Growing at different rates

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Abstract

I examine a two-country world. I endow each of the countries with externalities from both private and public capital that enable endogenous growth. I show conditions under which an equilibrium exists where both economies grow at constant but different rates.

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1 Introduction

Recent advances in growth theory broadly known as the "endogenous growth" literature have enabled economists to endogenize the long-run growth rate of the economy. This literature has now matured and a large volume is available for closed economy problems. Less attention has been given to open-economy applications. Here most models concern economies that grow at the same rate in the long run. This paper focuses on economies that grow at different rates.

One strand in the literature that does allow for persistent growth differentials despite perfect international capital mobility is Buiter and Kletzer (1991, 1993, 1995). In all three papers, the accumulation of human capital is crucial. All are based on a three-period overlapping generations model that allows for proper modeling both of the process of accumulation of human capital within a generation as well as the transmission of human capital between generations. For the purpose of this paper however, the most important message is that if production involves a non-traded input like human capital, and if endogenous growth occurs, then there can be steady-state divergence in the growth rate. However, within the one-commodity world of these models, steady-state growth differentials imply that one country has an infinite size vis-a-vis the other.

In this paper I argue that long run growth differentials can be sustained without one economy growing to infinite size or accumulating an infinite amount of foreign assets. The key to my result is to consider a world where each country specializes in the production of its own tradeable commodity. In that case, the difference in GDP growth can be "compensated" by a change in the terms of trade. I show conditions under which a balanced growth path with differential growth exists and derive some of its properties.

To generate growth differentials, I need a non-traded stock input in production. To keep matters as simple as possible, I consider that infrastructure is this non-traded stock. It appears convenient to use a two-country extension of the simple infrastructure model of Barro (1990). Such an extension is already available through the excellent work of Devereux and Mansoorian (1992). They endow two economies with a Barro (1990) technology, assume perfect capital mobility and examine in great detail the strategic interaction between symmetric economies. My focus here is different, I concentrate on ad hoc policies to examine the consequences on growth differentials.

The remainder of the paper is organised as follows. Section 2 describes the model. In Section 3 I examine the steady states of the model. I show the conditions for perpetual growth differentials and derive some simple comparative statics results of the impact of an increase in the foreign growth rate

$$
\begin{aligned}
Q(t) &= \bar{\epsilon}\, K(t-1)^{\gamma}\, S(t-1)^{1-\gamma} && (2.1)\\
K(t) &= (1-\delta)\, K(t-1) + I(t) && (2.2)\\
S(t) &= (1-\delta)\, S(t-1) + G(t) && (2.3)\\
G(t) &= \tau(t)\, Q(t) && (2.4)\\
\delta + r(t) &= \kappa\, (1-\tau(t+1))\, Q(t+1)/K(t) && (2.5)\\
C(t) &= \frac{W(t-1)\,(1+r(t-1))}{1+\xi(t)} && (2.6)\\
W(t-1) &= K(t-1) + F(t-1) + H(t-1) && (2.7)\\
\xi(t) &= \left(\frac{p(t)}{p(t+1)}\right)^{\frac{(1-\sigma)\,(1-\alpha)}{\sigma}} \frac{1+\xi(t+1)}{(1+r(t))^{1-1/\sigma}\,(1+d)^{1/\sigma}} && (2.8)\\
H(t) &= (1+r(t-1))\, H(t-1) + (1-\tau(t))\,(1-\kappa)\, Q(t) && (2.9)\\
F(t) &= (1+r(t-1))\, F(t-1) + (1-\tau(t))\, Q(t) - I(t) - C(t) && (2.10)\\
I(t) &= (1-\tau(t))\, Q(t) - \alpha\, C(t) - p(t)\,(1-\alpha^{*})\, C^{*}(t) && (2.11)\\
F^{*}(t) &= -F(t)/p(t) && (2.12)\\
p(t-1) &= \frac{p(t-1)\,(1+r(t))}{1+r^{*}(t-1)} && (2.13)
\end{aligned}
$$

Table 2.1: The Model

on the domestic growth rate. Section 4 concludes.

