THE WELFARE ECONOMICS OF RURAL TO URBAN MIGRATION: THE HARRIS-TODARO MODEL REVISITED

Thomas Krichel	Paul Levine
Department of Economics	Department of Economics
University of Surrey	University of Surrey
Guildford GU2 5XH	Guildford GU2 5XH
United Kingdom	United Kingdom
T.Krichel@surrey.ac.uk	P.Levine@surrey.ac.uk

December 17, 1997

Abstract

The Harris-Todaro model of rural to urban migration is extended to include urban agglomeration effects, some urban real wage flexibility and a government budget constraint. Without employment subsidies, laissez-faire migration is excessive unless real wage flexibility and agglomeration effects are high. Laissez-faire migration is too low compared with the first best outcome supported by a subsidies, if its financing involves no costs. Simulations suggest that such a program would imply a substantial increase in taxation. If, as seems likely, an increase of this magnitude involves economic costs, then the optimal outcome falls well short of first best.

JEL-Classification: F22, J61, O15

Keywords: Migration, Harris-Todaro, Welfare Economics, Employment Subsidies

The authors acknowledge comments on a earlier version of the paper by Subrata Ghatak and by participants at seminars at the Universities of Aberdeen and Loughborough. The usual disclaimer applies.

1 Introduction

Two contrasting views of the welfare economics of rural to urban migration are expressed by policymakers and the development literature. The first proposes the adoption of economic policies which would re-allocate labor from low productivity (rural) to high productivity (urban) areas and generally promote factor mobility. Indeed, the World Bank (1990) suggests that countries which adopted these policies fared better (in terms of aggregate and per-capita growth rate) than those which failed to affect labor transfer rapidly enough to high growth areas. In contrast to this prescription, there is a widespread view in the development literature that urban growth in the LDCs has been excessive and policy should be addressed at curbing an "urban bias".

This paper examines these views using a extended Harris-Todaro (HT) model which comprises a developed urban and less developed rural sector. The central feature of this genre of models is the existence of a "migration equilibrium" achieved through unemployment in the developed sector. Three extensions of the seminal contributions (Todaro (1969), Harris and Todaro (1970)) are introduced. First, following Shukla and Stark (1990)) an "agglomeration" or external economies of scale effect is introduced in the urban sector. Second, the HT assumption of real wage rigidity is relaxed in favor of some flexibility in response to urban unemployment. Third, in considering subsidy schemes, we do so in the context of a government budget constraint. Two types of policy exercises are carried out in the paper. The first contrasts laissez-faire migration with that chosen by a social planner with powers to control migration but with no use of subsidies to affect employment. This leads to some support that an urban bias exists and a lower rural to urban migration would be welfare-enhancing; but this result is critically dependent upon the degree of real wage flexibility. If real wages respond strongly enough (in a downward direction) to unemployment then laissez-faire migration is too low to realize the full potential benefit of a transfer of labor from the low productivity rural sector to the high productivity urban sector.[1]

The second exercise undertaken is the comparison between laissez-faire and a social optimum supported by employment subsidies in the two sectors. In another seminal contribution in this area, Bhagwati and Srinivason (1974) show that a social optimum which increases migration above the laissez-faire outcome can be supported by equal subsidies in the two sectors. Shukla and Stark (1990) show that if urban scale economies exist then a *higher* subsidy

[1] This idea is also pursued by Gehrig, Schmidt, and Zimmermann (1992) in a study motivated by actual and potential migration from Eastern Europe and the LDCs into Western Europe.

is needed in the urban sector. These findings support the view that increased mobility from the rural to the urban sectors may be desirable.
This paper addresses an important limitation of these studies: the insufficient attention paid to the public financing aspects. Subsidy programs have to be financed and this may require a significant increase in general taxation. Our simulation results on a calibrated model with a government constraint suggests that this indeed is the case. The paper shows further that the introduction of tax distortions in the rural sector and general costs of tax collection can lead to a social outcome which falls considerably short of the first-best, zero unemployment outcome of previous studies.
The rest of the paper is organized as follows. Section 2 sets out the generalized HT model with agglomeration effects, some degree of real wage flexibility and a government budget constraint. Section 3 considers the welfare economics of migration in the absence of employment subsidies. Section 4 examines optimal subsidy policies under a budget constraint and Section 5 provides some conclusions and suggestions for future research. Computational aspects have been confined to an Appendix A.

2 An Extended Harris-Todaro Model

2.1 Production

The production side of the model consists of the urban and rural sector. The capital stock in each sector is assumed fixed. The total labor supply $\bar{L}$ (denoting exogenous variables with a bar) for the two sectors together is also fixed and consists of a urban workforce L_{u} and a rural workforce L_{r}. Thus the global labor resource constraint is given by

$$\bar{N} = N_{\mathrm{u}} + N_{\mathrm{r}} \tag{2.1}$$

Both sectors are price-takers. We choose the price of manufacturing output as the numeraire and let P be the relative price of rural output. The representative firm i in the urban sector produces output with a production function Y_{ui} given by

$$Y_{\mathrm{ui}} = g(L_{\mathrm{u}})\, f(L_{\mathrm{ui}}), \quad f', g' > 0, \quad f'', g'' < 0 \tag{2.2}$$

where L_{ui} is labor employed by firm i and L_{u} is aggregate employment in the sector. The function $g(L_{\mathrm{u}})$ introduces external economies of scale in the manufacturing sector. An early example for such externalities can be found in Panagariya (1981); our formulation follows Shukla and Stark (1990). Firm i chooses employment L_{ui} to maximize its profits given the real wage W_{u} (net

of any subsidy S_{u}) and given aggregate employment. This leads to the first order condition

$$g(L_{\mathrm{u}})\, f'(L_{\mathrm{ui}}) = W_{\mathrm{u}} - S_{\mathrm{u}} \tag{2.3}$$

There are $\mathfrak{I}_{\mathrm{u}}$ identical firms so that $L_{\mathrm{ui}} = L_{\mathrm{u}}/\mathfrak{I}_{\mathrm{u}}$. The aggregate forms of (2.2) and (2.3) then become

$$Y_{\mathrm{u}} = \mathfrak{I}_{\mathrm{u}}\, g(L_{\mathrm{u}})\, f(L_{\mathrm{u}}/\mathfrak{I}_{\mathrm{u}}) \tag{2.4}$$

and

$$g(L_{\mathrm{u}})\, f'(L_{\mathrm{u}}/\mathfrak{I}_{\mathrm{u}}) = W_{\mathrm{u}} - S_{\mathrm{u}} \tag{2.5}$$

