large enough, a net exporter would actually *subsidize* carbon emissions.

Related literature. The main reference for the analysis is [Costinot et al. \(2014\)](#), who study intertemporal trade and capital controls in a multiple-country endowment economy without externalities, treating a policymaker who exhibits market power.⁴ In terms of the DICE model, I use a simplified version of the closed economy of [Goloso et al. \(2014\)](#). [Hillebrand and Hillebrand \(2019\)](#) have extended [Goloso et al. \(2014\)](#) in a multiple-country environment and studied cooperative policy. A contribution that shares the same interest in *unilateral* policies is the work of [Kortum and Weisbach \(2022\)](#), who study unilateral carbon policies in a static two-country setup.⁵ [Farrokhi and Lashkaripour \(2022\)](#) use a static quantitative trade model to explore the potential of trade policy for the mitigation of climate change.

Organization. Section 2 lays down a simple closed economy that is the building block of the multiple-country economy. It derives the efficient allocation and it contrasts it to the competitive equilibrium allocation. Section 3 sets up a multiple-country economy, derives the optimal cooperative climate policy, and shows how it can be implemented with a global carbon tax. Section 4 lays down the analysis for a large country that acts as a dynamic monopolist facing a passive rest of the world, and analyzes the interaction between climate policy and optimal capital controls. Section 5 concludes.

2 A simple closed economy

I start first with a simple closed economy, to set the stage for the analysis in the multi-country economy. The setup is a simplified version of the closed economy of [Goloso et al. \(2014\)](#).

⁴[Choi et al. \(2024\)](#) provide empirical evidence on market power in international capital markets of large participants as the United States. See also [Lloyd and Marin \(2024\)](#) for the analysis of optimal capital controls and trade policy.

⁵There is a substantial older literature that studies taxation and externalities in international settings- see [Markusen \(1975\)](#).

Time is discrete and the horizon is infinite. The economy is deterministic, without physical capital and without exhaustible resources. There is a final good that is produced by using labor n_t , and the intermediate input energy E_t (in carbon content) (or the flow of “emissions” - we use the two terms interchangeably). Energy is produced in an energy sector using labor $n_{E,t}$, and is ‘dirty’.

Resource constraint and production sectors. Climate enters the economy by creating *damages*. Damages are modeled as a reduction of total output that is available for consumption. The resource constraint in the economy reads

$$c_t = Y_t \equiv (1 - D(S_t))\hat{Y}_t \quad (1)$$

where c_t is consumption, Y_t net (of damages) output, $\hat{Y}_t$ gross output, and S_t a climate variable. The damage function is increasing in S and differentiable, so $D_S > 0$. Gross output $\hat{Y}_t$ is given by

$$\hat{Y}_t = A_t F(n_t, E_t), \quad (2)$$

where A_t an exogenous technology process and F a constant returns to scale production function that is increasing in both arguments and concave. The production technology for energy is

$$E_t = z_t f(n_{E,t}), \quad (3)$$

where z_t an exogenous forcing process and f increasing and concave.⁶

Law of motion of emissions. The climate module in this economy is captured by a law of motion for S_t , which we think of as the *stock* of emissions,

$$S_t = H(S_{t-1}, E_t), \quad (4)$$

where H is increasing and differentiable in both arguments, $H_E, H_S > 0$ and S_{-1} given.⁷ The law of motion (4) captures the dependence on the past, so the *durability* of emissions (and the subsequent increases in temperature). When I proceed to parametric assumptions for

⁶I allow full generality here for the production function f , but I assume a constant returns to scale production function when we proceed to the baseline example, so $E_t = z_t n_{E,t}$.

⁷The function H could also depend on time t . Any such dependence is implicit.

the dynamics of emissions, I allow for a richer dependence on the past that goes beyond the previous stock S_{t-1} , but I stick to (4) for the theoretical analysis throughout the paper. Increases in E_t increase gross output through (2), but increase also S_t and damages through (4) and (1) respectively, reducing net output that is available for consumption. This is the basic tradeoff in the heart of the social planner's problem.

Preferences. Lastly, the economy is populated by a representative household that derives utility from a stream of consumption $\{c_t\}$ and there is no disutility of labor. The preferences of the household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad (5)$$

where u an increasing, concave and twice differentiable period utility function that satisfies the Inada condition, and $\beta \in (0, 1)$ the subjective discount factor. The household is endowed with one unit of time that is allocated as labor between the two sectors, so $n_t + n_{E,t} = 1$.

2.1 First-best

To understand the nature of the climate externality and the need for corrective taxation, consider first a benevolent social planner who maximizes the utility of the representative household, taking into account how energy increases emissions, and, therefore, damages in the economy.

Problem 1. (*“First-best”*) *The social planner chooses sequences $\{c_t, n_t, E_t, S_t\}$ to maximize the utility of the representative household (5), subject to*

$$c_t = (1 - D(S_t))A_t F(n_t, E_t) \quad (6)$$

$$E_t = z_t f(1 - n_t) \quad (7)$$

$$S_t = H(S_{t-1}, E_t), \quad (8)$$

non-negativity and feasibility constraints $c_t \geq 0, n_t \in [0, 1]$, with S_{-1} given.

I have eliminated $n_{E,t}$ in problem 1, using $n_{E,t} = 1 - n_t$. Let μ_t and ξ_t denote the multipliers on (7) and (8), divided by the marginal utility of consumption, in order to be expressed in terms of the final good. The efficient allocation is characterized by the following optimality conditions.⁸

⁸Throughout the analysis, $F_{i,t}, i = n, E$, and $H_{i,t}, i = S, E$ is shorthand notation for the partial derivatives of F and H at t . Similarly, D_t and $D_{S,t}$ stand for the respective damages and marginal damages at t .

Efficient allocation of labor. The efficient allocation of labor between the final good sector and the energy sector satisfies⁹

$$\underbrace{(1 - D_t)A_t F_{n,t}}_{MP_n^{final}} = \underbrace{\mu_t}_{\text{shadow value of } E_t} \times \underbrace{z_t f_n(1 - n_t)}_{MP_n^{energy}}. \quad (9)$$

Thus, at the optimal allocation, the marginal product of labor in the final good sector is *equalized* to the marginal product of labor in the energy sector, times the shadow value of energy.

Shadow value of energy. The shadow value of energy μ_t is given by

$$\mu_t = \underbrace{(1 - D_t)A_t F_{E,t}}_{MP_E^{final}} - \underbrace{\xi_t H_{E,t}}_{\text{cost of emissions-externality}}, \quad (10)$$

The shadow value of energy has *two* components: an increase in E_t is beneficial, since it increases final good output ($F_E > 0$). However, higher energy increases the stock of emissions ($H_E > 0$), and therefore damages, which have shadow cost ξ_t . This the second component in the right-hand side of (10), and is exactly the production *externality* that is ignored by competitive firms in the market economy, as we will shortly see.

Shadow cost of the stock of emissions. The shadow cost of S_t is given by

$$\xi_t = \underbrace{D_{S,t} \hat{Y}_t}_{\text{current marginal damages}} + \underbrace{\beta \frac{u_{c,t+1}}{u_{c,t}} H_{S,t+1} \xi_{t+1}}_{\text{future marginal damages}}, \quad (11)$$

which we can solve forward to get

$$\xi_t = \underbrace{\sum_{i=0}^{\infty} \beta^i \frac{u_{c,t+i}}{u_{c,t}} X_{t,i} D_{S,t+i} \hat{Y}_{t+i}}_{\propto \text{social cost of carbon}} > 0, \quad (12)$$

where $X_{t,i} \equiv \prod_{j=1}^i H_{S,t+j}$, with $X_{t,0} \equiv 1$, a multiplicative factor that captures the depreciation (if any) of the emission stock. Thus, the shadow cost S_t is the present discounted value of current and future marginal damages. We dub the product $\xi_t \times H_{E,t}$ the *social cost of carbon*. This is the

⁹I use the terms efficient and first-best interchangeably.

social cost of the production externality, that generates the rationale for corrective (Pigouvian) carbon taxation.

2.2 Market economy

Consider a laissez-faire market economy, or else, the “business as usual” scenario.

Household’s problem. The household consumes, saves or borrows, gets labor income by providing its time endowment to the two sectors, and enjoys profits from the final good firm and the energy sector firm. The household’s dynamic budget constraint is given by

$$c_t + p_t b_{t+1} = w_t + \Pi_t + \pi_t + b_t, \quad (13)$$

where p_t the price of a discount bond that promises one unit of consumption next period, b_t the bond holdings in the beginning of the period, w_t the wage rate, and Π_t and π_t the profits of the final good sector firm and the energy sector firm respectively. Initial bonds are set to zero, $b_0 = 0$. The problem of the household is to choose consumption $c_t \geq 0$ and bond holdings b_{t+1} to maximize its utility (5) subject to (13) and an appropriate no-Ponzi game condition. As usual, the household equalizes the price of bonds to its intertemporal marginal rate of substitution,

$$p_t = \beta \frac{u_{c,t+1}}{u_{c,t}}. \quad (14)$$

Final good sector. The competitive firm in the final good sector chooses labor n_t and energy E_t to maximize profits, taking the stock of emissions S_t , the wage rate w_t and the price of energy $p_{E,t}$ as given. Profits are given by

$$\Pi_t = (1 - D(S_t))A_t F(n_t, E_t) - w_t n_t - p_{E,t} E_t. \quad (15)$$

Profit maximization leads to the equalization of the marginal products to the respective prices of inputs,

$$(1 - D(S_t))A_t F_{n,t} = w_t \quad (16)$$

$$(1 - D(S_t))A_t F_{E,t} = p_{E,t} \quad (17)$$

These two conditions determine the demand for labor and energy in the final good sector.

Energy sector. Taking as given the wage rate and the price of energy, the energy sector firm chooses $n_{E,t}$ to maximize its profits,

$$\pi_t = p_{E,t} z_t f(n_{E,t}) - w_t n_{E,t}. \quad (18)$$

Profit maximization leads to

$$p_{E,t} z_t f_n(n_{E,t}) = w_t, \quad (19)$$

which equalizes the marginal product labor in the energy sector to the relative wage $w_t/p_{E,t}$.

Equilibrium. A competitive equilibrium is a collection of prices $\{p, w, p_E\}$ and allocations, such that the household maximizes its utility, firms maximize profits, and markets clear, that is, the resource constraint (1) holds, bond markets clear, $b_{t+1} = 0, \forall t$, the labor market clears, $n_t + n_{E,t} = 1$, and the stock of emissions evolves according to (4).

