GROWTH, DEBT AND PUBLIC INFRASTRUCTURE.

Paul Levine*
University of Surrey,
London Business School
and CEPR.

Thomas Krichel
University of Surrey

December 1994: Revised version 2.

Abstract.

The paper presents a closed economy model of endogenous growth driven by capital externalities arising from both private capital and public infrastructure. The model is calibrated to fit data for India, an approxmiately closed economy. Simulations suggest that fiscal policy certainly matters and the choice of the income taxation rate, the mix of government spending between infrastructure and public consumption goods, and the long-run government debt/GDP ratio can all significantly affect the long-run growth rate. Intertemporal aspects of fiscal policy are also important and the precommitment (time-inconsistent) and non-precommitment policies differ substantially.

JEL Classification: C61, E21, E23, E62, O41.
Keywords: endogenous growth, fiscal policy, time inconsistency.

Correspondence to: Paul Levine
Department of Economics
University Surrey
Guildford
GU2 5XH, United Kingdom
E-mail: P.Levine@surrey.ac.uk

* The authors acknowledge useful comments by participants at the ESRC Development Economics Annual Conference, Leicester University, March 1994, especially those of Ali al-Nowaihi. The usual disclaimer applies.

1. **Introduction**.

The view that a country's growth rate is related to fiscal policy is by no means new to the development literature. Two theoretical developments in macroeconomics have provided a rigorous underpinning for this perception. The first, of course, is the endogenous growth 'revolution'; the second lies in the modelling of non-Ricardian effects of fiscal policy on private sector consumption and savings.

The overlapping generations model of Yaari (1965), Blanchard (1985), Frenkel and Razin (1987) and Weil (1989) show that even if households are intertemporal optimisers over the indefinite future with rational expectations, Ricardian equivalence fails if each household's probability of survival to the next period is below unity and/or there exists population growth. In the model presented in this paper there are two further reasons for the breakdown of Ricardian equivalence. First, all income, including income from wealth, is taxed which introduces a distortion reducing savings. Second, we divide consumers into a very low income group and the rest. The low income group own no wealth and consume out of current disposable income. We refer to this group as 'liquidity constrained'. The remaining consumers are unconstrained, own all the nation's wealth and make consumption and savings decisions derived from microfoundations.

It is useful to categorise the growth literature into four types of models.[1] First, there are the traditional Solow-type growth models which allow for growth through technological change, but make it exogenous. Second, there are models

[1] See Romer (1991), Buiter (1991) and Hamond and Rodriquez-Claire (1993) for recent surveys of the theoretical literature.

which support endogenous growth through the accumulation of human capital. (e.g., Lucas (1988), based on work originally carried out by Uzawa (1965)). Third, we have models which abandon price-taking by firms and assume monopolistic competition in which firms produce knowledge that is used in the production of a distinctive new good (e.g. Romer (1990)). The approach we follow models external effects of capital accumulation on labour productivity which, at the aggregate level, gives a production function with at least constant returns to scale in capital[2]. This genre of model goes back to Arrow (1962). We generalise over recent version of that approach by Romer (1986) and Barro (1990) models to allow for both private capital and public infrastructure raise labour productivity. We also provide a generalisation on the demand side of Barro who adopts a model with Barro-Ricardian equivalence. The demand side of our model is closest to those in Alogoskoufis and van der Ploeg (1991) and Saint-Paul (1992). They combine Yaari-Blanchard consumers with a production function which is linear in private aggregate capital, but do not consider tax distortions, government infrastructure effects and liquidity constraints.

We confine ourselves to a closed economy; for open economies, the mechanisms by which external debt affects domestic real interest rates and hence growth are different from those arising from domestic debt, raising new issues concerning risk premia etc. (See Rebello (1992) for a penetrating discussion of endogenous growth models of open economies). India fits our closed economy

[2] In fact the production function we use exhibits constant returns to scale in total private plus public capital. Increasing returns to scale can be handled but prevent analytical solutions. See Xie (1991) for a useful treatment of this issue.

assumption well. In section 3 we calibrate the model set out in the preceding section using macroeconomic data for India and we compute an order-of-magnitude feel for the effects on long-run growth of government debt, liquidity constraints and the division of government spending between consumption and infrastructure expenditure. Section 4 considers the extent and nature of the time inconsistency problem in the choice of debt and taxation paths to finance a given path for government spending. Section 5 provides brief conclusions.

2. The Model

Our economy is closed and inhabited by consumers who are divided into two groups: those who are liquidity-constrained, who consume current post-tax income and the remaining unconstrained consumers who own all the financial and private physical wealth. Unconstrained consumers choose consumption and savings to maximise an intertemporal utility function, subject to a budget constraint. Firms are competitive and maximise an intertemporal profit function. There exists an exogenously growing population and individual labour supply. The efficiency of the latter depends upon the economy-wide capital-labour ratio which generates the endogenous growth. Goods and labour markets clear instantly.[3]

Non-Ricardian effects of government debt are important in this paper and occur for four reasons. Following Yaari (1965) and Blanchard (1985), households leave no anticipated bequests to their heirs. A second reason for a breakdown of

[3] Alternatively labour markets may be subject to insider and/or efficiency wage effects which prevent the labour market from clearing. Our model can then be interpreted as one with unemployment permanently at its natural rate.

debt neutrality is the existence of population growth (Weil, 1989). The Yaari-Blanchard-Weil consumption function - set out below - combines those two features. The government finances public expenditure on goods and services by distortionary taxation (the third non-Ricardian effect) or borrowing. Finally, the existence of liquidity constraints is a fourth reason for the breakdown of Ricardian equivalence. Time is discrete[4]. The details of the model are as follows (See table 1 for a summary which includes the notation used).

Households

Our unconstrained consumers consist of overlapping generations, that are identical apart from age. Each faces a constant probability p per period of death. The single-period utility function of the consumer is logarithmic in private and public consumption and life-cycle aspects of labour income are ignored, labour income being the same for consumers of all ages.

Consider the unconstrained consumer born in period s. Her intertemporal expected utility function at time t≥s, in the absence of any uncertainty apart from death, is given by

$$U_{t,s}=\sum_{i=t}^{\infty}\left[\frac{1-p}{1+\theta}\right]^{i-t}[\gamma_1\log C_{i,s}+\gamma_2\log G^C_{i,s}] \tag{2.1}$$

where θ is the rate of time preference, and $C_{i,s}$ and $G^C_{i,s}$ denote consumption of the private and public good respectively over period [i,i+1]. Household labour supply is assumed to be exogenously fixed. Real consumer financial wealth at the

[4] Our use of discrete time differs from much of the literature in this area which uses continuous time. This choice is not fundamental but the time consistent policy, obtained by dynamic programming, turns out to be more transparent in discrete time. (See Appendix B).

beginning of period i is given by

$$V_{i,s}=D_{i,s}+K_{i,s} \tag{2.2}$$

where $D_{i,s}$ is (non-indexed) government debt held by domestic consumers and $K_{i,s}$ is capital stock owned by consumers. All stocks and flows are expressed in units of domestic output. Suppose that a constant tax rate is levied on all income. Labour supply is assumed to be growing at an exogenous rate so no distortions arise here. Consumers now receive an expected real return $R_i(1-\tau_i)$ on their assets where τ_i is the average tax rate.

The consumers budget identity is given by

$$V_{i+1,s}=(1+R_i(1-\tau_i)+r)V_{i,s}+\Omega_{i,s}-C_{i,s} \tag{2.3}$$

where $\Omega_{i,s}$ is labour income net of tax and $rV_{i,s}$ is a premium paid by insurance companies who inherit each consumer's non-human wealth on death. Solving (2.3) forward in time and imposing the usual tranversality condition transforms the identity into the following constraint

$$V_{t,s}=\sum_{i=0}^{\infty}\frac{C_{i+t,s}-\Omega_{i+t,s}}{(1+R_t(1-\tau_t)+r)(1+R_{t+1}(1-\tau_{t+1})+r)\cdot\cdot(1+R_{i+t}(1-\tau_{i+t})+r)} \tag{2.4}$$

The consumer maximises (2.1) subject to the intertemporal budget constraint (2.4). The problem is standard and leads to the solution

$$\frac{C_{i,s}}{C_{i-1,s}}=\frac{(1+R_i(1-\tau_i)+r)}{1+\mu} \tag{2.5}$$

where μ is the consumers 'mortality adjusted' rate of time preference defined by $\frac{1-p}{1+\theta}=\frac{1}{1+\mu}$. If p=0 then r=0, $\mu=\theta$ and we arrive at the familiar result for a representative agent model of consumption in which links between present and future generations are complete and consumers act as if they have infinite lives. Combining (2.4) and (2.5) leads to

$$C_{t,s}=\mu\,(V_{t+1,s}+H_{t+1,s}) \tag{2.6}$$

where $H_{t,s}$ is human capital defined by

$$H_{t,s}=\sum_{i=0}^{\infty}\frac{\Omega_{i+t,s}}{(1+R_t(1-\tau_t)+r)(1+R_{t+1}(1-\tau_{t+1})+r)\cdot\cdot(1+R_{i+t}(1-\tau_{i+t})+r)} \tag{2.7}$$

This completes the optimisation problem for the cohort of unconstrained consumers born in period s. We now turn to their aggregate behaviour.