It is worth noting at this point that a social planner would maximize *total* sector profits to yield

$$g'(L_{\mathrm{u}})\, f(L_{\mathrm{u}}/\mathfrak{I}_{\mathrm{u}}) + g(L_{\mathrm{u}})\, f'(L_{\mathrm{u}}/\mathfrak{I}_{\mathrm{u}}) = W_{\mathrm{u}} - S_{\mathrm{u}}$$

which results in a higher level of urban employment. This is the first externality that policy intervention must address.
There are no scale economies in the rural sector for which output of farm i is given by

$$Y_{\mathrm{ri}} = h(L_{\mathrm{ri}}) \quad h' > 0, \quad h'' < 0 \tag{2.6}$$

The first order profit-maximizing condition then gives, in aggregate form,

$$P\, h'(L_{\mathrm{r}}/\mathfrak{I}_{\mathrm{r}}) = W_{\mathrm{r}} - S_{\mathrm{r}} \tag{2.7}$$

where L_{r}, W_{r} and S_{r} are rural employment, the rural wage and the rural employment subsidy respectively. The aggregate form of (2.6) is

$$Y_{\mathrm{r}} = \mathfrak{I}_{\mathrm{r}}\, h(L_{\mathrm{r}}/\mathfrak{I}_{\mathrm{r}}) \tag{2.8}$$

The rural wage is completely flexible and the labor market clears in this sector. Hence $N_{\mathrm{r}} = L_{\mathrm{r}}$. However in the urban sector real wage rigidity (which becomes a constant real wage in the original HT model) leads to urban unemployment. This plays a central role in the migration equilibrium which we now consider.

2.2 The migration decision

Migrants are risk-neutral and compare their expected income after migrating with their certain rural income. Let Π be the probability of employment in the urban sector, $\bar{W}_a$ be the real income net of tax if the migrant is unemployed, let T be the income tax rate and let $\bar{C}$ be the cost per period of migration (e.g. the interest payments on debt incurred from migration). Then migration continues until an equilibrium is reached in which the expected income following migration (net of migration costs) equals the rural income. i.e.,

$$\Pi\, W_{\mathrm{u}}\,(1-T) + (1-\Pi)\,\bar{W}_a - \bar{C} = W_{\mathrm{r}}\,(1-T) \tag{2.9}$$

Migrants are assumed to be chosen at random for employment and they never enjoy the job security of an incumbent urban worker. This assumption allows to aggregate both the formal and informal sectors of the urban economy into one single state (Schaeffer 1984). The probability of employment in all periods is thus given by

$$\Pi = \frac{L_{\mathrm{u}}}{N_{\mathrm{u}}} = \frac{L_{\mathrm{u}}}{L_{\mathrm{u}} + U} \tag{2.10}$$

where U is unemployment and N_{u} is the urban workforce. Then given the total labor supply $\bar{N}$, the urban real wage W_{u} (assumed fixed in the HT model), the two subsidies, the alternative income $\bar{W}_a$ and the numbers of firms and farms ($\Im_{\mathrm{u}}$ and $\Im_{\mathrm{r}}$ respectively), the 7 equations (2.1), (2.4), (2.5), and (2.7) to (2.10) can be solved for Y_{u}, Y_{r}, N_{u}, $N_{\mathrm{r}} = L_{\mathrm{r}}$, L_{u}, Π and W_{r}, resulting in a migration equilibrium.
In the original HT model, the real wage in the urban sector is exogenous. The rural wage is endogenous and determined by labour demand and supply in the rural economy, with the latter dependent on the urban wage and unemployment. Our first major departure from that model is to introduce some degree of real wage flexibility in the urban sector. The assumption that the real wage is fixed is replaced with an endogenous determination of the real wage net of tax given by

$$\begin{aligned} W_{\mathrm{u}}(1-T) &= \frac{P_c}{P_{\mathrm{u}}}\,k(1-\Pi) \\ &= P^{1-\theta}\,k(1-\Pi) \\ k' < 0, &\qquad k(1) > P^{\theta-1}\,\bar{W}_a \end{aligned} \tag{2.11}$$

where $P = P_{\mathrm{r}}/P_{\mathrm{u}}$ is the relative price of urban and rural output, $P_c = P_{\mathrm{u}}^{\theta}\,P_{\mathrm{r}}^{1-\theta}$ is the consumer price index and $U = 1 - \Pi$ is the unemployment

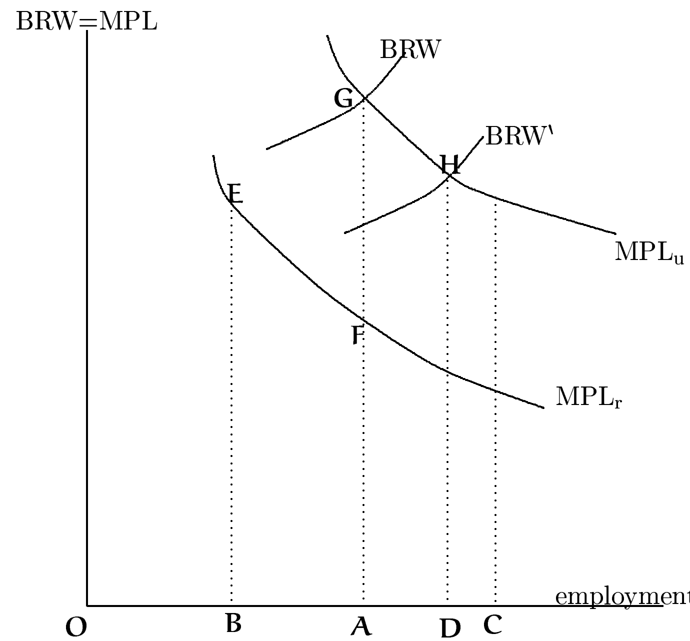

Figure 2.1: High wage flexibility

rate. The negative relationship between the unemployment rate and the post-tax real consumption wage postulated by (2.11) can be derived from microfoundations in a number of ways. It follows for instance from the "no shirking condition" in efficiency wage theory. Our preference is to appeal to bargaining theory and interpret (2.11) as a *bargained post-tax real wage* (BRW) relationship in which a monopoly union sets a mark-up on the post-tax reservation wage whose value falls as the unemployment rate rises (see Layard, Nickell, and Jackman (1991), for example, for a comprehensive treatment of wage equations of the form (2.11).
Whatever the interpretation one gives to (2.11), it adds a determination of urban wages that is absent in the standard HT model, which relies on an exogenous "subsidized" urban wage. An exogenous urban wage raises two questions. First the absence of agglomeration effects of Shukla and Stark (1990) there appears no *economic* justification for a subsidy to the urban sector. Second, in the original HT model the underlying subsidy has no effect on the government's budget.
In this paper, we rely on urban wage to be higher than rural wages not