Contrasting the market economy to the first-best. From the profit maximization conditions (16), (17) and (19), and using $n_{E,t} = 1 - n_t$, we get the following relationship between the marginal products of labor and energy in the two sectors.

$$\underbrace{(1 - D_t) A_t F_n(n_t, E_t)}_{MP_n^{final}} = \underbrace{(1 - D_t) A_t F_E(n_t, E_t)}_{MP_E^{final}} \times \underbrace{z_t f_n(1 - n_t)}_{MP_n^{energy}} \quad (20)$$

Contrast (20) with the respective conditions (9) and (10) at the first-best. The shadow value of energy μ_t includes *both* the marginal product of energy, *and* the costs associated with an increase in emissions, $\xi_t H_{E,t}$. These negative production externality is ignored by competitive firms, leading to the deviation of the market economy allocation from the efficient one.

We can rewrite (20) as

$$\frac{F_n(n_t, E_t)}{F_E(n_t, E_t)} = z_t f_n(1 - n_t), \quad (21)$$

which equalizes the marginal rate of technical substitution of n and E to the marginal product of labor in the energy sector. Instead, in the efficient allocation we have

$$\frac{F_n(n_t, E_t)}{F_E(n_t, E_t)} = \underbrace{\left[1 - \frac{\xi_t H_{E,t}}{(1 - D_t) A_t F_{E,t}}\right]}_{<1} z_t f_n(1 - n_t) < z_t f_n(1 - n_t), \quad (22)$$

so the marginal rate of technical substitution has to be *smaller* than the marginal product of labor in the energy sector. This leads to an amount of labor devoted to the final good sector that is larger in the first-best than in the competitive equilibrium. Analogously, the first-best production of energy is smaller than the competitive equilibrium amount of energy.

Market allocation. Using $E_t = z_t f(1 - n_t)$ in the *static* condition (21), we can solve for n_t *solely* as function of z_t , $n_t^{CE} = n(z_t)$. Thus, the market allocation of labor between the two sectors does not depend on the stock of emissions S_t and on A_t . The optimal allocation of labor determines the production of energy $E_t^{CE} = E(z_t)$, and therefore of gross output, $\hat{Y}_t^{CE} = \hat{Y}(z_t, A_t)$. Given energy E_t^{CE} we get from (4) the respective stock of emissions in the laissez-faire economy S_t^{CE} , which in the end determines net of damages output and therefore consumption through (1).

Decentralizing the first-best with carbon taxes on energy producers. A simple way to decentralize the first-best allocation is to internalize the externality by imposing a unit carbon tax on energy producers, τ_t , that is rebated lump-sum to consumers, $T_t \equiv \tau_t E_t$, and is equal to the social cost of carbon. To see that, consider the profits of the energy firm that pay the unit tax for any carbon unit E_t ,

$$\pi_t^{\text{tax}} = [p_{E,t} - \tau_t] z_t f(n_{E,t}) - w_t n_{E,t}, \quad (23)$$

leading to the profit maximization condition $[p_{E,t} - \tau_t] z_t f_n(n_{E,t}) = w_t$. Using (16), (17) and setting $\tau_t = \xi_t H_{E,t}$, delivers the first-best allocation.

2.3 Parametric example

For the baseline parametric example, I follow Golosov et al. (2014).

Law of motion of emissions. I use the *linear* specification of Golosov et al. (2014), which combines transitory and permanent components. In particular, the stock of emissions S_t is the sum of a permanent component x_t , and a transitory component y_t :

$$S_t = x_t + y_t \quad (24)$$

$$x_t = x_{t-1} + \phi_L E_t \quad (25)$$

$$y_t = (1 - \phi)y_{t-1} + (1 - \phi_L)\phi_0 E_t \quad (26)$$

The dynamics of emissions are characterized by the parameters (ϕ_L, ϕ_0, ϕ) . A ϕ_L share of emissions stays in the atmosphere *permanently*. Out of the remaining share $(1 - \phi_L)$, a fraction $(1 - \phi_0)$ dissolves immediately and therefore a share $(1 - \phi_L)\phi_0$ remains. This share is *transitory*, in the sense that it decays at the rate ϕ . The specification is flexible enough to include either only permanent ($\phi_0 = 0$) or only transitory emissions ($\phi_L = 0$).

Given these dynamics, the stock of emissions S_t is a linear function of past flow emissions,

$$S_t - \bar{S} = \sum_{i=0}^t (1 - d_i) E_{t-i}, \quad (27)$$

where

$$1 - d_i \equiv \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^i, \quad (28)$$

$y_{-1} \equiv 0, x_{-1} \equiv \bar{S}$, where $\bar{S}$ denotes the stock of emissions in pre-industrial times.

Social cost of carbon. The law of motion (4) assumes that the previous stock of emissions S_{t-1} is sufficient to capture the impact of past flow emissions, acting therefore as a *state* variable. Unless we have only permanent ($S_t = x_t$) or only transitory shocks ($S_t = y_t$), we need to keep track of both x_t and y_t to determine S_t . The analysis on the social cost of carbon in the previous section can be easily extended to that case. In particular, let ξ_t^x and ξ_t^y denote the respective multipliers (divided by marginal utility) on the permanent stock of emissions (25), and transitory stock of emissions (26). The shadow value of energy μ_t in (10) is given by

$$\mu_t = (1 - D_t)A_t F_{E,t} - \underbrace{[\phi_L \xi_t^x + (1 - \phi_L)\phi_0 \xi_t^y]}_{\xi_t H_{E,t}}, \quad (29)$$

that is the externality, which determines the carbon tax τ_t , is captured by a linear combination of ξ_t^x and ξ_t^y , capturing the term $\xi_t H_{E,t}$ in (10). The cost of the two stocks is determined by

$$\xi_t^x = D_{S,t}\hat{Y}_t + \beta \frac{u_{c,t+1}}{u_{c,t}} \xi_{t+1}^x \quad (30)$$

$$\xi_t^y = D_{S,t}\hat{Y}_t + \beta(1-\phi) \frac{u_{c,t+1}}{u_{c,t}} \xi_{t+1}^y \quad (31)$$

By solving forward (30) and (31), we finally get that the optimal carbon tax that corrects the climate externality is

$$\tau_t \equiv \phi_L \xi_t^x + (1-\phi_L)\phi_0 \xi_t^y = \sum_{i=0}^{\infty} \beta^i \frac{u_{c,t+i}}{u_{c,t}} D_{S,t+i} \hat{Y}_{t+i} (1-d_i). \quad (32)$$

Damage function. Let the damage function have the exponential form of Golosov et al. (2014),

$$D(S_t) = 1 - \exp(-\gamma(S_t - \bar{S})), \quad (33)$$

where γ a positive parameter that captures the damage sensitivity, $\gamma > 0$. If $\gamma = 0$, there are no damages, $D(S_t) = 0$, and therefore no externality. In that case, the social cost of emissions would collapse to zero, $\xi_t = \xi_t^x = \xi_t^y = 0$.

The exponential damage function (33) features the convenient property that $D_{S,t} = \gamma(1-D_t)$, so marginal damages are equal to a fraction γ of *net* output, $D_{S,t}\hat{Y}_t = \gamma(1-D_t)\hat{Y}_t = \gamma Y_t$. The optimal carbon tax (32) simplifies then to

$$\tau_t = \gamma \sum_{i=0}^{\infty} \beta^i \frac{u_{c,t+i}}{u_{c,t}} Y_{t+i} (1-d_i) \quad (34)$$

Power utility. Consider a power period utility function of consumption, that implies a constant elasticity of intertemporal substitution, $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho > 0$. Consumption is equal to output, so the optimal carbon tax as a share of output becomes

$$\frac{\tau_t}{Y_t} = \gamma \sum_{i=0}^{\infty} \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} (1-d_i) \quad (35)$$

For the logarithmic case, $\rho = 1$, the tax simplifies to a *constant* fraction of output,

$$\frac{\tau_t}{Y_t} = \gamma \left[\frac{\phi_L}{1 - \beta} + \frac{(1 - \phi_L)\phi_0}{1 - \beta(1 - \phi)} \right]. \quad (36)$$

This is the same formula as in the logarithmic case of [Goloso et al. \(2014\)](#) with full depreciation of capital, and shows for this special case the dependence of the carbon tax (as share of output) solely on the damage sensitivity γ and the climate parameters (ϕ_L, ϕ_0, ϕ) .¹⁰

3 A multiple-country setup

Let's move now to the main environment of interest, which is a multiple-country version of the economy in section 2. Assume that we have N countries and let the superscript l denote the country, $l = 1, \dots, N$.

Production sectors and damages. Each country l has a unit of labor that is allocated in the production of the final good (n_t^l) and in the production of energy ($n_{E,t}^l$), so $n_t^l + n_{E,t}^l = 1, \forall l$. Gross output in country l is denoted by $\hat{Y}_t^l$. Net of damages output is given by

$$Y_t^l = (1 - D^l(S_t))\hat{Y}_t^l = (1 - D^l(S_t))A_t^l F^l(n_t^l, E_t^l), l = 1, \dots, N. \quad (37)$$

The production technology for energy E_t^l is given by

$$E_t^l = z_t^l f^l(n_{E,t}^l), l = 1, \dots, N. \quad (38)$$

Note that production functions, technology processes and damage functions are indexed by the country l , and have the same properties as in the one country model we previously considered. Country-specific damages though depend on the *global* stock of emissions S_t , capturing the global nature of the negative externality that emissions impose. The total flow of emissions E_t is given by the sum of country-specific emissions, $E_t \equiv \sum_{l=1}^N E_t^l$. The global stock S_t follows the law of motion

$$S_t = H(S_{t-1}, \sum_{l=1}^N E_t^l), \quad (39)$$

¹⁰Full depreciation of capital and logarithmic utility imply that consumption is a constant fraction of net output in the economy of [Goloso et al. \(2014\)](#). This fact leads to the same carbon tax simplifications as in the current economy without capital.

with S_{-1} given, and H increasing in both arguments. Consequently, emissions in country l , E_t^l , increase S_t through (39) ($H_E > 0$), and generate damages in country $k \neq l$, $D_S^k > 0$.

Preferences and resource constraint. The preferences of the representative household in each country l are given by

$$\sum_{t=0}^{\infty} \beta^t u^l(c_t^l), \quad (40)$$

where c_t^l the consumption of final good in country l and u^l a period utility function for country l , that is increasing, concave and satisfies the Inada condition. The global resource constraint is given by

$$\sum_{l=0}^N c_t^l = \sum_{l=0}^N Y_t^l \equiv Y_t. \quad (41)$$

3.1 Optimal cooperative policy

Assign on each country Pareto weights $\eta^l > 0, l = 1, \dots, N$, with the normalization $\sum_{l=1}^N \eta^l = 1$.

