PHASES OF IMITATION AND INNOVATION IN A NORTH-SOUTH ENDOGENOUS GROWTH MODEL

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Abstract

In this paper, we develop a North-South endogenous growth model to examine three phases of development in the South: imitation of Northern products, imitation and innovation and finally, innovation only. In particular, the model has the features of catching up (and potentially overtaking) which are of particular relevance to the Pacific Rim economies. We show that the possible equilibria depend on cross-country assimilation effects and the ease of imitation. We then apply the model to analyse the impact of R&D subsidies. There are some clear global policy implications which emerge from our analysis. Firstly, because subsidies to Southern innovation benefit the North as well, it is beneficial to the North to pay for some of these subsidies. Secondly, because the ability of the South to assimilate Northern knowledge and innovate depends on Southern skills levels, the consequent spillover benefits on growth make the subsidising of Southern education by the North particularly attractive.

Keywords: Innovation, imitation, subsidies, growth.

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Two key features that determine trading patterns and growth in the global economy are the speed with which the poorer South absorbs scientific and technical knowledge, and management and commercial skills from the North, and the relative cost of imitating compared to innovating in the South. Knowledge transfers can be increased or impeded by international institutional arrangements and specific policies that affect education and training in the South. The incentive to innovate rather than imitate in the South will depend on the international protection of intellectual property rights and on government policies, including subsidies, to encourage innovation in the North and South.

This paper examine these issues by developing an endogenous growth North-South model driven by knowledge spillovers which is then used to analyse the impact of various R&D subsidies. Our model builds on the work of Grossman and Helpman (1991) (henceforth G&H), but extends it in an important direction. G&H develop North/South models in which the South imitates the North. Such models incorporate the traditional relationship between North and South; but they are less appropriate for the analysis of relationships between the mature industrial economies (OECD) and the rapidly growing economies of the Pacific Rim which are becoming innovators in their own right.

Our model is the first two-bloc endogenous growth model to examine and integrate three phases of development: innovation in the North with imitation in the South; innovation in the North with both imitation and innovation in the South; and finally innovation in both blocs. The key factors that determine which phase is appropriate are, first, the degree to which copying is easier than innovation for a given stock of knowledge capital and, second, the relative stocks of knowledge capital per worker. The latter in turn depends on the rate of assimilation of one region's innovations into the knowledge capital of the other. Two parameters in the model capture these effects: a_c/a which is the relative cost of copying compared to innovating in the South and κ defined as the rate of assimilation of Northern innovations into Southern knowledge capital. These two parameters, alongside the other parameters characterizing consumer preferences, also endogenize the balance in the South between innovation and imitation and enable us to address the impact of government in the form of subsidies.

The main result on subsidies is that, in the phase of development with both

innovation and imitation in the South, a subsidy to imitation in the South or innovation in the North can lower the global steady-state growth rate. A subsidy to Southern innovation, however, raises the global steady-state growth rate and can bring about a transition to the phase with innovation in both the North and the South.

There are some clear global policy implications which emerge from our analysis. Firstly, because the subsidies to Southern innovation benefit the North as well, it may well be in the interests of the North to pay for some of these subsidies, perhaps through transfer programs organized through the IMF and/or World Bank. Secondly, the benefits of education should feed directly into the ability of the South to assimilate Northern knowledge and innovate; the consequent spillover effects to growth and welfare make the subsidising of Southern education by the North particularly attractive.

But because of these knowledge spillovers and the associated externalities, policies aimed at fostering innovation and growth may be inefficient or suboptimal if they remain uncoordinated at an international level. This means that the institutions and associated rules that govern these policies matter for the growth performance of the international economy. Since inefficiencies may influence the rate of growth as well as the level of output, the costs of inefficient policies may be very high. These issues are the subject of continuing research by the authors.

Notation Summary

Throughout the paper, we use:

Superscript:	N	North	S	South
Subscript:	i	innovation	c	imitation

For the variables and parameters:

n	number of varieties
x	demand for variety
p	price
α	taste parameter
ε	elasticity of substitution
E	total expenditure
π	profits
w	wage rate
L	total labour employed
K	knowledge capital
$1/a$	efficiency parameter for innovation
$1/a_c$	efficiency parameter for copying
θ	constant of proportionality to density of varieties
κ	rate of assimilation
v	value of firm
r	rate of returns on riskless bond
ω	relative wage ratio (w^S/w^N)
ξ	product share
c	rate of copying
g	rate of growth
k	ratio of Southern innovative knowledge capital in the North (K^S/K^N)
ρ	discount factor

1. Introduction

Two key features that determine trading patterns and growth in the global economy are the speed with which the poorer South absorbs scientific and technical knowledge, and management and commercial skills from the North, and the relative cost of imitating compared to innovating in the South. Knowledge transfers can be increased or impeded by international institutional arrangements and specific policies that affect education and training in the South. The incentive to innovate rather than imitate in the South will depend on the international protection of intellectual property rights and on government policies, including subsidies, to encourage innovation in the North and South.

This paper examine these issues by developing an endogenous growth North-South model driven by knowledge spillovers which is then used to analyse the impact of various R&D subsidies. Our model builds on the work of Grossman & Helpman (1991) (henceforth G&H), but extends it in an important direction. G&H develop North/South models in which the South imitates the North. Such models incorporate the traditional relationship between North and South; but they are less appropriate for the analysis of relationships between the mature industrial economies (OECD) and the rapidly growing economies of the Pacific Rim. Table 1 indicates how South Korea and Taiwan, two leading Pacific Rim economies, are now emerging as centres of R&D activity and new product development. By comparing the ratio of R&D expenditure in GDP, the annual compound growth in R&D expenditure, and patents granted to residents of Taiwan and South Korea with two other typical advanced OECD countries, USA and UK, we can see that the former group is becoming a focus of innovation in its own right.[1] To analyse the interaction between Northern innovators and Southern imitators and innovators thus requires a model that allows for the possibility of both imitation and innovation in the latter region; developing a model that has this feature is the central contribution of this paper.

Various other models along 'new' growth theory lines have been developed to describe North/South interactions. Initially these took the traditional focus of the South as not being an innovator (G&H, Chapter 11, Segerstrom, Anant & Dinopolous (1990)). This has been followed by 'leapfrogging models' which assume that the South, while starting off as an imitator, may become an innovator

[1] See Chui, Pearlman, Levine & Sentance (1996) for further discussion of these developments.

Table 1. Innovation Performance

	R&D/GDP 1993 (%)	Ranking	Growth in R&D[†] 1989–1993 (%)	Ranking	Patents Granted[‡] 1992–1993	Ranking
Korea	2.08	11	9.72	10	9.25	10
Taiwan	1.78	13	16.71	4	66.46	1
UK	2.11	10	−1.86	29	7.86	12
US	2.77	4	−1.05	28	20.53	6

[†]Annual real compound percentage growth in Total Expenditure on R&D.

[‡]Average annual number of patents granted to residents per 100,000 inhabitants.

Sources: The World Competitiveness Report 1995.

instead (Chou & Shy (1991), Barro & Sala-i-Martin (1995), Brezis, Krugman & Tsiddon (1993)). Segerstrom (1991), in a closed-economy model, shows that an equilibrium with both imitation and innovation involving collusion between firms can exist. The prospect of collusion then provides an incentive to engage in costly imitation. In the North-South model of G&H it is the absence of factor price equalization and the ability of Southern firms with lower wage costs to price Northern firms out of the markets that provide this incentive, and we retain this structure.