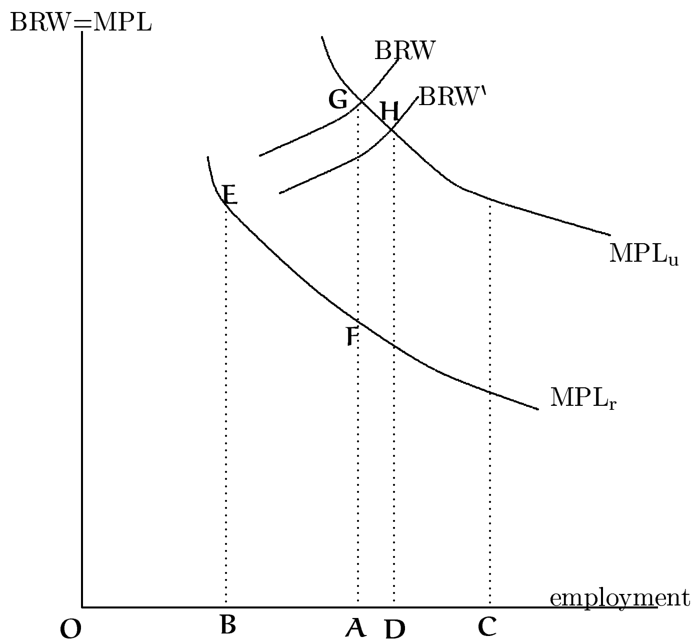

Figure 2.2: Low wage flexibility

through the invisible hand of the government but through the intrinsic characteristics of the urban labor market as expressed in (2.11). If we think of (2.11) as a "no-shirking" condition, then we can motivate it though the observation that urban work is more easy to shirk because monitoring is more difficult for sophisticated jobs. But again it is more convenient to view (2.11) as a bargain real wage relationship. Clearly in the urban area union activity is more easily organized. Within a larger density of population it is easier to get a critical group of activists together, it is easier to stage demonstrations, pickets etc.

The consequences of real wage flexibility for migration is illustrated in Figures 2.1 and 2.2. Assume for the moment that in an initial migration equilibrium the size of the workforce in the urban and rural sectors is equal (**OA** in the figures). Suppose that a number **BA** leave the countryside and migrate to the cities increasing the urban workforce by **AC** = **BA**. With some real wage flexibility the BRW curve shifts to BRW' and urban employment increases by **AD** < **AC** increasing unemployment by **DC** in that sector.

The welfare implications of migration can be assessed by comparing the increase in urban output, area **ADHG**, with the drop in rural output, area **BAFE** (both in terms of units of urban output). Starting from this initial migration equilibrium, rural to urban migration by an increment **BA** is welfare enhancing iff **ADHG** > **BAFE** + costs of migration. If this is not satisfied there is "urban bias" and some reverse migration back to the countryside is welfare enhancing. The two figures compare the case of a relatively high degree of responsiveness of the urban real wage to unemployment (see Figure 2.1) with a very low degree of responsiveness (see Figure 2.2). In the latter case (which is close to that of a fixed real wage as in HT) we have **ADHG** < **BAFE** and an urban bias exists. The next section establishes an expression for the degree of real wage flexibility that is required for a migration equilibrium to yield urban bias in the absence of employment subsidies.

The second change we make to previous studies of policy using a HT type model is to introduce a government budget constraint. We consider the simplest form—a balanced budget—and we therefore preclude the possibility of deficit financing. Let $\bar{W}_c$ be the direct cost to the government of an urban unemployed, again fixed. Let be remaining government expenditure (other than employment subsidies) which we hold fixed. Let T be the income tax rate which is assumed to be uniform for all income generated in the urban and rural sectors. Then the budget constraint becomes

$$T\,(Y_{\rm u} + Y_{\rm r}) = \bar{G} + \bar{W}_c\,U + L_{\rm r}\,S_{\rm r} + L_{\rm u}\,S_{\rm u}$$

The left-hand-side of (2.2) gives total government revenue. The right-hand-side consists of the components of government expenditure. This completes our two-sector model with real wage flexibility and a budget constraint.

For the analysis that follows it is convenient to specialize the functional forms $g(\cdot)$, $f(\cdot)$, and $h(\cdot)$ used in the production functions. Cobb-Douglas technology with $g(x) = x^{\gamma}$, $h(x) = x^{\beta}$ and $f(x) = x^{\alpha}$ is assumed[1] enabling us to rewrite (2.4) and (2.8) as

$$Y_{\mathrm{u}}(L_{\mathrm{u}}) = A\, L_{\mathrm{u}}^{\gamma+\alpha} \tag{2.12}$$

$$Y_{\mathrm{r}}(L_{\mathrm{r}}) = B\, L_{\mathrm{r}}^{\beta} \tag{2.13}$$

where $A = \mathfrak{I}_{\mathrm{u}}^{1-\alpha}$ and $B = \mathfrak{I}_{\mathrm{r}}^{1-\beta}$. Table 2.1 summarizes the generalized HT model with some real wage flexibility, Cobb-Douglas technology and budget constraint.

3 The Welfare Economics of the Migration Decision in the Absence of Subsidies

Consider an initial migration equilibrium resulting in an allocation of the total fixed labor force $\bar{N} = \tilde{N}_{\mathrm{u}} + \tilde{N}_{\mathrm{r}}$ between the urban and rural sectors without subsidies. We now examine whether this level of migration is excessive or insufficient from a welfare viewpoint by considering a further migration of $M\,\tilde{N}_{\mathrm{r}}$ where M is the migration rate as a proportion of the laissez-faire rural workforce. Then

$$N_{\mathrm{r}} = (1-M)\tilde{N}_{\mathrm{r}}; \quad N_{\mathrm{u}} = \tilde{N}_{\mathrm{u}} + M\tilde{N}_{\mathrm{r}}$$

give the urban and rural workforces after this further migration. Consider the social welfare function

$$\begin{aligned} Z(M) &= Y_{\mathrm{u}} + P\,Y_{\mathrm{r}} - (\tilde{N}_{\mathrm{r}} - N_{\mathrm{r}})\,\bar{C} \\ &= Y_{\mathrm{u}}(L_{\mathrm{u}}(M)) + Y_{\mathrm{r}}(N_{\mathrm{r}}(M)) - M\,\tilde{N}_{\mathrm{r}}\,\bar{C} \end{aligned}$$

The first order condition for a maximum is given by

$$\frac{\mathrm{d}Z}{\mathrm{d}M} = \frac{\mathrm{d}Y_{\mathrm{u}}}{\mathrm{d}L_{\mathrm{u}}}\frac{\mathrm{d}L_{\mathrm{u}}}{\mathrm{d}M} - P\,\frac{\mathrm{d}Y_{\mathrm{r}}}{\mathrm{d}N_{\mathrm{r}}}\,\tilde{N}_{\mathrm{r}} - \tilde{N}_{\mathrm{r}}\,\bar{C} = 0$$