Problem 2. *The problem of the global social planner is to choose sequences $\{c_t^l, n_t^l, E_t^l, S_t\}_{\forall l, t}$ to maximize the weighted utility*

$$\sum_{l=1}^N \eta^l \sum_{t=0}^{\infty} \beta^t u^l(c_t^l) \quad (42)$$

subject to

$$\sum_l^N c_t^l = \sum_{l=0}^N (1 - D^l(S_t)) A_t^l F^l(n_t^l, E_t^l), \forall t \quad (43)$$

$$E_t^l = z_t^l f^l(1 - n_t^l), \forall l, t \quad (44)$$

$$S_t = H(S_{t-1}, \sum_l^N E_t^l), \forall t \quad (45)$$

with S_{-1} given, and $c_t^l \geq 0, n_t^l \in [0, 1], \forall l, t$.

The solution to the global planner's problem features the following optimality conditions.

Consumption efficiency. The first-order condition with respect to consumption at country l delivers $\eta^l u_c^l(c_t^l) = \lambda_t, \forall l$, where λ_t the shadow value of the world's output. This implies the standard condition for Pareto-optimal allocations between countries k and l ,

$$\frac{u_{c,t}^l}{u_{c,t}^k} = \frac{\eta^k}{\eta^l}, \forall k, l, t. \quad (46)$$

Thus, the ratio of marginal utilities is *constant* for each t and inversely related to the ratio of Pareto weights. Consequently, intertemporal marginal rates of substitution are equalized across countries, $\beta \frac{u_{c,t+1}^l}{u_{c,t}^l} = \beta \frac{u_{c,t+1}^k}{u_{c,t}^k} \forall k, l$. Condition (46), together with the global resource constraint (41) implies that consumption for each country is a function only of the world output Y_t .

Efficient intersectoral allocation of labor and shadow value of energy in country l . The optimal allocation intersectoral allocation of labor in country l satisfies

$$(1 - D_t^l) A_t^l F_{n,t}^l = \mu_t^l \times z_t^l f_n^l (1 - n_t^l), \forall l \quad (47)$$

where the *country-specific* shadow value of energy is given by

$$\mu_t^l = \underbrace{(1 - D_t^l) A_t^l F_{E,t}^l}_{MP_E^{final} \text{ in } l} - \underbrace{\xi_t^{global} H_{E,t}}_{\text{global externality}}, \forall l. \quad (48)$$

The shadow value of energy E_t^l is equal to the marginal product of energy in the final good sector, a component which depends on country l , minus the shadow cost of the stock of emissions, a component that is *global*, ξ_t^{global} .¹¹ This feature necessitates a *global* carbon tax on energy producers in each country.

Shadow cost of the global stock of emissions. The shadow cost of the stock of emissions in the multiple-country setup is given by

$$\xi_t^{global} = \underbrace{\sum_{l=1}^N D_{S,t}^l \hat{Y}_t^l}_{\text{global current marginal damages}} + \underbrace{\beta \frac{\lambda_{t+1}}{\lambda_t} H_{S,t+1} \xi_{t+1}^{global}}_{\text{future global marginal damages}}, \quad (49)$$

where $\lambda_{t+1}/\lambda_t = u_{c,t+1}^l/u_{c,t}^l = u_{c,t+1}^k/u_{c,t}^k, \forall k, l$. Thus, the social cost of carbon is the present

¹¹Both μ_t^l and ξ_t^{global} are scaled by the shadow value of global output λ_t .

discounted value of current and future *global* marginal damages.

3.2 Market equilibrium

Household. The household in each country l chooses $c_t^l \geq 0, b_{t+1}^l$ to maximize (40) subject to the respective country budget constraint,

$$c_t^l + p_t b_{t+1}^l = w_t^l + \Pi_t^l + \pi_t^l + b_t^l, \quad (50)$$

and the no-Ponzi-game condition, where initial holdings are set to zero for each country, $b_0^l = 0, \forall l$. The variables w_t^l, Π_t^l, π_t^l are the country specific wage rate, final good sector profits and energy sector profits respectively. The budget constraint (50) is the country-specific version of (13). Household optimality requires that

$$p_t = \beta \frac{u_{c,t+1}^l}{u_{c,t}^l}, \forall l. \quad (51)$$

Final good sector and energy sector firms. The profits of the two firms in country l are respectively

$$\Pi_t^l = (1 - D^l(S_t)) A_t^l F^l(n_t^l, E_t^l) - w_t^l n_t^l - p_{E,t}^l E_t^l \quad (52)$$

$$\pi_t^l = p_{E,t}^l z_t^l f^l(n_{E,t}^l) - w_t^l n_{E,t}^l. \quad (53)$$

Profit maximization in the two sectors leads to equalization of marginal products to the respective country-specific prices,

$$(1 - D^l(S_t)) A_t^l F_{n,t}^l = w_t^l \quad (54)$$

$$(1 - D^l(S_t)) A_t^l F_{E,t}^l = p_{E,t}^l \quad (55)$$

$$p_{E,t}^l z_t^l f_n^l(n_{E,t}^l) = w_t^l. \quad (56)$$

Competitive equilibrium in the global economy with externalities. The competitive equilibrium is a collection of bond prices p , country-specific wage rates and energy prices $\{w^l, p_E^l\}_{l=1}^N$, allocations $\{c^l, n^l, E^l\}$, stock of emissions S and bond holdings b^l , such that the household in each country maximizes utility, firms in each country maximize profits, and markets clear, that is, the global resource constraint holds (41), bond markets clear $\sum_{l=1}^N b_{t+1}^l = 0$,

labor markets clear $n_t^l + n_{E,t}^l = 1 \forall l$, and the global stock of emissions evolves as (39).

Analysis of the competitive equilibrium. Using (54), (55) and (56) we get

$$\frac{F_n^l(n_t^l, E_t^l)}{F_E^l(n_t^l, E_t^l)} = z_t^l f_n^l(1 - n_t^l), \forall l \quad (57)$$

which equalizes the marginal rate of technical substitution in the final good sector of country l to the marginal product of labor in the energy sector in l . Condition (57) extends the respective conditions we found in the closed economy in (20) and (21) and is contrasted to the efficiency conditions (47) and (48). Firms do not internalize the damages that are associated with the production of energy. Condition (57) allows us to solve for labor in the final good sector of country l as function only of z_t^l , $n_t^l = n(z_t^l)$, which makes also country-specific energy a function of z_t^l , $E_t^l = E(z_t^l)$.

Moreover, the equalization of intertemporal marginal rates of substitution to the price of bonds p_t in (51), makes the ratio of marginal utilities in the competitive equilibrium *constant*, $u_{c,t}^l/u_{c,t}^k = \text{constant}, \forall l, k$, which, using (41), implies that the consumption of country l is function *solely* of the world net output Y_t . This property is the *same* as in the Pareto efficient allocation (where the consumption share depends on the Pareto weights- see next section), *but* the level of the global output in the efficient allocation is obviously different.

Implementing the planner's allocation as a competitive equilibrium. The after-tax profits in the energy sector of country l , if a unit tax τ_t^l is imposed, are

$$\pi_t^{l,\text{tax}} = [p_{E,t}^l - \tau_t^l] z_t^l f(n_{E,t}^l) - w_t^l n_{E,t}^l. \quad (58)$$

To decentralize the global planner's allocation, we set a common unit tax $\tau_t^l = \tau_t^{\text{global}} \forall l$, that is equal to the global externality, $\tau_t^{\text{global}} = \xi_t^{\text{global}} H_{E,t}$.

3.3 Parametric example and the global carbon tax

We make similar parametric assumptions as in the respective closed economy of section 2 and analyze the Pareto allocation and the global carbon tax.

Preferences and consumption. Assume that the preferences of each country are given by $u(c) = c^{1-\rho}/(1-\rho)$. The consumption efficiency condition in (46) together with the world resource constraint (41) implies that consumption in each country is a *constant* fraction of world's output Y_t ,

$$c_t^l = \theta^l Y_t, \quad \theta^l \equiv \frac{(\eta^l)^{\frac{1}{\rho}}}{\sum_{l=1}^N (\eta^l)^{\frac{1}{\rho}}}. \quad (59)$$

This is a well-known property of Pareto allocations in conventional environments without externalities. As a result, the discounting of future damages in the determination of the shadow cost of emissions in (49) is determined by world output, $\frac{\lambda_{t+1}}{\lambda_t} = (\frac{Y_{t+1}}{Y_t})^{-\rho}$.

Emissions. As Hillebrand and Hillebrand (2019) do, I use the multiple-country extension of the parametric assumptions of Golosov et al. (2014). Emissions in the world economy have a permanent (x_t) and transitory (y_t) component, $S_t = x_t + y_t$, with laws of motion

$$x_t = x_{t-1} + \phi_L \sum_{l=1}^N E_t^l \quad (60)$$

$$y_t = (1 - \phi)y_{t-1} + (1 - \phi_L)\phi_0 \sum_{l=1}^N E_t^l \quad (61)$$

As in the analysis of the closed economy, the shadow value of energy in each country in (48) with permanent and transitory emissions takes the form

$$\mu_t^l = (1 - D_t^l)A_t^l F_{E,t}^l - [\phi_L \xi_t^x + (1 - \phi_L)\phi_0 \xi_t^y], \quad (62)$$

where ξ_t^x and ξ_t^y the (scaled) multipliers on (60) and (61), with evolution

$$\xi_t^x = \sum_{l=1}^N D_{S,t}^l \hat{Y}_t^l + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^x \quad (63)$$

$$\xi_t^y = \sum_{l=1}^N D_{S,t}^l \hat{Y}_t^l + \beta(1 - \phi) \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^y. \quad (64)$$

The global tax that corrects the negative externality is

$$\tau_t^{\text{global}} \equiv \phi_L \xi_t^x + (1 - \phi_L)\phi_0 \xi_t^y = \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} (1 - d_i) \sum_{l=1}^N D_{S,t+i}^l \hat{Y}_{t+i}^l, \quad (65)$$

where d_i defined in (28).