Let $L_{t,s}$ be the size of the cohort born during period [s,s+1] who are still alive at the beginning of period t. Then $L_{t,s}=(1-p)^{t-s}L_{s,s}$. Let β be the birth rate defined by $L_{t,t}=(1+\beta)L_t$ where L_t is the population size. If g denotes population growth, then $L_t=(1+g)^t L_0$. Hence we must have that

$$L_t=(1+g)^t L_0=\beta\sum_{s=-\infty}^{t-1}L_{t,s}=\beta L_0\sum_{s=-\infty}^{t-1}(1+g)^s(1-p)^{t-s} \tag{2.8}$$

Performing the summation leads to

$$\frac{1}{1+\beta}=\frac{1-p}{1+g} \tag{2.9}$$

which determines the birth rate β. Aggregate variables are defined as follows: For **flows** such as the consumption of unconstrained consumers C_t^u we have

$$C_t^u=\sum_{s=-\infty}^{t}L_{t,s}C_{t,s} \tag{2.10}$$

whilst for stocks such as human capital

$$H_{t+1}=\sum_{s=-\infty}^{t}L_{t,s}H_{t+1,s} \tag{2.11}$$

Taking first differences we have

$$H_{t+1}-H_t=L_{t,t}H_{t+1,t}+\sum_{s=-\infty}^{t-1}[L_{t,s}H_{t+1,s}-L_{t-1,s}H_{t,s}] \tag{2.12}$$

The first term in on the right-hand-side of (2.12) is equal to βH_{t+1} since human wealth of all age groups is equal (the 'perpetual youth' assumption). From (2.7) human capital accumulates according to

$$H_{i+1,s}=(1+R_i(1-\tau_i)+r)H_{i,s}-\Omega_{i,s}^u \tag{2.13}$$

The premium r for a competitive insurance industry must satisfy the zero profit condition $r(1-p)=p$. Hence (2.12) and (2.13) give

$$(1-\beta)H_{t+1}=(1+R_t(1-\tau_t)(1-p))H_t-\Omega_t^u \tag{2.14}$$

where Ω_t^u is the human capital of the unconstrained consumers. Aggregating financial wealth similarly, the first term in the summation is $L_{t,t}V_{t+1,t}=0$ because the newly born consumers (born in period [t,t+1]) inherit no financial wealth which can only start accumulating from t+1 onwards. Hence, corresponding to (2.14) we have

$$V_{t+1}=(1+R_t(1-\tau_t)(1-p))V_t+\Omega_t^u-C_t^u \tag{2.15}$$

where $V_t=D_t+K_t$. From (2.6), (2.14) and (2.15) we arrive at the discrete-time Yaari-Blanchard consumption function

$$C_t^u=\frac{1+R_t(1-\tau_t)(1-p)}{1+\mu-\beta}C_{t-1}^u-\frac{\beta\mu}{1+\mu-\beta}V_{t+1} \tag{2.16}$$

which describes the consumption and savings behaviour of the unconstrained consumers.

The constrained consumers consume current post-tax (and possibly post-transfers) income and accumulate no wealth. Their consumption is therefore given by $C_t^c=\Omega_t^c(1-\tau_t)$ where Ω_t^c is their pre-tax labour income. Total pre-tax labour income is given by $\Omega_t=Y_t-(R_t+\delta)K_t$. Suppose that constrained consumers receive a portion λ of post-tax (and benefit) labour income. Then we must have that $C_t^c=\lambda(Y_t-(R_t+\delta)K_t)(1-\tau_t)$ which with (2.16) determines total consumption $C_t=C_t^u+C_t^c$.

Private Sector Output and Investment.

The representative firm f produces homogeneous (value-added) output with the following Cobb-Douglas constant returns to scale production function at time t

$$Y_{t,f}=F(K_{t,f},J_{t,f})=K_{t,f}^{\alpha}J_{t,f}^{1-\alpha} \tag{2.17}$$

where K_{ft} is private physical capital and J_{ft} is labour input in efficiency units. Write

$$J_{t,f}=\epsilon_{t,f}L_{t,f} \tag{2.18}$$

where $\varepsilon_{t,i}$ is a measure of the efficiency of raw labour input L_t. The crucial assumption that drives endogenous growth in this model is that this efficiency measure is a function of the **economy-wide** capital-labour ratio (see Buiter (1991) for an interesting discussion of this formulation). Let K_t be aggregate private capital. In addition to the externality from private capital, the government affects labour efficiency by providing physical capital in the form of infrastructure which may be broadened to include education, health etc, accumulated out of the economy's single output. This is captured by

$$\epsilon_{t,f}=A_f\frac{(K_t^G)^{\gamma_1}(K_t)^{1-\gamma_1}}{L_t} \tag{2.19}$$

Note that for an exogenous growth model with Harrod-neutral productivity growth at a rate ξ say, we would replace (2.19) by $\epsilon_{t,i}=(1+\xi)\epsilon_{t-1,i}$. Assuming identical firms, and aggregating, we arrive at the aggregate production function

$$Y_t=\boldsymbol{B}K_t^{\gamma_2}(K_t^G)^{1-\gamma_2} \tag{2.20}$$

where $\gamma_2=\alpha+(1-\alpha)(1-\gamma_1)$ and $\boldsymbol{B}=\boldsymbol{A}^{1-\alpha}$. Note that if $\gamma_1=0$, $\gamma_2=1$, (2.20) reduces to the Romer (1986) model with only private sector externalities and we have $Y_t=\boldsymbol{B}K_t$.

Firm f ignores the externality in choosing capital stock. Equating the **private** marginal post-tax product of capital to the post-tax cost of capital (including depreciation at a rate δ), and assuming that profits net of depreciation are taxed gives

$$\frac{K_t}{Y_t}=\frac{\alpha}{R_t+\delta} \tag{2.21}$$

This completes the behaviour of the private sector. We are now turning to the public sector.

The Government.

The government provides an amount G_t^C of public consumption goods using the same technology as for the privately produced good, and purchases an amount G_t^I of the latter to invest in infrastructure. Total government expenditure is then $G_t = G_t^C + G_t^I$ which is financed by a combination of taxation (T_t) and borrowing (D_t). (We ignore or rule out seigniorage). The government borrowing identity is given by

$$D_{t+1} = (1+R_t)D_t + G_t - T_t \tag{2.22}$$

and public sector capital accumulates according to

$$K_{t+1}^G = (1-\delta)K_t^G + G_t^I \tag{2.23}$$

assuming the same depreciation rate as in the private sector.

Output Equilibrium.

Equilibrium in the output market gives

$$Y_t = C_t + I_t + G_t \tag{2.24}$$

where gross private investment is given by

$$I_t = K_{t+1} - (1-\delta)K_t \tag{2.25}$$

The supply and demand sides of the economy are now completely determined given the government choice of fiscal policy variables G_t^C, G_t^I and τ_t. It is convenient to express all macroeconomic stock and flow variables as ratios of GDP.[5] The model is summarised in this form in table 1 below. Note that (2.21) assumes that only profits net of depreciation are taxed at the rate τ_t. Total receipts are therefore $\tau_t(Y_t - \delta K_t)$. Thus T_t which now denote total receipts as a proportion of output is given by $T_t = \tau_t(1-\delta K_t)$.

The Steady-State.

[5] To ease the notational burden we retain the same symbols to denote these ratios. Thus K_t/Y_t in (2.21) becomes K_t etc.

We seek a balanced growth steady-state in which all stocks and flows are growing at the same endogenous rate N, the steady-state value of N_t. To facilitate the subsequent analysis we choose the time interval constituting a period such that rates N, R, p and θ are much smaller than unity. The second order terms can then be ignored. With this approximation the steady-state forms of the model summarised in table 1 becomes

$$(\boldsymbol{R}(1-\tau)-\theta-N+\boldsymbol{g})C^{u}=(\boldsymbol{p}+\boldsymbol{g})(\boldsymbol{p}+\theta)(\boldsymbol{D}+K) \tag{2.26}$$

$$(\boldsymbol{R}-N)\boldsymbol{D}=T-\boldsymbol{G}=(1-\delta K(\boldsymbol{R}))\tau-\boldsymbol{G} \tag{2.27}$$

$$K=\frac{\alpha}{\boldsymbol{R}+\delta}=K(\boldsymbol{R}) \tag{2.28}$$

$$log\boldsymbol{B}+\gamma_2\log(K(\boldsymbol{R}))+(1-\gamma_2)\log(G^{I}/(N+\delta))=0 \tag{2.29}$$

$$C^{u}=1-(N+\delta)K(\boldsymbol{R})-\boldsymbol{G}-\lambda(1-\alpha)(1-\tau)\ ; \quad C^{c}=\lambda(1-\alpha)(1-\tau) \tag{2.30}$$

Yaari-Blanchard and Liquidity-Constrained Consumers:

$$C_t^u = \frac{1+R_t(1-\tau_t)(1-p)}{(1+\mu-\beta)(1+N_{t-1})} C_{t-1}^u - \frac{\beta\mu(1+N_t)}{1+\mu-\beta} V_{t+1} \quad ; \quad C_t^c = \lambda(1-\tau_t)(1-\alpha) \qquad \text{(i)}$$

$$V_t = D_t + K_t \qquad \text{(ii)}$$

Private sector Output and Investment:

$$1 = BK_t^{\gamma_2}(K_t^G)^{1-\gamma_2} \qquad \text{(iii)}$$

$$K_t = \frac{\alpha}{R_t+\delta} \qquad \text{(iv)}$$

$$I_t = (1+N_t)K_{t+1} - (1-\delta)K_t \qquad \text{(v)}$$

The Government:

$$(1+N_t)D_{t+1} = (1+R_t)D_t + G_t - T_t \qquad \text{(vi)}$$

$$(1+N_t)K_{t+1}^G = (1-\delta)K_t^G + G_t^I \qquad \text{(vii)}$$

Output Equilibrium:

$$1 = C_t + I_t + G_t = C_t^u + C_t^c + I_t + G_t \qquad \text{(viii)}$$

where $C_t = C_t^u + C_t^c$ =consumption of unconstrained and constrained consumers respectively, V_t=private sector financial wealth, K_t,K_t^G=private and public capital stock, I_t,G_t^I=private and public investment, G_t^C=public consumption, $G_t = G_t^C + G_t^I$=total government expenditure, D_t=government debt, T_t=taxation, all as ratios of GDP. R_t=the real interest rate. $N_t=(Y_{t+1}-Y_t)/Y_t$=growth rate. $\tau_t=T_t/(1-\delta K_t)$=tax rate. Important parameters are: p=probability per period of death, $\beta=(p+g)/(1-p)$=birth rate where g is the population growth rate, $\mu=(p+\theta)/(1-p)$=mortality adjusted rate of time preference, δ=depreciation rate. γ_2=socially efficient private sector share of capital stock and $\alpha \le \gamma_2 = \alpha + (1-\alpha)(1-\gamma_1)$ is the corresponding share in a market equilibrium. γ_1 and $1-\gamma_1$ are measures of the external effects of private and public capital on labour efficiency respectively.

Table 1. Summary of Model.

3. Fiscal Policy and Long-Run Growth.

This section studies the steady-state of the economy given by (2.26) to (2.30). Given fiscal instruments, τ, G^C and G^I we have five equations in five endogenous variables R,C,D,K and, of course, growth N. Since we wish to focus on the effects of government debt D, the distortionary tax rate τ and the mix of government spending as between consumption and investment, we will characterise fiscal policy in terms of D and τ making total government spending endogenous. Let $G^I=\gamma G$ making the third fiscal instrument the proportion γ.

