FISCAL AND MONETARY POLICY IN A MONETARY UNION:

CREDIBLE INFLATION TARGETS OR MONETISED DEBT?

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Abstract

The paper examines the interrelationship between fiscal and monetary policy in a two-country monetary union. The worst scenario occurs when an independent central bank (CB) sets the nominal interest rate and responds to rising government debt/GDP ratios by monetisation. The result is high inflation, high debt/GDP ratios and a large public sector. Government debt and inflation are contained if the governments bear sole responsibility for solvency, but the public sector remains excessively large. The best scenario occurs if the CB removes the incentive for the governments to engineer surprise inflation by credible inflation targeting.

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1.INTRODUCTION

The case for fiscal policy co-ordination within a monetary union, as proposed in the European context by the Maastricht agreement, remains a highly controversial issue. Two aspects of this question can be distinguished. The first aspect is 'purely fiscal', arises in a non-monetary economy, and focuses on possible externalities associated with the uncoordinated conduct of fiscal policy for a given inflation rate. The second - our main focus - arises from the interdependence of monetary and fiscal policy and the concern that decentralised fiscal policy will undermine the successful conduct of a low-inflation monetary policy by the central bank.

In examining the relationship between monetary and fiscal policy, a useful point of departure is the seminal study by Sargent and Wallace (1981) of the monetary-fiscal coordination problem which faces any central bank. Their title, 'Unpleasant Monetarist Arithmetic' alludes to the fact that a central bank cannot successfully control inflation if the fiscal authorities persist in running primary budget deficits. More specifically they assume a quantity theory demand schedule for money (embodying 'unadulterated monetarism') and that the real interest rate is constant and higher than the GDP growth rate. Government debt as a proportion of GDP can grow until some time $\bar{t}$, after which it must be stabilised. This is achieved by the central bank monetising the deficit. They conclude that tighter monetary policy up to time $\bar{t}$ causes higher inflation after that date.

Sargent and Wallace represent the actions of fiscal authorities as a series of deficits excluding seigniorage. A central feature of our paper is the use of dynamic game equilibria to characterise a number of strategic relationships between policymakers of which 'unpleasant monetary arithmetic' is just one possible scenario. We follow recent approaches to macro-modelling in constructing a two-country model of monetary union which is based on rigorous micro-foundations. Section 2 sets out the model which consists of two economies with identical economic structures but producing two goods which are imperfect substitutes. A central bank sets the common, nominal rate of interest. As an alternative to the two-good version of monetary union we also consider the one-good case in order to examine the implications of complete economic integration.

Section 3 sets out the game-theoretic framework in which monetary union is a game played between two national governments, the ’fiscal authorities’, and one central bank (CB). The possible gains from the full coordination of fiscal and monetary policy are examined by comparing a cooperative equilibrium, in which the global welfare function consists of the average of the three players’ individual welfare functions, with the non-cooperative Nash equilibrium. Similarly we examine the gains from fiscal policy coordination alone, with an independent CB, by treating the fiscal authorities as one coordinated player in a two-player non-cooperative equilibrium. The emphasis is on time-consistent equilibria, but the socially optimal regime (full cooperation between all three players where policymakers enjoy a reputation for precommitment) provides a benchmark. Section 4 provides the main results in the form of simulations of the calibrated model and section 5 concludes the paper.

2. THE MODEL.

We first set out the two-good, two-country model of monetary union. Each economy consists of a large number of competitive firms. Population and labour supply grow at the same constant exogenous rate. The goods market clears instantly, but unions representing ’insiders’ prevent a similar outcome in the labour market. Nominal wage contracts provide the familiar output gains from inflation surprises. Labour-augmenting technical change occurs at a constant exogenous rate. Consumers’ wealth consists of government bonds issued by both domestic and foreign governments, capital and real money balances. Government spending is financed by borrowing, distortionary taxes and seigniorage subject to a government solvency constraint. Domestic and overseas bonds are perfect substitutes. Time is discrete[1]. The details of the model are as follows:

Households and Government

Each economy is composed of overlapping generations of identical consumers, each of whom faces a constant probability p of death.(See table 2.1 for a summary of all notation). The single-period utility function of the consumer is logarithmic in private and public consumption and money is held for the purposes of purchasing both domestic and imported goods. For the consumer in the 'domestic' country, born in period s, the expected utility at time t≥s, in the absence of any uncertainty apart from death, is then given by

$$U_{t,s} = \sum_{i=t}^{\infty} \left[\frac{1-p}{1+\theta}\right]^{i-t} [\gamma_1 \log C_{di,s} + \gamma_2 \log C_{mi,s} + \gamma_3 \log G_{i,s} + \gamma_4 \log M_{i-1,s}/P_i + \gamma_5 \log M_{i-1,s}/P_i^*] \tag{2.1}$$

where γ_i are positive parameters satisfying $\gamma_1+\gamma_2+\gamma_4+\gamma_5=1$. $C_{di,s}$, $C_{mi,s}$ and $G_{i,s}$ denote consumption of the domestically produced good, the foreign imported good and exhaustive government expenditure respectively, at time i. The latter is assumed to consist of the domestic good only. $M_{i-1,s}$ denotes the nominal (high-powered) money stock at the beginning of the period i, P_t is the price of domestic output, P_t^* is the price of foreign output and θ is the consumer's pure rate of time preference.

Following Yaari (1965), Blanchard (1985) and Frenkel and Razin (1987), life-cycle aspects of labour income are ignored and household labour supply is assumed to be fixed. Households leave no bequests or debts to their heirs. Instead, life insurance companies inherit each consumer's non-human wealth (or debt) and pay out (or receive) a premium during their life-time. This latter feature of the model together with population growth causes Ricardian equivalence to break down. Consumers' non-human wealth $V_{t,s}$ consists of government bonds issued by both governments, capital and real money balances; i.e.,

[1] Our use of discrete time follows Frenkel and Razin (1987), but differs from much of the literature in this area which uses continuous time. This choice is not fundamental but the dynamic game concepts turn out to be more transparent in discrete time.

$$V_{t,s}=M_{t,s}/P_t+D_{t,s}+F_{t,s}+K_{t,s} \tag{2.2}$$

where $D_{t,s}$ is (non-indexed) government debt held by domestic or foreign consumers. $F_{t,s}$ is net overseas assets and $K_{t,s}$ is domestic capital stock assumed to owned by domestic residents. All assets are measured at the end of the period and are expressed in units of domestic output. We assume the simplest residence-based tax structure: a constant tax rate is levied on all income of residents. Consumers receive an expected return $R_t^e(1-\tilde{T}_t)$ on their assets where $\tilde{T}_t$ is the average tax rate. Assuming that real returns are taxed, the effective expected nominal rate is $R_{nt}(1-\tilde{T}_t)+\tilde{T}_t\Pi^e_{t+1,t}$ where R_{nt} is the nominal interest rate and $\Pi_t=(P_t-P_{t-1})/P_{t-1}$ is the inflation rate.