Our model is the first two-bloc endogenous growth model to examine and integrate three phases of development: innovation in the North with imitation in the South; innovation in the North with both imitation and innovation in the South; and finally innovation in both blocs. The key factors that determine which phase is appropriate are, first, the degree to which copying is easier than innovation for a given stock of knowledge capital and, second, the relative stocks of knowledge capital per worker. The latter in turn depends on the rate of assimilation of one region's innovations into the knowledge capital of the other. Figure 1 below (p. 11) summarizes this dependency. The parameter κ is the rate of assimilation of Northern innovations by the South into Southern knowledge capital and $\kappa \to \infty$ is the extreme of instant assimilation. The parameter a_c/a is lower the easier it is to copy rather than innovate in the South. Thus for sufficiently low κ and a_c/a, a region of imitation dominates. For an intermediate range, a region of both innovation and imitation dominates. These two parameters, alongside the other parameters characterizing consumer preferences, also endogenize the balance in the South between innovation and imitation and enable us to address the impact of government in the form of subsidies.

The structure of the rest of the paper is as follows. Section 2 sets out our model and focuses particularly on the treatment of knowledge capital and the spillover effects. Section 3 examines the steady-state equilibria, and the transitional dynamics are discussed in section 4. Section 5 examines the effect of subsidies to innovation or imitation in all three phases of development and section 6 provides conclusions and suggestions for future research.

2. The Model

There are two regions: North and South. Both economies consist of a monopolistic competitive production sector and a competitive sector conducting R&D. n^N varieties were invented and are now produced in the North. n^S are produced in the South and of these n_i^S originated in the South from innovation and n_c^S were copied from the North (where the subscripts i and c represent innovation and copying respectively). Thus the total number of varieties available to consumers in both regions is $n = n^N + n^S = n^N + n_i^S + n_c^S$. The demand side of the model is entirely standard: consumers have identical preferences world-wide and maximize a utility function which is logarithmic in a Dixit-Stiglitz index with elasticity of substitution $\varepsilon = 1/(1-\alpha) > 1$ where $\alpha \in (0, 1)$ is the taste parameter.[2] Aggregating over regions and writing world nominal consumption as E gives total world demand for variety j wherever it is produced as[3]

$$x_j = \frac{Ep_j^{-\varepsilon}}{\int_0^n p_{j'}^{1-\varepsilon} dj'}; \qquad j \in (0, n]. \tag{1}$$

The novel features of the model are on the supply side. Labour is the only factor of production and one unit produces one unit of output. Then the operating profits of firm j producing variety j in bloc b is given by $\pi_j^b = (p_j^b - w_j^b)x_j^b$; $b = N, S$. The Northern firm's profit-maximizing price and profits subject to

[2]A lower value of α represents a greater taste for variety by consumers. See Appendix A for further details.

[3]The general notation adopted throughout denotes supply of variety j by bloc b by x_j^b; $b = N, S$ at a price p_j^b or p_j if the origin is irrelevant. G&H choose the normalization $E = 1$. We prefer the normalization that sets the Northern wage to unity. (See Appendix A.2.)

demand given by (1) are then given by

$$
\begin{aligned}
p_j^N &= p^N = \frac{w^N}{\alpha}, \\
\pi_j^N &= \pi^N = (1-\alpha)p^N x^N,
\end{aligned} \tag{2}
$$

i.e., all varieties in the North have equal prices and profits. For Southern innovating firms pricing and profits are analogous to (2) and given by

$$
\begin{aligned}
p_{i,j}^S &= p_i^S = \frac{w^S}{\alpha}, \\
\pi_{i,j}^S &= \pi_i^S = (1-\alpha)p_i^S x_i^S.
\end{aligned} \tag{3}
$$

Following G&H for Southern copying firms we distinguish the **narrow-gap** case, $\alpha w^N < w^S < w^N$, where there exists only a small cost advantage in the South and the **wide-gap** case where $w^S < \alpha w^N$. For the latter case copying firms can charge the full monopoly price giving price and profits

$$
\begin{aligned}
p_{c,j}^S &= p_c^S = \frac{w^S}{\alpha}, \\
\pi_{c,j}^S &= \pi_c^S = (1-\alpha)p_c^S x_c^S.
\end{aligned} \tag{4}
$$

In the narrow-gap case the Southern firm charges a limit price just sufficient to eliminate a potential Northern rival, i.e.,

$$
\begin{aligned}
p_{c,j}^S &= p_c^S = w^N, \\
\pi_{c,j}^S &= \pi_c^S = (w^N - w^S)x_c^S.
\end{aligned} \tag{5}
$$

Now consider the R&D sectors. The rates of production of new goods in the North and South, and the transfer of production to the South through copying are given by

$$
\begin{aligned}
\dot{n}_c^S + \dot{n}^N &= \frac{L^N K^N}{a}, \\
\dot{n}_i^S &= \frac{L_i^S K^S}{a}, \\
\dot{n}_c^S &= \frac{L_c^S K^S}{a_c}; \qquad a_c < a
\end{aligned} \tag{6}
$$

where L^N, L_i^S and L_c^S, and K^N and K^S denote labour employed and knowledge capital per unit of labour in North and South respectively. The assumption $a_c < a$ captures the idea that copying is easier than innovation for a given stock of knowledge capital. We shall eventually assume that a_c is a function of the relative size of the copying sector, n_c^S/n. This relationship is discussed at the end of section 3, but until then we treat it as constant.

The treatment of knowledge capital is central both in general to endogenous growth models of this genre, and in particular in our model where we allow for the possible co-existence of innovation and copying (imitation) in the South. The general idea is that each activity in the R&D sector gives rise to both a new blueprint and an addition to society's stock of knowledge capital, which contains new ideas and information that will be useful to later generations of innovators and in our context, imitators.[4] Before adopting specific functional forms it is useful to classify some of the results of G&H. Our first comment is a general one: G&H proxy knowledge capital by the measure of varieties of differentiated goods. This leads to the result that growth increases with increasing population size. General observation as well as empirical work (Jones (1995)) rebuts this result. A more realistic conclusion arises from the observation that innovative research arises not merely from reading journals and the details of patents, but also from interaction between potential inventors. The larger the total population, the smaller is the probability of meeting everyone involved in innovation.[5] Thus it is reasonable to assume that knowledge capital is proportional to the **density** of varieties within the population. If we define this constant of proportionality as θ, then all the results of G&H remain unaltered apart from replacing population size by θ. If θ is the same for all countries, then growth is identical for all countries and the scale effect, with growth increasing as population size increases, disappears.

Turning now to the functional forms used for knowledge capital, it is useful to contrast the effects of knowledge capital with and without cross-region spillovers when two regions engage in innovation and production. If there are immediate spillovers of knowledge capital, so that knowledge capital in each region depends on the total measure of all different varieties, then in equilibrium both regions grow at the same rate. If there are no spillovers, so that knowledge capital depends on the measure of varieties in each region, then in general one region will dominate and the other will grow more slowly. These are features analysed by G&H and Devereux & Lapham (1994). Thus to ensure that an equilibrium

[4]The general approach to model knowledge international spillovers is taken from Rivera-Batiz and Romer (1991a, b).