[1]There are two reasons for this choice. First is is analytically tractable. Second, the results based on it remain true as long as the elasticity of output with respect to factor usage remains constant or within tight bounds for all levels of production. Thus our result remain valid if the elasticity remains constant,

Table 2.1: The Extended HT Model

Urban Output: $Y_{\mathrm{u}} = A\, L_{\mathrm{u}}^{\alpha+\gamma}$ (i)
where L_{u} denotes urban employment

Rural Output: $Y_{\mathrm{r}} = P\,B\,L_{\mathrm{r}}^{\beta}$ (ii)
where L_{r} denotes rural employment and P the rural/urban terms of trade

Labor Resource Constraint: $\bar{N} = \bar{N}_{\mathrm{u}} + \bar{N}_{\mathrm{r}} = L_{\mathrm{u}} + L_{\mathrm{r}}$ (iii)
where $\bar{N}_{\mathrm{u}}$, $\bar{N}_{\mathrm{r}}$ denote initial workforce allocations before migration

Labor Markets: $N_{\mathrm{u}} = L_{\mathrm{u}} + U; \quad N_{\mathrm{r}} = L_{\mathrm{r}}$ (iv)
where U is urban unemployment

Labor Demand: $\alpha\, A\, L_{\mathrm{u}}^{\alpha+\gamma-1} = W_{\mathrm{u}} - S_{\mathrm{u}}$ (v)
$\beta\, P\, B\, L_{\mathrm{r}}^{\beta-1} = W_{\mathrm{r}} - S_{\mathrm{r}}$ (vi)

The Bargained Real Wage: $W_{\mathrm{u}}\,(1-T) = P^{1-\theta}\,k(u)$ (vii)
where T is the taxation rate, $u = U/(L_{\mathrm{u}} + U)$ is the unemployment rate and $k' < 0$

Migration Equilibrium: $W_{\mathrm{r}}\,(1-T) = \Pi\, W_{\mathrm{u}}\,(1-T) + (1-\Pi)\,\bar{W}_a - \bar{C}$ (viii)
where $\Pi = L_{\mathrm{u}}/N_{\mathrm{u}} = 1 - u$ is the probability of employment

Budget Constraint: $T\,(Y_{\mathrm{u}} + Y_{\mathrm{r}}) = \bar{G} + \bar{W}_c\, U + L_{\mathrm{r}}\, S_{\mathrm{r}} + L_{\mathrm{u}}\, S_{\mathrm{u}}$ (ix)
where $\bar{W}_c$ is the direct budgetary cost of unemployment and $\bar{G}$ is government expenditure

which can be written

$$\frac{\mathrm{d}Y_\mathrm{u}}{\mathrm{d}L_\mathrm{u}}\,\frac{\mathrm{d}L_\mathrm{u}}{\mathrm{d}(\tilde{N}_\mathrm{r}\,M)} = \frac{\mathrm{d}Y_\mathrm{r}}{\mathrm{d}N_\mathrm{r}} + \bar{C} \tag{3.1}$$

The left hand side of (3.1) is the marginal product of the migrants employed in the urban sector. The right hand side is the rural marginal product plus the cost of further migration.
Assume Cobb-Douglas technology as in (2.12) and (2.13). Then the demand for labor schedules (v) and (vi) in Table 2.1, with no subsidies, become

$$W_\mathrm{u} = \alpha\,A\,L_\mathrm{u}^{\alpha+\gamma-1}; \qquad W_\mathrm{r} = \beta\,B\,P\,L_\mathrm{u}^{\beta-1}$$

Hence at the laissez-faire equilibrium we have

$$\frac{\mathrm{d}Z}{\mathrm{d}M} = \tilde{W}_\mathrm{u}\left(1+\frac{\gamma}{\alpha}\right)\frac{\mathrm{d}L_\mathrm{u}}{\mathrm{d}M} - (\tilde{W}_\mathrm{r}+\bar{C})\,\tilde{N}_\mathrm{r} \tag{3.2}$$

Put $\xi = \mathrm{d}L_\mathrm{u}/\mathrm{d}M/\tilde{L}_\mathrm{u}$, the semi-elasticity of urban employment with respect to the migration rate measured at the migration equilibrium. Then laissez-faire migration is too great relative to the social optimum if and only if $\mathrm{d}Z/\mathrm{d}M < 0$ or from (3.2) if and only if

$$\tilde{W}_\mathrm{u}\left(1+\frac{\gamma}{\alpha}\right)\xi\,\frac{\tilde{L}_\mathrm{u}}{\tilde{N}_\mathrm{r}} < \tilde{W}_\mathrm{r} + \bar{C} \tag{3.3}$$

To find the elasticity ξ, equate labor demand with labor supply (using (v) and (vii)) to give

$$\begin{aligned} A\,\alpha\,L_\mathrm{u}^{\alpha+\gamma-1} &= \frac{1}{1-T}\,k\left(\frac{U}{N_\mathrm{u}}\right) \\ &= \frac{1}{1-T}\,k\left(1-\frac{L_\mathrm{u}}{\tilde{N}_\mathrm{u}+M,\tilde{N}_\mathrm{r}}\right) \end{aligned} \tag{3.4}$$

Equation (3.4) is a relationship between urban employment L_u and the migration rate M given the parameters determining the production technology and real wage behavior. Let $\eta = k'/W_\mathrm{u} \leq 0$ be the semi-elasticity of the real wage with respect to the unemployment rate. Then differentiating (3.4) implicitly, a little algebra gives the semi-elasticity of urban employment with respect to the migration rate measured at the laissez-faire migration equilibrium as

$$\xi = \frac{1}{\tilde{L}_\mathrm{u}}\frac{dL_\mathrm{u}}{\mathrm{d}M} = \frac{\eta\,\tilde{N}_\mathrm{r}\,\tilde{L}_\mathrm{u}}{\tilde{N}_\mathrm{u}((\alpha+\gamma-1)\,\tilde{N}_\mathrm{u}+\eta\tilde{L}_\mathrm{u})}$$

Put $L_\mathrm{u}/N_\mathrm{u} = 1-U$ where U is the urban unemployment rate. Then substituting for ξ in (3.3) and performing some algebra yields a cut-off point for the modulus of the real wage-unemployment elasticity η at which laissez-faire migration becomes too *small* relative to the social optimum. This is expressed as

PROPOSITION 1 The laissez-faire level of migration is too high relative to the social optimum if and only if

$$|\eta| < \frac{(\tilde{W}_\mathrm{r}+\bar{C})\,\alpha\,(1-\alpha-\gamma)}{\tilde{W}_\mathrm{u}(\alpha+\gamma)\,(1-\tilde{U})^2 - (\tilde{W}_\mathrm{r}+\bar{C})\,\alpha\,(1-\tilde{U})} \tag{3.5}$$

if the denominator is positive. If the denominator is less than or equal to zero then the laissez-faire level is higher for *all* $|\eta|$.