The individual consumer born in period s maximises (2.1) given her budget constraint, the tax structure and rational expectations of for the nominal and real interest rate. Let 'total' consumption including and excluding foregone expected interest payments on money balances be denoted by $C_{t,s}$ and $\hat{C}_{t,s}$ respectively. Carrying out the individuals's optimisation problem and aggregating over the all generations leads to the following aggregate consumption and demand for money functions:

$$C^e_{t+1,t}=(1+R^e_{t,t}(1-\tilde{T}_t)-\theta+g)C_t-(p+g)(p+\theta)V^e_{t+1,t} \tag{2.3}$$

$$M_{t-1}/P_{t-1}=(\gamma_5+\gamma_6)C^e_{t,t-1}/(R_{nt-1}(1-\tilde{T}_{t-1})+\tilde{T}_{t-1}\Pi^e_{t,t-1}) \tag{2.4}$$

$$C_{dt}=\gamma_1 C_t \;;\; C_{mt}=\gamma_2 C_t/E_t \;;\; \hat{C}_t=(\gamma_1+\gamma_2)C_t \tag{2.5}$$

where $V_t=M_t/P_t+D_t+F_t+K_t$ is aggregate non-human end-of-period wealth and $E_t=P^*_t/P_t$ is the relative price of foreign to domestic output.[2] In (2.3), $C^e_{t+1,t}$ denotes rational expectations of C_{t+1} formed at time t. Under perfect foresight $C^e_{t+1,t}=C_{t+1}$ in equilibrium, but we retain the expectational superscript to emphasize that C_t is a forward-looking 'jump' variable. The expected or **ex ante** real interest rate $R^e_{t,t}=R_{nt}-\Pi^e_{t+1,t}$ differs from the **ex post** real interest rate (R_t), appearing in the budget

[2]This is the discrete time analogue of a result derived in Blanchard (1985) with added tax distortions. Second order terms in p,θ, R, g and n have been ignored.

identities, an important distinction when we consider credibility.

Foreign assets accumulate according to

$$\Delta F_t = \boldsymbol{R}_{t-1} F_{t-1} + \boldsymbol{B}_t \qquad (2.6)$$

where $B_t = C^*_{mt} - E_t C_{mt} + I^*_{mt} - E_t I_{mt}$ is the trade balance, I_{mt} is investment out of imported goods and I^*_{mt} is overseas investment out of domestic output. Throughout the paper, starred variables such as C^*_{mt} refer to the 'foreign' country. Government debt accumulation is given by the government budget identity introduced in section 3.

Firms and Labour Market

Aggregate output is assumed to be given by a Cobb-Douglas production function

$$Y_t = K^{\beta_1}_{dt-1} K^{\beta_2}_{mt-1} (A_t L_t)^{1-\beta_1-\beta_2} \qquad (2.7)$$

where K_{dt} (K_{mt}) is end-of-period capital stock accumulated out of domestic (foreign) output $A_t = A_0(1+\mu)^t$ represents Harrod-neutral technical change where μ is the productivity growth rate. In the steady-state, the GDP growth rate is n=μ+g where g is employment growth. Employment is in effect determined by unions who set the nominal wage to achieve a desired employment target $\hat{L}_t$ over the a one-period contract, based upon expectations P_t and given the firms' demand for labour function.

The representative firm's optimisation problem at time t is to choose an investment plan $\{I_{dt}\}$ and $\{I_{mt}\}$ out of domestic and foreign output respectively to maximise:

$$\sum_{i=o}^{\infty} \lambda_{t+i} \, (Y_t - W_{rt} L_t - I_{dt} - E_t I_{mt}) \qquad (2.8)$$

where $\lambda_{t+i} = 1/(1+\boldsymbol{R}_{t+i-1})\lambda_{t+i-1},\ i \geq 1$; $\lambda_t = 1$ defines λ_{t+i} and W_{rt} is the real wage, subject to

$$K_{dt} = (1-\delta)K_{dt-1} + I_{dt} \ ; \qquad K_{mt} = (1-\delta)K_{mt-1} + I_{mt} \qquad (2.9)$$

where δ is the depreciation rate which is assumed to be equal for the two types of capital. In carrying out this optimisation problem the firm takes the real interest rate $\{R_t\}$, the real wage $\{W_{rt}\}$, the labour supply $\{L_t\}$ equal to $\{\hat{L}_t\}$ and the relative price $\{E_t\}$ over the planning period as given. Then the first order conditions for profit-maximisation are:

$$\frac{K_{dt-1}}{Y_t} = \frac{\beta_1}{R^e_{t-1,t-1} + \delta} \quad ; \quad \frac{E_{t-1}K_{mt-1}}{Y_t} = \frac{\beta_2}{R^e_{t-1,t-1} + (E_{t-1} - E^e_{t,t-1}(1-\delta))/E_{t-1}} \tag{2.10}$$

Thus the capital-output ratios for the two types of capital are negatively related to the user cost of capital which includes depreciation; for imported capital the user cost must include the expected capital gains following a real exchange rate depreciation. We assume that profits and the opportunity cost of capital are taxed at the same rate. Then provided that profits are taxed net of depreciation costs, taxation does not affect the investment decisions of the firm and (2.10) still applies.

Turning to the labour market, the nominal wage W_{nt} is chosen in period t-1 to achieve the employment target $\hat{L}_t$ given expectations of P_t. Then in period t, given K_{dt-1},

K_{mt-1} and W_{nt}, actual employment satisfies the first order condition for profit-maximisation:

$$\frac{W_{nt}}{P_t} = \frac{\partial Y_t}{\partial L_t} = (1-\beta_1-\beta_2)K^{\beta_1}_{dt-1}K^{\beta_2}_{mt-1}A_t^{1-\beta_1-\beta_2}L_t^{-\beta_1-\beta_2} \tag{2.11}$$

given the current observed price level P_t. It follows that the desired employment satisfies

$$\frac{W_{nt}}{P^e_t} = (1-\beta_1-\beta_2)K^{\beta_1}_{dt-1}K^{\beta_2}_{mt-1}A_t^{1-\beta_1-\beta_2}\hat{L}_t^{-\beta_1-\beta_2} \tag{2.12}$$