Derivation of Multipliers.

Consider the Yaari-Blanchard consumption relationship (2.26) and the production function (2.29). Eliminating C, K and G using the remaining equations leads to the following two relationships determining R and N given D,τ and γ.

$$\begin{aligned} f(N,\boldsymbol{R},\boldsymbol{D},\tau) = & (\boldsymbol{R}(1-\tau)-\theta+\boldsymbol{g}-N)(1-(N+\delta)K(\boldsymbol{R})+(\boldsymbol{R}-N)\boldsymbol{D}-\tau(1-\delta K(\boldsymbol{R})-\lambda(1-\tau)(1-\alpha))) \\ & -(\boldsymbol{p}+\boldsymbol{g})(\boldsymbol{p}+\theta)(\boldsymbol{D}+K(\boldsymbol{R}))=0 \end{aligned} \tag{3.1}$$

$$\begin{aligned} \boldsymbol{g}(N,\boldsymbol{R},\boldsymbol{D},\tau,\gamma) = & \gamma_2 logK(\boldsymbol{R})+(1-\gamma_2)(\log\gamma+\log(\tau(1-\delta K(\boldsymbol{R}))-(\boldsymbol{R}-N)\boldsymbol{D})-\log(N+\delta)) \\ & +log\boldsymbol{B}=0 \end{aligned} \tag{3.2}$$

The relationship f(N,R,D,τ)=0 describes the locus of interest and growth rates consistent with Yaari-Blanchard consumption behaviour, output equilibrium, private sector investment and the government budget constraint. We term this locus the Yaari-Blanchard (YB) curve. In a Ricardian world associated in this model with p=g=0, it becomes the 'golden rule' $\boldsymbol{R}(1-\tau)-N=\theta$; the real post-tax growth-adjusted real interest rate equals the private sector's rate of time preference and this relationship in unaffected by changes in government debt. In our non-Ricardian economy with p+g>0 this is no longer the case. The relationship g(N,R,D,τ,γ)=0 is the locus consistent with balanced growth and our linear technology, private sector investment and the government budget constraint. We call the relationship the linear technology (LT) curve. For the Romer (1986) model $\gamma_2=1$ and the LT curve is vertical; for an exogenous growth model the LT curve is horizontal.

Consider incremental changes in the exogenous fiscal variables dD, dτ, dγ and the corresponding incremental changes dN and dR along the YB and LT curves. Differentiating, these incremental changes satisfy

$$f_N dN+f_R dR+f_D dD+f_\tau d\tau=0\ ;\ g_N dN+g_R dR+g_D dD+g_\tau d\tau+g_\gamma d\gamma=0 \tag{3.3}$$

Hence keeping fiscal policy fixed, the slopes of the YB and LT curves are given by

$$\left(\frac{\partial N}{\partial R}\right)_{f=0}=-\frac{f_R}{f_N}\ ;\ \left(\frac{\partial N}{\partial R}\right)_{g=0}=-\frac{g_R}{g_N} \tag{3.4}$$

From (3.1) we have

$$f_N=-C^u-\phi(K+D)<0 \tag{3.5}$$

Here $\phi=R(1-\tau)-\theta+g-N>0$ (because C>0 and V>0). Let $\eta=(p+g)(p+\theta)$. Then from (3.1)

$$f_R=\phi(D-(N+\delta(1-\tau))K'(R))+C(1-\tau)-\eta K'(R)>0 \tag{3.6}$$

and it follows from (3.4), (3.5) and (3.6) that the **YB curve is upward-sloping**.

Turning to the LT curve, from (3.2) we have

$$g_N=(1-\gamma_2)\left(\frac{D}{G}-\frac{1}{N+\delta}\right)\ ;\ g_R=\gamma_2\frac{K'(R)}{K(R)}-(1-\gamma_2)\left(\frac{D}{G}+\frac{\tau\delta K'(R)}{G}\right) \tag{3.7}$$

Hence $g_N<0$ if $D<G/(N+\delta)$. For G=0.3 and N+δ<0.1, this condition is satisfied if D<3 (i.e., a debt/GDP ratio less than 300%), which is not a stringent condition. Then a sufficient (by no means necessary) condition for $g_N<0$ is that $G>\tau\delta\alpha(N+\delta)/(R+\delta)^2$

Here the numeratore is a 4th order whilst the denomiator is of the second order therefore we can reasonable assume that this condition is satisfied. For example, if the parameters appearing in the condition as .5, .1, .8, .1 and .15 respectively, we would need that G>.18, which is a reasonable requirement.

From these considerations and (3.4) we deduce that, under very lax conditions, the **LT curve is downward-sloping**.

The remaining partial derivatives are

$$f_D = \phi(R-N) - \eta \;\; ; \; f_\tau = -RC^u - \phi(1-\delta K + \lambda(1-\alpha)) \tag{3.8}$$

$$g_\tau = \frac{(1-\gamma_2)(1-\delta K)}{G} \;\; ; \; g_D = -\frac{(1-\gamma_2)(R-N)}{G} \;\; ; \; g_\gamma = \frac{(1-\gamma_2)}{\gamma} \tag{3.9}$$

From these results we can unambiguously sign $g_D<0$ and $g_\gamma>0$. We require $(1-\delta K)>0$ for to obtain positive consumption which gives $f_\tau<0$ and $g_\tau>0$. Finally along f=0 we have that $f_D = (R-N)\eta V/C - \eta$. Hence $f_D<0$ if $R-N<C/V$ which requires extraordinarily large capital/GDP and debt/GDP ratios to violate. To summarise we expect:

$$f_R, g_\tau, g_\gamma > 0 \;\; ; \; f_N, f_D, f_\tau, g_N, g_R, g_D < 0 \tag{3.10}$$

Consider next the effect of changes in fiscal variables on the YB and LT curves. From (3.3) keeping R and τ fixed, $\partial N/\partial D = -f_D/f_N < 0$. Hence the YB curve shifts to the right as a result of an increase in the debt/GDP ratio D. Similarly keeping R, τ and γ fixed, $\partial N/\partial D = -g_D/g_N < 0$. Hence the LT curve must shift downwards if D rises. The combined effect is that an increase in D reduces growth but has an ambiguous effect on the real interest rate. These results are illustrated in figure 1 below.[6]

We can confirm this result by solving (3.3) to obtain dN and dR as functions

[6] Note that in an exogenous growth model, the LT curve is horizontal and the effect of increasing D is to unambiguously raise the real interest rate.

of dD, dτ, and dγ. This gives the following multipliers which can be signed using (3.10).

$$\left[\frac{\partial N}{\partial D}\right]_{\tau,\gamma\ fixed} = \frac{g_R f_D - g_D f_R}{\Theta} < 0 \tag{3.11}$$

$$\left[\frac{\partial N}{\partial \tau}\right]_{D,\gamma\ fixed} = \frac{g_R f_\tau - g_\tau f_R}{\Theta} \text{ (ambiguously signed)} \tag{3.12}$$

$$\left[\frac{\partial N}{\partial \gamma}\right]_{D,\tau\ fixed} = \frac{-g_\gamma f_R}{\Theta} > 0 \tag{3.13}$$

where $\Theta = f_r g_N - g_R f_N < 0$. Hence we arrive at the proposition

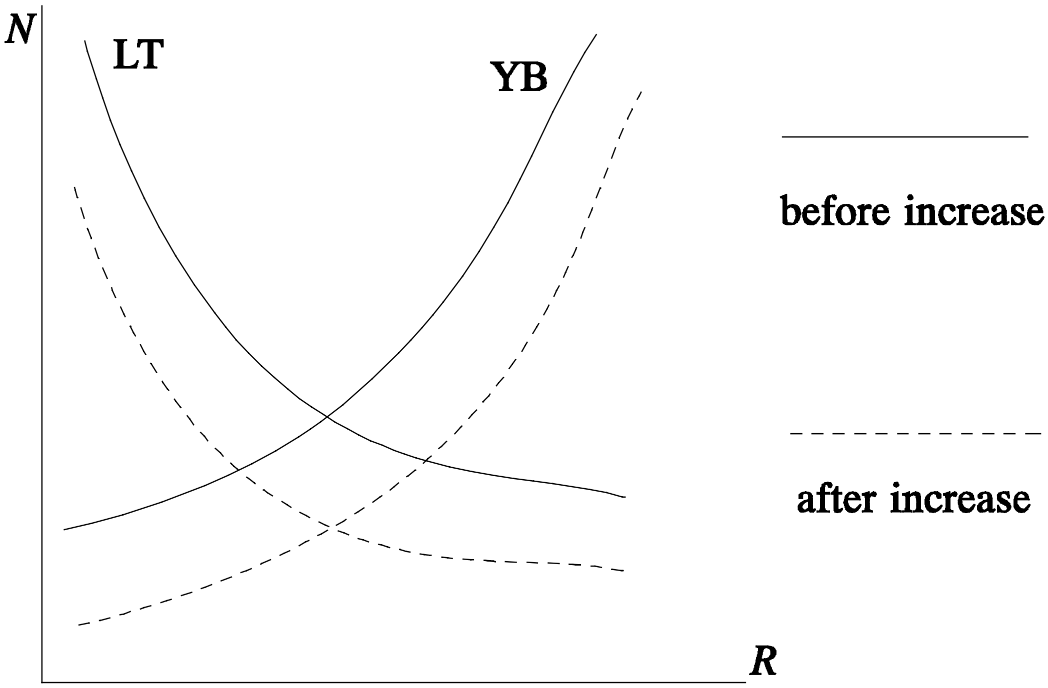
<u>Proposition 1</u>

Steady-state growth can be increased by reducing the debt/GDP ratio D, and/or increasing the proportion γ of government expenditure devoted to public investment.

The Optimal Growth Rate

All this accords with economic intuition. The sign of $\frac{\partial N}{\partial \tau}$ however is ambiguous. The reason for this is that as the tax rate increases, given the debt/GDP ratio, a higher government spending-GDP ratio consistent with the government budget constraint can be reached. Part of this additional spending goes on infrastructure which enhances growth. However taxes are distortionary and, for a given real interest rate R, an increase in the tax rate reduces savings as a proportion of GDP which depresses growth. An optimal growth rate is achieved at the tax rate

Figure 1: Shifts in the LT and YB curve from increasing debt



where $\frac{\partial N}{\partial \tau}=0$ which from (3.12) is given by $g_R f_\tau = g_\tau f_R$. Substituting in for the derivatives leads to a complicated relationship which provides no useful insights. However an intuitively plausible analytical result can be obtained for the case studied by Barro(1990) which is (Barro-)Ricardian ($p=g=\eta=0$) with no depreciation ($\delta=0$), and a balanced budget ($D=0$). Then, from (2.26) and (2.27), $\phi=R(1-\tau)-\theta-N=0$ and $G=\tau$. Substituting into the condition $g_R f_\tau = g_\tau f_R$ leads to the proposition:

Proposition 2.