Proposition 1 tells us that in the absence of other instruments (considered later) migration controls that reduce migration below the laissez-faire level cease to become socially desirable if the urban real wage is sufficiently responsive to unemployment. It would be of interest to assess the magnitude of the cut-off point for the elasticity η. But first we must 'calibrate' the model.
A problem with the condition (3.5) is that it is expressed in terms of the cost of migration $\bar{C}$ which is not directly observable. However given data for the urban/rural wage ratio, the urban unemployment rate and a guestimate of the formal/informal urban wage ratio, the cost of migration can be deduced from HT migration equilibrium condition (2.9). To proceed along these lines, first express $\bar{C}$ and the alternative disposable wage $\bar{W}_a$ as proportions of the rural wage net of tax and the urban wage net of tax respectively. i.e., put

$$\bar{C} = \phi\,\tilde{W}_\mathrm{r}\,(1-T); \qquad \bar{W}_a = \mu\,\tilde{W}_\mathrm{u}\,(1-T)$$

Then from the migration equilibrium (2.9) we have that

$$\phi = \frac{\tilde{W}_\mathrm{u}}{\tilde{W}_\mathrm{r}}\left[1-\tilde{U}\,(1-\mu)\right]-1 \tag{3.6}$$

and (3.5) therefore becomes

$$|\eta| < \frac{\alpha\,(1-\alpha-\gamma)\,(1+\phi(1-T))}{\tilde{W}_\mathrm{u}/\tilde{W}_\mathrm{r}(\alpha+\gamma)\,(1-\tilde{U})^2 - \alpha\,(1-\tilde{U})\,(1+\phi\,(1-T))} \tag{3.7}$$

again as long as the denominator is positive (if not, the cut-off point is infinity). Equations (3.6) and (3.7) enable us to calibrate the cut-off point

Table 3.1: Calibration of Variables and Parameters

Variable	Range or Value	Source
$\tilde{W}_u/\tilde{W}_r$	1.3 – 1.4	Government of India (1991b)
$\tilde{U}$	0.11 – 0.12	Government of India (1991b)
μ	$0.33\,\tilde{W}_r/\tilde{W}_u$	Papanek (1988)
T	0.18	Government of India (1991a)
α	0.6 – 0.8	Ahluwalia (1985)
γ	0.1	Shukla and Stark (1990)
β	0.5	Shukla and Stark (1990)
$\tilde{N}_u/\tilde{N}_r$	0.33	Government of India (1991b)

for η at which migration controls are not socially desirable. We first need to provide values for $\tilde{W}_u/\tilde{W}_r$, $\tilde{U}$, μ, T, α and γ. These are provided in Table 3.1 for Indian data with sources shown.[2] Also shown are assumed values for β and $\tilde{N}_u/\tilde{N}_r$ needed for the later simulations. Table 3.2 displays the upper bound for $|\eta|$ below which the laissez-faire level of migration is excessive compared with the social optimum and the case for policies to discourage migration holds. The table shows that whether $|\eta|$ reaches this critical value is crucially dependent upon the size of the agglomeration effect (γ) and the output-employment elasticity as perceived by the firm, α. As $\alpha + \gamma \to 1$ and we approach constant returns to scale in the urban sector, then rather small values of the real wage/unemployment rate elasticity are sufficient for laissez-faire migration to become too small[3].

4 Optimal Tax and Subsidy Policies under a Budget Constraint.

4.1 The social optimum

This section considers the socially optimal allocation of urban and rural workers supported by employment subsidies. We first prove the result due

[2] It is assumed that observed data for India constitutes a HT migration equilibrium. We are grateful to Subrata Ghatak for supplying these sources.

[3] Little work on estimating the elasticity η in developing countries has been done and this is an important area for research. Alogoskoufis and Manning (1988) and Layard, Nickell, and Jackman (1991) find values for OECD countries which in general exceed unity and in some cases are far higher.

Table 3.2: Critical values for $|\eta|$ as given by (3.7)

	$\gamma = 0$			$\gamma = 0.1$			$\gamma = 0.15$		
α	0.6	0.7	0.8	0.6	0.7	0.8	0.6	0.7	0.8
$\tilde{W}_u/\tilde{W}_r$									
1.2	∞	∞	∞	2.4	1.9	1.1	1.3	0.9	0.3
1.3	∞	∞	∞	2.2	1.7	1.0	1.2	0.8	0.3
1.4	60	45	30	1.9	1.5	0.9	1.1	0.8	0.3
1.5	23	18	12	1.8	1.4	0.8	1.0	0.7	0.3
1.6	15	11	8	1.7	1.3	0.7	1.0	0.7	0.2

to Shukla and Stark (1990) that there exists a pair of subsidies (S_u^*, S_r^*) that will support the socially optimal allocation of labor between the two sectors and hence the optimal rate of migration. The social planner would choose labor allocations to maximize total income minus the costs of migration, i.e.,

$$Z = Y_u + Y_r - (\bar{N} - N_r)\,\bar{C} \tag{4.1}$$

In the "first best" social optimum there cannot be any unemployment because the marginal product of the unemployed, zero, is less than that in the rural sector. Putting $N_u = L_u$ and using the global labor resource constraint (2.1) we can then write S as

$$Z(L_u) = A\,L_u^{\alpha+\gamma} + B\,P\,(\bar{N} - L_u)^{\beta} - L_u\,\bar{C}$$

The first order condition for a maximum is

$$A\,(\alpha+\gamma)\,L_u^{\alpha+\gamma-1} - B\,P\,\beta\,(\bar{N} - L_u)^{\beta-1} - \bar{C} = 0 \tag{4.2}$$

This condition equates the marginal product of labor in the urban sector *taking into account the economies of scale externality* minus the cost per migrant of moving with the marginal product of the rural worker. The concavity of the production functions ensures that the second order conditions are satisfied. The solution to (4.2) gives the optimal labor allocation denoted by L_u^* and $L_r^* = \bar{N} - L_u^*$.