$C_t, (\hat{C}_t)$	aggregate consumption including (excluding) foregone interest payments
C_{dt}, (C_{mt})	aggregate real consumption of domestic (imported) goods
R_{nt}	nominal interest rate
$\Pi_t=1-P_{t-1}/P_t$	inflation rate over period [t-1,t], where P_t=price level
$R_t=R_{nt}-\Pi_{t+1,t}$	expected real interest rate over period [t,t+1]
$E_t=P_t^*/P_t$	relative price of foreign to domestic output
M_t	end-of-period money balances
D_t	end-of-period non-indexed real government debt
K_t	end-of-period capital stock
K_{dt}, K_{mt}	end-of-period domestic, imported capital stock

I_{dt}, I_{mt}	domestic, imported investment goods
F_t	net overseas assets
V_t	total consumer real wealth
T_t,($\tilde{T}_t$)	total taxation (the tax rate)
G_t	exhaustive government spending
Q_t	primary deficit including seigniorage receipts
Y_t	real output
L_t	employment
W_{nt}, W_{rt}	nominal wage and real wage
ϕ	proportion of indexed wage contracts
B_t	trade balance
g	population growth
p	probability of death
θ	consumers' rate of time preference
$n=\mu+g$	long-run GDP growth, where μ is productivity growth
δ	depreciation rate of capital

Table 2.1. Summary of Notation.

The general notation is: $\tilde{X}_t=X_t/Y_t$ (for GDP ratios). Lower case variables, such as y_t in table 4.1, are **proportional** deviations about the trend, (e.g. $y_t=(Y_t-\bar{Y}_t)/\bar{Y}_t$) where $\bar{Y}_t$ is the trend, and **absolute** deviations (e.g., $\tilde{g}_t=(\tilde{G}_t-\tilde{G})$ where $\tilde{G}$ is the long-run value of $\tilde{G}_t$) which are used for per GDP ratios, the inflation rate and interest rates.

Hence from (2.11) and (2.12) we have $(L_t/\hat{L}_t)^{(\beta_1+\beta_2)}=P_t/P_t^e$. If a proportion ϕ of contracts are indexed to the domestic price level this is generalised to

$$(L_t/\hat{L}_t)^{(\beta_1+\beta_2)}=(P_t/P_t^e)^{1-\phi} \tag{2.13}$$

Given $\hat{L}_t$, P_t, E_{t-1} and expectational variables, (2.13) then determine employment L_t. This completes the supply-side. Equating demand and supply and using the definition of the trade balance B_t after (2.6) then gives the following output market equilibrium condition

$$Y_t = C_{dt} + E_t C_{mt} + I_{dt} + E_t I_{mt} + G_{dt} + E_t G_{mt} + \boldsymbol{B}_t = C_{dt} + I_{dt} + G_{dt} + C^*_{mt} + I^*_{mt} + G^*_{mt} \tag{2.14}$$

Market Structure Within The Monetary Union: Models MU1 and MU2

Up to now we have assumed that monetary union will leave the two countries producing distinct goods. For this case, referred to as MU2, the dynamics of the relative price, $E_t = P^*_t / P_t$, are given by

$$\Delta E_t / E_{t-1} = (1 + \Pi^*_t)/(1 + \Pi_t) - 1 \tag{2.15}$$

Analogous equations for the foreign country then completes the model of a two-country, two-good monetary union. Now suppose that economic integration within the monetary union results in the two economies producing an identical product. We refer to this one-good monetary union as MU1. Then $E_t = P^*_t / P_t = 1$ and $R_t = R^*_t$. Let $\hat{C}_t$ be total consumption, excluding foregone interest payments on holdings of real money balances, as before. Then $\hat{C}_t = (\gamma_1 + \gamma_2) C_t$, which replaces C_{dt} and C_{mt} in (2.5), with total consumption (including foregone interest payments) C_t still given by (2.3). Then

$$Y_t + Y^*_t = \hat{C}_t + \hat{C}^*_t + I_t + I^*_t + G_t + G^*_t \tag{2.16}$$

describes output equilibrium. Aggregate demand can exceed aggregate supply leading to 'trade balances' given by $\boldsymbol{B}_t = Y_t - \hat{C}_t - I_t - G_t = -\boldsymbol{B}^*_t$. Otherwise model MU1 is as MU2.

Calibration

Dynamic games based on models with structural dynamics almost inevitably lead to solutions which are analytically intractable. Our model is quite rich in detail and analysis is only possible at most in the steady-state for ad hoc forms of government behaviour. By contrast, the dynamic game equilibria concepts employed in this paper are not analytically tractable and require numerical solutions. 'Deep' parameter values are chosen by first, solving for the steady-state given baseline values for fiscal and monetary instruments and second, calibrating the steady-state values of selected readily observed variables of the model to observed data.[3] There are of course many ways of doing this; in our calibration the observed variables and their values are chosen to be: n=Π=0.03, $\tilde{K}$=2.5, R=0.05, $\tilde{G}$=0.2, $\tilde{D}$=0.5, β=0.25. Then $\tilde{C}$, θ, δ, $\tilde{K}$, $\tilde{T}$ and $\tilde{V}/\tilde{C}$ can be determined endogenously. Table 2.2 summarises the details of the calibration and include the steady-state properties used.

Observed Variable	Value	Source
$n=\Pi=\tilde{M}$	0.03	OECD
R	0.05	OECD
$\tilde{K}$	2.50	OECD
$\tilde{G}$	0.20	OECD
$\tilde{D}$	0.50	OECD
$\beta_1=\beta_2$	0.125	OECD
$\gamma_1=\gamma_2$	0.5	Imposed
g	0.005	OECD
p	1/35	Imposed
ϕ	1/3	Imposed
E	1	Normalisation
Derived Parameter	**Value**	**Equation**
μ	0.025	$\mu=n-g$
δ	0.05	$\delta=\beta/\tilde{K}-R$
$\tilde{F}$	0	By symmetry
$\tilde{C}$	0.61	$\tilde{C}=1-(\delta+n)/(1+n)\tilde{K}-\tilde{G}$
$\tilde{T}$	0.21	$\tilde{D}=1/(R-n)[\tilde{T}-\tilde{G}+\tilde{M}]$
$\tilde{V}$	2.57	$\tilde{V}=\tilde{M}+\tilde{D}+\tilde{K}$
θ	0.02	$\theta=\frac{[R(1-\tilde{T})-n+g]\tilde{C}.\tilde{V}-(p+g)p}{\tilde{C}.\tilde{V}+p+g}$

Table 2.2 Details of the Calibration.

Source: OECD (1993) average of France, Germany, Italy and the UK.

Understanding the Model: Fiscal Expansion in One Country

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[3] This corresponds to the approach of Shoven and Whalley (1992).