[5]Porter (1990) suggests that face-to-face contact is an important factor for the spreading of innovative ideas. (See also Brezis et al. (1993) for a discussion of this point.)

growth path is attained, with each region inventing a non-zero proportion of new goods, it is necessary to incorporate a spillover effect. How can this be accomplished in such a way as to account for a Pacific Rim country possibly catching up and overtaking? A general representation for innovation knowledge capital (incorporating the density effect outlined above) is the following, adapted from Grossman & Helpman (1990):

$$
\begin{aligned}
K^N &= \theta \left[\frac{n^N + n_c^S}{L^N} + \frac{\kappa^N \int_{-\infty}^{t} e^{-\kappa^N (t-\tau)} n_i^S(\tau) d\tau}{L^S} \right], \\
K^S &= \theta \left[\frac{n_i^S + n_c^S}{L^S} + \frac{\kappa^S \int_{-\infty}^{t} e^{-\kappa^S (t-\tau)} n^N(\tau) d\tau}{L^N} \right],
\end{aligned} \tag{7}
$$

so that there are spillovers from one region to another, but the rate of assimilation of one region's innovations into the knowledge capital of the other, captured by $0 < \kappa^N, \kappa^S < \infty$, is not immediate. In particular there are gradual spillovers from Southern innovative activity to the North and from varieties invented and produced in the North, to the South. In the limit as $\kappa^N = \kappa^S \to \infty$, assimilation is instant and if populations are the same, both Northern and Southern knowledge capital is proportional to the total number of varieties in the world, $n = n^N + n_c^S + n_i^S$. Clearly whichever region assimilates the other's inventions the faster will be able to produce more new goods, and will have a higher stock of capital in the steady state. If one can satisfactorily describe how government policy in the South can be used to raise the value of κ^S then one has a more comprehensive model of catching up and overtaking. But this question is not pursued in this paper.

We now assume that the North is characterized by an advantage in assimilating knowledge capital from the rest of the world, i.e., $\kappa^N > \kappa^S$. We further simplify and accentuate this feature by letting κ^N tends to infinity, i.e., the North assimilates knowledge capital instantly.[6] In order to reduce subsequent notation, we set $\kappa^S = \kappa$. With this assumption our representation for knowledge capital in the North replaces (7) with

$$
K^N = \theta \left[\frac{n^N + n_c^S}{L^N} + \frac{n_i^S}{L^S} \right]. \tag{8}
$$

[6]With this assumption catching up by the South occurs as κ^S increases, but overtaking can never happen.

Our formulation of knowledge capital largely removes population size effects on growth; it is straightforward to show there are also no size effects on levels of real variables such as the terms of trade when $\kappa \to \infty$. With little loss of generality we therefore assume equal populations, i.e., $L^N = L^S$ in the subsequent sections of the paper. This simplification is unimportant when innovation only takes place, as a transformation of variables from n^b to n^b/L^b $(b = S, N)$ will readily demonstrate. When imitation occurs, the population ratio L^S/L^N plays a role in the results; this is unsurprising, because the greater this ratio, the greater is the likelihood of imitation.

Now consider the financial sectors. Let the stock market value of the typical R&D firm in the production sectors producing innovative goods in the North and South and imitated goods in the South be denoted as v^N, v_i^S and v_c^S respectively. A new blueprint in the North costs $(w^N a)/K^N$ and value maximization requires this to be equated with v^N. The same argument applies to innovation and imitation in the South giving

$$v^N = \frac{w^N a}{K^N}, \quad v_i^S = \frac{w^S a}{K^S}, \quad v_c^S = \frac{w^S a_c}{K^S}. \tag{9}$$

We assume perfect capital mobility between production and R&D sectors in each region, but financial autarky between regions. In the North the typical firm in production must take into account that, during the period of time dt, it will be imitated by the South and forced out of business with probability $\dot{n}_c^S/n^N dt$. This gives the no-arbitrage condition

$$\frac{\pi^N}{v^N} + \frac{\dot{v}^N}{v^N} - \frac{\dot{n}_c^S}{n^N} = r^N \tag{10}$$

where r^b denotes the interest rate in bloc b. In the South we must take into account arbitrage between three assets: riskless bonds, and the equity of firms producing innovative and imitated goods. The no-arbitrage condition is then

$$\frac{\pi_i^S}{v_i^S} + \frac{\dot{v}_i^S}{v_i^S} = r^S = \frac{\pi_c^S}{v_c^S} + \frac{\dot{v}_c^S}{v_c^S}. \tag{11}$$

The model is closed with a labour market equilibrium condition for each region:

$$\begin{aligned} &\frac{a}{K^N}(\dot{n} - \dot{n}_i^S) + n^N x^N = L^N, \\ &\frac{a_c}{K^S}\dot{n}_c^S + \frac{a}{K^S}\dot{n}_i^S + n_c^S x_c^S + n_i^S x_i^S = L^S. \end{aligned} \tag{12}$$

3. The Equilibria in Three Phases of Development

The model set out above characterizes one of the three phases of development we are studying: innovation in the North and innovation alongside imitation (copying) in the South and imitation only in the South. The phase with imitation only in the South is obtained by putting $n_i^S = 0$ and suppressing the arbitrage condition (11). Similarly innovation only in the South is obtained by putting $n_c^S = 0$. The Northern innovation-Southern imitation case is examined in G&H but ignores international spillovers in knowledge capital, i.e., $K^N = \theta(n^N + n_c^S)/L^N$, and $K^S = \theta n_c^S/L^S$ in our formulation.[7] Grossman & Helpman (1990) extend their model to include lags in the diffusion of knowledge. A two-country model with instant assimilation of knowledge capital ($\kappa \to \infty$) and with innovation only in both countries is studied by Wälde (1995) who provides an interesting treatment of global stability.

First we have a proposition that disposes of one possible equilibrium:

Proposition 1. *There is no wide-gap equilibrium with both imitation and innovation in the South.*

This follows because in a wide-gap equilibrium the prices and therefore the profits and valuation of imitated and innovative goods in the South would then be the same. However, Southern innovation is more costly than imitation and would therefore not occur.

Now consider the remaining narrow-gap equilibrium with both imitation and innovation in the South. From the demand and pricing equations, (1), (2), (3)

[7]The more general case $K^S = K^S(n^N, n_c^S)$ is briefly examined in an Appendix to G&H Chapter 11.

and (5) we can write down the following relative demand relationships

$$\text{(13)} \qquad \begin{aligned} \frac{x_c^S}{x^N} &= \left(\frac{p_c^S}{p^N}\right)^{-\varepsilon} = \alpha^{-\varepsilon}, \\ \frac{x_i^S}{x^N} &= \left(\frac{p_i^S}{p^N}\right)^{-\varepsilon} = \left(\frac{w^S}{w^N}\right)^{-\varepsilon}. \end{aligned}$$

Now let us introduce some more notation. Let $\omega = w^S/w^N$ be the South/North wage ratio which from (13) is equal to the terms of trade in innovative goods. Let $\xi^N = n^N/n$, and $\xi_i^S = n_i^S/n$ and $\xi_c^S = 1 - \xi^N - \xi_i^S$ be the shares of the three types of products in the two regions. Let $c = \dot{n}_c^S/n^N$ be the rate at which the South copies the North.

Let $k = K^S/K^N$ be the ratio of knowledge capital in the South to that of the North. We now solve for a balanced-growth steady-state in which the product shares and k are constant, and the total market value of each production sector is constant. Then

$$\text{(14)} \qquad \frac{\dot{n}}{n} = \frac{\dot{n}^N}{n^N} = \frac{\dot{n}_c^S}{n_c^S} = \frac{\dot{n}_i^S}{n_i^S} = -\frac{\dot{v}^N}{v^N} = -\frac{\dot{v}_c^S}{v_c} = -\frac{\dot{v}_i^S}{v_i^S} = g,$$

$$\text{(15)} \qquad 1 - \xi_i^S = \frac{c+g}{g} \cdot \xi^N$$

follow. Differentiating K^S w.r.t. time using the definition in (7) and putting $\dot{k} = 0$ gives the steady-state of k as

$$\text{(16)} \qquad k = 1 - \frac{\xi^N g}{g + \kappa}.$$

In the steady state we have $r^N = r^S = \rho$ where ρ is the rate of time preference of consumers. (See Appendix A.) Then combining the no-arbitrage conditions (10) and (11), the NPV rules for R&D investment (9), the pricing equations (2), (3) and (5) with (13) and (14) gives

$$\text{(17)} \qquad \frac{\pi_i^S/v_i^S}{\pi^N/v^N} = \frac{K^S x_i^S}{K^N x^N} = k\omega^{-\varepsilon} = \frac{\rho+g}{\rho+g+c},$$

$$\text{(18)} \qquad \frac{\pi_c^S/v_c^S}{\pi^N/v^N} = \frac{K^S(1-\omega)\alpha x_c^S a}{K^N \omega(1-\alpha) x^N a_c} = \frac{k(1-\omega)\alpha\alpha^{-\varepsilon} a}{\omega(1-\alpha)a_c} = \frac{\rho+g}{\rho+g+c}.$$

Then equating (17) and (18) we arrive at

$$\alpha^{\varepsilon}\frac{(1-\alpha)}{\alpha}\frac{a_c}{a}=\omega^{\varepsilon}\frac{(1-\omega)}{\omega}. \tag{19}$$

From (19), this leads to a relative wage $\omega = w^S/w^N$, which will turn out to be the same over a range of values of κ, provided that a_c/a is fixed.