Unlike the mythical social planner, the government cannot choose this allocation directly but must use subsidies (S_u, S_r) such that private sector behavior by firms and migrants captured by the model summarized above results in this outcome. Let us ignore budgetary considerations for the moment and put the tax rate $T = 0$. Then from (viii) at the full employment level $\Pi = 1$ and $W_u^* = W_r^* + \bar{C}$. From (v) and (vi) in Table 2.1 the global efficiency condition (4.2) is satisfied at $L_u = L_u^*$ if and only if

$$A\,\alpha\,L_u^{*\alpha+\gamma-1} = W_u^* - S_u^* \tag{4.3}$$

$$P\,B\,\beta\,L_r^{*\beta-1} = W_r^* - S_r^* \tag{4.4}$$

where the real wage $W_u^* = q^{1-\theta}\,k(0)$ given by (vii), and

$$S_u^* - S_r^* = A\,\gamma\,L_u^{\alpha+\gamma-1} \tag{4.5}$$

The resource constraint $L_r^* + L_u^* = \bar{N}$, the migration equilibrium condition $W_u^* = W_r^* + \bar{C}$ and equations (4.3) to (4.6) give L_u^*, L_r^*, W_r^* and the subsidies that will support this efficient outcome S_u^* and S_r^*. In the absence of scale economies $\gamma = 0$, resulting in *equal* subsidies supporting the social optimum (Bhagwati and Srinivason 1974). If scale economies exist then from (4.4) a *higher* subsidy is needed in the urban sector (Shukla and Stark 1990).
How does the socially optimal urban workforce L_u^* compare with that under the laissez-faire (no subsidies), $L_u = \tilde{L}_u$ say? To answer this differentiate (4.1) and use the labor demand equations (v) and (vi) from Table 2.1 to give

$$\begin{aligned} \left.\frac{\mathrm{d}Z}{\mathrm{d}L_u}\right|_{L_u=\tilde{L}_u} &= A\,\gamma\,\tilde{L}_u^{\alpha+\beta-1} + \tilde{W}_u - \tilde{W}_r - \bar{C} \\ &= A\,\gamma\,\tilde{L}_u^{\alpha+\beta-1} + (1-\Pi)\,(\tilde{W}_u - \tilde{W}_a) \\ &> 0 \end{aligned} \tag{4.6}$$

Hence at the urban employment level $L_u = \tilde{L}_u$, $\mathrm{d}Z/\mathrm{d}L_u > 0$ provided that $\gamma > 0$ (i.e., there exists agglomeration effects) and/or $\Pi < 1$ (i.e., unemployment exists in the laissez-faire equilibrium). Thus the laissez-faire level of urban population (and hence migration rate) is *less* than the social optimum reached by appropriate subsidies. Our results can be summarized in the following proposition.

PROPOSITION 2 The social optimum can be reached by a pair of subsidies (S_u^*, S_r^*) where $S_u^* \geq S_r^*$. The laissez-faire level of urban population is less than the social optimum.

4.2 Budgetary consequences

The budgetary consequences of the subsidy scheme is found by using the budget constraint (ix) in Table 2.1. In general the policy will require an

Table 4.1: First Best (Zero Unemployment) Outcome

	subsidies	taxes	output	employment	real disp. wage
urban	$s_u^* = 44$	$t = 24$	$y_r^* = 28$	$l_u = 35$	7
rural	$s_r^* = 46$	$t = 24$	$y_r^* = -4$	$l_r = -7$	29

All figures are percentages. Notation: $s^* = S^*/W^*$, $y^* = (Y^* - \tilde{Y})/\tilde{Y}$, $l^* = (L^* - \tilde{L})/\tilde{L}$, $t^* = T^* - \tilde{T}$, $s_r^* = S_r^*/W_r^*$

increase in the taxation rate from $\tilde{T}$ in the no-subsidy, laissez-faire case to $\tilde{T}^*$ where

$$T^* - \tilde{T} = \frac{\bar{G} + L_r^*\,S_r^* + L_u^*\,S_u^*}{Y_u^* + Y_r^*} - \frac{\bar{G} + \bar{W}_c\,\tilde{U}}{\tilde{Y}_u + \tilde{Y}_r}$$

We now turn to the numerical evaluation of the tax rate increase necessary to finance the optimal subsidy pair (S_u^*, S_r^*). In the simulations reported parameter values are set at central values reported in Table 3.1. In addition we put $\eta = 0.5$, a rather low value by OECD standards, unemployment costs are expressed as $\bar{W}_c = \zeta\,\tilde{W}_u(1 - \tilde{T})$ and we put $\zeta = .2$. Details of the numerical calculations are provided in the appendix.
Table 4.1 shows the social optimum measured in deviation form about the baseline laissez-faire equilibrium with $\tilde{S}_u = \tilde{S}_r = 0$. In the new equilibrium there is no urban unemployment. Urban employment increases by 35% and urban output increases. By contrast rural employment falls by 7% and rural output by 4%. The main beneficiaries of the increase in total output are the former urban unemployed and the rural workers whose real disposable income rise by 29%. This is due to the HT migration condition which now requires that the rural disposable wage should equal that in the urban sector net of the cost of migration.
The most important result in Table 4.1 is the increase in the income tax rate required to finance the substantial subsidies of 44% of the baseline real urban wage and 46% of the baseline rural wage. (Note that because $\tilde{W}_u = 1.4\,\tilde{W}_r$ the total subsidy per worker in the urban sector exceeds that in the rural sector as required by Proposition 2). The increase is 24% which means that the tax rate rises from 18% in the baseline (see Table 3.1) to 42%! An increase of this magnitude would be expected to involve other social costs not explicitly modeled. These costs include collection costs and global supply-side effects (recall that the resource constraint (2.1) in the HT model assumes a fixed total labor supply).
The effect of a substantial increase in the income tax rate can be captured

Table 4.2: Optimal Subsidies with Costs of Taxation

weight ω	u^*	t^*	s_u^*	s_r^*
30	−10	18	38	33
35	−8	13	30	23
40	−6	9	25	15
45	−5	6	20	9
50	−4	4	16	4
55	−3	2	13	0
60	−2	0	11	−3

All figures are percentages. Notation: $u^* = U^* - \tilde{U}$, $t^* = T^* - \tilde{T}$, $s_u^* = S_u^*/W_u^*$, $s_r^* = S_r^*/W_r^*$

in an ad hoc fashion by introducing an additional term in the social welfare function (4.1) of the form $-\omega T^2$, i.e., rising exponentially as the tax rate increases. The first-best outcome reported in table 4 is no longer supported by a subsidy/tax scheme and Table 4.2 shows that a trade-off now emerges between the tax increase and the unemployment decrease.

At the low extreme a weight ω of 0.3 gives an outcome close to the first best of Table 3.2 with most of the initial unemployment of 12% eliminated. As ω increases, the optimal income tax rate decreases and the drop in the unemployment rate decreases substantially. At the high extreme of ω = 0.6, we have a *self-financing* subsidy scheme in which the urban subsidy is financed by revenue increases which do not require a change in the income tax rate but do require a rural employment *tax* (i.e., a negative subsidy).