In the model described there are three forms of fiscal spillovers. The first is that one country's public sector deficit or public expenditure or distortionary taxation can all exert permanent upward pressure on real interest rates and crowd out community-wide investment. This is in a sense a free-rider problem which only exists in a non-Ricardian world. The second externality is beggar-thy-neighbour in character and arises in the case of MU2 where each country specialises in the production of its own 'home' good. Then an improvement in either country's terms of trade can be engineered by a unilateral fiscal expansion. The third externality arises from labour market distortions. If the central bank cannot achieve credible inflation targets, then both countries can experience an increase in output from a surprise fiscal expansion in **either** country which causes global surprise inflation.

In order to understand these spillovers it is instructive to first consider some ad hoc policy changes on the calibrated model. Consider a 1% change in the government spending/GDP ratio relative to the steady-state (i.e., $\tilde{g}=1\%$) for the domestic country keeping the same ratio for the foreign country constant (i.e., $\tilde{g}^*=0$). For the ad hoc policy exercises of this section we introduce the following stabilisation rules, assumed to be credible, for taxes and the nominal interest rate:

$$\tilde{t}_t=\tau\tilde{d}_{t-1}\ ;\quad \tilde{t}_t^*=\tau\tilde{d}_{t-1}^* \qquad r_{nt}=\beta(\pi_t+\pi_t^*)/2 \quad \beta>1 \tag{2.18}$$

where τ>R-n is chosen to ensure government solvency. According to the interest rate rule in (2.18) the CB responds to an increase in inflation by increasing the real interest rate.

Figure 1 displays the trajectories for consumption, investment, government spending and the trade balance as proportions of GDP ($\tilde{c}$, $\tilde{i}$, $\tilde{g}$ and $\tilde{b}$ respectively), inflation (π) and the nominal interest rate (r_n) under MU1 and MU2. From the trajectories for consumption, investment, the trade balance and government spending shares it can be seen that fiscal expansion in the domestic country (an increase in $\tilde{g}$ and $\tilde{d}$) crowds out its consumption, exports and, after one year, investment. In the very short term inflation rises above the nominal interest rate, real interest rates fall and investment rises. However, the rule (2.18) ensures that the real interest rate quickly rises, crowding out investment. Households in the foreign country increase their savings ratio and acquire more overseas assets. In a rational expectations equilibrium, an inflation surprise can only take place in the first period when π_t^e is predetermined and set at zero. Output thus rises sharply at first but eventually falls as the capital stock falls in response to a rise in the real interest rate.

For the case of MU2, inflation rises in the foreign country and quickly converges to that of the domestic country. The main difference is that an improvement in the domestic terms of trade (a fall in E) occurs. This makes imported consumption goods cheaper for households in the domestic country and their welfare improves. In addition imported capital is cheaper bringing further supply-side gains. A fiscal expansion in the domestic country imposes considerable external costs on the foreign country because of a contraction of world savings, a higher real interest rate, a deterioration in the terms of trade (for the foreign country) and higher inflation. This externality is important for understanding the later simulation results for the two-good non-cooperative game.[4]

FIGURE 1 ABOUT HERE

3. THE MONETARY UNION POLICY GAME.

In our two-country model the monetary union policy game has three players - the two fiscal authorities and the CB. We consider these in turn.

The Fiscal Authorities.

The instruments of the fiscal authorities are the tax rate and government spending chosen subject to a budget constraint. To derive the latter we assume a simple distributional formula in which the total seigniorage accruing to the CB is distributed equally to each country in the form of fiscal transfers. Then the primary deficit of the domestic authority is

[4] The simulation model is available on request from the authors in the form of an executable program with facilities to change parameters and the rule (2.18).

$$Q_t = G_t - T_t - \frac{1}{2}\left[\frac{\Delta M_t + \Delta M_t^*}{P_t}\right] \tag{3.1}$$

where the last term are the seigniorage receipts. The government budget identity is then:

$$\tilde{D}_t = \left(\frac{1+R_{t-1}}{1+n_{t-1}}\right)\tilde{D}_{t-1} + \tilde{Q}_t \tag{3.2}$$

in per GDP form where n_t is the GDP growth rate over the interval [t,t+1]. Define the growth-adjusted real interest rate over the interval [t,t+1] by $1+\rho_t = (1+R_t)/(1+n_t)$. Then solving (3.2) forward in time the budget **identity** becomes the **solvency constraint**:

$$\tilde{D}_t = -\sum_{i=0}^{\infty} \frac{\tilde{Q}_{t+i+1}}{(1+\rho_t)(1+\rho_{t+2}) \;..\; (1+\rho_{t+i})} \tag{3.3}$$

provided that the tranversality condition holds - that is, in the long run $\tilde{D}_t$ grows at a rate less than ρ_t. In (3.3) we assume that eventually $\rho_t > 0$. This is a feature of the consumption/savings model in section 2 and rules out 'dynamic inefficiency'. According to (3.3) a government in debt with $\tilde{D}_t > 0$ must, sometime in the future, run primary surpluses to be solvent. Notice that solvency does not require a **stable** debt/GDP ratio $\tilde{D}_t$ but merely that, in the long run, it does not increase faster than the growth adjusted real interest rate. However in a world with even very small departures from perfectly functioning capital markets, the notion of unbounded government debt/GDP ratios does not appeal. A stronger concept of solvency - adopted in this paper - is that debt/GDP ratios stabilise in the long run. [5]

Following Calvo and Obstfeld (1988) we measure social welfare in terms of total private and public consumption aggregated over all consumers of different ages. The fiscal authority in the domestic country chooses paths for spending and taxes to maximise a national welfare function of the form

$$U_t = \sum_{i=t}^{\infty}\left[\frac{1-p}{1+\theta}\right]^{i-t}\left[\gamma_1^F logC_{di} + \gamma_2^F logC_{mi} + \gamma_3^F \log G_i - \tfrac{1}{2}(\gamma_4^F \Pi_i^2 + \gamma_5^F \tilde{T}_i^2 + \gamma_6^F \tilde{D}_i^2)\right] \tag{3.4}$$

Direct externalities in government spending are ignored. (See, however, Levine and Brociner (1994) for an assessment of these externalities). (3.4) differs from an

[5] Buiter and Patel (1990) provide an interesting discussion of this distinction.

aggregate welfare function based on (2.1) in that demand for money in the utility function is replaced by a more general term capturing inflation costs and we include a quadratic term that penalises high taxation capturing the costs of collection. The final term in (3.4), penalising a high government debt/GDP ratio is introduced to ensure solvency. In fact a small value for γ_6 is sufficient for this purpose.