Equations (15), (16), (17) and (19) together with the steady states of the equations in (12), i.e.

$$\begin{aligned}\theta &= a(1-\xi_i^S)g+\frac{a\alpha\xi^N}{1-\alpha}(\rho+g+c),\\ \theta &= \frac{a_c}{k}(1-\xi^N-\xi_i^S)\left[g+(\rho+g)\frac{\omega}{1-\omega}\right]+\frac{a\xi_i^S}{k}\left[g+(\rho+g)\frac{\alpha}{1-\alpha}\right]\end{aligned} \tag{20}$$

yield six equations for the steady state of the six endogenous variables k, ξ^N, ξ_i^S, ω, g and c.

To examine the possibility of transition between the three equilibria of (i) Southern imitation only, (ii) Southern imitation and innovation and (iii) Southern innovation only, it is useful to think in terms of changes in the parameters a_c/a and κ, reflecting the relative cost of copying and the speed at which the South absorbs knowledge capital from the North. Figure 1 illustrates the domain of these equilibria where the boundary between imitation only and imitation/innovation are those values of a_c/a and κ such that $\xi_i^S = 0$ in equations (15)–(20), and the boundary between imitation/innovation and innovation only corresponds to $\xi_c^S = c = 0$.

In the absence of imitation in the South, the rate of imitation $c = 0$ and from (17) $k = \omega^{\varepsilon}$. When κ tends to infinity, $k = 1$ (see (16)), the differences between the two blocs disappear. We then arrive at factor price equalization as one expects. However, in general if the assimilation of international knowledge capital by the South is slower than that in the North ($\kappa < \kappa^N < \infty$) our model exhibits a lower wage in the South even in the phase of innovation only in the South. At the other extreme as κ tends to 0, Southern imitation dominates and Southern innovation disappears.

The evolution of the South/North wage ratio with increasing κ as we pass from phase (ii) (Southern imitation and innovation) to phase (iii) (Southern innovation only) can be found from (17) and (19). For simplicity, first consider the case when a_c is constant; from (19) we see that ω remains the same irrespective of the value

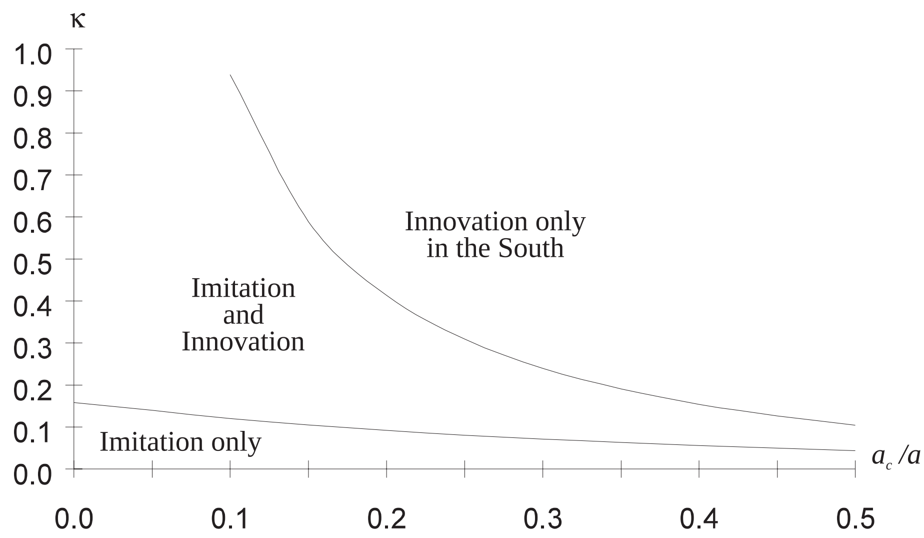


FIGURE 1. Equilibria for Varying a_c and κ.

of κ during phase (ii). When the boundary in Figure 1 is reached, the copying sector no longer exists in equilibrium, and (17) applies with $c = 0$; since relative Southern knowledge capital, k, increases with increasing κ, it follows that the relative wage increases too.

Our more general formulation has $a_c = a_c(\xi_c^S)$, where this relationship between the cost of copying and the size of the imitative sector may be positive or negative. A positive relationship represents a diminishing returns to copying effect as in Barro & Sala-i-Martin (1995), where imitators copy those products which are easiest to copy first, so that the more difficult products to copy are associated with a higher a_c. This obviously raises the question as to why the same argument does not apply to innovation (see Stokey (1995)). A negative relationship implies that copying becomes easier as the copying sector increases in size. The rationale for this is that monitoring patents by innovators in the North becomes more difficult as the copying sector becomes bigger. If this effect outweighs the diminishing returns to copying effect then the relationship $a_c = a_c(\xi_c^S)$ is negative. We can investigate these effects using Figure 2. Considering the case when $da_c/d\xi_c^S < 0$, an increase in κ encourages innovation in the South, decreases the size of the copying sector increases a_c, and we move from A to B, with a *fall* in the relative wage in the South. If on the other hand the diminishing returns effect to copying

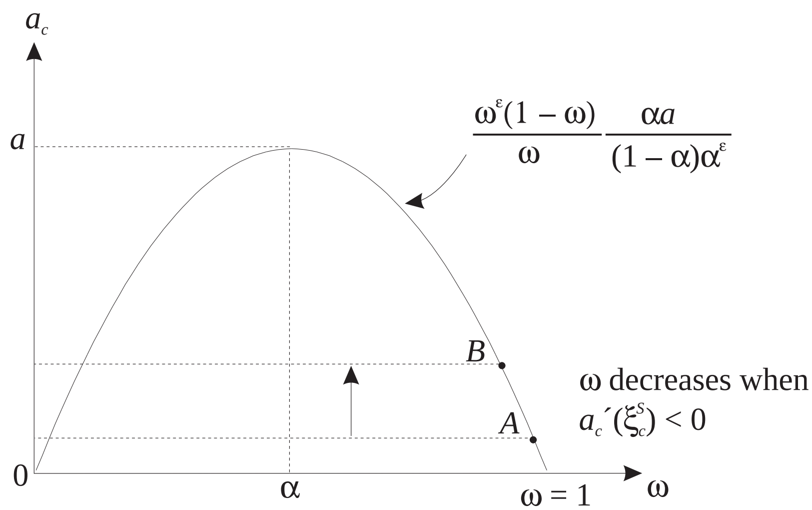


FIGURE 2. Changes in the South/North Wage Ratio ω as κ increases in Phase (ii).

dominates, then we move down the curve and the relative wage *rises.*

The results are summarized in the following proposition.