5 Conclusions

The results of the paper can be summarized as follows. If migration control (or the discouragement of migration by unspecified policies) is the only available instrument of policy, then a welfare case for restricting migration can be made from the standpoint of society. However a low degree of real wage flexibility in the urban sector in response to unemployment combined with urban external economies of scale can overturn this conclusion.

Employment subsidies can support a first-best social optimum with no unemployment for which migration now *exceeds* the laissez-faire level. Simulations suggest that the public financing of the subsidy programs requires a large increase in general taxation. When distortionary effects and collection costs of the latter are included in the social welfare then the optimal subsidy

and taxation policy falls far short of the first-best outcome.

There are a number of possible directions for future research. The HT model provides a theory of migration equilibrium levels, but not flows. These could be introduced into the model by costs of adjustment for employment at both the level of the firm and as an externality in which excessive growth of urban areas is penalized in the social welfare function. A dynamic model exhibiting migration flows would emerge from these modifications. Policy analysis would now be intertemporal in character and may involve a serious time-inconsistency problem.

Migration issues will grow in importance in a European context as a result of German unification and changes in Eastern Europe.[4] However there are a number of respects in which the analysis of migration between European countries and within Germany needs to go beyond the HT framework presented in this paper. First, European migration largely takes place between industrial sectors which are characterized by large differences in the initial capital stocks. The costs of changing both capital and labor are an important component of the analysis (Burda and Wyplosz (1991) and Burda and Funke (1992)). Second, neither sector has a labor market that clears, unlike the HT case of a market-clearing rural sector. Indeed there is likely to be higher unemployment in the donor than in the host country. Third, human capital considerations are of crucial importance. If the composition of migrants is biased towards more highly skilled and better educated workers, then migration may have a seriously negative impact on the growth of the donor countries. The endogenous growth literature will play an important role in understanding these phenomena.

References

Ahluwalia, Isher. 1985. *Indian Industrial Performance*. Delhi: Oxford University Press.

Alogoskoufis, George and Allan Manning. 1988. On the Persistence of Unemployment. *Economic Policy 7*, 427–469.

Bhagwati, Jagdish and T. N. Srinivason. 1974. On Re-Analysing the Harris-Todaro Model: Policy Ranking in the Case of Sector Specific Sticky Wages. *American Economic Review 64*, 502–508.

Burda, Micheal. and Michael Funke. 1992. Trade Unions,Wages and Structural Adjustment in the New German States. CEPR Discussion Paper No. 1092.

[4]See Straubhaar and Zimmermann (1992) for a perceptive survey of the issues.

Burda, Michael and Charles Wyplosz. 1991. Human Capital Investment and Migration in an Integrated Europe. CEPR Discussion Paper No. 614.

Gehrig, Annette, Christoph M. Schmidt, and Klaus F. Zimmermann. 1992. Mass Migration, Unions and Fiscal Migration Policy. mimeo, Universtität München.

Government of India. 1991a. *Economic Survey.* Delhi: C.S.O.

Government of India. 1991b. *Statistical Abstract.* Delhi: C.S.O.

Harris, John R. and Michael P. Todaro. 1970. Migration, Unemployment and Development. *American Economic Review 60*, 126–144.

Layard, Richard, Stephen Nickell, and Robert Jackman. 1991. *Unemployment.* New York: Oxford University Press.

Lucas, Robert E.B. and Gustav F. Papanek (Eds.). 1988. *The Indian Economy.* Oxford University Press.

Panagariya, Arvind. 1981. Variable Returns to Scale in Production and Patterns of Specialization. *American Economic Review 71*, 221–230.

Papanek, Gustav. F.. 1988. Poverty in India. See Lucas and Papanek (1988).

Schaeffer, Peter. 1984. Barriers and Forsight in International Labour Migratino. *Environment and Planning A 16*, 451–466.

Shukla, Vibhooti and Oded Stark. 1990. Policy Comparisons with an Agglomeration Effects Augmented Dual Economy Model. *Journal of Urban Economics 27*, 1–27.

Straubhaar, Thomas and Klaus F. Zimmermann. 1992. Towards a European Migration Policy. CEPR Discussion Paper No.641.

Todaro, Micheal P.. 1969. A Model of Labour Migration and Urban Unemployment in Less Developed Countries. *American Economic Review 59*, 138–48.

World Bank. 1988, 1989, 1990. *World Development Reports.* New York / Oxford: Oxford University Press.

A Computational Aspects

A.1 Linearized model

Consider an initial migration equilibrium with subsidies $\tilde{S}_{\mathrm{u}}, \tilde{S}_{\mathrm{r}}$, and other variables denoted by a tilde. The laissez-faire equilibrium examined in Sec-

tion 3 puts $\tilde{S}_{\mathrm{u}} = \tilde{S}_{\mathrm{r}} = 0$. Our numerical results use the following linearization about this initial equilibrium.

$$y_{\mathrm{u}} = (\alpha + \gamma)\, l_{\mathrm{u}}$$

$$y_{\mathrm{r}} = \beta\, l_{\mathrm{r}}$$

$$l_{\mathrm{r}} = -\frac{\tilde{L}_{\mathrm{u}}}{\tilde{L}_{\mathrm{r}}\,(1-\tilde{U})}\left(l_{\mathrm{u}} + \frac{1}{1-\tilde{U}}\,u\right) \tag{A.1}$$

$$l_{\mathrm{u}} = -\frac{w_{\mathrm{u}} - s_{\mathrm{u}}}{(1-\alpha-\gamma)\,(1-\tilde{S}_{\mathrm{u}})} \tag{A.2}$$

$$l_{\mathrm{r}} = -\frac{w_{\mathrm{r}} - s_{\mathrm{r}}}{(1-\beta)\,(1-\tilde{S}_{\mathrm{r}})} \tag{A.3}$$

$$w_{\mathrm{u}} = \frac{\eta}{1-\tilde{T}}\,\pi + \frac{t}{1-\tilde{T}} \tag{A.4}$$

$$\pi = -u \tag{A.5}$$

$$w_{\mathrm{r}} = (1-\tilde{U})\,\frac{\tilde{W}_{\mathrm{u}}}{\tilde{W}_{\mathrm{r}}}\,w_{\mathrm{u}} + \frac{\tilde{W}_{\mathrm{u}}}{\tilde{W}_{\mathrm{r}}}\,(1-\mu)\,\pi - \frac{\theta}{1-\tilde{T}}\,t \tag{A.6}$$