Country-specific damages. Assume an exponential damage function for each country,

$$D^l(S_t) = 1 - \exp(-\gamma^l(S_t - \bar{S})), \quad (66)$$

where $\gamma^l > 0$ the *country-specific* damage sensitivity. Under this parametric assumption, the optimal carbon tax (65) simplifies to

$$\tau_t^{\text{global}} = \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} (1 - d_i) \sum_{l=1}^N \gamma^l Y_{t+i}^l. \quad (67)$$

Using now the power utility assumption, we get the final expression for the global carbon tax, as share of global output,

$$\frac{\tau_t^{\text{global}}}{Y_t} = \sum_{i=0}^{\infty} \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} (1 - d_i) \cdot \underbrace{\gamma_{t+i}^{\text{global}}}_{\text{global damage sensitivity}}, \quad (68)$$

where $\gamma_t^{\text{global}} \equiv \sum_{l=1}^N s_t^l \gamma^l$, the *global* damage sensitivity, and $s_t^l \equiv \frac{Y_t^l}{Y_t}$, the *share* of country's l output in global output. The global damage sensitivity captures the global marginal damages as share of world's output is just an output-weighted average of each country's sensitivity γ^l .

Some remarks are due: a) Output shares s_t^l are time-varying, so, in contrast to the closed-economy case in (36), the tax rate as share of output is not constant in the logarithmic case $\rho = 1$. b) If damages are common across countries, $\gamma^l = \gamma, \forall l$, then $\gamma_t^{\text{global}} = \gamma$ and formula (68) reduces to the formula of the closed economy (35) (with the understanding that Y_t refers to global output at the planner's allocation). c) If the share of each country's output in global output is *constant*, $s_t^l = s^l \forall l, t$, that is, if each country grows at the *same* rate, $\frac{Y_{t+1}^l}{Y_t^l} = \frac{Y_{t+1}}{Y_t}, \forall l$, then the global damage sensitivity is constant, $\gamma_t^{\text{global}} = \gamma^{\text{global}} \equiv \sum_{l=1}^N s^l \gamma^l$, and formula (68) reduces again to (35), by replacing γ with γ^{global} . Assume furthermore that we are at a balanced growth path (BGP henceforth), so that this common growth rate is *constant*. Let $\tilde{\beta} \equiv \beta(1+g)^{1-\rho}$, and assume that $\beta^{-1} > (1+g)^{1-\rho}$ so that $\tilde{\beta} < 1$. Then, the global tax becomes¹²

$$\frac{\tau_t}{Y_t} = \gamma^{\text{global}} \cdot \underbrace{\left[\frac{\phi_L}{1 - \tilde{\beta}} + \frac{(1 - \phi_L)\phi_0}{1 - \tilde{\beta}(1 - \phi)} \right]}_{\sum_{i=0}^{\infty} \tilde{\beta}^i (1 - d_i)}. \quad (69)$$

¹²See also Hillebrand and Hillebrand (2019) who emphasized this type of formula at the BGP in an economy with capital.

4 Beyond cooperation

Consider now a setup beyond cooperation, where a country, that understands *both* the domestic *and* the international dimension of the climate externality, trades intertemporally with the rest of the world, and chooses its own policy. The country is *large*, *both* as an emitter, *and* as a participant in international capital markets, and understands how equilibrium interest rates are formed, exhibiting market power.

4.1 Prelude: A price-taking policymaker

As a first step, consider a restricted case, where the country-specific policymaker takes into account the domestic and international dimension of the climate externality, but acts as a *price-taker* in international capital markets.

Problem 3. *Taking as given the energy use of other countries $E_t^j, j \neq l$, and international prices p , the policymaker in country l chooses $\{c_t^l, n_t^l, E_t^l, b_{t+1}^l, S_t\}$ to maximize¹³*

$$\sum_{t=0}^{\infty} \beta^t u^l(c_t^l) \quad (70)$$

subject to

$$c_t^l + p_t b_{t+1}^l = (1 - D^l(S_t)) A_t^l F^l(n_t^l, E_t^l) + b_t^l \quad (71)$$

$$E_t^l = z_t^l f^l(1 - n_t^l) \quad (72)$$

$$S_t = H(S_{t-1}, E_t^l + \sum_{j \neq l} E_t^j) \quad (73)$$

with S_{-1} and b_0^l given, $c_t^l \geq 0$ and $n_t^l \in [0, 1]$.

Equilibrium. The equilibrium in this setup is determined by having all countries solving problem 3, and having prices p that clear markets, that is, the global resource constraint (41) holds and bonds markets clear, $\sum_{l=1}^N b_{t+1}^l = 0$.

Analysis. Trading in international markets leads to the equalization of each country's marginal rate of substitution to the price of bonds, as in (51). Thus, the ratio of marginal utilities across countries is *constant*.

¹³Note the setup above considers a policymaker that trades with the rest of the world on behalf of the household. We decentralize the allocation later.

Let μ_t^l and ξ_t^l denote the multipliers on energy and the stock of emissions, divided by the marginal utility of consumption of country l . The optimal intersectoral allocation of labor in country l is given by

$$(1 - D_t^l)A_t^l F_{n,t}^l = \mu_t^l \times z_t^l f_n^l (1 - n_t^l), \quad (74)$$

where the shadow value of energy for country l for problem 3 is given now by

$$\mu_t^l = (1 - D_t^l)A_t^l F_{E,t}^l \underbrace{- \xi_t^l H_{E,t}}_{\text{country-specific shadow cost}}. \quad (75)$$

In contrast to the case of a global planner, the shadow cost of emissions ξ_t^l is *country-specific*. In particular, it satisfies the recursion

$$\xi_t^l = \underbrace{D_{S,t}^l \hat{Y}_t^l}_{\text{current country marginal damages}} + \underbrace{\beta \frac{u_{c,t+1}^l}{u_{c,t}^l} H_{S,t+1} \xi_{t+1}^l}_{\text{future country marginal damages}}. \quad (76)$$

A policymaker that cares only for the utility of the household in country l , considers only the *country-specific* damages of emissions and *not* the global marginal damages that we saw in (49). The discounting of future country-specific damages is conducted at the *same* rate across countries, due to the equalization of intertemporal marginal rates of substitution.

Country-specific carbon tax and parametric example. The analysis above implies that the policymaker in each country l can decentralize the allocation with a *country-specific* carbon tax on energy producers, $\tau_t^l = \xi_t^l H_{E,t}$, with proceeds that are rebated lump-sum to the household, $T_t^l = \tau_t^l E_t^l$. Using the same parametric assumptions as in the cooperative solution, we get that the country-specific carbon tax in country l is

$$\frac{\tau_t^l}{Y_t} = \sum_{i=0}^{\infty} \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} (1 - d_i) \underbrace{\gamma^l s_{t+i}^l}_{\text{damages of } l} \quad \forall l, \quad (77)$$

which contrasts to the cooperative solution in (68), since only the country-specific marginal damages matter for the determination of the optimal carbon tax.

4.2 A dynamic monopolist

Assume now that we have two countries ($N = 2$). We use H (Home) for country 1, and F (Foreign) for country 2.¹⁴ We may think of Home as a large emitter (China or the U.S.) and of Foreign as the rest of the world. Home acts as a *dynamic monopolist*, exhibiting *market power*, following Costinot et al. (2014). The foreign country instead, is a *passive* price-taker.

Problem of F. F is not implementing any climate policy, following a “business as usual” scenario. The household and firms in F acts as price-takers, so I can use the optimality conditions we found in the analysis of the market economy in section 3.2.

The household’s budget constraint in F is

$$c_t^2 + p_t b_{t+1}^2 = w_t^2 + \Pi_t^2 + \pi_t^2 + b_t^2, \quad (78)$$

with initial debt set to zero, $b_0^2 = 0$ (so we have also $b_0^1 = 0$). Maximization of utility of the household in F leads to

$$p_t = \beta \frac{u_c^2(c_{t+1}^2)}{u_c^2(c_t^2)}. \quad (79)$$

Moreover, recall that profit maximization in the final food sector and the energy sector in F, together with equilibrium in the labor market of F leads to

$$\frac{F_n^2(n_t^2, E_t^2)}{F_E^2(n_t^2, E_t^2)} = z_t^2 f_n^2(1 - n_t^2). \quad (80)$$

As previously noted, given that $E_t^2 = z_t^2 f(1 - n_t^2)$, equation (80) can be solved for labor n_t^2 as function solely of z_t^2 , $n_t^2 = n^2(z_t^2)$. Similarly, foreign energy production is $E_t^2 = E^2(z_t^2)$, and therefore, foreign *gross* output $\hat{Y}_t^2$ is function only of the exogenous variables (A_t^2, z_t^2) , $\hat{Y}_t^2 = \hat{Y}^2(A_t^2, z_t^2)$.

Finally, the foreign resource constraint is

$$c_t^2 + p_t b_{t+1}^2 = Y_t^2 + b_t^2. \quad (81)$$

¹⁴To keep the notation comparable to the previous sections, I avoid currently the use of the customary asterisk to denote the foreign country.

Implementability constraint and Ramsey problem. Consider now H, that acts as a dynamic monopolist who faces a price-taker.¹⁵ The monopolist maximizes at $t = 0$ the utility of the domestic representative household, subject to the foreign optimality conditions, budget and resource constraints, the global resource constraint, and the law of motion of emissions. To set up the dynamic monopolist's problem, it is convenient to work with the $t = 0$ intertemporal budget constraint of F, which given (78) and (81), becomes

$$\sum_{t=0}^{\infty} q_t (c_t^2 - Y_t^2) = 0, \quad (82)$$

where $q_t \equiv \Pi_{j=1}^t p_j$, the respective time zero price of an Arrow-Debreu contract, with $q_0 \equiv 1$. Given (79), equilibrium q_t is

$$q_t = \beta^t \frac{u_c^2(c_t^2)}{u_c^2(c_0^2)}. \quad (83)$$

H understands how equilibrium prices are related to foreign consumption c_t^2 through (83). As Costinot et al. (2014) do, I follow the primal approach of Lucas and Stokey (1983) and eliminate equilibrium prices q_t by using intertemporal marginal rates of substitution in (83) to rewrite (82) as

$$\sum_{t=0}^{\infty} \beta^t u_c^2(c_t^2) [c_t^2 - (1 - D^2(S_t)) \hat{Y}_t^2] = 0. \quad (84)$$

Equation (84) acts as an *implementability* constraint for H. We are ready now to state the problem of H.