Proposition 2. *In the transition from innovation and imitation in the South (phase (ii)) to innovation only in the South (phase (iii)) brought about by an increase in the rate of North/South knowledge flows (an increase in* κ*) then if* $da_c/d\xi_c^S < 0$*, the relative wage in the South,* ω*, must fall. If* $da_c/d\xi_c^S > 0$ *then* ω *must rise.*

The result is a consequence of equilibrium in output and financial markets, i.e., the no-arbitrage conditions between Northern innovation, Southern imitation and innovation and the forgone returns on riskless bonds. As innovation becomes easier in the South, the incentive to imitate and therefore the rate of imitation, c, falls. Limit pricing requires a rise in profits in imitating Southern firms relative to Northern firms because of the reduction in the risk premium c. (See (18)). This forces down the Southern relative wage ω. Equivalently, the same relative Northern profit can be maintained with a higher Northern relative wage rate because of the reduction in risk.

However, when imitation disappears in the South so does this latter effect. Then rates of return, π^b/v^b, in Northern and Southern innovating firms are equalized (see (17) with $c = 0$). As Southern innovation becomes easier this forces up

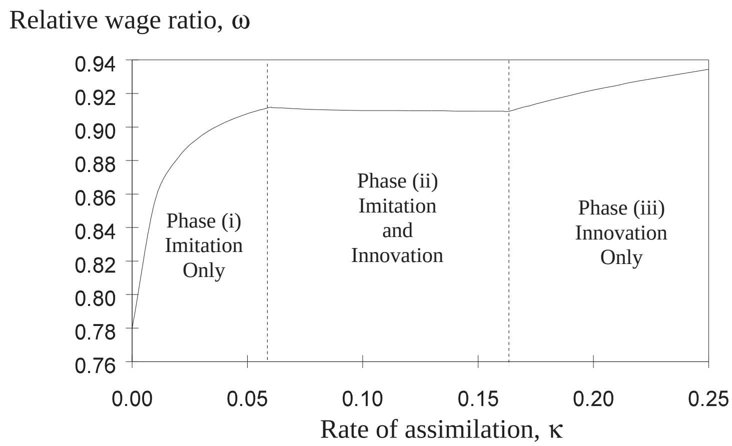


FIGURE 3. Changes in the South/North Wage Ratio ω as κ increases.

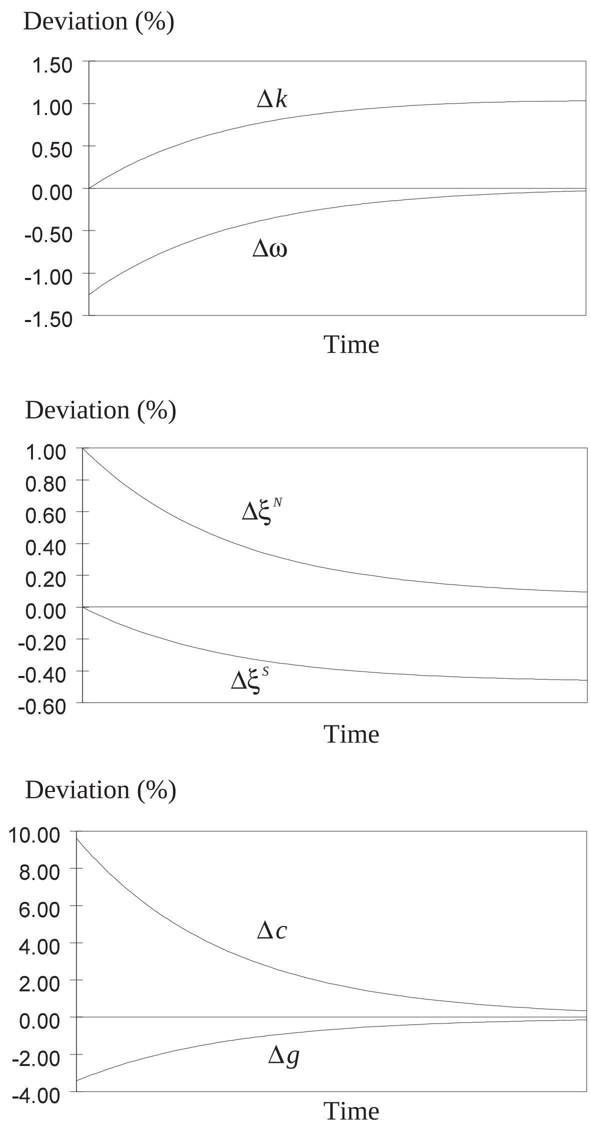
the Southern relative wage. The path of ω for differing values of κ for the case $da_c/d\xi_c^S < 0$ is shown in Figure 3.

4. Stability and Transitional Dynamics

We now turn to the stability of the three equilibria. Assuming that the trade balance is zero, we can write the full dynamic model in terms of dynamic equations in g, k, ξ^N, ξ_i^S and ω, with g and ω as jump variables. Taking deviations about their equilibrium values we obtain an approximation to the dynamics by evaluating the Jacobian matrix. (See Appendix A.) Numerical simulations based on the calibration described in Appendix C show that the system is saddle-path stable for phase (i) (Southern imitation only) and phase (iii) (Southern innovation only), and these result proves robust for a wide range of parameter values about the central calibrated values. Table 2 summarizes the details of the calibration and the range of parameters adopted in the sensitivity analysis, each being changed keeping other parameters at their central values.

For phase (ii), Southern innovation and imitation, we consider two cases. First consider the case where the effect of increasing costs of monitoring $da_c/d\xi_c^S < 0$. We find that the system is saddle-path stable for the range of parameter values in Table 2. In fact this result holds even for a very small value of $da_c/d\xi_c^S$. In the subsequent dynamic simulations of phase (ii) we replace a_c with $\bar{a}_c + \psi\xi_c^S$ with ψ close to zero so that the equilibrium results remain virtually the same.

The stability result for $da_c/d\xi_c^S < 0$ can be explained as follows. Consider the



Notation: $k = K^S/K^N$; $\omega = w^S/w^N$; ξ^N, ξ_i^S are the shares of Northern and Southern innovative goods respectively; g is the rate of growth; and c is the rate of copying. Δk is deviation from steady state; $\Delta\omega$ etc. are defined similarly.

FIGURE 4. Trajectories of the Variables, all parameters are at their central values.

Table 2. Calibration and Sensitivity Analysis

Parameter	Central Values	Range
α	0.67	0.50–0.80
a_c/a	0.20	0.05–0.34
κ	0.20	0.10–0.40
ρ	0.05	0.02–0.10

dynamic arbitrage value for the relative wage of equation (19). In Figure 2 its equilibrium value is indicated by point A. As an example of the dynamics of the system, suppose that ξ^N starts above and ξ_c^S below their equilibrium levels. a_c is therefore above its equilibrium level, since $da_c/d\xi_c^S < 0$ and the initial arbitrage level of ω is therefore at B where the relative wage, ω, has fallen. This makes imitation more attractive because it raises the profit rate relative to innovation and the rate of copying rises. The share of the Southern imitation sector therefore rises and that of Northern innovation falls. The relative wage ω gradually rises which eventually leads to the share of Southern innovation increasing back to is equilibrium value. Meanwhile the overall growth rate which dropped initially, also returns to its equilibrium value. (See Figure 4.)

What happens if we instead assume increasing or constant costs of imitation? Now the relative wage ω cannot fall as in the previous case and this equilibrating effect when imitation declines is absent. Summarizing these results, we have:

Numerical Result 1. *Phases (i) (Southern imitation only) and (iii) (Southern innovation only) are saddle-path stable for a wide range of parameter values about the central calibration. Phase (ii) is saddle-path stable if and only if* $da_c/d\xi_c^S < 0$, *i.e., the increasing costs of monitoring patents dominates diminishing returns to copying.*

The details of the dynamics when saddle-path stability is absent, are complicated and treated fully elsewhere (Pearlman (1996)). However, in essence what happens is that the South temporarily shifts to a regime of imitation only (together with production of previously imitated and innovative Southern products), which is coupled with a drop in the relative wage. The latter increases until such a point where there is an incentive for both imitation and innovation to take place. For the case of a_c constant, this is exactly when the relative wage has reached its dynamic arbitrage (constant) value.