A consequence of using a welfare criteria of the form (3.4) is that it embodies the policymaker's desired distribution across present and future generations and is dependent upon the authorities' discount factor. Coordination is required for two identical countries to achieve their desired distribution and the lack of coordination causes **pecuniary** externalities, i.e., externalities which work through the price mechanism. Both the public debt and the real exchange rate externalities are pecuniary. They give rise to Pareto-inefficiency with respect to our chosen social welfare criteria, but not with respect to individual household utilities across different generations. The existence of labour market imperfections and distortionary taxes also result in **technological externalities** and here decentralised fiscal policy has implications for Pareto-efficiency, even with respect to individual household preferences.[6]

The Central Bank

The CB is concerned with the global economy and chooses its instrument, the nominal interest rate, to maximise a global welfare function of the form[7]

$$U_t^{CB} = \frac{1}{2}\sum_{i=t}^{\infty}\left[\frac{1-p}{1+\theta}\right]^{i-t}[\gamma_1^F logC_{di} + \gamma_2^F logC_{mi} + \gamma_3^F \log G_{di} - \tfrac{1}{2}(\gamma_4^{CB}\Pi_i^2 + \gamma_5^F \tilde{T}_i^2 + \gamma_6^{CB}\tilde{D}_i^2)]$$

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[6] See Buiter and Kletzer (1991).

[7] In the dynamic game equilibria set out in the Appendix we use a linearised model and a Taylor series quadratic approximation to (3.4) and (3.5), around the baseline steady-state, with bliss points $\hat{c}_d = \hat{c}_m = \hat{g} = 1$; $\hat{\pi} = -\Pi$, $\tilde{t} = -\tilde{T}$, $\tilde{d} = -\tilde{D}$; in deviation form. i.e., actual bliss values of inflation, taxation and government debt are zero.

$$+\frac{1}{2}\sum_{i=t}^{\infty}\left[\frac{1-p}{1+\theta}\right]^{i-t}[\gamma_1^{*F} logC_{di}^{*}+\gamma_2^{*F} logC_{mi}^{*}+\gamma_3^{*F}\log G_{di}^{*}-\tfrac{1}{2}(\gamma_4^{CB}\Pi_i^{*\,2}+\gamma_5^{*F}\tilde{T}_i^{*\,2}+\gamma_6^{CB}\tilde{D}_i^{*\,2})] \quad (3.5)$$

Thus $\gamma_i^{CB}=\gamma_i^{F}$ except for the following differences: the CB is more 'conservative' than the fiscal authorities, i.e., primarily concerned with inflation, so that $\gamma_4^{CB}>>\gamma_4^{F}$. In the first set of results the CB is not responsible for ensuring the solvency of the fiscal authorities. This is captured by putting $\gamma_6^{CB}=0$ whereas γ_6^{F} and γ_6^{*F} must be given small values to guarantee solvency.[8] In a further exercise we consider the effects of a 'weak' CB which concerns itself with global solvency, puts $\gamma_6^{CB}>0$ and hence monetises the debt. This corresponds to the Sargent-Wallace 'unpleasant monetary arithmetic'.

Reputation and Cooperation

The credibility of policies and the associated problem of time inconsistency is a major issue in the debate over EMU. In our model time inconsistency originates from three sources. First, it arises from the optimal choice of consumption, savings and demand for money by each household where taxes are distortionary (see, for example, Lucas and Stokey (1983)). The second source of time inconsistency is that treasuries issue nominal rather than indexed bonds and as a consequence there arises an incentive to erode the debt/GDP ratio by engaging in surprise inflation. Thirdly, there exists the familiar role for surprise inflation in the labour market.

Given these features of the model (and rational expectations) we can distinguish between the cases when an authority has or does not have a reputation for precommitment. A fiscal or monetary authority which enjoys reputation can exercise the greatest leverage over the private sector because an announced path of instrument settings would be credible and would affect private sector behaviour

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[8] Chosen parameter values are: $\gamma_i^{F}=\gamma_i^{*F}$ (identical governments); $\gamma_1^{F}=\gamma_2^{F}=0.5$; $\gamma_3^{F}=\bar{G}_t/\bar{C}_t$ (the latter assumes that the observed government spending/private consumption ratios correspond to household preferences), $\gamma_4^{F}=1$, $\gamma_4^{CB}=30$, $\gamma_5^{F}=0.2$ and $\gamma_6^{F}=0.1$.

immediately in the desired way. For instance the announcement of lower government spending and taxes in the future will immediately raise consumption. But how relevant are reputational (time inconsistent) regimes? A large of volume literature now exists on how a time inconsistent policy may be enforced despite the incentive to renege. Mechanisms suggested include constitutional constraints (Kydland and Prescott (1977)) and ’punishment strategies’ on the part of the private sector (Barro and Gordon (1983); Levine and Currie (1987)). There are a number of problems with the latter not least of which how atomistic economic agents might coordinate on the precise punishment scheme that supports a particular reputational equilibrium.[9]

When a government cannot precommit itself to a future policy, it must act each period to maximise its welfare function, given that a similar optimisation problem will be carried out in the next period. Formally, the policymaker maximises at time t a welfare function U_t

$$U_t = u_t + \lambda U_{t+1} \tag{2.4}$$

where u_t is the single-period welfare, λ is the discount factor and U_t is evaluated on the assumption that an identical optimisation exercise is carried out from time t+1 onwards. The solution to this problem is found by dynamic programming and, unlike the reputational policy, leads to a time consistent trajectory or rule for instruments. Details of all the solution concepts are to be found in the Appendix and in Currie and Levine (1993), Chapter 6.

With this distinction between reputational and non-reputational policies in mind we now return to the question of cooperation. The socially optimal policy is for the CB and national governments to agree on a global welfare function and to be able to precommit in their joint choice of all policy instruments in its maximisation. The global welfare function under full cooperation is assumed to be $0.5U_t^{CB} + 0.25U_t^F + 0.25U_t^{*F}$

[9] See al-Nowaihi and Levine (1993) for a discussion of this problem.

thus giving the CB the same bargaining power as the two combined national governments. We call this regime CR (cooperation with reputation) and we use it as a benchmark against more realistic regimes.

Suppose that both the CB and the fiscal authorities lack a reputation for precommitment to the private sector. This leaves three possibilities. The first is where the CB is not independent and monetary and fiscal policies are fully coordinated without reputation. This is regime CNR (cooperation without reputation). The second possibility is with an independent CB and uncoordinated fiscal policies. This is the non-cooperative equilibrium in the three-player game, NCNR (non-cooperation with no reputation). We also consider an intermediate regime where fiscal authorities alone cooperate but the CB remains independent. This is in effect a two-player non-cooperative equilibrium which we refer to as NCNRF (non-cooperation between fiscal and monetary authorities but fiscal cooperation). Under NCNRF we assume the fiscal authorities jointly maximise $0.5(U_t^F + U_t^{*F})$.