$$\begin{aligned}\left(\frac{1}{\alpha}\,(1-\tilde{S}_{\mathrm{u}}) + \frac{\Theta}{\beta}\,(1-\tilde{S}_{\mathrm{r}})\right) t + \frac{\tilde{T}\,(1-\tilde{S}_{\mathrm{u}})}{\alpha}\,y_{\mathrm{u}} + \frac{\tilde{T}\,(1-\tilde{S}_{\mathrm{r}})}{\beta}\,\Theta\,y_{\mathrm{r}} \\ = \Theta s_{\mathrm{r}} + s_{\mathrm{u}} + \frac{\zeta\,\tilde{U}}{1-\tilde{U}}\left(l_{\mathrm{u}} + \frac{1}{\tilde{U}\,(1-\tilde{U})}\,\tilde{u}\right) + \Theta\,\tilde{S}_{\mathrm{r}}\,l_{\mathrm{r}} + \tilde{S}_{\mathrm{u}}\,l_{\mathrm{u}}\end{aligned} \tag{A.7}$$

$$\begin{aligned}S - \tilde{S} = \frac{\beta\,(1-\tilde{S}_{\mathrm{u}})}{2\,\Theta\,\alpha\,(1-\tilde{S}_{\mathrm{r}})}[2\,(\alpha+\gamma)\,l_{\mathrm{u}} - (\alpha+\gamma)\,(1-\alpha-\gamma)\,l_{\mathrm{u}}^2] \\ + \frac{1}{2}\left[2\,\beta\,(1+\phi)\,l_{\mathrm{r}} - \beta\,(1-\beta)\,l_{\mathrm{r}}^2\right]\end{aligned} \tag{A.8}$$

where $\Theta = \tilde{W}_{\mathrm{r}}\,\tilde{L}_{\mathrm{r}}/\tilde{W}_{\mathrm{u}}/\tilde{L}_{\mathrm{u}}$ and we have put $\bar{C} = \phi\,\tilde{W}_{\mathrm{r}}\,(1-\tilde{T})$, $\bar{W}_a = \mu\,\tilde{W}_{\mathrm{u}}\,(1-\tilde{T})$ as in (3) and, in addition, unemployment costs are expressed as $\bar{W}_c = \zeta\,\tilde{W}\,(1-\tilde{T})$. The unemployment rate, the tax rate and the probability of urban employment are defined by $u = U - \tilde{U}$, $t = T - \tilde{T}$ and $\pi = \Pi - \tilde{\Pi}$, and i.e., as *absolute* deviations about the baseline equilibrium. Remaining variables such as y_{u} are defined by $y_{\mathrm{u}} = (Y_{\mathrm{u}} - \tilde{Y}_{\mathrm{u}})/\tilde{Y}_{\mathrm{u}}$ i.e., as *proportional* deviations about the baseline equilibrium.

A.2 The first best allocation in the command economy

A social planner would allocate labor between rural and urban employment subject to the resource constraint (A.1) with $U = 0$. Then $u = -\tilde{U}$ and the first best allocation (l_u^*, l_r^*) satisfies

$$l_r^* = -\frac{\tilde{L}_u}{\tilde{L}_r\,(1-\tilde{U})}\left(l_u^* - \frac{\tilde{U}}{1-\tilde{U}}\right)$$

Then minimizing (A.8) with respect to l_u subject to (A.2) gives

$$\begin{aligned} 0 = &\frac{\beta\,(1-\tilde{S}_u)}{\Theta\,\alpha\,(1-\tilde{S}_r)}\left[(\alpha+\gamma) - (\alpha+\gamma)\,(1-\alpha-\gamma)\,l_u^*\right] \\ &- \frac{\tilde{L}_u}{\tilde{L}_r\,(1-\tilde{U})}\left[\beta\,(1+\phi) - \beta\,(1-\beta)\,l_r^*\right] \end{aligned} \tag{A.9}$$

Solving (A.2) and (A.9) gives the socially optimal allocation of labor. A more accurate solution can be obtained by linearizing about the new equilibrium and re-computing the optimum.

A.3 The first best allocation in the market economy without tax distortions

For a given tax rate t and given the first best outcome (l_u^*, l_r^*), after putting $\pi = -\tilde{U}$ in (A.4) and eliminating w_u and w_r from (A.4) and (A.6) we can solve (A.2) and (A.3) to give the pair of subsidies that will support this allocation as functions of t i.e., $s_u^* = (1-\alpha-\gamma)\,l_u^* - w_u^*(t) = s_u^*(t)$ and $s_r^* = (1-\beta)\,l_r^* - w_r^*(t) = s_r^*(t)$. Then substituting into the budget identity (A.7) gives the tax rate t^* required to finance the subsidies as reported in Table 4.1.

A.4 The socially optimal allocation in the market economy with tax distortions

The first best allocation with zero unemployment is now no longer attainable. To find the social optimum define the row vector $\mathbf{x} = [l_u, l_r, w_r, u, \pi, t]$. Then (A.1) to (A.7) gives 7 equations in the 7 variables of $\mathbf{x}$ in terms of the subsidies $\mathbf{s} = [s_u, s_r]$. We can write this as

$$\mathbf{x}\,\mathbf{A} = \mathbf{s}\,\mathbf{B}$$

where $\mathbf{A}$ is a 7×7 non-singular matrix and $\mathbf{B}$ is a 2×7 matrix. Solving for $\mathbf{x}$ in terms of $\mathbf{s}$ gives

$$\mathbf{x} = \mathbf{s}\,\mathbf{B}\,\mathbf{A}^{-1} = \mathbf{s}\,\mathbf{C}$$

say. The social welfare function (A.8) with an extra term $-\omega\,T^2 = -\omega\,(\tilde{T}+t)^2$ capturing tax distortions can be expressed as the quadratic form

$$S - \tilde{S} = \mathbf{x}\,\mathbf{D}\,\mathbf{x}^\top + \mathbf{x}\,\mathbf{E}$$

where $\mathbf{D}$ is a positive definite 7×7 matrix, $\mathbf{E}$ is a 7×2 matrix, and the superscript $^\top$ denotes the transform. Substituting for $\mathbf{x}$ from (A.4) and differentiating with respect to $\mathbf{s}$ gives the first order condition for a maximum in $\mathbf{s}^*$ as

$$\mathbf{s}^*\,\mathbf{C}\,\mathbf{D}\,\mathbf{C}^\top + \mathbf{C}\,\mathbf{E} = 0$$

Hence the socially optimal pair of subsidies is given by

$$\mathbf{s}^* = -\mathbf{C}\,\mathbf{E}\,[\mathbf{C}\,\mathbf{D}\,\mathbf{C}^\top]^{-1}$$

These are reported in Table 4.2.