2 The Model

The model is based on Devereux and Mansoorian (1992).[1] There are two countries. Table 2.1 summarises the model for the domestic country, the equations for the foreign country are analogous and are not reproduced here to conserve space. Time is discrete, and stocks are noted as end-of-period magnitudes. Each country is specialised in the production of a tradeable commodity. I assume that domestic output Q is a function of the private capital stock K and the public infrastructure stock S as in (2.1). The assumption of endogenous growth driven by fiscal externalities could be replaced by other production functions exhibiting endogenous growth. As long as one of the productive stocks is non-traded, the possibility of different growth rates arises. The capital and infrastructure stocks in each country are made up of the domestic commodity of each country. Both productive stocks depreciate at a rate $\delta \leq 1$. This expressed in (2.2), where I is private investment, and (2.3), where G is government spending on augmenting the infrastructure stock. According to (2.4) infrastructure investment is financed through a balanced-budget source-based tax. Government consumption and debt could be introduced without altering the main message of the paper.

[1] I add three features. First the shares of consumption of the domestic and foreign commodity in utility do not need to be 1/2. Second and more importantly, I allow for incomplete depreciation of stocks at rate δ. Last, I model infrastructure as a stock rather than a flow. I neglect government consumption, because it is irrelevant to the main argument of the paper.

Both commodities are used for consumption both at home and abroad; the domestic commodity is also used for capital accumulation in its respective country. Let C_d be the consumption of the home commodity, and C_f the consumption of the foreign commodity in the home country. Let there be a single infinitely-lived representative consumer in both the domestic and the foreign economy. Both have the same type of utility function. For the domestic economy I assume

$$U(t) = \sum_{t'=0}^{\infty} \left(\frac{1}{1+d}\right)^{t'} \frac{\left(C_d(t)^{\alpha}\, C_f(t)^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma} \tag{2.14}$$

As is standard is this type of literature, (2.14) implicitly assumes additive separability of the utility in different periods. Utility U is a discounted sum of the felicity derived from consumption in different periods, $d > 0$ is the discount rate. The isoelastic nature of the felicity function in (2.14) is also a standard feature; the elasticity of substitution is given by $1/\sigma > 0$.

The consumer's problem is to maximise (2.14) under an intertemporal budget constraint. For later reference, I note that the Euler equation is

$$C(t+1) = \left(\frac{p(t)}{p(t')}\right)^{(1-\sigma)\,(1-\alpha)/\sigma} C(t)\, \frac{(1+r(t))^{1/\sigma}}{(1+d)^{1/\sigma}} \tag{2.15}$$

where C is consumption expenditure and $r_t(t')$ is the interest rate that prevails between period t and time $t' \geq t$; of course $r_t(t) = r(t)$. The consumer's optimal behaviour is characterized by (2.6), where W is total wealth, as defined in (2.7) as the sum of capital, human wealth and foreign assets. $1/(1+\xi(t))$ is the marginal propensity to consume out of wealth, understood here as principal plus interest. The evolution of ξ is given by (2.8). Human wealth evolves according to (2.9), and foreign assets evolve according to (2.10), where κ is capital's share in the production of firms. (2.11) is the market clearing condition for the domestic commodity. The model is closed by either (2.12) which states that domestic assets are foreign liabilities or (2.13), which requires the uncovered interest rate parity. Both are equivalent.

3 The steady state of the model

I depart from the assumption that permanent growth differentials are possible. Consider (2.11) and assume that all domestic expenditure elements grow at rate n, but that foreign consumption expenditure grows at rate n^*. Alternatively, consider (2.12) and assume that each country's foreign assets per GDP remains stable. Both approaches immediately lead to

$$\frac{p(\mathrm{t})}{p(\mathrm{t}+1)} = \frac{1+n^*}{1+n} \quad \text{if} \quad f \neq 0 \quad \forall \mathrm{t} \tag{3.1}$$

This is my first result. There can be no steady state with an imbalance in the rate of growth unless there is a continuous change in the terms of trade. The change in the terms of trade forces the domestic terms of trade deteriorate if the domestic economy is to grow faster than the foreign economy. It is the key relationship that enables differential growth in the steady state of the economy. Using (3.1) in the Euler equation (2.15), I obtain

$$(1+r) = (1+d)\,(1+n)^{\sigma+(1-\sigma)\,(1-\alpha)}\,(1+n^*)^{(\sigma-1)\,(1-\alpha)} \tag{3.2}$$