Problem 4. (*'Dynamic monopolist'*) *The problem of the policymaker in H is to choose $\{c_t^1, c_t^2, n_t^1, E_t^1, S_t\}$ to maximize*

$$\sum_{t=0}^{\infty} \beta^t u^1(c_t^1) \quad (85)$$

subject to

¹⁵Note the similarities to settings where a large firm faces a competitive fringe.

$$c_t^1 + c_t^2 = (1 - D^1(S_t))A_t^1 F^1(n_t^1, E_t^1) + (1 - D^2(S_t))\hat{Y}_t^2 \quad (86)$$

$$E_t^1 = z_t^1 f^1(1 - n_t^1) \quad (87)$$

$$S_t = H(S_{t-1}, E_t^1 + E_t^2) \quad (88)$$

$$\sum_{t=0}^{\infty} \beta^t u_c^2(c_t^2) [c_t^2 - (1 - D^2(S_t))\hat{Y}_t^2] = 0, \quad (89)$$

where $c_t^1, c_t^2 \geq 0$, $n_t^1 \in [0, 1]$, and S_{-1} is given.

Foreign labor and energy are functions only of z_t^2 , and foreign gross output a function of only (A_t^2, z_t^2) . Consequently, E_t^2 and $\hat{Y}_t^2$ in problem 4 are *not* controllable by the dynamic monopolist, since they depend on the exogenous processes (A_t^2, z_t^2) . Instead, *net* foreign output $Y_t^2 = (1 - D_t^2)\hat{Y}_t^2$ is controlled by the monopolist through the stock of emissions S_t .

The dynamic monopolist maximizes the utility of the domestic household subject to the global resource constraint (86), the production function of domestic energy (87), the law of motion of emissions (88), and the implementability constraint (89). By choosing c_t^2 , the monopolist is effectively choosing equilibrium prices. There are *two* margins that are relevant for the problem, which necessitate two instruments to decentralize the optimal allocation of the monopolist problem: a) the *intersectoral* margin of the optimal allocation of labor, that is, the conventional margin where climate externalities enter, decentralized by a *domestic* carbon tax on energy production, τ_t b) the *intertemporal* margin, which is coming from the fact that Home is *large* and manipulates equilibrium prices. This margin is decentralized with a tax or subsidy on borrowing/savings, τ_t^b , as we will shortly see.

4.3 Consumption allocation and time-varying shares

Consider the consumption allocation of the dynamic monopolist's problem.¹⁶ Let $\Phi > 0$ denote the multiplier on the implementability constraint (89). The choice of foreign consumption c_t^2 satisfies the following optimality condition,

$$\underbrace{u_{c,t}^1}_{\text{MC of increasing } c_t^2} = \Phi \left[u_{c,t}^2 - u_{cc,t}^2 \underbrace{(c_t^1 - Y_t^1)}_{\text{net buyer (+)/seller (-)}} \right] \quad (90)$$

The left-hand side of (90) denotes the marginal cost of increasing foreign consumption, which is measured by the marginal utility of domestic consumption that is sacrificed. The right-hand

¹⁶See the Appendix for the respective Lagrangian and the derivation of the first-order conditions.

side denotes the marginal benefit of increasing foreign consumption. At first, an increase in foreign consumption relaxes mechanically the domestic budget constraint, an effect measured the equilibrium price q_t . This is the first term in the right hand side, since equilibrium prices are proportional to $u_{c,t}^2$, as we see from (83).

The second term in the right-hand side of (90), reflects the intertemporal price manipulation incentives of a large country, which can be either *positive* or *negative*. In particular, the monopolist takes into account how an increase in foreign consumption affects equilibrium prices. An increase in c_t^2 reduces foreign marginal utility and therefore *decreases* the respective price of consumption at t . The reduction in prices q_t is *beneficial* to Home, when Home is a net *buyer* of goods, $c_t^1 > Y_t^1$, since the home budget constraint is relaxed. Instead, a reduction in prices is *harmful* to Home, when Home is a net *seller* of goods, $c_t^1 < Y_t^1$.

Thus, the *net* position in final goods at t determines the incentives for price manipulation by H. Note that by using $c_t^2 = Y_t - c_t^1$, and given a Φ , we can solve from (90) for domestic consumption as a function of world's output *and* domestic output, that is, $c_t^1 = c^1(Y_t^1, Y_t; \Phi)$. To understand better the implications for the consumption allocation, rewrite (90) as

$$\begin{aligned} \frac{u_{c,t}^1}{u_{c,t}^2} &= \Phi \left[1 + \epsilon_{cc,t}^2 \frac{c_t^1 - Y_t^1}{c_t^2} \right] = \Phi \left[1 + \epsilon_{cc,t}^2 \frac{\frac{c_t^1 - Y_t^1}{Y_t}}{\frac{c_t^2}{Y_t}} \right] \\ &= \Phi \left[1 + \epsilon_{cc,t}^2 \frac{\theta_t^1 - s_t^1}{1 - \theta_t^1} \right], \end{aligned} \quad (91)$$

where $\epsilon_{cc,t}^2 \equiv -\frac{u_{cc}(c_t^2)}{u_c^2(c_t^2)} c_t^2$, the *elasticity* of the marginal utility of consumption of F, $\theta_t^1 \equiv c_t^1/Y_t$ and $s_t^1 \equiv Y_t^1/Y_t$, the share of domestic consumption and domestic output in world's output.

The price wedge and time-varying “Pareto” weights. Following the terminology of Costinot et al. (2014), define as the *price wedge* the term $\chi_t \equiv \frac{u_{c,t}^1}{\Phi u_{c,t}^2} - 1$. From (91) we see that the optimal price wedge is a function of Home's net position in final goods and $\epsilon_{cc,t}^2$,

$$\chi_t = \epsilon_{cc,t}^2 \frac{\theta_t^1 - s_t^1}{1 - \theta_t^1}. \quad (92)$$

We can rewrite the optimal consumption allocation condition (91) as

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \Phi_t, \quad (93)$$

where $\Phi_t \equiv \Phi(1 + \chi_t)$. Condition (93) show that the ratio of marginal utilities across countries

is *time-varying*, in contrast to the property of the Pareto-optimal allocation in (46) (or any of the competitive allocations we considered), since the price wedge depends on the *time-varying* net trading position $\theta_t^1 - s_t^1$ of Home, even when elasticities are constant. We can interpret Φ_t as an *inverse* “Pareto” weight, that varies over time due to *market power* of H.¹⁷

Power utility. We can make more progress in the analysis by assuming that both countries have a power utility function, $u(c) = (c^{1-\rho} - 1)/(1 - \rho)$. The homotheticity assumption allows as to express (91) in terms of time-varying *shares* in global output,

$$\left(\frac{\theta_t^1}{1 - \theta_t^1}\right)^{-\rho} = \Phi \underbrace{\left(1 + \rho \frac{\theta_t^1 - s_t^1}{1 - \theta_t^1}\right)}_{\chi_t}, \quad (94)$$

which allows to solve for the optimal share of Home consumption as a function of Home’s share in the world’s output, $\theta_t^1 = \theta^1(s_t^1)$. The following proposition shows how the consumption share depends on the Home’s output share, and solves for the consumption share in closed-form for $\rho = 1$.

Proposition 1. (*‘Optimal time-varying shares of consumption’*)

1. *The optimal share of Home’s consumption is an increasing function of the Home’s output share, $\frac{\partial \theta_t^1}{\partial s_t^1} > 0$.*
2. *The price wedge is a decreasing function of Home’s output share, $\frac{\partial \chi_t}{\partial s_t^1} < 0$.*
3. *For $\rho = 1$ we have*

$$\theta^1(s_t^1) = 1 - \frac{\Phi(1 - s_t^1)}{2} \left[\sqrt{1 + \frac{4}{\Phi(1 - s_t^1)}} - 1 \right]. \quad (95)$$

Proof. See Appendix. □

In contrast to a *constant* consumption share, that we would have at the cooperative solution, the unregulated case, or the case of a price-taking country-specific policymaker, proposition 1 shows that a larger share in the world’s output leads to a larger Home consumption. The reason is intuitive: a larger share in global output reduces the marginal benefit of increasing

¹⁷To see that, define the time-varying Pareto weights $\eta_t^1 \equiv \frac{1}{1+\Phi_t}$ and $\eta_t^2 \equiv \frac{\Phi_t}{1+\Phi_t}$, and express the ratio of marginal utilities as $\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{\eta_t^2}{\eta_t^1}$.

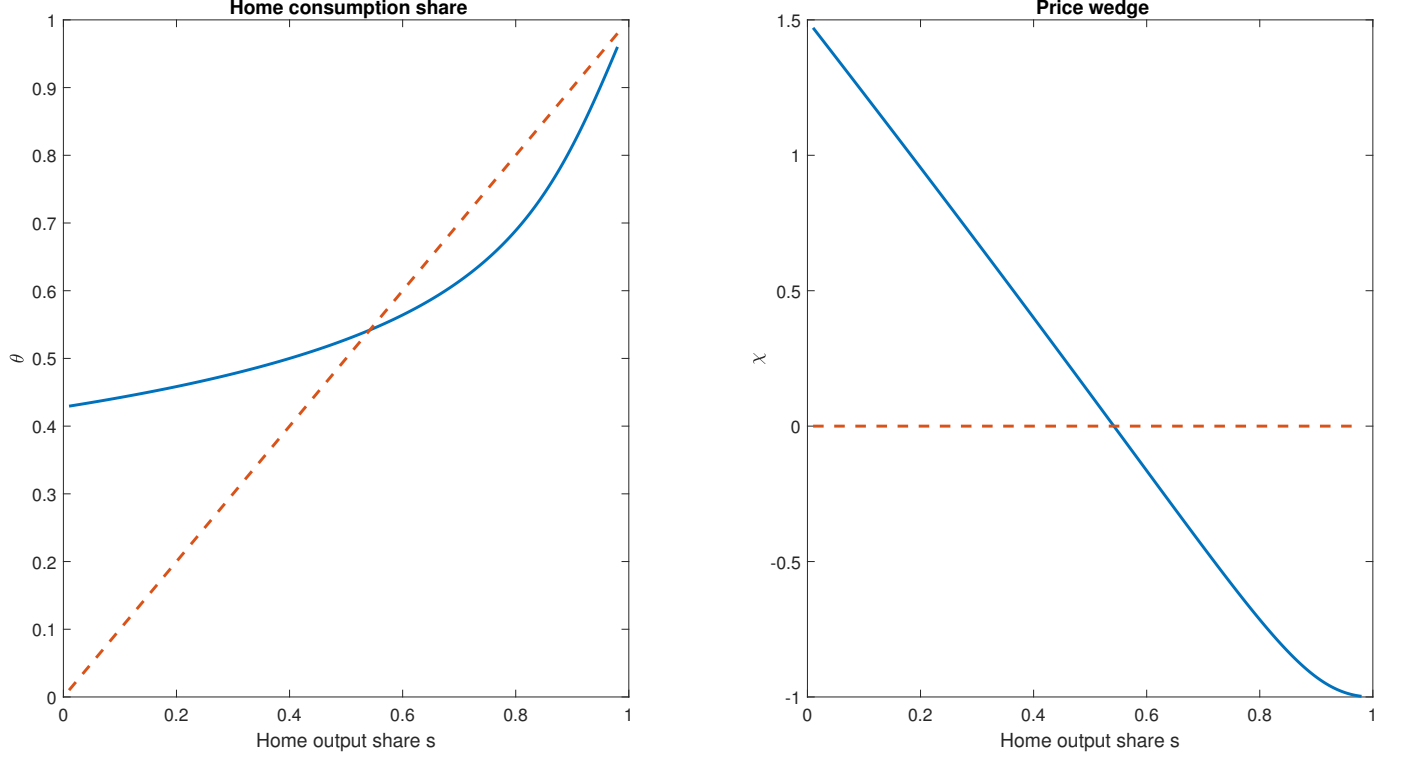


Figure 1: Let $\rho = 2$ and assume that $\Phi = 0.7141$. The left panel depicts the optimal consumption share θ as function of the Home output share s . The right panel depicts the respective price wedge χ .

foreign consumption to reduces prices, since the net trading position of Home becomes smaller. As a result, the share of foreign consumption falls and the share of domestic consumption increases. Figure 1 provides an illustration of the increasing consumption share (left panel) and the decreasing price wedge (right panel) of Proposition 1 for a given Φ .