5. Subsidies to Innovation or Imitation

First consider an equilibrium with high κ when there is no imitation in the South. Suppose that the government finances a fraction ϕ^N of all Northern research expenses by a lump-sum tax on residents; this lowers the private cost of invention to $(1-\phi^N)a/n$. This in turn raises the profit/valuation ratio which, assuming the no-arbitrage condition, is equal to $g+\rho$. The subsidy to innovation raises the return from innovation, but leaves the return to production unchanged, and this induces a shift in labour resources from production to innovation. The positive spillover effects from knowledge capital raise the return from innovation in the South as well, so that there is a shift in labour there as well in the same direction. The combined effect is to raise the steady-state global growth rate. Similar arguments apply to subsidies to innovation in the South as well.

We can summarize this as follows:

Proposition 3. *In an equilibrium with no Southern imitation (phase (iii)), a subsidy to either Northern or Southern innovation raises the steady-state growth rate.*

Proof: *Changes to the steady state of the model when subsidies to innovation are given in Appendix B. To obtain the phase of development with innovation in North and South, set $c=0$, $\xi_i^S = 1-\xi^N$ and suppressing (18). The proposition then follows by differentiating the resulting steady-state conditions with respect to ϕ^N or ϕ^S.*

Likewise, in an equilibrium with low κ, when there is no innovation activity in the South, a subsidy to Northern innovation raises the steady-state growth rate because of the shift in labour resources from production to innovation. In a narrow-gap equilibrium a subsidy to Southern imitation however, has no effect on relative prices of the consumption goods produced by each region because of the strategic limit pricing assumption. Thus relative demands remain the same. Furthermore, unlike Northern subsidies to innovators which affect Northern strategic interactions and hence prices and wage rates in both North and South, Northern prices and wages are unaffected by Southern imitation subsidies. Hence growth remains unchanged. The only effect is on Southern wages which increase. In a wide-gap equilibrium, which may exist for sufficiently small κ when Southern imitation knowledge capital is less than Northern innovation knowledge capital, Southern firms charge a mark-up on Southern wages. Then a subsidy to imitation leads to relative demand changes and an increase in both imitation and growth rates. We summarize these results as:

Proposition 4. *In an equilibrium with imitation only in the South (phase (i)), a subsidy to Northern innovation raises the steady-state growth rate. A subsidy to Southern imitation raises steady-state growth on a narrow-gap equilibrium, but has no effect on the steady-state growth in a wide-gap equilibrium.*
Proof: *This phase of development is obtained by* $\xi_i^S = 0$ *and suppressing* (17)*. The proof is then similar to Proposition* 3*.*

Consider now an equilibrium with both innovation and imitation in the South (phase (ii)). Analytical results are impossible to obtain, so we present numerical results in Tables 3 and 4. The results of subsidizing the North, the Southern innovators and imitators are presented in Figure 5. In this situation anything which induces more imitation represents a waste of resources and is likely to lower global knowledge capital and welfare. A subsidy to Northern innovation raises the return to innovation as before and thereby raises the relative wage rate in the North. However this raises the return to imitation in the South and induces a shift in labour resources from innovation to imitation in the South. The overall effect is seemingly perverse in that the growth rate drops, although the share of Northern innovations and Southern imitations increases at the expense of Southern innovations. The effects of a subsidy to imitation in the South has similar effects. Again overall growth decreases and imitation increases while the share of Northern innovations and Southern imitations increases. However benefits are derived from a subsidy to Southern innovation. In this case there is nothing which induces imitation in the South; indeed the imitation rate declines. Overall growth increases in both South and North (because of the spillover effects), and the behaviour is similar to those seen in the equilibrium discussed above with no imitation in the South. Indeed for sufficiently high subsidies to Southern innovation, imitation completely vanishes. These results are summarized as:

Numerical Result 2. *In an equilibrium with imitation and innovation in the South (phase (ii)) a subsidy to imitation in the South or innovation in the North can lower the steady-state growth rate. A subsidy to Southern innovation however raises the steady-state growth rate and can lead to a transition to phase (i) with no imitation in the South.*

TABLE 3. Simulation Results of Varying a_c.

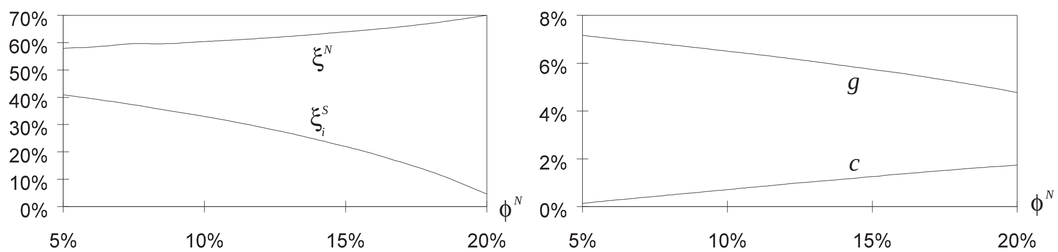
a_c	0.0500	0.1000	0.1500	0.2000	0.2500	0.3000	0.3400
k	0.8660	0.8611	0.8566	0.8522	0.8479	0.8436	0.8401
ω	0.9926	0.9849	0.9770	0.9688	0.9603	0.9514	0.9441
ξ^N	0.6267	0.6021	0.5835	0.5691	0.5578	0.5489	0.5434
ξ_i^S	0.2183	0.2775	0.3258	0.3667	0.4022	0.4338	0.4568
g	0.0544	0.0600	0.0652	0.0702	0.0750	0.0797	0.0834
c	0.0135	0.0120	0.0101	0.0079	0.0054	0.0025	0.0000

$\alpha = 0.67$, $a = 1.00$, $\rho = 0.05$, $\kappa = 0.20$, and $\theta = 0.19$.

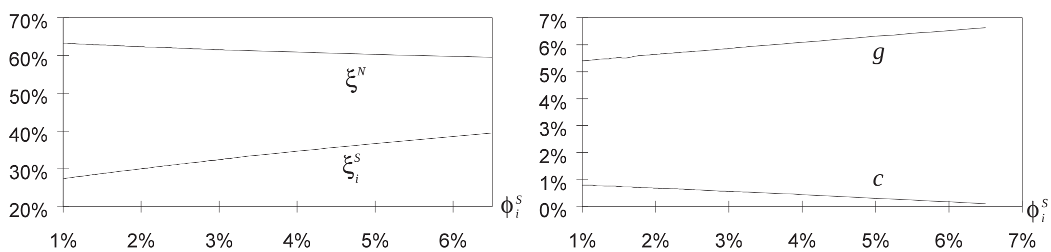
TABLE 4. Simulation Results of Varying κ.