In the (n,r) plane, this expression defines an intertemporal "demand" curve. Such a curve first appeared in Krichel and Levine (1995)—within a single-country model—and will be labeled the "KL" curve here. The KL curve for the foreign economy is

$$(1+r^*) = (1+d^*)\,(1+n^*)^{\sigma^*+(1-\sigma^*)\,(1-\alpha^*)}\,(1+n)^{(\sigma^*-1)\,(1-\alpha^*)} \tag{3.3}$$

But note that (2.13) and (3.1) require that in the steady state

$$\frac{1+r^*}{1+r} = \frac{1+n^*}{1+n} \tag{3.4}$$

Dividing (3.2) by (3.3) and making use of (3.4), I find

$$\frac{(1+n)^{\varphi}}{(1+n^*)^{\varphi^*}} = \frac{1+d}{1+d^*} \tag{3.5}$$

where $\varphi = \alpha\,(1-\sigma) - (1-\sigma^*)\,(1-\alpha^*)$ and $\varphi^* = \alpha^*(1-\sigma^*) - (1-\sigma)\,(1-\alpha)$. Equation (3.5) *must hold as an identity* for *all* n^* and n. Otherwise the system would be overdetermined. In a conventional neoclassical growth model, $n = n^* = 0$ and the condition (3.5) reduces to the well-known requirement that the discount rates in the two economies must be equal for an international steady state to exist. If growth is symmetric, then I need to assume that $\varphi = \varphi^*$. It is straightforward to see that this requires that $\sigma = \sigma^*$. If there is a growth differential I need the more stringent condition that $\varphi = \varphi^* = 0$. The equality to zero requires that $\alpha + \alpha^* = 1$, in addition to the previous conditions. I can summarize

PROPOSITION 3.1 There is no steady state in the model unless $d = d^*$. There is no steady state with positive growth unless $\sigma = \sigma^*$. There is no steady state with different growth rates unless $\alpha + \alpha^* = 1$.

The KL curves of (3.2) and (3.3) can be thought of as intertemporal demand curves. For each country, a supply curve can be derived from the steady state of (2.1), in conjunction with (2.2) and (2.3). Using the steady-state of (2.5) for the private capital stock and (2.4) for the infrastructure stock, I obtain

$$\delta + n = \bar{\epsilon}^{1/(1-\gamma)} \left(\frac{(1-\tau)\,\kappa}{r+\delta}\right)^{\gamma/(1-\gamma)} \tau \tag{3.6}$$

This relationship also holds for the foreign economy. Since these curves are derived from the Cobb-Douglas production function, I will label them the "CD" curves. They are downward sloping in the (n,r) space. Since the KL curves (3.2) and (3.3) are upward sloping, a unique equilibrium will exist in each economy. This equilibrium is parameterized by the foreign growth rate.

To formally investigate the comparative statics impact of one country's growth rate on the other's, differentiate

$$\mathrm{d}n = \frac{n+\delta}{r+\delta}\,\frac{\gamma}{1-\gamma}\,\mathrm{d}r \tag{3.7}$$

$$\mathrm{d}r = \frac{(\sigma+(1-\sigma)\,(1-\alpha))\,(1+r)}{1+n}\,\mathrm{d}n + \frac{(\sigma-1)\,(1-\alpha)\,(1+r)}{1+n^*}\,\mathrm{d}n^* \tag{3.8}$$

Since all parameters are strictly positive and $\alpha < 1$, it can be shown that the sign of $\mathrm{d}n/\mathrm{d}n^*$ is equal to the sign of $1-\sigma$. Theoretical studies suggest that a high rate of substitution leads to counterintuitive results, see for example Dornbusch (1983). More generally, the received wisdom is that $\sigma \approx 2$. This implies that an increase in the growth rate in the foreign country leads to a decline in the domestic growth rate, in comparative static terms. I illustrate this effect in Figure 3.1. To calculate the curves, I have used the calibration $\delta = 4\%$, $\gamma = 36\%$, $\alpha = .5$. It should be noted that the almost