Since we have adopted identical objectives and model structures for the two countries, the relative price of the two goods under MU2 remain at their initial equilibrium values. Consequently the equilibria of CR, CNR and NCNRF, involving fiscal cooperation, are the same for MU1 and MU2. However, for NCNR, each national government perceives a welfare gain from improving its terms of trade and so the equilibria, denoted by NCNR1 and NCNR2, differ under MU1 and MU2. This leaves us five regimes summarised in Table 3.1.

	Full Cooperation	**Fiscal Cooperation**	**Non-Cooperation**
Reputation	CR		
Non-Reputation	CNR	NCNRF	NCNR1 (MU1) NCNR2 (MU2)

Table 3.1 Summary of the Five Regimes.

4. SIMULATION RESULTS

No Monetisation of Debt by the CB

In our first set of results the CB plays no part in ensuring the solvency of the fiscal authorities. Thus $\gamma_6^{CB}=0$ whereas γ_6^{F} must be given a small value to guarantee strong solvency in the two countries. The long-run equilibria of the five regimes are given in table 4.1 relative to the original baseline steady-state about which all variables are measured. In order to explain the nature of the time inconsistency of regime CR, the trajectories of CR and CNR are compared in figure 2. Consider first regime CR. According to table 4.1, CR involves a long-run lowering of the government spending/GDP ($\tilde{g}$) and the debt/GDP ($\tilde{d}$) ratios. In our non-Ricardian economies this must result in a long-run fall in the real interest rate. Investment is crowded in and output rises.

The dynamic paths that lead to this long-run for CR are shown in figure 3. There is an immediate **increase** in government spending and taxation, the latter dominating so that the debt/GDP ratio falls towards its desired value $\tilde{d}=-0.5$. In the long-run this drop in government debt allows for a **fall** in the primary surplus and the taxation/GDP ratio falls by more than the government spending/GDP ratio, reducing tax distortions. Inflation increases sharply in the first period by around 3% contributing to the erosion of government debt and output rises in response to this once-and-for-all surprise. Thereafter inflation drops to its desired level of -3% (relative to a baseline of 3% implying an actual zero value of inflation)

FIGURE 2 ABOUT HERE

A promise of a long-run fall in taxation coupled with an immediate increase is the initially optimal policy for reducing the distortionary effects of taxation on consumption, savings and the real interest rate, consistent with the government budget constraint. But this policy is time inconsistent. Similarly an immediate increase in inflation followed by a permanent fall is also time inconsistent. Loosely speaking the initial settings of instruments under CR are the values the combined fiscal and monetary authority would return to at any time after t>0. Under CNR the private sector anticipates re-optimisation and the initial rise in $\tilde{g}$ observed under CR continues indefinitely. Inflation also remains persistently high and provides an extra source of seigniorage permitting the tax ratio $\tilde{t}$ to rise by less than $\tilde{g}$.

REGIME	$\tilde{g}$	$\tilde{t}$	$\tilde{m}$	π	$\tilde{d}$	r	y	$\tilde{i}$	$\tilde{c}$	U^F
CR	**-0.4**	**-1.3**	**2.3**	**-3.0**	**-41**	**-0.2**	**0.8**	**0.4**	**0.0**	**44**
CNR	**1.4**	**0.5**	**-2.4**	**3.3**	**-41**	**-0.05**	**0.2**	**0.1**	**-1.5**	**28**
NCNR1	**12.5**	**12.7**	**-1.9**	**1.3**	**-15**	**0.9**	**-3.7**	**-2.1**	**-10.4**	**-281**
NCNR2	**16.4**	**16.7**	**-2.3**	**1.4**	**-16**	**1.2**	**-4.9**	**-2.9**	**-13.5**	**-474**
NCNRF	**20.7**	**21.0**	**-2.7**	**1.6**	**-24**	**1.4**	**-6.2**	**-3.6**	**-17.1**	**-608**

Table 4.1. Long-Run Equilibria of the Regimes See table 2.1 for notation.

Now consider the non-cooperative regime NCNR1 under MU1. The first point to notice about all the non-cooperative regimes is that the independence of the CB, which places a priority on lowering inflation, ensures that inflation is less compared with CNR where the CB has to compromise over its objective function. However, this gain in terms of lower inflation is bought at a price. The CB fixes the nominal interest rate. Given the nominal interest rate rule, the fiscal authorities can increase inflation through a fiscal expansion in the form of an approximately balanced budget increase in the ratio $\tilde{g}$. This was clearly seen in our earlier simulations of ad hoc rules. These inflation increases are perceived by the fiscal authorities as surprises which increase output and reduce the real value of debt offering a nominal return. In a rational expectations equilibrium of course these benefits do not materialise, as inflation is anticipated, and the resulting equilibrium is one where there is simply an upward bias in government spending $\tilde{g}$.

The inefficient outcome is the result of a conflict between fiscal authorities who attempt to engineer surprise inflation by raising $\tilde{g}$ and a conservative CB who respond by raising nominal and hence real interest rates to squeeze out inflation. Going back to table 4.2, if fiscal authorities alone cooperate (regime NCNRF) then their incentive to inflate increases because the combined fiscal expansion has a greater impact on the common inflation rate. The result is that the upward bias in government spending worsens. Thus with an independent CB setting the nominal interest rate, **fiscal policy coordination is counterproductive.**[10] The fiscal externality arising from each country's public sector deficit or government spending or distortionary taxation can only be successfully internalised if fiscal coordination is accompanied by fiscal and monetary policy coordination, preferably with reputation (regime CR), but also without reputation (regime CNR).

γ_4^{CB}	$\tilde{g}$	$\tilde{t}$	$\tilde{m}$	π	$\tilde{d}$	**r**	**y**	$\tilde{i}$	$\tilde{c}$	U^F
5	**10.5**	**10.9**	**-19.0**	**25.3**	**12.0**	**0.8**	**-3.2**	**-2.0**	**-8.5**	**-313**
10	**11.7**	**12.2**	**-8.4**	**10.4**	**2.3**	**0.9**	**-3.6**	**-2.1**	**-9.6**	**-292**
30	**12.5**	**12.7**	**-1.9**	**1.3**	**-15**	**0.9**	**-3.7**	**-2.1**	**-10.4**	**-281**
100	**12.8**	**12.8**	**0.8**	**-1.7**	**-26.9**	**0.9**	**-3.7**	**-2.1**	**-10.7**	**-227**

Table 4.2. The Effect of Conservative Bankers: Regime NCNR1 (1-good)

Table 4.2 shows the effect of changing the degree of conservatism of the CB. Inflation falls but the upward bias in government spending rises. The last column of the table shows that the positive welfare effects of the former outweigh the negative effects of the latter so that increasing the degree of conservatism is welfare-enhancing. Notice that a more conservative CB results in a **lower** steady-state debt/GDP ratio $\tilde{d}$. The reason for this is that the fiscal authorities offset the rise in the real interest rate, resulting from the tightening of monetary policy by an increasingly conservative CB, by raising taxation sufficiently to lower $\tilde{d}$ The combined effect is to leave the real interest rate almost unchanged.