Growth rates. To see what this result entails, assume that the share in world's output is becoming larger, $s_{t+1}^1 > s_t^1$, so Home's output grows at a faster rate than the world, and Foreign's output grows at a slower rate than the world, $\frac{Y_{t+1}^1}{Y_t^1} > \frac{Y_{t+1}^1 + Y_{t+1}^2}{Y_t^1 + Y_t^2} = s_t^1 \frac{Y_{t+1}^1}{Y_t^1} + (1 - s_t^1) \frac{Y_{t+1}^2}{Y_t^2} > \frac{Y_{t+1}^2}{Y_t^2}$. Proposition 1 implies that $\theta_{t+1}^1 > \theta_t^1$, so consumption growth at Home will be larger than consumption growth at Foreign, $\frac{c_{t+1}^1}{c_t^1} > \frac{Y_{t+1}^1 + Y_{t+1}^2}{Y_t^1 + Y_t^2} > \frac{c_{t+1}^2}{c_t^2}$.

4.4 Optimal capital controls

To dig deeper in the dynamics of the consumption allocation and its decentralization, consider the intertemporal marginal rates of substitution in the two countries. We have

$$p_t = \beta \frac{u_{c,t+1}^2}{u_{c,t}^2} = \frac{\Phi_t}{\Phi_{t+1}} \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} = \underbrace{\frac{1 + \chi_t}{1 + \chi_{t+1}}}_{\text{markup/markdown}} \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \quad (96)$$

The *change* in the price wedge χ_t introduces effectively a “markup” or a “markdown” of the international prices p_t over the domestic intertemporal marginal rate of substitution. The domestic policymaker can decentralize the allocation of problem 4 by using a tax/subsidy on borrowing τ_t^b , introducing effectively *capital controls*, together with the carbon tax τ_t .¹⁸ In particular, the household’s budget constraint for Home with these two instruments (τ_t^b, τ_t) takes the form

$$c_t^1 + p_t b_{t+1}^1 = w_t^1 + \Pi_t^1 + \pi_t^{1,\text{tax}} + T_t + (1 + \tau_t^b) b_t^1 + T_t^b, \quad (97)$$

where $\pi_t^{1,\text{tax}}$ the after-tax profits in the energy sector as in (58), $T_t = \tau_t E_t^1$, $T_t^b = -\tau_t^b b_t^1$, the proceeds of the two taxes that are rebated lump-sum. The Euler equation of the domestic household reads

$$p_t = (1 + \tau_{t+1}^b) \beta \frac{u_{c,t+1}^1}{u_{c,t}^1}. \quad (98)$$

When $\tau_{t+1}^b > 0$ we have a tax on borrowing (subsidy to saving). In the opposite case of $\tau_{t+1}^b < 0$, we have a subsidy to borrowing (tax on saving). To implement the allocation of the dynamic monopolist in (96), the Home policymaker can set

$$\tau_{t+1}^b = \frac{\chi_t - \chi_{t+1}}{1 + \chi_{t+1}}. \quad (99)$$

A decrease in the price wedge, $\chi_{t+1} < \chi_t$, implies a tax on borrowing/ subsidy to saving, $\tau_{t+1}^b > 0$. In contrast, an increase in the price wedge, $\chi_{t+1} > \chi_t$ implies a tax on saving/ subsidy to borrowing, $\tau_{t+1}^b < 0$. Given our results before for the power utility case, we have then the following: if Home grows faster than Foreign, ($s_{t+1}^1 > s_t^1$), then price wedges fall, $\chi_{t+1} < \chi_t$, so the policymaker uses a tax on borrowing $\tau_{t+1}^b > 0$. If Home grows slower than Foreign, then price wedges increase, $\chi_{t+1} > \chi_t$, and Home uses a tax on domestic savings $\tau_{t+1}^b < 0$. If the two

¹⁸Costinot et al. (2014) have emphasized optimal capital controls for a large country in an endowment economy without externalities. Note that we could have an equivalent decentralization in terms of *consumption* taxes τ_t^c , by setting $\tau_t^c = \chi_t$.

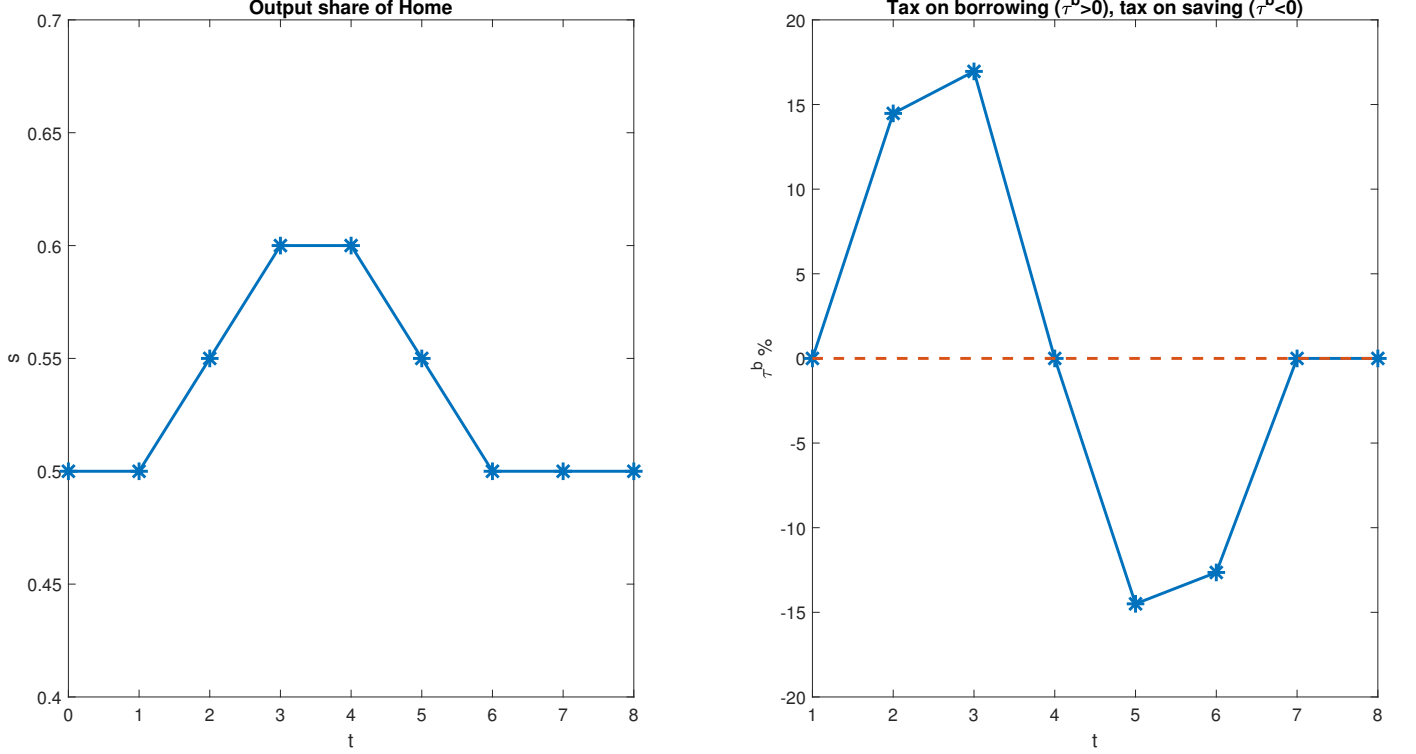


Figure 2: Indicative path of taxes (right panel) on borrowing ($\tau^b > 0$) or saving ($\tau^b < 0$) for a path of s (left panel) for $\rho = 2$ and $\Phi = 0.7141$.

countries grow at the same rate, then $\chi_t = \chi_{t+1}$ and $\tau_{t+1}^b = 0$. The same result holds if a BGP is reached in the long-run, $\tau^b = 0$.

Power utility and borrowing tax. Turning to the power utility case, we can express the borrowing tax as the difference between the log-change in the home share consumption share and the foreign consumption share,

$$\tau_{t+1}^b \simeq \ln(1 + \tau_{t+1}^b) = \rho \cdot \left[\ln \frac{\theta_{t+1}^1}{\theta_t^1} - \ln \frac{1 - \theta_{t+1}^1}{1 - \theta_t^1} \right] \quad (100)$$

Illustration. To get a sense about the dynamics of the tax on borrowing τ_t^b , figure 2 plots an indicative path for the Home's share of output and calculates the respective tax. The left panel of figure 2 denotes the evolution of the home output share. The home output share s_t^1 stays constant for one period, then increases for two periods, stays constant again for one period, and then decreases for two periods, till it becomes constant again. Equivalently, the *output growth* rate of H is the same as F's for one period. For two periods H grows *faster* than F, then it grows the same for one period, and then H grows *slower* than F for two periods, till it reaches again the

same growth rate. The right panel of 2 denotes the respective borrowing tax. In period where the growth rate of H is the same as F, there is a zero tax. In periods when H grows faster, there is a borrowing tax ($\tau^b > 0$), and in periods where H grows slower than F there is a borrowing subsidy ($\tau^b < 0$), or else, a tax on saving.