κ	0.0920	0.1000	0.1500	0.2000	0.3000	0.3500	0.4100
k	0.7824	0.7888	0.8261	0.8522	0.8855	0.8969	0.9078
ω	0.9688	0.9688	0.9688	0.9688	0.9688	0.9688	0.9688
ξ^N	0.7310	0.6953	0.6024	0.5691	0.5395	0.5314	0.5244
ξ_i^S	0.0005	0.0785	0.2883	0.3667	0.4379	0.4577	0.4751
g	0.0390	0.0436	0.0609	0.0702	0.0808	0.0843	0.0875
c	0.0143	0.0142	0.0110	0.0079	0.0034	0.0017	0.0001

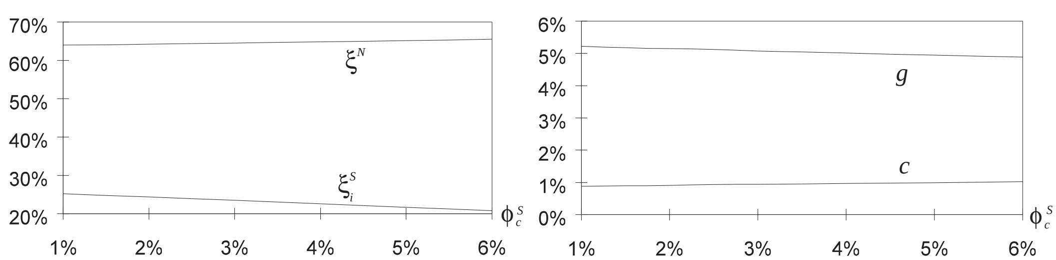
$\alpha = 0.67$, $a = 1.0$, $a_c = 0.2$, $\rho = 0.05$, and $\theta = 0.19$.



(a) Subsidising the North.



(b) Subsidising the Southern Innovators.



(c) Subsidising the Southern Imitators.

FIGURE 5. Effects of Subsidy on Products Shares, ξ^N, ξ_i^S, and the Rates of Growth g and Copying c

6. Conclusions

We have presented a model of imitation and innovation which has the features of catching up (and potentially overtaking) which is most clearly relevant to the Pacific Rim economies. We have shown that equilibria including innovation and/or imitation depend on cross-country assimilation effects and the ease of imitation. We have discussed the effects of various different types of subsidy and have found the global benefits occur when there is a subsidy to Southern innovation. There are some clear global policy implications which emerge from our analysis. Firstly, because the subsidies to Southern innovation benefit the North as well, it may well be in the interests of the North to pay for some of these subsidies, perhaps through transfer programs organized through the IMF and/or World Bank. Secondly, the benefits of education should feed directly into the ability of the South to assimilate Northern knowledge and innovate; the consequent spillover effects to growth and welfare make the subsidising of Southern education by the North particularly attractive.

But because of these knowledge spillovers and the associated externalities, policies aimed at fostering innovation and growth may be inefficient or suboptimal if they remain uncoordinated at an international level. This means that the institutions and associated rules that govern these policies matter for the growth performance of the international economy. Since inefficiencies may influence the rate of growth as well as the level of output, the costs of inefficient policies may be very high. These issues are the subject of continuing research by the authors.

Appendix A. Transitional Dynamics

In this appendix, we provide the details of the dynamics of the model. With little loss of generality, we assume $L^S = L^N = 1$.

A.1. The Model. We define the intertemporal preferences of bloc b as,

$$U_t^b = \int_0^\infty e^{-\rho(\tau - t)} \log D^b(\tau) d\tau, \tag{A1}$$

where ρ is the subjective discount rate, and $D^b(\tau)$ takes the following form

$$D^b = \left\{ \int_0^{n^b} [x^b(j)]^\alpha dj \right\}^{1/\alpha}. \tag{A2}$$

The budget constraints for the North and South are given by,

$$\begin{aligned} \dot{A}^N &= r^N A^N + w^N - E^N; \qquad A^N = n^N v^N, \\ \dot{A}^S &= r^S A^S + w^S - E^S; \qquad A^S = n_c^S v_c^S + n_i^S v_i^S, \end{aligned} \tag{A3}$$

where E^b is the nominal expenditure for bloc b, and A^b is the total wealth. Maximising (A1) subject to (A3) and imposing the usual transversality conditions gives

$$\frac{\dot{E}^b}{E} = r^b - \rho; \qquad \text{for } b = N, S. \tag{A4}$$

A.2. Transitional Dynamics. First we consider the North. We normalize w^N at unity, and write E^N as

$$E^N = p^N n^N x^N = \frac{1}{\alpha} n^N x^N, \tag{A5}$$

From the labour market clearing condition, we have

$$L^N = \frac{a}{K^N}(\dot{n} - \dot{n}_i^S) + n^N x^N. \tag{A6}$$

Since $L^N = L^S = 1$ the Northern knowledge capital becomes $K^N = \theta n / L^N = \theta n$. Therefore, we have

$$L^N = \frac{a(\dot{n} - \dot{n}_i^S)}{\theta n} + n^N x^N. \tag{A7}$$

Substituting from (A7) into (A5), we have

$$E^N = \frac{1}{\alpha\theta}\left[\theta - ag + a\xi_i^S g_i^S\right], \tag{A8}$$

where $g = \dot{n}/n$, and $g_i^S = \dot{n}_i^S / n_i^S$.

To find out r^N, we make use of the no-arbitrage condition,

$$\begin{aligned}\dot{v}^N &= r^N v^N - \pi^N + \frac{\dot{n}_c^S}{n^N} v^N \\ &= r^N v^N - \frac{(1-\alpha)}{\alpha} x^N + \frac{\dot{n}_c^S}{n^N} v^N.\end{aligned} \tag{A9}$$

Also, value maximization means that $v^N = w^N a / K^N$, or $nv^N = a/\theta$, so rearranging (A9),

$$\begin{aligned}r^N &= \frac{\dot{v}^N}{v^N} + \frac{(1-\alpha)}{\alpha} \frac{x^N}{v^N} - \frac{\dot{n}_c^S}{n^N} \\ &= -g + \frac{(1-\alpha)}{\alpha a} \left[\frac{\theta - ag + a\xi_i^S g_i^S}{\xi^N} \right] - \frac{1}{\xi^N} (g - \xi_i^S g_i^S - \xi^N g^N).\end{aligned} \tag{A10}$$

Substituting (A8) and (A10) into (A4), we have

$$\begin{aligned}\frac{d(\theta - ag + a\xi_i^S g_i^S)}{dt} &= (\theta - ag + a\xi_i^S g_i^S) \Bigg\{ -g - \rho \\ &+ \frac{(1-\alpha)}{\alpha a} \left[\frac{\theta - ag + a\xi_i^S g_i^S}{\xi^N} \right] - \frac{1}{\xi^N} (g - \xi_i^S g_i^S - \xi^N g^N) \Bigg\}.\end{aligned} \tag{A11}$$

By definition, $k = K^S / K^N$, using $K^N = \theta n$ and K^S as in (7) yields

$$\dot{k} = \frac{\dot{K}^S}{K^N} - kg = (g + \kappa)(1 - k) - g^N \xi^N. \tag{A12}$$

Similarly, from the definition of the shares of innovative goods produced in the North and the South, we have

$$\dot{\xi}^N = \frac{\dot{n}^N}{n} - \frac{n^N}{n} \frac{\dot{n}}{n} = \xi^N g^N - \xi^N g, \tag{A13}$$

$$\dot{\xi}_i^S = \xi_i^S g_i^S - \xi_i^S g. \tag{A14}$$

In the South, maximizing the utility function subject to the budget constraint, and then using (9), (11) and (13) gives,

$$\frac{\dot{E}^S}{E^S} = r^S - \rho = \frac{\dot{\omega}}{\omega} - \frac{\dot{k}}{k} - g + (1-\alpha) \frac{\omega^{-\varepsilon} k}{\alpha a \xi^N} [\theta - ag + a\xi_i^S g_i^S] - \rho, \tag{A15}$$

where $\omega = w^S/w^N$. E^S can be expressed as follows using (13) and (A5):

$$\text{(A16)} \quad \begin{aligned} E^S &= p_i^S n_i^S x_i^S + p_c^S n_c^S x_c^S = \frac{\omega}{\alpha} n_i^S x_i^S + n_c^S x_c^S \\ &= \frac{E^N}{\xi^N}[\omega^{1-\varepsilon}\xi_i^S + \alpha^{1-\varepsilon}(1 - \xi^N - \xi_i^S)]. \end{aligned}$$