Unpleasant Monetary Arithmetic

We now relax the earlier assumption that solvency is the sole concern of the fiscal authorities and examine a scenario in the spirit of Sargent and Wallace. In

[10] This provides a fiscal analogy to the well-known result by Rogoff(1985a) that **monetary** policy coordination can be counterproductive.

table 4.3 the CB concerns itself with rising debt/GDP ratio in the two countries by adopting a weight $\gamma_6^{CB}>0$ on the deviation of the debt/GDP ratio about a bliss point. γ_6^F is set at a small value just sufficient to ensure strong solvency. Experiments lead to the adoption $\gamma_6^F = 0.1$. The effect of forcing the CB to share responsibility for solvency is striking. As γ_6^{CB} increases from 0.1 (equal to γ_6^F) to 0.5, inflation rises significantly and debt/GDP ratios (and with them the real interest rate) rise substantially.

ω	$\tilde{g}$	$\tilde{t}$	$\tilde{m}$	π	$\tilde{d}$	**r**	**y**	$\tilde{i}$	$\tilde{c}$	U^F
0	**12.5**	**12.7**	**-1.9**	**1.3**	**-15**	**0.9**	**-3.7**	**-2.1**	**-10.4**	**-281**
1	**12.2**	**13.3**	**-2.4**	**1.9**	**28**	**1.0**	**-4.3**	**-2.5**	**-9.7**	**-335**
2	**12.1**	**14.8**	**-3.2**	**4.0**	**100**	**1.2**	**-5.3**	**-3.1**	**-9.0**	**-416**
3	**11.8**	**18.9**	**-6.9**	**7.1**	**307**	**2.0**	**-8.4**	**-5.0**	**-6.8**	**-632**

Table 4.3. Unpleasant Monetary Arithmetic: Regime NCNR1 (1-good); $\omega=\gamma_6^{CB}/\gamma_6^F$

Credible Inflation Targeting

The inefficiency of the non-cooperative regimes arises, in part, from the conflict between the fiscal authorities who perceive of benefits from surprise inflation originating from their own expansionary policies, and the CB who respond by tightening monetary policy. We now examine the extent to which this may be due to the choice of monetary instrument - the nominal interest rate.[11] It is well-known from oligopoly theory that the choice of instrument can crucially affect the nature of a Nash equilibrium. In an oligopoly, the Cournot-Nash equilibrium for which output is the decision variable is totally different from the Bertrand-Nash equilibrium for which prices are the instruments. An analogous result holds in our

[11] See also Levine (1993) for an analogous analytical result using a much simpler model with an IS/LM demand side.

monetary-fiscal policy game.

Suppose that the CB sets credible inflation targets to be achieved by appropriate monetary instruments. The precise nature of these instruments do not concern us. What is important are the perceptions of each player in the Nash equilibrium. As long as the private sector and the fiscal authorities believe that inflation is low and, in the case of the latter, beyond their control we may put $\pi_t=\pi_t^e$ and fiscal authorities will no longer attempt to engage in surprise inflation. The remaining externalities are the 'purely fiscal' ones discussed in the introduction. Their effect on the non-cooperative outcome of what is now a two-player game can be seen in table 4.4. The main result is that there is now a **welfare case for fiscal policy coordination** with an independent CB, especially in a two-good world where the terms of trade externality exists. But these gains are far less than those from full fiscal **and** monetary policy coordination reported before with the nominal interest rate as the instrument.

REGIME	$\tilde{g}$	$\tilde{t}$	$\tilde{m}$	π	$\tilde{d}$	r	y	$\tilde{i}$	$\tilde{c}$	U^F
CR	**-0.4**	**-1.3**	**2.3**	**-3.0**	**-41**	**-0.2**	**0.8**	**0.4**	**0.0**	**32**
CNR	**1.9**	**1.5**	**2.3**	**-3.0**	**-2.6**	**0.05**	**-0.2**	**-0.2**	**-1.7**	**24**
NCNR 1	**4.3**	**3.6**	**2.3**	**-3.0**	**-41**	**0.2**	**-0.7**	**-0.4**	**-3.9**	**6**
NCNR 2	**10.8**	**10.2**	**2.3**	**-3.0**	**-46**	**0.6**	**-2.7**	**-1.7**	**-9.1**	**-141**

Table 4.4 Credible Inflation Targets.

5. CONCLUSIONS.

The paper has emphasised the distinction between the purely fiscal reasons for fiscal policy coordination given a credible low-inflation policy by the CB and the spillover effects of an uncoordinated fiscal policy on monetary policy. Our worst scenario (table 4.3), 'the unpleasant monetary arithmetic', is where an independent CB sets the common nominal interest rate and responds to a rising government debt/GDP ratio in either of the two countries with a looser monetary stance - in effect monetising the debt. This results in high inflation, high debt/GDP ratios and an excessively large public sector.

Our intermediate scenario (table 4.2), ’no monetisation of debt by the CB’, is one where the CB sets the nominal interest rate and fiscal authorities bear sole responsibility for their own solvency. The result is again excessively large public sector spending; but government debt is contained and inflation kept low. For these first two scenarios fiscal policy coordination with an independent CB is counterproductive because it increases the incentive for fiscal authorities to engage in surprise inflation.

The best scenario (table 4.4) occurs with credible inflation targeting by the CB. This removes the incentive for the fiscal authorities to cause surprise inflation. There are now welfare gains from fiscal coordination with an independent CB, but these are only substantial in a two-good world where there exists an incentive to attempt beggar-thy-neighbour improvements in the terms of trade by fiscal expansion.

There are number of directions for future research. We have arbitrarily imposed various degrees of conservatism on the CB whereas the type of banker can be seen as a strategic variable chosen optimally in a ’delegation game’. Stabilisation policy is absent in our deterministic model, but the choice of conservative banker should balance the gains of lower average inflation against a less effective monetary stabilisation rule (Rogoff, 1985b). Another development would be the re-examination of the fiscal-monetary policy games in the context of an endogenous growth model, drawing upon the recent vast literature in this area.

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APPENDIX. Dynamic Game Solution Concepts.