4.5 Optimal carbon taxes

The optimal intersectoral allocation of labor is determined as usual by

$$(1 - D_t^1)A_t^1 F_{n,t}^1 = \mu_t^M \times z_t^1 f_n^1(1 - n_t^1), \quad (101)$$

where μ_t^M captures the shadow value of energy for the monopolist,¹⁹

$$\mu_t^M = (1 - D_t^1)A_t^1 F_{E,t}^1 \underbrace{- \xi_t^M H_{E,t}}_{\text{externality}}. \quad (102)$$

Shadow cost of emissions. The carbon tax that decentralizes the allocation is $\tau_t^M \equiv \xi_t^M H_{E,t}$. The *crucial* difference in the dynamic monopolist problem is that the shadow cost of the stock of emissions ξ_t^M follows the recursion

$$\xi_t^M = \underbrace{D_{S,t}^1 \hat{Y}_t^1}_{\text{current damages of H (pricetaker)}} + \underbrace{\left(1 - \Phi \frac{u_{c,t}^2}{u_{c,t}^1}\right) D_{S,t}^2 \hat{Y}_t^2}_{\text{interest rate manipulation} \times \text{current damages of F}} + \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \xi_{t+1}^M H_{S,t+1} \quad (103)$$

Recall that in the cooperative solution (49), the planner takes into account the *global* marginal damages. Moreover, at the other extreme, a *price-taking* country-specific policymaker, takes into account only the *country-specific* marginal damages, as we saw in (76).

Here instead, the shadow cost ξ_t^M has *two* components, as displayed in the first two terms on the right-hand side of (103): a) the current domestic marginal damages, as in the case of a price-taking policymaker, b) a second term that is associated with *foreign* marginal damages. This is so, even if H cares only for its own utility.

Why does the domestic policymaker takes into account the damages of F? Foreign damages *reduce* foreign net output, and therefore affect the *trading* position of H with F. A large country that understands how prices are formed, cares also for the net amount of goods that it will

¹⁹As usual, the shadow values μ_t^M and ξ_t^M are the respective multipliers on energy production and the stock of emissions in the monopolist problem, divided by the marginal utility of domestic consumption.

exchange with F. To see clearly the implications for this fact for the optimal carbon tax, use the definition of the price wedge to rewrite (103) as

$$\xi_t^M = D_{S,t}^1 \hat{Y}_t^1 + \underbrace{\frac{\chi_t}{1 + \chi_t}}_{+/-} D_{S,t}^2 \hat{Y}_t^2 + \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \xi_{t+1}^M H_{S,t+1}, \quad (104)$$

where the price wedge χ_t satisfies (92), and is positive (negative) if Home is a net buyer (seller) of goods. Thus, a *fraction* $\chi_t/(1 + \chi_t)$ of foreign damages enter the determination of the domestic shadow cost of emissions, and therefore, the domestic carbon tax.

Carbon taxes or subsidies. If there is no intertemporal trade between the two countries at t , $c_t^1 = Y_t^1$, we have $\chi_t = 0$. In that special case, (104) instructs us that only the *domestic* current marginal damages matter for the determination of the current period component of the shadow cost of emissions, as in the case of the price-taking policymaker. If, instead, Home is a net *buyer* of goods, $c_t^1 > Y_t^1$, we have $\chi_t > 0$. In that case, foreign damages enter with a *positive* sign the shadow cost of emissions. The reason is intuitive: emissions reduce net foreign output. This is *harmful* for H, if H is a net *importer* of goods, $c_t^1 > Y_t^1$. As a result, H partially *internalizes* the externality of the foreigners, and imposes a carbon tax that is *larger* than its own domestic damages.

In contrast, foreign damages enter *negatively* the shadow cost of emissions ξ_t^M , if H is a net *seller* of goods, $c_t^1 < Y_t^1$, since in that case the price wedge becomes negative, $\chi_t < 0$. Thus, the reduction in net foreign output is *beneficial* for H, if H is a *net exporter* of goods. Consequently, the total current component of ξ_t^M is *below* the respective country-specific damages, $D_{S,t}^1 \hat{Y}_t^1$. So, the incentives for carbon taxation are *mitigated* for a net exporter. Interestingly enough, if the benefit of carbon emissions (for a net exporter) through the reduction of foreign output exceeds the domestic damages, the carbon tax becomes *negative*, that is, a carbon *subsidy*.

The discussion above takes into account only the contribution of the *current* domestic and foreign damages to the carbon tax, muting the fact that emissions are durable. Solving (104) forward, we see that the carbon tax will be determined by the present discounted value of domestic damages, and the present discounted value of a fraction of foreign damages, that is determined by the current and *future* trading position with F.

Zero domestic damages. To see clearer the potential for a subsidy, assume the extreme case where the home country has *no* damages, so $D^1(S_t) = 0 \forall t$, and therefore $D_{S,t}^1 \equiv 0 \forall t$. In that case, (104) reduces to

$$\xi_t^M = \underbrace{\frac{\chi_t}{1 + \chi_t} D_{S,t}^2 \hat{Y}_t^2}_{+/-} + \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \xi_{t+1}^M H_{S,t+1}. \quad (105)$$

In contrast to a price-taking policymaker, who would set a zero carbon tax, the shadow cost of the stock of emissions for the monopolist is *not* zero, as long as there is intertemporal trade. If H is *persistently* selling goods abroad, then there is the incentive to subsidize energy, shifting labor to the energy sector, and generating more carbon emissions.

Parametric example. Turning to our parametric example, the carbon tax (or subsidy) we can see clearly how domestic damages, foreign damages, and the current and future trading positions affect the optimal carbon tax as a share of global output:

$$\begin{aligned} \frac{\tau_t^M}{Y_t} = & \underbrace{\gamma^1 \sum_{i=0}^{\infty} \beta^i \left(\frac{\theta_{t+i}^1}{\theta_t^1} \right)^{-\rho} \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} (1 - d_i) s_{t+i}^1}_{\text{PV of domestic damages}} \\ & + \underbrace{\gamma^2 \sum_{i=0}^{\infty} \beta^i \left(\frac{\theta_{t+i}^1}{\theta_t^1} \right)^{-\rho} \left(\frac{Y_{t+i}}{Y_t} \right)^{1-\rho} (1 - d_i) (1 - s_{t+i}^1) \frac{\chi_{t+i}}{1 + \chi_{t+i}}}_{\text{PV of trading positions} \times \text{foreign damages}}, \end{aligned} \quad (106)$$

where all variables are evaluated at the solution of the monopolist's problem. The first term in (106) reflects the present value of domestic marginal damages (see (77) for the price-taker's carbon tax), and the second term the amplification of mitigation of incentives for carbon taxation depending on the current and future trading positions with F, as captured by $\frac{\chi_{t+i}}{1 + \chi_{t+i}}$.²⁰ If we reach a BGP with a common growth rate g , and recalling that $\tilde{\beta} \equiv \beta(1 + g)^{1-\rho} < 1$, the carbon tax/subsidy in (106) simplifies to

$$\frac{\tau_t^M}{Y_t} = \underbrace{\left(s^1 \gamma^1 + \frac{\chi}{1 + \chi} (1 - s^1) \gamma^2 \right)}_{\neq \gamma^{global}} \cdot \left[\frac{\phi_L}{1 - \tilde{\beta}} + \frac{(1 - \phi_L) \phi_0}{1 - \tilde{\beta}(1 - \phi)} \right]. \quad (107)$$

The monopolist's carbon tax at the BGP is not reflecting the global damage sensitivity $\gamma^{global} = \sum_l s^l \gamma^l$ that we saw in (69). Moreover, in contrast to the case of the tax on borrowing or saving τ^b , which is *zero* at the BGP, the domestic carbon tax/subsidy reflects the marginal damages of

²⁰Note that the discounting in (106) reflects the fact that the consumption share θ_t^1 is time-varying, in contrast to the respective formulas for the cooperative case in (67), or the price-taker's case in (77).

F not only at the transition, but also in the *long run*, as long as there is intertemporal trade at the BGP.

4.6 An altruistic domestic policymaker

In the previous section we analyzed a large domestic policymaker that maximizes the utility of the *domestic* household, ignoring the repercussions of the domestic carbon tax and capital controls on the welfare of F. Given the wide-ranging implications of climate change, it is of interest to consider also a situation where the domestic policymaker exhibits *altruism*, and maximizes a weighted average of the utility of H and F.²¹

Problem 5. (*‘A large altruistic domestic policymaker’*)

Let η and $1 - \eta$ denote the welfare weights on H and F respectively, $\eta \in (0, 1)$. The problem of the altruistic policymaker in H is to choose $\{c_t^1, n_t^1, E_t^1, c_t^2, S_t\}$ to maximize

$$\eta \sum_{t=0}^{\infty} \beta^t u^1(c_t^1) + (1 - \eta) \sum_{t=0}^{\infty} \beta^t u^2(c_t^2) \quad (108)$$

subject to

$$c_t^1 + c_t^2 = (1 - D^1(S_t))A_t^1 F^1(n_t^1, E_t^1) + (1 - D^2(S_t))\hat{Y}_t^2 \quad (109)$$

$$E_t^1 = z_t^1 f^1(1 - n_t^1) \quad (110)$$

$$S_t = H(S_{t-1}, E_t^1 + E_t^2) \quad (111)$$

$$\sum_{t=0}^{\infty} \beta^t u_c^2(c_t^2) [c_t^2 - (1 - D^2(S_t))\hat{Y}_t^2] = 0, \quad (112)$$

where $c_t^1, c_t^2 \geq 0$, $n_t^1 \in [0, 1]$, and S_{-1} is given.

The constraints in the altruist’s problem 5 are the same as the constraints in the monopolist’s problem 4. Problem 4 is recovered if we put a unitary weight on the utility of H, $\eta = 1$.