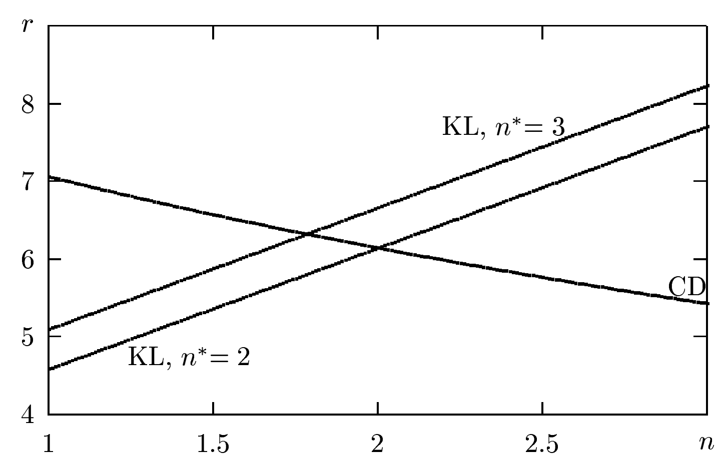

Figure 3.1: Growth and interest rates, all figures are percentages

linear appearance of the KL and CD curves is robust to many calibration values. The value of the impact of foreign on domestic growth turns out to be −22% in this case. Thus a one percent increase in the foreign growth rate reduces the domestic growth rate by about −.22%. More generally the importance of the slowdown of domestic growth as a response faster foreign growth depends on three factors. The smaller is the elasticity of substitution $1/\sigma$, the more an increase in the foreign growth rate is leading to an increase in the interest rate. The larger the share of the foreign commodity in the world economy, the more the increase in the foreign interest rate leads to an increase in the domestic interest rate. The higher the impact of interest rate rise on the supply of capital, the more important will be the adverse effect of an interest rate raise on growth rate.

It is interesting to note that the growth rate in the steady state can be related to exogenous parameters and policy variables. It does not depend on foreign assets. When a country has foreign assets, it will be able to have an expenditure that exceeds income. Consumption, but not investment will depend on the foreign assets. Using lower-case letters for aggregates as a ratio of GDP, I find

$$c = (1-\gamma)(1-\tau) + \frac{\gamma(1-\tau)(r-n)}{r+\delta} + \frac{(r-n)f}{(1+n)}$$

In this model, f is an exogenous variable. Growth rates and interest rates can not be used to explain foreign asset accumulation, it is determined from the past and has no impact on growth. This is my last result.

4 Conclusions

There are two interpretations of this paper. On the one hand, one could argue that all I have shown is that a situation exists where the domestic and foreign growth rates are different forever. To enable such a scenario, I need to ensure that the size of the slow-growing economy is not zero in the long run. I show that with a permanent change in the terms of trade, this is indeed possible. In this situation, the domestic output will grow at the domestic growth rate if expressed in units of the domestic commodity, and it will grow at the foreign growth rate if expressed in units of the foreign commodity. Under those circumstances, it does not matter for a consumer if output grows at a slow or a fast rate, since all the benefits of high growth in the domestic commodity output are lost through the decline in the terms of trade. Growth does not matter.

An alternative view of my findings comes from the impact on domestic growth of a change in foreign growth. I have shown that for common parameter values, an increase in foreign growth reduces domestic growth. This is a quite general result. Assume that the consumer's intertemporal substitution is smaller than one, i.e. when percentage change in relative prices of consumption in different periods leads to a change in the relative consumption that is smaller. In this realistic case there is a positive long-run relationship between interest rates and growth rates. It does matter little if it is domestic or foreign growth rate I am referring to, any increase in the growth rate will lead to an increase in the required interest rate. It is also true under very general conditions that an increase in the interest rate depresses private capital accumulation. Again under very general conditions a reduction in capital accumulation reduces the rate of growth. Therefore an increase of growth in one country raises the world interest rate, and depresses growth in another country.

All my findings depend crucially on the assumption that the demand shares remain fixed. That implies that the consumption demand addressed to domestic producers always has a fixed share in world consumption. Clearly this is not realistic when growth is driven by the accumulation of knowledge. In that case the share of each country in the world economy is endogenous. There is a large volume of such models, but they all share a logarithmic felicity function that rules out the growth externalities that I have considered in this paper. Bringing together both strands is a challenge that has yet to be taken up, but I am convinced that it will lead to further insight into the process of relative development of different countries.

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