In a Ricardian economy with no depreciation and a balanced budget, the optimal rate
of growth is achieved at the tax rate $\tau=1-\gamma_2$.

Calibration and Numerical Results for India.

The calibration strategy is as follows: 'deep' parameter values θ and α are chosen by calibrating the steady-state values of selected variables of the model; namely, the return on capital and the consumption/output, government spending ratios, to observed data. Thus we construct a microfoundations model, in which consumers maximise utility and producers maximise profits, which is consistent with observed data.[7].

Rates of return for projects financed by the World Bank average around 15%. This we identify with the marginal product of capital which has to be divided between the two components of the cost of capital, the real interest rate, R and the depreciation rate, δ. For physical capital and advanced industrial countries a rough division would be R=5% and δ=10% on an annual basis, the latter reflecting depreciation through technological obsolescence as well as wear and tear. Our concept of capital in this paper is broader and incorporates human as well as physical capital. We expect a rather lower depreciation rate for human capital and reflecting this we choose the division R=10% and δ=5%. There are a number of studies which estimate Yaari-Blanchard consumption functions with liquidity constraints. Haque and Montiel (1989) obtain an estimates of 0.344 for the proportion of liquidity constrained consumers in India and rather higher estimates for some other LDC's.

This does not exactly correspond to our parameter λ but, nonetheless, we put λ=0.3.

---

[7] This procedure corresponds to the approach of Shoven and Whalley (1992). A more sophisticated approach uses stochastic simulations to compare observed historical moments with population moments from the simulation model (see, Gregory and Smith (1991)).

IMF figures for India give consumption and total government spending on goods, services and investment at 64% and 22% of GDP respectively, leaving 14% of GDP for private investment (assuming an approximately closed economy). From (2.30) this implies

a private capital/output ratio of 1.87. This is on the high side for physical capital, especially for a LDC (see, Obstfeld (1994)); but since we are including human capital in this measure, this figure seems reasonable enough.

Growth rates of GDP (per capita) and population over 1980-91 for India were 2.5 and 2% respectively. Life-expectancy of a new-born child is reported by the World Bank as 61 years. However new-born children do not immediately start making consumption and savings decision. We also need to correct for the fact that the model imposes a constant probability of death. For people that make decisions in the middle of their lives, a constant probobility would underestimate the risk of death. To correct for both the timeing of savings decisions and non exponential length of life, we believe that an average probability of death per year p=1/40 is an accurate representation of reality. The IMF figure for the government debt/GDP in 1990 is 53%. To sustain G=22%, the budget constraint (2.27) implies that the taxation rate must rise to τ=29% in a sustainable steady-state. Finally World Bank figures for 1990 gives government investment as a proportion of spending at 30% and we therefore choose $G_t^I = 0.3 G_t$. We divide the externality effect of private and public capital equally choosing γ_1=0.5. From these calibrations and using (2.26) (2.28) and (2.29) the 'deep' parameters α, A and θ are revealed as α=0.28, B=0.7 and θ=0.06. Details of our chosen calibration are summarised in table 2.

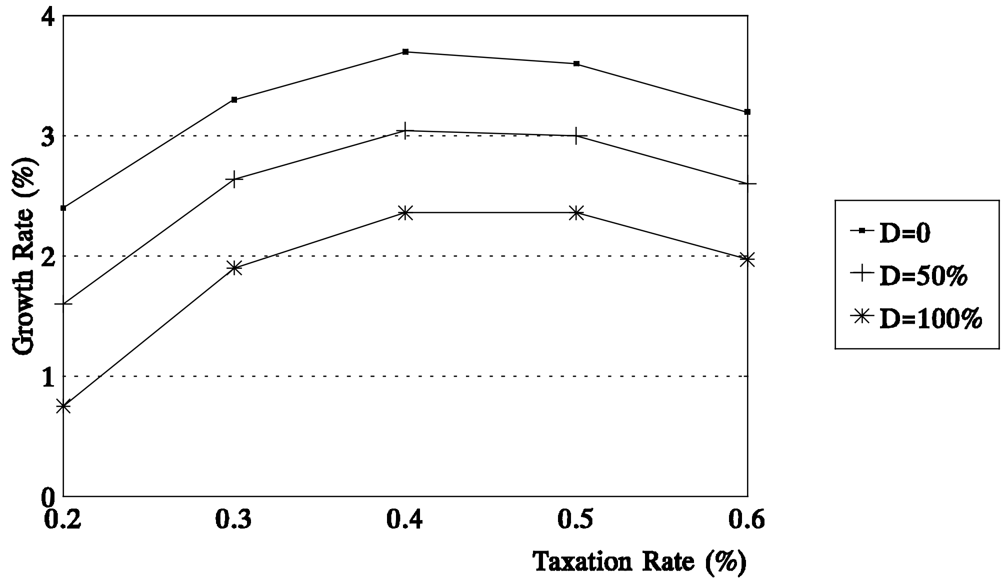
With this calibration the multipliers (3.11) to (3.13) have the following values at the sub-optimal tax rate τ=0.29.

$$\left[\frac{\partial N}{\partial D}\right]_{\tau,\gamma} = -0.0094 \ ; \quad \left[\frac{\partial N}{\partial \tau}\right]_{D,\gamma} = 0.063 \ ; \quad \left[\frac{\partial N}{\partial \gamma}\right]_{D,\tau} = 0.076 \qquad (3.14)$$

Thus our calibrated model says that 50% decrease in the government debt/GDP ratio from its present 53% will increase the per capita growth rate of India by almost 0.5% raising it to around 3%. Raising the proportion of infrastructure spending from γ=30% to 40% will raise growth by 0.8%. The present rate of taxation τ and total government spending G is below that which will generate the highest growth rate with γ=30%.

Now let τ and G increase for a given D and γ, consistent with the steady-state budget constraint. Figures 2 to 5 show how growth rates increase to their maxima for different values of D (figure 2), γ (figure 3), γ_1 (figure 4) and λ (figure 5). The effects of changing D and γ reported above for the sub-optimal taxation rate still approximately hold at the optimal rate which is around τ=0.4. This is close to $\tau=1-\gamma_2=0.36$ as predicted by proposition 2 for the Ricardian case. The effect on growth of changing the liquidity parameter λ is rather small. Making the poor even poorer, because the 'rich save', will not increase the long-run growth rate of the Indian economy significantly.

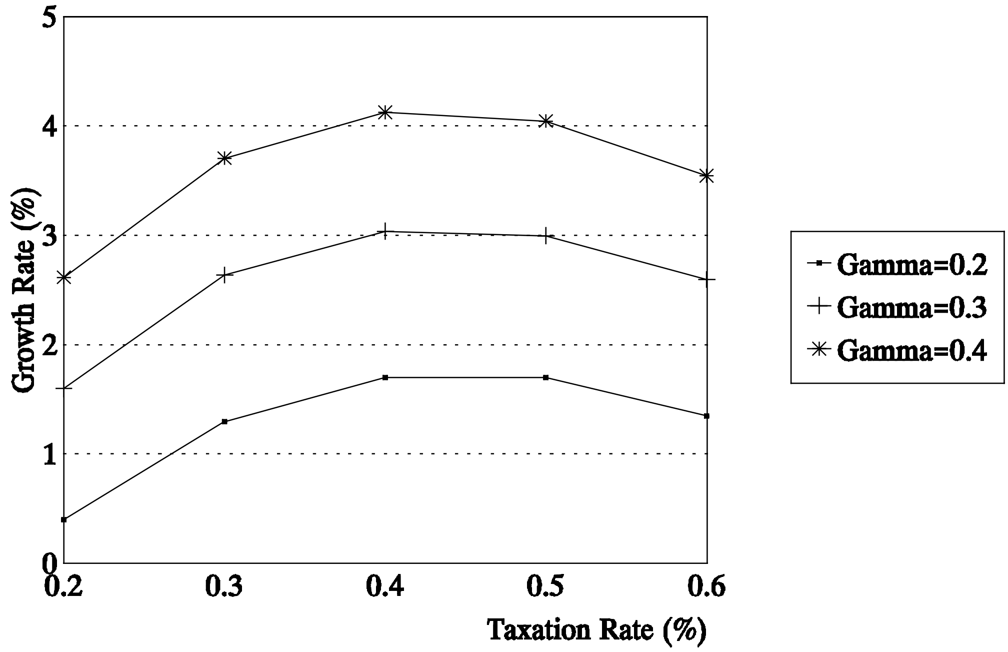
Figure 2 Optimal Growth and the Debt/GDP Ratio



D=Government Debt/GDP Ratio

Parameter	**Value**	**Source**
Marg. Prod. Capital=R+δ	0.15	World Bank
Depreciation Rate=δ	0.05	Imposed
Real Rate of Interest, R	0.10	Imposed
Consumption/GDP=C	0.64	IMF
Gov. Exp./GDP=G	0.22	IMF
Gov. Debt/GDP=D	0.53	IMF
p=1/life-expectancy	1/40	World Bank
λ=liquidity parameter	0.3	Haque and Montiel (1989)
GDP growth (per capita)	0.025	IMF
Population growth	0.02	IMF