Differentiating the log of (A16) with respect to time gives

$$\text{(A17)} \quad \frac{\dot{E}^S}{E^S} = \frac{\dot{E}^N}{E^N} - \frac{\dot{\xi}^N}{\xi^N} + \frac{1}{\omega^{1-\varepsilon}\xi_i^S + \alpha^{1-\varepsilon}(1 - \xi^N - \xi_i^S)} \times [(1-\varepsilon)\omega^{-\varepsilon}\xi_i^S\dot{\omega} + \omega^{1-\varepsilon}\dot{\xi}_i^S - \alpha^{1-\varepsilon}\dot{\xi}^N - \alpha^{1-\varepsilon}\dot{\xi}_i^S].$$

Now substitute (A10) and (A15) into (A17) to give

$$\text{(A18)} \quad \begin{aligned} &\frac{\dot{\omega}}{\omega} - \frac{\dot{k}}{k} + (1-\alpha)\frac{\omega^{-\varepsilon}k}{\alpha a \xi^N}[\theta - ag + a\xi_i^S g_i^S] = \\ &\frac{(1-\alpha)}{\alpha a}\left[\frac{\theta - ag + a\xi_i^S g_i^S}{\xi^N}\right] - \frac{1}{\xi^N}(g - \xi_i^S g_i^S - \xi^N g^N) - \frac{\dot{\xi}^N}{\xi^N} \\ &+ \frac{(1-\varepsilon)\omega^{-\varepsilon}\xi_i^S\dot{\omega} + \omega^{1-\varepsilon}\dot{\xi}_i^S - \alpha^{1-\varepsilon}\dot{\xi}^N - \alpha^{1-\varepsilon}\dot{\xi}_i^S}{\omega^{1-\varepsilon}\xi_i^S + \alpha^{1-\varepsilon}(1 - \xi^N - \xi_i^S)}. \end{aligned}$$

Since $K^S = kn\theta$, Southern labour market equilibrium implies that,

$$\text{(A19)} \quad \begin{aligned} n_c^S x_c^S + n_i^S x_i^S &= L^S - \frac{a_c \dot{n}_c^S}{K^S} - \frac{a\dot{n}_i^S}{K^S} \\ &= 1 - \frac{a_c}{k\theta}(g - \xi_i^S g_i^S - \xi^N g^N) + \frac{a}{k\theta}\xi_i^S g_i^S. \end{aligned}$$

The ratio of consumption of the South to the North is,

$$\text{(A20)} \quad \frac{n_c^S x_c^S + n_i^S x_i^S}{n^N x^N} = \frac{(1 - \xi_i^S - \xi^N)\alpha^{-\varepsilon} + \xi_i^S\omega^{-\varepsilon}}{\xi^N}.$$

Combining (A19) and (A20), we have

$$\text{(A21)} \quad \frac{\theta - \frac{a_c}{k}(g - \xi_i^S g_i^S - \xi^N g^N) - \frac{a}{k}\xi_i^S g_i^S}{\theta - ag + a\xi_i^S g_i^S} = \frac{(1 - \xi_i^S - \xi^N)\alpha^{-\varepsilon} + \xi_i^S\omega^{-\varepsilon}}{\xi^N}.$$

Furthermore, from the two no-arbitrage conditions (11) in the South, we have

$$\text{(A22)} \quad \frac{\dot{v}_c^{S,e}}{v_c^S} + \frac{(1-\omega)x_c^S}{v_c^S} = \frac{\dot{v}_i^{S,e}}{v_i^S} + \frac{(1-\alpha)}{\alpha}\frac{\omega x_i^S}{v_i^S}$$

From (9) we can deduce that $v_c^S = v_i^S a_c/a$ where $a_c = a_c(\xi^N + \xi_i^S)$ and hence using (13), (A22) implies

$$\text{(A23)}\quad \frac{a_c'}{a_c}(\dot{\xi}^N + \dot{\xi}_i^S) + \left[\frac{(1-\omega)\alpha^{-\varepsilon}}{a_c\omega} - \frac{(1-\alpha)\omega^{-\varepsilon}}{a\alpha}\right]\frac{k}{\xi^N}(\theta - ag + a\xi_i^S g_i^S) = 0.$$

Substituting $\dot{\xi}^N$ and $\dot{\xi}_i^S$ from (A13) and (A14), we see that (A21) and (A23) represent linear identities for $\xi^N g^N$ and $\xi_i^S g_i^S$ in terms of the five other variables: g, k, ξ^N, ξ_i^S, and ω. The dynamics of these are represented by (A11)–(A14) and (A18).

Appendix B. Subsidies

In this appendix, we present the changes to the model when subsidies are granted by the government to Northern innovation, Southern innovation and Southern imitation respectively.

B.1. Subsidy to Northern Innovation. Let ϕ^N be the rate of subsidy government granted to the Northern innovators, thus

$$\frac{K^N v^N}{a} = (1-\phi^N)w^N$$

replaces the first equation in (9). This leads to changes to equations (17) and (20):

$$\text{(17}'\text{)}\qquad \frac{\rho+g}{\rho+g+c} = k\omega^{-\varepsilon}(1-\phi^N),$$

$$\text{(20}'\text{)}\qquad \rho+g+c = \frac{1-\alpha}{\alpha}\left[\theta - (1-\xi_i^S)ag\right]\frac{1}{a\xi^N(1-\phi^N)}.$$

B.2. Subsidy to Southern Innovation. Similarly, if we assume the rate of subsidy to Southern innovators be ϕ_i^S, then we have

$$\frac{K^S v_i^S}{a} = (1-\phi_i^S)w^S$$

replaces the first equation in (9). This leads to changes to equations (17), (19) and (20):

$$\text{(17}''\text{)}\qquad \frac{\rho+g}{\rho+g+c} = \frac{k\omega^{-\varepsilon}}{(1-\phi_i^S)},$$

$$\text{(19}''\text{)}\qquad \frac{a_c}{a}\frac{1}{1-\phi_i^S}\frac{\alpha^{\varepsilon}(1-\alpha)}{\alpha} = \omega^{\varepsilon}\frac{1-\omega}{\omega},$$

(20″)

$$\theta = \frac{a_c}{k}(1 - \xi^N - \xi_i^S)\left[g + (\rho + g)\frac{\omega}{1-\omega}\right] + \frac{a\xi_i^S}{k}\left[g + (\rho + g)\frac{\alpha}{1-\alpha}(1 - \phi_i^S)\right].$$

B.3. Subsidy to Southern Imitation. Finally, if we assume government is subsidising Southern imitation at the rate of ϕ_c^S, then

$$\frac{K^S v_c^S}{a_c} = (1 - \phi_c^S)w^S$$

replaces the first equation in (9). This leads to change in equations (19) and (20):

(19‴)
$$(1 - \phi_c^S)\frac{a_c}{a}\frac{\alpha^\varepsilon(1-\alpha)}{\alpha} = \omega^\varepsilon \frac{1-\omega}{\omega},$$

(20‴)

$$\theta = \frac{a_c}{k}(1 - \xi^N - \xi_i^S)\left[g + (\rho + g)\frac{\omega}{1-\omega}(1 - \phi_c^S)\right] + \frac{a\xi_i^S}{k}\left[g + (\rho + g)\frac{\alpha}{1-\alpha}\right].$$

Appendix C. Calibration

We calibrate parameters using the single country version of (20), $\theta = ag + \frac{\alpha}{1-\alpha}(\rho + g)$ and by assuming that the growth rate $g = 0.03$ per annum and the discount rate $\rho = 0.05$. The ratio of the efficiency parameters a_c/a is assumed to be 0.2, and a is normalized at unity. According to Krugman (1991), a plausible mid-range estimate for the elasticity of substitution will be around 3, so we choose the value for α to be 0.67. This yields a value for $\theta = 0.19$.

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