Optimal consumption and taxes on borrowing/saving. The optimal consumption allocation of H and F is determined by

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \underbrace{\frac{1 - \eta}{\eta}}_{\text{ratio of weights}} + \frac{\Phi}{\eta} \left[1 + \underbrace{\epsilon_{cc,t}^2 \frac{\theta_t^1 - s_t^1}{1 - \theta_t^1}}_{\text{price manipulation}} \right], \quad (113)$$

²¹See the Appendix for the derivations behind the results in this section.

which, for $\eta = 1$, reduces to the case we considered in (91). An increase in consumption c_t^2 increases now the utility of the altruistic home country ($\eta < 1$), and reduces prices, which may be beneficial or harmful for H depending on the trade balance. As a result, the ratio of marginal utilities is determined now by the *ratio* of welfare weights $(1 - \eta)/\eta$ and the price manipulation incentives that we identified previously. Define as the price wedge χ_t

$$\chi_t \equiv \left[\frac{u_{c,t}^1}{u_{c,t}^2} - \frac{1 - \eta + \Phi}{\eta} \right] \frac{\eta}{\Phi}, \quad (114)$$

which reduces to the definition of the price wedge for $\eta = 1$, $\chi_t \equiv \frac{u_{c,t}^1}{\Phi u_{c,t}^2} - 1$. From (113) we get the same optimal price wedge as in (92), $\chi_t = \epsilon_{cc,t}^2 \frac{\theta_t^1 - s_t^1}{1 - \theta_t^1}$. The optimal tax on borrowing/saving is then

$$\tau_{t+1}^b = \frac{\Phi(\chi_t - \chi_{t+1})}{1 - \eta + \Phi(1 + \chi_{t+1})}. \quad (115)$$

Even in the altruistic case, the intertemporal distortion is determined by the variation in χ_t . If $\chi_t > (<) \chi_{t+1}$, we have a tax on borrowing (saving), $\tau_{t+1}^b > (<) 0$.

Optimal carbon tax. The shadow value of the altruistic policymaker, μ_t^A takes the same form as in (102), implying an optimal carbon tax equal to $\tau_t^A = \xi_t^A H_{E,t}$, where ξ_t^A the shadow cost of emissions for the altruistic policymaker. The shadow cost of emission ξ_t^A obeys the forward recursion

$$\xi_t^A = D_{S,t}^1 \hat{Y}_t^1 + \underbrace{\frac{1 - \eta + \Phi \chi_t}{1 - \eta + \Phi(1 + \chi_t)}}_{+/-} D_{S,t}^2 \hat{Y}_t^2 + \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \xi_{t+1}^A H_{S,t+1}. \quad (116)$$

Equation (116) nests the case of a selfish policymaker in (104) for $\eta = 1$. Foreign marginal damages now matter both because the policymaker cares about the utility of F, and because of the manipulation of intertemporal prices, captured by χ_t . We may still have the case where foreign damages enter the determination of the shadow cost of emissions negatively, creating an incentive for a carbon *subsidy*. However, the price wedge has to be negative enough, $\chi_t < (\eta - 1)/\Phi$. That is, the cost of foreign damages, which reduce the utility of the policymaker by $1 - \eta$ has to be smaller than the benefit of increasing prices as an exporter of goods ($\Phi \chi_t$).

Furthermore, even if there is zero trade at time t , $\chi_t = 0$, the altruistic policymaker takes into account the foreign damages for the determination of period t component of the cost of

emissions. In that case, we see from (116) that the altruist puts a weight $\frac{1-\eta}{1-\eta+\Phi} < 1$ on foreign marginal damages, which is *smaller* than the unitary weight of the global social planner in (49). Thus, the large altruist that faces a passive rest of the world would value less current foreign damages than the global planner, even if there is no intertemporal trade at t .

5 Concluding remarks

In this paper, I study the optimal climate policy in a global economy, taking into account the international dimension of climate externalities. I start with the optimal cooperative policy, that prescribes a global carbon tax that corrects the climate externality. I move beyond cooperation and analyze the optimal unilateral carbon taxes and capital controls of a large country that faces a passive rest of the world. The desire to manipulate global interest rates may diminish the incentives of large net exporters to impose a carbon tax.

A Dynamic monopolist's problem

Assign multipliers $\{\beta^t \lambda_t^M, \beta^t \tilde{\mu}_t^M, \beta^t \tilde{\xi}_t^M\}$ and $\Phi \geq 0$ on (86), (87), (88) and (89) respectively. The Lagrangian of the monopolist's problem is

$$\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t u^1(c_t^1) - \sum_{t=0}^{\infty} \beta^t \lambda_t^M [c_t^1 + c_t^2 - (1 - D^1(S_t))A_t^1 F^1(n_t^1, E_t^1) - (1 - D^2(S_t))\hat{Y}_t^2] \\
& - \sum_{t=0}^{\infty} \beta^t \tilde{\mu}_t^M [E_t^1 - z_t^1 f^1(1 - n_t^1)] + \sum_{t=0}^{\infty} \beta^t \tilde{\xi}_t^M [S_t - H(S_{t-1}, E_t^1 + E_t^2)] \\
& + \Phi \sum_{t=0}^{\infty} \beta^t u_c^2(c_t^2) [c_t^2 - (1 - D^2(S_t))\hat{Y}_t^2].
\end{aligned} \tag{A.1}$$

First-order necessary conditions are

$$c_t^1 : \quad u_{c,t}^1 = \lambda_t^M \tag{A.2}$$

$$n_t^1 : \quad \lambda_t^M (1 - D_t^1) A_t^1 F_{n,t}^1 = \tilde{\mu}_t^M z_t^1 f_n^1(1 - n_t^1) \tag{A.3}$$

$$E_t^1 : \quad \tilde{\mu}_t^M = \lambda_t^M (1 - D_t^1) A_t^1 F_{E,t}^1 - \tilde{\xi}_t^M H_{E,t} \tag{A.4}$$

$$S_t : \quad \tilde{\xi}_t^M = \lambda_t^M D_{S,t}^1 \hat{Y}_t^1 + (\lambda_t^M - \Phi u_{c,t}^2) D_{S,t}^2 \hat{Y}_t^2 + \beta \tilde{\xi}_{t+1}^M H_{S,t+1} \tag{A.5}$$

$$c_t^2 : \quad \lambda_t^M = \Phi [u_{c,t}^2 + u_{cc,t}^2(c_t^2)(c_t^2 - Y_t^2)] \tag{A.6}$$

Use (A.2) to eliminate λ_t^M from (A.6), and use the fact that $c_t^2 - Y_t^2 = Y_t^1 - c_t^1$, to get (90). Divide (A.3), (A.4) and (A.5) with $\lambda_t^M = u_{c,t}^1$, and define $\mu_t \equiv \tilde{\mu}_t^M / \lambda_t^M$, $\xi_t^M \equiv \tilde{\xi}_t^M / \lambda_t^M$, to get respectively (101), (102) and (103).

B Proof of proposition 1

I drop here the time indices and the superscripts from (94). We have

$$\left(\frac{\theta}{1 - \theta} \right)^{-\rho} = \Phi [1 + \rho \frac{\theta - s}{1 - \theta}], \tag{B.1}$$

where θ and s correspond to the consumption share and production share of H in the world's output respectively. Equation (B.1) determines the optimal share θ as function of s , $\theta = \theta(s)$ (given the constant Φ). Differentiate implicitly with respect to s and simplify to get

$$\theta'(s) = \frac{\Phi(1-\theta)}{\Phi(1-s) + \left(\frac{\theta}{1-\theta}\right)^{-\rho-1}} > 0 \quad (\text{B.2})$$

Similarly, the optimal price wedge χ is a function of s through θ ,

$$\chi(s) = \rho \frac{\theta(s) - s}{1 - \theta(s)} \quad (\text{B.3})$$

with

$$\chi'(s) = \rho \frac{\theta'(s)(1-s) - (1-\theta)}{(1-\theta)^2} < 0, \quad (\text{B.4})$$

since $\theta'(s) < \frac{1-\theta}{1-s}$. To see that, note that $\theta'(s) = \frac{\Phi(1-\theta)}{\Phi(1-s) + \left(\frac{\theta}{1-\theta}\right)^{-\rho-1}} < \frac{\Phi(1-\theta)}{\Phi(1-s)} = \frac{1-\theta}{1-s}$.

Logarithmic case. For $\rho = 1$, (B.1) simplifies to $\left(\frac{\theta}{1-\theta}\right)^{-1} = \Phi \frac{1-s}{1-\theta}$, which reduces to finding the root of a quadratic in θ ,

$$Q(\theta) \equiv \theta^2 - (2 + \Phi(1-s))\theta + 1 = 0. \quad (\text{B.5})$$

We have $Q(0) = 1 > 0$ and $Q(1) = -\Phi(1-s) < 0$. The root in $(0, 1)$ is given by

$$\theta(s) = 1 - \frac{\Phi(1-s)}{2} \left[\sqrt{1 + \frac{4}{\Phi(1-s)}} - 1 \right], \quad (\text{B.6})$$

which delivers the expression in the proposition.

C Altruist's problem

Assign multipliers $\{\beta^t \lambda_t^A, \beta^t \tilde{\mu}_t^A, \beta^t \tilde{\xi}_t^A\}$ and $\Phi \geq 0$ on (109), (110), (111) and (112) respectively, and form a similar Lagrangian as in (A.1). Besides the obvious change in the superscripts of the multipliers from M to A , the first-order necessary conditions with respect to n_t^1 , E_t^1 , and S_t are the same as conditions (A.3), (A.4) and (A.5), and will not be repeated. The first-order conditions with respect to c_t^1 and c_t^2 are:

$$c_t^1 : \quad \eta u_{c,t}^1 = \lambda_t^A \quad (\text{C.1})$$

$$c_t^2 : \quad \lambda_t^A = (1 - \eta)u_{c,t}^2 + \Phi[u_{c,t}^2 + u_{cc,t}^2(c_t^2 - Y_t^2)] \quad (\text{C.2})$$

Use (C.1) to eliminate λ_t^A from (C.2), express the resulting condition in terms of the elasticity $\epsilon_{cc,t}^2$, and use $c_t^2 - Y_t^2 = Y_t^1 - c_t^1$ to get (113) in the text. Given the definition of the price wedge in (114), we get $\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{1-\eta+\Phi(1+\chi_t)}{\eta}$, which is used to get the tax on borrowing/saving in (115).

Lastly, the shadow cost of emissions $\xi_t^A \equiv \tilde{\xi}_t^A / \lambda_t^A$ follows the recursion

$$\xi_t^A = D_{S,t}^1 \hat{Y}_t^1 + \left(1 - \Phi \frac{u_{c,t}^2}{\eta u_{c,t}^1}\right) D_{S,t}^2 \hat{Y}_t^2 + \beta \frac{u_{c,t+1}^1}{u_{c,t}^1} \xi_{t+1}^A H_{S,t+1} \quad (\text{C.3})$$

Since $\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{1-\eta+\Phi(1+\chi_t)}{\eta}$, we have

$$1 - \frac{\Phi u_{c,t}^2}{\eta u_{c,t}^1} = 1 - \frac{\Phi}{1 - \eta + \Phi(1 + \chi_t)} = \frac{1 - \eta + \Phi \chi_t}{1 - \eta + \Phi(1 + \chi_t)} \quad (\text{C.4})$$

Use (C.4) in (C.3) to get (116).

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