Figure 3 Optimal Growth and Infrastructure Expenditure

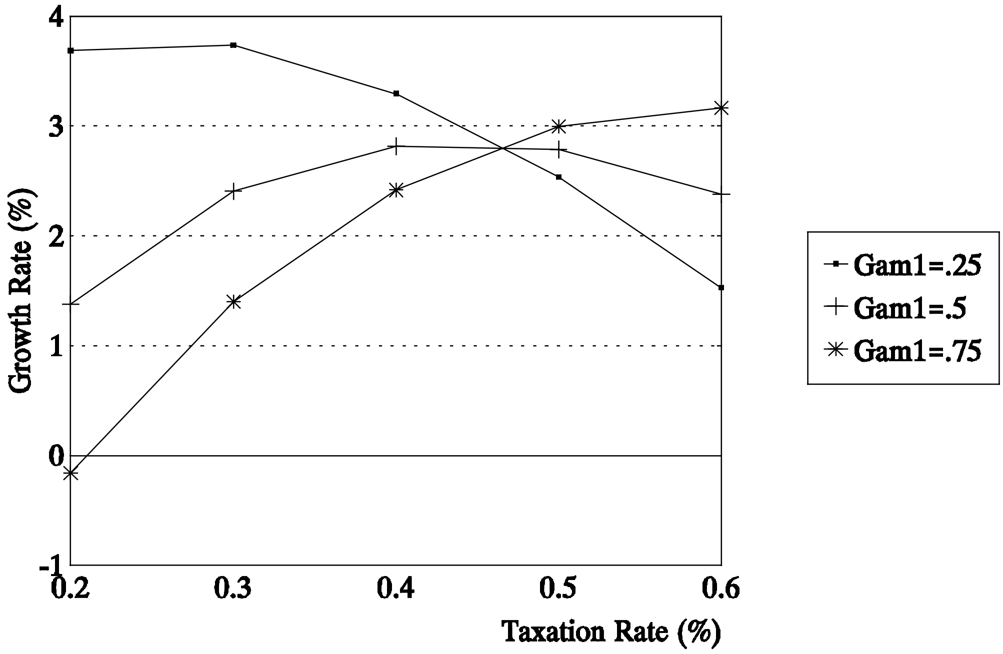


Gamma=Infrastructure/Total Government Expenditure

Gov. Investment/G=γ	0.30	IMF
Capital Externality=γ_1	0.5	Imposed
	$\gamma_2=\alpha+(1-\alpha)(1-\gamma_1)$	0.64
Capital/GDP=K	1.87	Equation (2.30)
α	0.28	Equation (2.28)
θ	0.06	Equation (2.26)
A=$B^{(1-\alpha)}$	0.70	Equation (2.29)

Table 2. Summary of Calibration. Sources: Data for India from The World

Figure 4 Optimal Growth and Efficiency of Public Capital



Gam1 = Elasticity of productivity to public capital

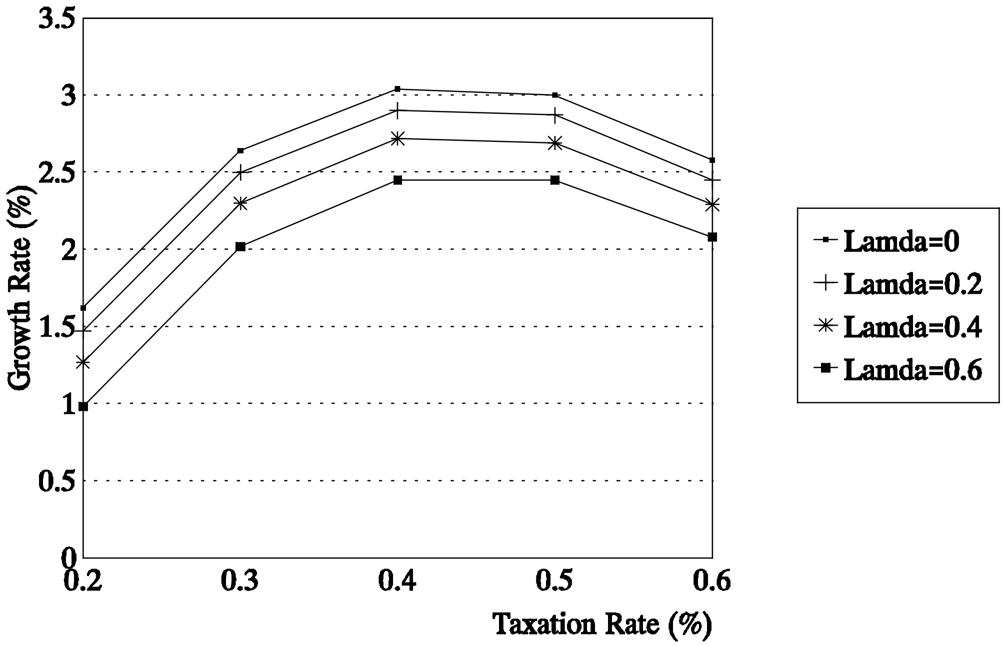
Bank (1992), IMF (1992).[8]

4. Intertemporal Aspects of Fiscal Policy: Is There a Time-Inconsistency Problem?

Until now the paper has focused on the steady-state of an economy in which consumers are intertemporal optimisers. Fiscal policy has been introduced in an ad hoc fashion ignoring the consequences of treating the government too as an intertemporal optimiser. Suppose that the government is benevolent and chooses a utility function which reflects that of a representative consumer. An immediate problem is that there is no representative consumer in our overlapping generations

[8] We are grateful to Ila Patnaik for assistance in collecting this data.

Figure 5 Optimal Growth and Liquidity Constraints



Lamda=Proportion of Liquidity Constrained Consumers.

model, but rather a spectrum of young and old consumers and those yet to be born. If we base our social welfare function on aggregate consumption, a natural choice based on (2.1) would be

$$U_t = \sum_{i=0}^{\infty} \left[\frac{1-p}{1+\theta} \right]^i [\gamma_1 \ln C_{t+i} + \gamma_2 \ln G^C_{t+i}] \tag{4.1}$$

at time t. (Note that C_t and G_t^C in (4.1) are actual values and not in per GDP form).

By using a social welfare function which aggregates consumption across all households of different ages we can formulate the optimisation problem in a linear-quadratic form and so utilise the equilibrium concepts developed in Currie and Levine (1993) and sketched out in Appendix B. However a consequence of using this welfare criterion is that it embodies the policymaker's desired distribution across present and future generations and is dependent upon the authorities' discount factor. Welfare-improvement with respect to our chosen social welfare function is not necessarily Pareto-improving with respect to present and future generations; for example, an increase in long-run growth can be at the expense of the current generation (See Saint-Paul (1992)). In our model with private capital externalities and tax distortions, there are potential efficiency gains, but these cannot be disentangled from an increase in social welfare measured by (4.1) which arises from redistribution between generations. Bearing in mind the distinction between welfare improvement using (4.1) as the criterion, and Pareto improvement across all generations, we formulate the governments problem as the maximisation of (4.1) with respect to fiscal instruments given the model summarised in table 1.

Solvency Considerations.

Let $\rho_t = R_t - N_t$ be the 'growth-adjusted' real interest rate over [t,t+1] and let $X_t = G_t - T_t$ be the 'primary deficit'. Then solving (vi) in table 1 forward in time we transform the budget **identity** into a **solvency constraint** at time t

$$D_t = -\sum_{i=0}^{\infty} \frac{X_{t+i}}{(1+\rho_t)(1+\rho_{t+2})\ ..\ (1+\rho_{t+i})} \qquad (4.2)$$

provided that the tranversality or 'no-Ponzi' condition

$$\lim_{i \to \infty} \frac{D_{t+i}}{(1+\rho_t)(1+\rho_{t+2})\ ..\ (1+\rho_{t+i})} = 0 \qquad (4.3)$$

holds. In (4.2) and (4.3) we assume that eventually $\rho_t > 0$. This is a feature of the Yaari-Blanchard consumption/savings model and rules out 'dynamic inefficiency'. According to (4.2) a government in debt with $D_t > 0$ must, sometime in the future, run primary surpluses to be solvent.

It should be noted that the transversality condition (4.3) does not require a **stable** debt/GDP ratio but merely that, in the long run, it does not increase faster than the growth adjusted real interest rate ρ_t. Stability of D_t is sufficient but not necessary to ensure solvency. However in a world with even very small departures from perfectly functioning capital markets, the notion of unbounded government debt/GDP ratios does not appeal. A stronger concept of solvency is that debt/GDP ratios do stabilise. We shall refer to the transversality condition (4.3) and the latter stability condition as **weak** and **strong** solvency conditions respectively.[9] In this paper we adopt the strong condition.

Expectations and Time Inconsistency

The credibility of policies and the associated problem of time inconsistency is potentially a major issue in any policy debate. In the model of this paper time inconsistency originates from three sources. First, it arises in our Yaari-Blanchard model of intertemporal optimal choice of consumption, savings and demand for money by each household where taxes are distortionary. Then when taxes are distortionary the consequences for time inconsistency are well-known (see, for example, Lucas and Stokey (1983), Stokey (1989) and Kydland and Prescott (1980)). The second source of time inconsistency lies in the private capital externality which can be corrected by public investment and the manipulation of forward-looking savings decisions using the tax instrument.

Given these features of the model and rational expectations we can distinguish between the cases when an authority has or does not have a reputation for precommitment. A fiscal or monetary authority which enjoys reputation in this

[9] Buiter and Patel (1990) provide an interesting discussion of this distinction.

sense can exercise the greatest leverage over the private sector in that an announced path of instrument settings would be credible and would effect private sector behaviour immediately in the desired way. For instance the announcement of a low taxes in the distant future will immediately raise savings, lower the real interest rate and increase private investment.

When a government cannot precommit itself to a future policy, it must act each period to maximise its welfare function, given that a similar optimisation problem will be carried out in the next period. Formally, the policymaker maximises at time t a welfare function U_t (or $E_t(U_t)$ in a stochastic setting) such that

$$U_t = u_t + \left[\frac{1-p}{1+\theta}\right] U_{t+1} \tag{4.4}$$

where $u_t = \gamma_1 \ln C_t + \gamma_2 \ln G_t^C$ is the single-period welfare given in (4.1) and U_t is evaluated on the assumption that an identical optimisation exercise is carried out from time t+1 onwards. Optimisation is subject to the constraint of the model and the strong solvency condition. The solution to this problem is found by dynamic programming and, unlike precommitment policy leads to a time consistent trajectory or rule for instruments. More details of both the precommitment and time-consistent solution procedures are to be found in Appendix B.

Simulation Results.