The model in linearised form has the following state-space representation:

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} z_t \\ x_t \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} w_t \\ w_t^* \\ r_{nt} \end{bmatrix} = \boldsymbol{A} y_t + \boldsymbol{B} \omega_t \qquad \text{(B1)}$$

say, where z_t is an (n - m) x 1 vector of predetermined variables, x_t is an m x 1 vector of non-predetermined variables, $x^e_{t+1,t}$ denotes rational expectations of x_{t+1} formed at time t on the basis knowledge of the model (B1), w_t and w^*_t are 2x1 vectors of fiscal instruments. All capital letters (such as A and B above) indicate matrices. The initial conditions at t=0 are given by z_0. All time-varying variables are measured as proportional or absolute deviations from the baseline steady-state (see the general notation defined after table 2.1).

A quadratic approximation to a welfare **loss** (minus the utility) associated with the utility (3.4) at time t is $W_t = ½\sum_{i=t}^{\infty} \lambda^{i-t}[\gamma_1 f(c_{di}) + \gamma_2 f(c_{mi}) + \gamma_3 f(g_i) + \gamma_4 g(\pi_i) + \gamma_5 g(\tilde{t}_i) + \gamma_6 g(\tilde{d}_i)]$ where $\lambda = (1-p)/(1+\theta)$, $f(x_i)=g(x_i)-\hat{x}^2$ and $g(x_i)=(x_i-\hat{x})^2$. This uses a Taylor series expansion: $\log(X_i/\bar{X}_i) \approx (1-(x_i-1)^2)/2$ where $x_i = (X_i - \bar{X}_i)/\bar{X}_i$. Then $\hat{c}_d = \hat{c}_m = \hat{g} = 1$ are targets set at 100% above the baseline. Other targets are set at $\hat{\pi} = -\pi$, $\tilde{t} = -\tilde{T}$, $\tilde{d} = -\tilde{D}$; thus the domestic fiscal authority's welfare loss can be written in the general form:

$$W_0=\frac{1}{2}\sum_{t=0}^{\infty}\lambda^t[y_t^T Q_F y_t+2y_t^T U_F w_t+w_t^T R_F w_t] \qquad \text{(B2)}$$

at time t=0, where superscript T denotes the transform. A analogous welfare loss can be written for the second fiscal authority, the CB and the economies as a whole.

Regime CR.

Under CR the three players cooperate and are able to precommit. A joint welfare loss of the form (B2) is chosen and minimised, at time t=0, with respect to fiscal and monetary instruments ω_t. By the standard theory of Lagrangian multipliers, a function

$$L_0=\sum_{t=0}^{\infty}\lambda^t[1/2(y_t^T Q y_t+2y_t^T U\omega_t+\omega_t^T R\omega_t+p_{t+1}(Ay_t+B\omega_t-y_{t+1})]$$

is defined where p_t is a vector of costate variables. L_0 is then minimised with respect to all state and costate variables and the instruments. Partitioning $p_t=\begin{bmatrix}p_{1t}\\p_{2t}\end{bmatrix}$ conformably with $y_t=\begin{bmatrix}z_t\\x_t\end{bmatrix}$ gives a solution for the economy under control of the form

$$\omega_t=G\begin{bmatrix}z_t\\p_{2t}\end{bmatrix}\ ;\quad \begin{bmatrix}z_{t+1}\\p_{2t+1}\end{bmatrix}=H\begin{bmatrix}z_t\\p_{2t}\end{bmatrix}\ ;\quad x_t=-N\begin{bmatrix}z_t\\p_{2t}\end{bmatrix} \qquad \text{(B3)}$$

Boundary conditions: $p_{20}=0$; $\lim_{t\to\infty}\lambda^t p_t=0$ complete the solution. (B3) gives the feedback form of the cooperative, optimal policy. Partitioning H conformably with z_t and p_{2t} so that H_{21} and H_{22} are mx(n-m) and mxm respectively, $p_{2t+1}=H_{21}z_t+H_{22}p_{2t}$, Then (B3) becomes

$$\omega_t=G_1 z_t+G_2 H_{21}\sum_{\tau=1}^{\infty}(H_{22})^{\tau-1}z_{t-\tau} \qquad \text{(B4)}$$

where $G=[G_1\ G_2]$, partitioned conformably with z_t and p_{2t}. The rule then consists of a feedback on the lagged predetermined variables extending back to time t=0, the time of the formulation and announcement of the policy, with geometrically declining weights.

The welfare loss from time t onwards (the "cost-to-go") is given by

$$W_t=-½\mathrm{tr}(N_{11}(z_t z_t^T+N_{22}p_{2t}p_{2t}^T) \qquad \text{(B5)}$$

Regime CNR.

The precommitment solution takes the feedback form of a rule (B3) which, as we have seen from (B4) is a rule with memory. The time-inconsistency of this equilibrium can be best seen by examining the ’cost-to-go’ (B5). Re-optimisation at time t, and reneging on the commitment given at time 0, then involves putting $p_{2\tau}=0$. Thus the gains from reneging are $-\frac{1}{2}tr(N_{22}p_{2\tau}p_{2\tau}^T)$. It can be shown that $N_{22}<0$ (i.e., negative definite) and it follows that everywhere along the optimal trajectory at which $p_{2\tau}\neq 0$, there will be gains from reneging and the ex ante optimal policy is sub-optimal ex post.

In order to construct a time-consistent policy we employ dynamic programming and seek a Markov-perfect equilibrium in which instruments are still allowed to depend on past history, but only through a feedback on the **current** value of the state variables. This precludes a feedback as in (B3) which involves memory. Thus we seek a stationary solution $w_t=Gz_t$ in which W_t is minimised at time t subject to the model (B1) in the knowledge that an identical procedure will be used to minimise W_{t+1} at time t+1. Other features of the solution are that $x_t=-Nz_t$, which we know is true of saddlepath stable solutions to rational expectations models under a rule $w_t=Gz_t$, and that $W_\tau=z_\tau^T Sz_\tau$ where S is a 'Riccati' matrix. **Regimes NCNR and NCNRF.**

These are closed-loop Nash equilibria found by iterating between the policymakers and the private sector in a Cournot-like adjustment process. Then, given initial values for G, G^* and N, defined as in the regime CNR above, we arrive at the equilibria NCNR (a game with policymakers) or NCNRF (2 policymakers).[12]

Figure 1. Fiscal Expansion in Country 1. $\tilde{g}$=1%.

MU1 **MU2**

[12] The software used to compute the equilibria is described Gaines and Levine (1989). Full details of the solution procedures are in Currie and Levine (1993), chapter 6.

Figure 2. Trajectories for Regimes CR and CNR

CR **CNR**