How relevant are reputational (time inconsistent) regimes? A large literature now exists on how a time inconsistent policy may be enforced despite the incentive to renege. Mechanisms suggested include constitutional constraints (Kydland and Prescott (1977)), trigger-strategy punishment mechanisms on the part of the private sector (Barro and Gordon (1983), Currie and Levine (1993) and intricate asset-management schemes for government debt (Lucas and Stokey (1983), Persson,

Persson and Svensson (1987)). However these mechanisms for enforcing time-inconsistent policies are not without their problems. The constitutional constraint solution assumes away the problem . For trigger-strategy mechanisms there exists the problem of how atomistic agents might coordinate on the precise punishment scheme that supports a particular reputational equilibrium. Organized agents, in some social contract, might achieve this co-ordination, but this still leaves a further problem. There is a sense in which Friedman-type trigger strategies lack credibility because carrying out the punishment is costly. Recent advances in game theory, in particular the renegotiation-proofness literature, address this problem at an abstract level; but the application of these concepts to the problem at hand is by no means straightforward.[10]

$U_t = u_t + \left[\frac{1-p}{1+\theta}\right] U_{t+1}$ Simluation Results $u_t = \gamma_1 \ln C_t + \gamma_2 \ln G_t^C$

The simulations reported use a linearised form of the model and a Taylor series quadratic approximation to the social welfare function valid in the vicinity of the original steady-state corresponding to the calibration in table 2. Details are provided in Appendix A. We consider the traditional time-inconsistency issue: the financing of a given path of government spending by a combination of borrowing and taxation. Thus both G_t^C and G_t^I are held fixed in this exercise and either the taxation rate τ or the debt/GDP ratio D can be considered as the fiscal instrument. Table 3 reports the values for selected variables at times t=1 and t=∞ for the policy with precommitment (P) and the time-consistent policy (TC) where the P solution

[10] al-Nowaihi and Levine (1994) discuss this literature and provide a resolution for the Barro-Gordon monetary policy game. The application of their solution to more general dynamic models, such as that presented in this paper, remains to be tackled.

cannot be enforced.

A number of features of these results are worthy of comment. First, maximising utility (or, in our procedure, minimising a welfare loss) does **not** result in long-run growth maximisation. The TC policy is sub-optimal but results in a **higher** long-run growth rate. Second, the welfare loss under TC is only slightly greater than under the P policy, but the means by which this is achieved is quite different. Both policies involve a large increase in the taxation rate in the first few periods which that wipe out government debt and turn the government into a creditor that owns a part of the capital stock. This happens when the debt/GDP ratio, in deviation form, falls below the baseline steady-state value D=53%. From figure 5, this occurs after about 3 years. Thereafter the two regimes radically depart. Under TC the government continues to accumulate assets and finances its government expenditure from the proceeds. In the long-run from table 2 it owns an equity stake in private capital equal to 91-53=28% of GDP compared with a total capital/GDP ratio of 1.87. There is now no longer an incentive to tax away the accumulated debt - there isn't any! -and the policy is time consistent.[11] By contrast, under the P regime, the government eventually becomes a debtor again and finances its expenditure in the conventional way. The policy is time-inconsistent because everywhere along the trajectory at t>0 there exists an incentive to re-optimise with a new burst of high taxation.

Variable	P Regime	TC Regime

[11] Obstfeld (1991) obtains the same result in an exogenous growth model with seigniorage as well as taxes.

n	(-0.24,0.28)	(-0.69,0.15)
r	(0.02,-0.025)	(0.03,-0.14)
c	(.74,-0.08)	(1.66,-0.47)
d	(-16,-35)	(-22,-91.25)
τ	(18,-.49)	(30,-2.65)
k	(-0.29,0.47)	(-0.43,2.5)
k^G	(0.31,0.32)	(0.76,-1.75)
Welfare loss (x10^6)	(1.124,1.123)	(1.126,1.124)

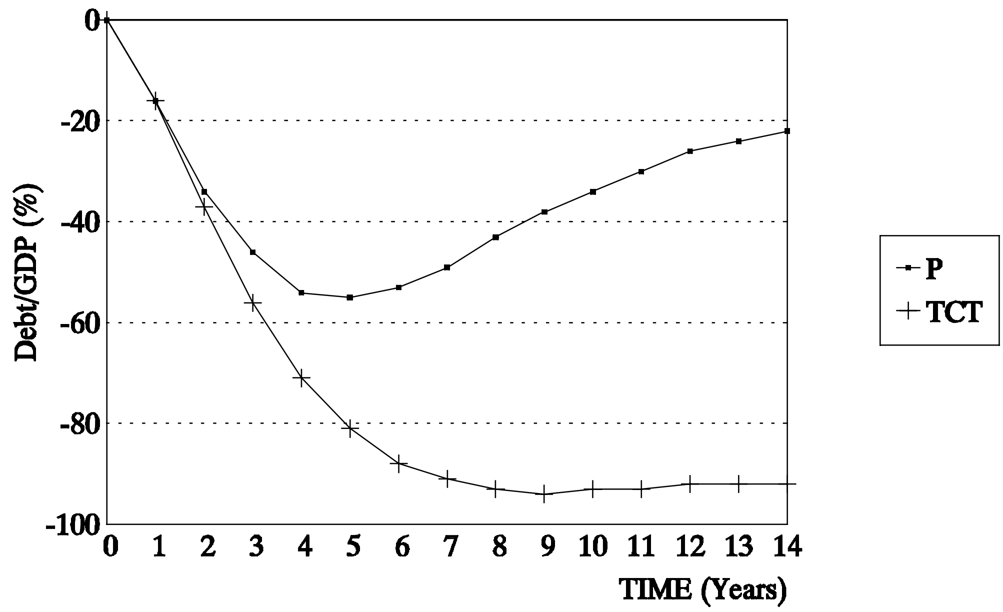
Table 3. The Precommitment (P) and Time-Consistent (TC) Financing Policies.

Notes: Variables are in % and measured as deviations about the original steady-state. For example, $n_t=N_t-N$ where N is the original steady-state growth rate reported in table 2 (i.e., N=4.5%). Values given are then (n_1, n_∞) for the two regimes.

5. Conclusions.

The results of the paper certainly suggest that for an approximately closed economy such as India, fiscal policy matters and the choice of taxation rates, the mix between infrastructure and total government expenditure and the level at which the government debt/GDP ratio stabilises can all significantly affect the long-run growth rate. Intertemporal aspects of policy are also important: the precommitment regime which is initially optimal but time-inconsistent, and the non-precommitment (time-consistent) regime lead to quite different outcomes in terms of the path for

Figure 6 Precommitment and Time Consistent Policies



P=Precommitment Solution
TCT=Time Consistent Solution

government debt; but the welfare differences are not great. Although sub-optimal in terms of the intertemporal welfare function, the time-consistent regime actually leads to a higher long-run growth rate. This is associated with long-run asset (rather than debt) accumulation by the government which enables it to use the proceeds to finance both spending and savings subsidies and avoid distortionary taxes.

Areas which will benefit from further research include, on the modelling side, the development of an endogenous model of liquidity constraints and the introduction of adjustment costs into investment. An examination of growth and debt in the context of an open economy (see Rebello (1992)) would enable fiscal policy to be examined in more open export-oriented LDC's and NIC's. A deeper look at time inconsistency, extending the analysis to expenditure as well as financing issues would merit further attention, as would as would an examination of government myopia, originating from political instability.

References.

al-Nowaihi, A. and P. Levine (1993), "Can Reputation Resolve the Monetary Policy Credibility Problem?", **Journal of Monetary Economics**, forthcoming.

Alogoskoufis, G.S. and van der Ploeg, F. (1991), 'On budgetary policies, growth and external deficits in an interdependent world', **Journal of Japanese and International Economies**,5,pp305-324.

Arrow, K.J. (1962),'The economic implications of learning by doing', **Review of Economic Studies**, 39, pp155-173.

Barro, R.J. (1990),' Government spending in a simple model of endogenous growth', **Journal of Political Economy**, Vol. 98, No. 5, pp103-125.

Barro, R.J. and Gordon, D. (1983), "Rules, Discretion and Reputation in a Model of Monetary Policy", **Journal of Monetary Economics**, 12, 101-21.

Blanchard, O.J. (1985), 'Debt, deficits and finite horizons', **Journal of Political Economy**,93, pp223-247

Buiter, W.H. (1991), 'Savings and endogenous growth: a survey of theory and policy', CEPR Discussion Paper No. 606.

Buiter, W.H. and Patel, U.R. (1990), "Debt, Deficits and Inflation: An Application to the Public Finance of India", CEPR Discussion Paper no. 408.

Currie, D.A. and Levine, P. (1993), **Rules, Reputation and Macroeconomic Policy Coordination**, Cambridge University Press.

Frenkel, J.A. and Razin, A. (1987), **Fiscal Policies and the World Economy**, MIT Press, Cambridge, Massachusetts.

Gregory, A.W. and G.W. Smith (1991)." Calibration as Testing: Inference in Simulated Macroeconomic Models", **Journal of Business and Economic Statistics**, Vol. 9, no. 3, 297-303.

Hamond, R.J. and Rodriquez-Claire, A. (1993), 'On endogenizing long-run

growth',**Scandanavian Journal of Economics**, 95(4), pp391-425.
Haque, N.U. and Montiel, P. (1989), 'Consumption in Developing Countries: Tests for Liquidity Constraints and Finite Horizons',**Review of Economics and Statistics**, pp408-415.
Kydland, F.W. and Prescott, E.C. (1977). "Rules Rather than Discretion: The Inconsistency of Optimal Plans", **Journal of Political Economy**, 85, 474-491.
Kydland, F.W. and Prescott, E.C.(1980)."Dynamic optimal Taxation, Rational expectations and Control", **Journal of Economic Dynamics and Control,** 2, 79-91
Lucas, R.E. (1988), 'On the mechanics of economic development',**Journal of Monetary Economics**, 22, pp3-42.
Lucas, R.E. and Stokey, N.L. (1983), "Optimal Fiscal and Monetary Policy in an Economy without Capital", **Journal of Monetary Economics**, 12, 55-93.
Obstfeld, M. (1991), "A Model of Currency Depreciation and the Debt-Inflation Spiral", **Journal of Economic Dynamics and Control**, 15(1), 151-178.
Obstfeld, M. (1994), "International Capital Mobility in the 1990's" , CEPR Discussion paper No. 902.
Persson, M, Persson, T. and Svensson, L.E.O. (1987), "Time Consistency of Fiscal and Monetary Policy", **Econometrica**, 55, 1419-1431.
Romer, P.M. (1986), 'Increasing returns and long-run growth', **Journal of Political Economy**, Vol. 94, pp1002-37.
Romer, P.M. (1990), 'Endogenous technical change', **Journal of Political Economy**, Vol. 98, pp71-103.
Romer, P.M. (1991),'Increasing returns and new developments in growth theory', in Barnett,W.A, Cornet, B., D'Aspremont, C., Gabszewicz, J.J. and Mas-Colell, A. (eds),**Equilibrium Theory and Applications**, Cambridge University Press.
Rebello, S. (1992), 'Growth in open economies', CEPR Discussion Paper No. 667.
Saint-Paul, G. (1992), 'Fiscal policy in an endogenous growth model', **Quarterly Journal of Economics**, Nov., pp1243-1259.
Shoven, J.B. and J. Whalley (1992), **Applying General Equilibrium**, Cambridge

Stokey, N.L. (1989), "Reputation and Time-Consistency", **American Economic**

Review, 79, 134-139.

Uzawa, H. (1965), 'Optimal technical change in an aggregative model of economic growth', **International Economic Review**, 6, pp18-31.

Weil, P. (1989), 'Overlapping families of infinitely lived agents', **Journal of Public Economics**, 38, pp183-198.

The World Bank (1992), **World Development Report**, Oxford University Press.

Xie, D. (1991), 'Increasing returns and increasing rates of growth', **Journal of Political Economy**, Vol. 99, No. 2, pp.429-435.

Yaari, M. (1965),' Uncertain lifetime, life insurance and the theory of the consumer', **Review of Economic Studies**, 32, pp137-150.

APPENDIX A. The Linear-Quadratic Form of the Optimisation Problem.

Employing the rates of change approximation preceding (2.26), the linearisation of the dynamic model set out in table 1 about the baseline steady-state corresponding to table 2 is given by

$$(1+p+\theta)(1+N)c_t^{u}=(1+p+g+R(1-\tau))c_{t-1}^{u}-CR\tau_t-(1+p+\theta)n_{t-1}+C(1-\tau)r_t-(p+g)(p+\theta)v \tag{A1a}$$

$$c_t^{c}=-\lambda(1-\alpha)\tau_t \tag{A1b}$$

$$v_t=d_t+k_t \tag{A2}$$

$$\gamma_2\frac{k_t}{K}+(1-\gamma_2)\frac{k_t^G}{K^G}=0 \tag{A3}$$

$$k_t=-K/(R+\delta)r_t \tag{A4}$$

$$(1+N)k_{t+1}=(1-\delta)k_t-Kn_t+i_t \tag{A5}$$

$$(1+N)d_{t+1}=(1+R)d_t+Dr_t-Dn_t+g_t-(1-\delta K)\tau_t-\tau\delta k_t \tag{A6}$$

$$(1+N)k_{t+1}^G=(1-\delta)k_t^G-K^G n_t+g_t^I \tag{A7}$$

$$c_t+i_t+g_t=c_t^{u}+c_t^{c}+i_t+g_t=0 \tag{A8}$$

where $c_t=C_t-C$ etc and all variables are either ratios per GDP or rates of change.

To obtain a quadratic approximation to (4.1),[13] first note that C_t and G_t are not expressed in per GDP form. With this in mind, write the government's discount factor as

$\kappa=(1-p)/(1+\theta)$ and write the first summation in (4.1) at t=0 as

$$\sum_{i=0}^{\infty}\kappa^i[\log C_i/Y_i+\log Y_i] \tag{A9}$$

Putting $N_t=(Y_{t+1}-Y_t)/Y_t$, the second term in (A9) can be written

$$\sum_{i=0}^{\infty}\kappa^i\log Y_{t+i}=\frac{\log(Y_t)}{1-\kappa}+\sum_{i=0}^{\infty}\frac{\kappa^i}{1-\kappa}\log(1+N_{t+i+1}) \tag{A10}$$

---

[13] We are indebted to Joe Pearlman who worked out this approximation.

Then using the approximation

$$\log(1+x)\approx x-x^2/2 \tag{A11}$$

we obtain our required quadratic form:

$$U_0=\sum_{i=0}^{\infty}\kappa^i\left[\gamma_1\left(\frac{\tilde{C}_t}{\tilde{C}}-\frac{\tilde{C}_t^2}{2\tilde{C}^2}\right)+\gamma_2\left(\frac{\tilde{G}_t^C}{\tilde{G}^C}-\frac{(\tilde{G}_t)^2}{2(\tilde{G}^C)^2}\right)+\frac{\kappa(\gamma_1+\gamma_2)}{1-\kappa}\left(\frac{n_t}{1+N}-\frac{n_t^2}{2(1+N)^2}\right)\right]+\text{a constant} \tag{A12}$$

where the notation $\tilde{C}=C/Y$ is used in this section only.

APPENDIX B. The Precommitment and Time-Consistent Regimes.

Replace the subscript t in (A1) with t+1 and take expectations at time t. Then the model can be expressed in state-space form

$$\begin{bmatrix} z_{t+1} \\ x_{t+1,t}^e \end{bmatrix}=A\begin{bmatrix} z_t \\ x_t \end{bmatrix}+Bw_t+C\epsilon_t \tag{B1}$$

$$s_t=E_1\begin{bmatrix} z_t \\ x_t \end{bmatrix}+E_2w_t \tag{B2}$$

where z_t is an (n-m)×1 vector of predetermined variables at time t, x_t is an m×1 vector of predetermined variables, s_t is an r×1 vector of target variables and $x_{t+1,t}^e$ denotes rational expectations of x_{t+1}. In our model there is only one non-predetermined variable, c_t, and m=1. Using (B2) we can write the policymaker's welfare **loss** at time τ is given by W_τ where

$$W_\tau=\frac{1}{2}\sum_{t=0}^{t=\infty}\kappa^t s_{\tau+t}^T\tilde{Q}s_{\tau+t} \tag{B3}$$

where $\tilde{Q}$ is symmetric and positive definite and κε(0,1) is a discount factor. The policymaker's optimisation problem at time t=τ is to minimise W_τ subject to the model (B2) and the initial value of the predetermined vector z_τ. Substituting (B2) into (B3) gives the following form of the welfare loss used in the subsequent analysis.

$$W_\tau = \frac{1}{2}\sum_{t=0}^{\infty} \kappa^t [y_{\tau+t}^T Q y_{\tau+t} + 2y_{\tau+t}^T U w_{\tau+t} + w_{\tau+t}^T \boldsymbol{R} w_{\tau+t}] \tag{B4}$$

where $y_t^T = [z_t^T \; x_t^T]$, $Q = E_1^T \tilde{Q} E_1$, $U = E_1^T \tilde{Q} E_1$ and $\boldsymbol{R} = E_2^T \tilde{Q} E_2$.

The Optimal Policy with Precommitment.

Consider the policymaker's ex ante optimal policy at τ=0 under the assumption that precommitment is possible. By standard theory of Lagrangian multipliers, we then minimise the Lagrangian

$$L_0 = \sum_{t=0}^{\infty} \kappa^t [\frac{1}{2}(y_t^T Q y_t + 2y_t^t U w_t + w_t^T \boldsymbol{R} w_t) + \boldsymbol{p}_{t+1}(Ay_t + \boldsymbol{B}w_t - y_{t+1})] \tag{B5}$$

with respect to $\{y_t\}$,$\{p_t\}$,and $\{w_t\}$ given z_0. This gives the first order conditions

$$w_t = -\boldsymbol{R}^{-1}(\kappa \boldsymbol{B}^T \boldsymbol{p}_{t+1} + U^T y_t) \tag{B6}$$

$$\kappa A^T \boldsymbol{p}_{t+1} - \boldsymbol{p}_t = -Qy_t - Uw_t \tag{B7}$$

together with the original constraint

$$y_{t+1} = \boldsymbol{A}y_t + \boldsymbol{B}w_t \tag{B8}$$

all holding for t ≥ 1. The condition

$$\boldsymbol{p}_0^T \delta y_0 = 0 \tag{B9}$$

completes the first order conditions. Equations (B6), (B7) and (B8) can be written in state-space form

$$\begin{bmatrix} I & \kappa \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T \\ 0 & \kappa(A^T - U\boldsymbol{R}^{-1}\boldsymbol{B}^T) \end{bmatrix} \begin{bmatrix} y_{t+1} \\ \boldsymbol{p}_{t+1} \end{bmatrix} = \begin{bmatrix} A - \boldsymbol{B}\boldsymbol{R}^{-1}U^T & 0 \\ -Q + U\boldsymbol{R}^{-1}U^T & I \end{bmatrix} \begin{bmatrix} y_t \\ \boldsymbol{p}_t \end{bmatrix} \tag{B10}$$

The solution to (B10) requires 2n boundary conditions. The initial values z_0 gives n-m of these conditions. The **tranversality conditions**

$$\lim_{t\to\infty} \kappa^t \boldsymbol{p}_t = 0 \tag{B11}$$

provide n more conditions. The remaining m conditions are found from (B9). Since z_0 is given and x_0 are 'free', (B9) reduces to

$$\boldsymbol{p}_{20} = 0 \tag{B12}$$

where $\boldsymbol{p}_t^T = [\boldsymbol{p}_{1t}^T \; \boldsymbol{p}_{2t}^T]$ is partitioned so that p_{1t} is of dimension (n-m)x1.

Now seek a solution of the form

$$p_t = S y_t \tag{B13}$$

Substituting into (B7) we get

$$w_t = -(\boldsymbol{R} + \kappa \boldsymbol{B}^T S \boldsymbol{B})^{-1} (\boldsymbol{B}^T S \boldsymbol{A} + U^T) y_t = -F y_t \tag{B14}$$

say, where S is the solution to the **Ricatti** matrix equation

$$S = \boldsymbol{Q} - UF - F^T U^T + F^T \boldsymbol{R} F + \kappa (\boldsymbol{A} - \boldsymbol{B} F)^T S (\boldsymbol{A} - \boldsymbol{B} F) \tag{B15}$$

To complete the solution we express the non-predetermined variables in (A10), $[\boldsymbol{p}_{1t}^T \boldsymbol{p}_{2t}^T]^T$ in terms of the predetermined variables $[z_t^T \boldsymbol{p}_{2t}^T]^T$. From (B14) we obtain

$$\begin{bmatrix} \boldsymbol{p}_{1t} \\ x_t \end{bmatrix} = \begin{bmatrix} S_{11} - S_{12} S_{22}^{-1} S_{21} & S_{12} S_{22}^{-1} \\ -S_{22}^{-1} S_{21} & S_{22}^{-1} \end{bmatrix} \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} = -N \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} \tag{B16}$$

Substituting into (B14) gives

$$w_t = -F \begin{bmatrix} I & 0 \\ -N_{21} & -N_{22} \end{bmatrix} \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} = G \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} \tag{B17}$$

say, and combining (B8), (B14) and (B16) gives

$$\begin{bmatrix} z_{t+1} \\ \boldsymbol{p}_{2t+1} \end{bmatrix} = T(\boldsymbol{A} - \boldsymbol{B} F) T^{-1} \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} = H \begin{bmatrix} z_t \\ \boldsymbol{p}_{2t} \end{bmatrix} \tag{B18}$$

say, where $T = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix}$. Given the solution S to the Ricatti equation (B15), equations (B16) to (B18) completely characterise the solution to the optimisation problem.

The welfare loss along the trajectory of the optimal policy or ’cost to go’ can now be evaluated. From the ’maximum principle’ and the first order condition (B9) we have that

$$\frac{dW_0}{dy_0} = \frac{dL_0}{dy_0} = \boldsymbol{p}_0^T \tag{B19}$$

Hence from (B13) on integration we have

$$W_0 = \frac{1}{2} y_0^T S y_0 \tag{B20}$$

at time τ=0. At time τ this becomes

$$W_\tau = \frac{1}{2} y_\tau^T S y_\tau \tag{B21}$$

Another way of expressing, W_τ which will prove useful, is found by eliminating x_τ in (B21) using (B16). We obtain

$$W_\tau = -\frac{1}{2}(tr(N_{11}Z_\tau) + tr(N_{22}p_{2\tau}p_{2\tau}^T)) \tag{B22}$$

where $Z_\tau = z_\tau z_\tau^T$ which, using (B12), becomes $W_0 = -\frac{1}{2}tr(N_{11}Z_0)$ at τ=0.

The Optimal Rule with Precommitment.

The feedback form of the optimal policy with precommitment (at time τ=0) is given by (B17). Write $T^{-1}(A-BF)T=H$ in (B18) and partition $H=\begin{bmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{bmatrix}$ so that H_{11} is mxm.

Similarly partition $G=[G_1 \;\; G_2]$ in (B17) conformably with $\begin{bmatrix} z_t \\ x_t \end{bmatrix}$. Then from (B18) we have

$$p_{2t+1} = H_{21}z_t + H_{22}p_{2t} \tag{B23}$$

Solving (B23) with $p_{20}=0$ (see (B12)) gives

$$p_{2t+1} = H_{21}\sum_{i=1}^{t}(H_{22})^{i-1}z_{t-i} \tag{B24}$$

Hence the feedback form of the rule, $w_t = G_1 z_t + G_2 p_{2t}$ can be expressed solely in terms of the observable (at time t) predetermined variables z_t.

The Time-Consistent (Markov-Perfect) equilibrium.

The precommitment solution takes the feedback form of a rule (B17) which, as we have seen from (B23) is a rule with **memory**. The time-inconsistency of this equilibrium can be best seen by examining the 'cost-to-go' (B21) . Re-optimisation at time τ and reneging on the commitment given at time 0 then involves putting $p_{2\tau}=0$. Thus the gains from reneging are $-\frac{1}{2}tr(N_{22}p_{2\tau}p_{2\tau}^T)$. Since it can be shown that $N_{22}<0$ (i.e., negative definite)[14] it follows that everywhere along the optimal trajectory at which $p_{2\tau}\neq 0$, there will be gains from reneging and the ex ante optimal policy is sub-optimal ex post.

In order to construct a time-consistent policy we employ dynamic programming and seek a Markov-perfect equilibrium in which instruments are still allowed to depend on past history , but only through a feedback on the **current** value of the state variables. This precludes a feedback as in (B23) which involves memory. Thus we seek a stationary solution $w_t=-Gz_t$ in which W_τ is minimised at time τ subject to the model (B2) in the knowledge that an identical procedure will be used to minimise $W_{\tau+1}$ at time $\tau+1$. Other features of the solution are that $x_t=-Nz_t$, which we know is true of saddlepath stable solutions to rational expectations models under a rule $w_t=-Fz_t$, and $W_\tau=z_\tau^T S z_\tau$. Notice that all these solution features follow from the precommitment solution with $p_{2t}=0$ for all t.

The solution is completely characterised by the matrices F, N and S. We now derive an iterative procedure and sequences $\{F_\tau\}$, $\{N_\tau\}$ and $\{S_\tau\}$ which (if convergent) converge to these stationary values. Suppose from time $\tau+1$ onwards,

$$x_{\tau+t}=-N_{\tau+t}z_{\tau+t} \;, \quad t\geq 1 \tag{B25}$$

Then from (B1)

$$x_{\tau+1}=-N_{\tau+1}(\boldsymbol{A}_{11}z_\tau+\boldsymbol{A}_{12}x_\tau+\boldsymbol{B}^1 w_\tau)=\boldsymbol{A}_{21}z_\tau+\boldsymbol{A}_{22}x_\tau+\boldsymbol{B}^2 w_\tau \tag{B26}$$

partitioning $\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{21}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix}$ and $\boldsymbol{B}=\begin{bmatrix}\boldsymbol{B}^1\\ \boldsymbol{B}^2\end{bmatrix}$ conformably with $\begin{bmatrix}z_t\\ x_t\end{bmatrix}$. Thus

[14] See Currie and Levine, Chapter 5, p145 for a formal proof.

$$x_\tau = J_\tau z_\tau + K_\tau w_\tau \tag{B27}$$

where

$$J_\tau = -(\boldsymbol{A}_{22} + N_{\tau+1}\boldsymbol{A}_{12})^{-1}(N_{\tau+1}\boldsymbol{A}_{11} + \boldsymbol{A}_{21}) \tag{B28a}$$

$$K_\tau = -(\boldsymbol{A}_{22} + N_{\tau+1}\boldsymbol{A}_{12})^{-1}(N_{\tau+1}\boldsymbol{B}^1 + \boldsymbol{B}^2) \tag{B28b}$$

Write

$$W_\tau = \frac{1}{2}(y_\tau^T Q_\tau y_\tau + 2y_\tau^T U w_\tau + w_\tau^T \boldsymbol{R} w_\tau) + \kappa W_{\tau+1} \tag{B29}$$

Then putting $W_{\tau+1} = \frac{1}{2} z_{\tau+1}^T S_{\tau+1} z_{\tau+1}$ and substituting for x_τ from (B28) we obtain

$$W_\tau = \frac{1}{2}(z_\tau^T \bar{\boldsymbol{Q}}_\tau z_\tau + 2z_\tau^T \bar{U}_\tau w_\tau + w_\tau^T \bar{\boldsymbol{R}}_\tau w_\tau + \kappa z_{\tau+1}^T S_{\tau+1} z_{\tau+1}) \tag{B30}$$

where $\bar{\boldsymbol{Q}}_\tau = \boldsymbol{Q}_{11} + J_\tau^T Q_{21} + \boldsymbol{Q}_{12} J_\tau + J_\tau^T Q_{22} J_\tau$, $\bar{U}_\tau = U^1 + \boldsymbol{Q}_{12} K_\tau + J_\tau^T U^2 + J_\tau^T Q_{22} J_\tau$ and $\bar{\boldsymbol{R}}_\tau = \boldsymbol{R} + K_\tau^T Q_{22} K_\tau + U^{2T} K_\tau + K_\tau^T U^2$. In (B30) Q and U are partitioned conformably with $\begin{bmatrix} z_t \\ x_t \end{bmatrix}$ as for A and B in (B26). Similarly eliminate x_τ from (B1) to obtain

$$z_{\tau+1} = \bar{\boldsymbol{A}}_\tau z_\tau + \bar{\boldsymbol{B}}_\tau w_\tau \tag{B31}$$

where $\bar{\boldsymbol{A}}_\tau = \boldsymbol{A}_{11} + \boldsymbol{A}_{12} J_\tau$ and $\bar{\boldsymbol{B}}_\tau = \boldsymbol{B}^1 + \boldsymbol{A}_{12} K_\tau$. Hence substituting (B31) into (B30) we arrive at

$$W_\tau = \frac{1}{2}[z_\tau^T(\bar{\boldsymbol{Q}}_\tau + \kappa \bar{\boldsymbol{A}}_\tau S_{\tau+1} \bar{\boldsymbol{A}}_\tau) z_\tau + 2z_\tau^T(\bar{U} + \lambda \bar{\boldsymbol{A}}_\tau^T S_{\tau+1} \bar{\boldsymbol{B}}_\tau) w_\tau + w_\tau^T(\bar{\boldsymbol{R}}_\tau + \kappa \bar{\boldsymbol{B}}_\tau^T S_{\tau+1} \bar{\boldsymbol{B}}_\tau) w_\tau] \tag{B32}$$

The control problem is now to minimise W_τ with respect to w_τ given the current state z_τ and given $S_{\tau+1}$ and $N_{\tau+1}$, which are determined by subsequent reoptimisa-tions. The first order condition is then

$$w_\tau = (\bar{\boldsymbol{R}}_\tau + \kappa \bar{\boldsymbol{B}}_\tau^T S_{\tau+1} \bar{\boldsymbol{B}}_\tau)^{-1}(U_\tau^T + \kappa \bar{\boldsymbol{B}}_\tau^T S_{\tau+1} \bar{\boldsymbol{A}}_\tau) z_\tau = G_\tau z_\tau \tag{B33}$$

say. Then combining (B27) and (B33) we have

$$x_\tau = (J_\tau - K_\tau F_\tau) z_\tau = -N_\tau z_\tau \tag{B34}$$

say. Substituting (B33) into (B32) and equating the quadratic terms in z_τ gives

$$S_\tau = \bar{\boldsymbol{Q}}_\tau + \bar{U}_\tau G_\tau + G_\tau^T \bar{U}_\tau^T + G_\tau^T \bar{\boldsymbol{R}}_\tau G_\tau + \kappa(\bar{\boldsymbol{A}}_\tau + \bar{\boldsymbol{B}}_\tau G_\tau)^T S_{\tau+1}(\bar{\boldsymbol{A}}_\tau + \bar{\boldsymbol{B}}_\tau G_\tau) \tag{B35}$$

Given $S_{\tau+1}$ and $N_{\tau+1}$, equations (B33), (B34) and (B35) give $F_{\tau,}$,N_τ and S_τ defining our iterative process. If these converge [15] to stationary values F, N and S, then we have a time-consistent optimal rule $w_t=Gz_t$ with cost to go

$$W_\tau = \frac{1}{2} z_\tau{}^T S z_\tau = \frac{1}{2} tr(SZ_\tau) \qquad \text{(B36)}$$

[15] I have not found any problems with convergence for a wide range of models, including that in this paper.