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Non-linear technological progress and the substitutability of energy for capital: an application using the translog cost function

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ABSTRACT

This paper analyses the production process of four industries over four separate time periods using datasets taken form Berndt and Wood (1975, 1979), Hunt (1984a, 1986), Norsworthy and Harper (1981) and Jorgensen and Stiroh (2000). In their initial paper Berndt and Wood failed to explore the alternative options available to them to represent technological progress, a deficiency noted by Hunt (1986) who tested for alternative representations of technology (inter alia) using the Berndt and Wood data. This paper extends this line of reasoning/research by allowing technological progress to take more flexible non-linear forms using both deterministic and stochastic trend models. The results reveal that 'non-linear trend' models are generally preferred to 'linear trend' or 'no trend' models hence raising a question over the validity of assumptions used in much previous empirical research. Further the results reveal that the different assumptions lead to different results for the energy-capital elasticity of substitution.

JEL Classification: O33; O47; Q49.

Key Words: Translog, energy-capital substitution, productivity

NON-LINEAR TECHNOLOGICAL PROGRESS AND THE SUBSTITUTABILITY OF ENERGY FOR CAPITAL: AN APPLICATION USING THE TRANSLOG COST FUNCTION

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1. Introduction

Based upon an inspection of around 100 empirical papers on production functions featuring energy as an included factor, approximately 35-40% of studies did not attempt to feature the effects of technological progress (see Broadstock *et al*, 2007). As Koetse *et al* (2007) indicated in a meta-study of elasticities of substitution between energy and capital (σ_{ke}), the effects of capturing technological progress can significantly change the value of the measured elasticities. There is thus a need for production analysts to reconsider the empirical measurement of technological progress effects.

Berndt and Wood (1975, 1979) generated the seminal results within the energy literature, using a Translog model of production to place an empirical value on the elasticity of substitution between energy and capital [for US Manufacturing]. Hunt (1986) extended these results[2] to test for the role of technological progress in production with the inclusion of non-neutral (or factor augmenting) technological progress, achieved by the inclusion of a deterministic linear trend. For the Berndt and Wood data (US Manufacturing) this was rejected, whilst conversely for the UK [Industrial sector], linear technological progress was statistically preferred. This study extends the efforts of these previous studies by testing the hypothesis that *Technological progress is not a linear function of time* in the context of a

[1] I am grateful for comments and questions received at the 9th annual conference of the International Association of Energy Economists (IAEE), Florence June 2007. Also I would like to thank Lester C. Hunt (SEEC) for comments received on an earlier draft of this paper. I alone am responsible for all omissions and errors.

[2] Although this was not the only, or first piece empirically reviewing the results of Berndt and Wood.

Translog cost function. The focus of the results is directed towards their impact upon the derived empirical elasticity of substitution between energy and capital (σ_{KE}) when applying different representations of technology. This is pertinent due to the prominent role its understanding plays in the development of sustainability conscious policy measures, and the importance such measures have within wider analytical models such as general equilibrium systems.

The order of the paper is as follows; the next section outlines the estimation methodology, outlining the approaches to defining the linear and non-linear technological progress terms followed by a brief description on how the elasticities of substitution are derived from these models. Section 3 provides a short note on the data used in this study, while section 4 presents the results of the analysis, focussing upon the empirical shape of the underlying effects of technological progress and subsequent effects on the elasticity of substitution. Concluding remarks are then offered in section 5.

2. Methodology

This section discusses the key methodological considerations, as well as the empirical methods used. The most commonly used functional form for empirically estimating production functions is the Translog specification originally due to Christensen *et al* (1973), which offers increased flexibility over other forms. Alternative flexible functional forms do exist, such as the generalised Leontief due to Diewert (1971), though these are not considered in the present application due to their relative lack of empirical implementation. The present application's research hypothesis is therefore tested in the context of the Translog model featuring four factor inputs, namely ***K****apital*, ***L****abour*, ***E****nergy* and ***M****aterials*. i.e. $y=f(K,L,E,M)$ where y=output. The exposition of the Translog function (including hicks neutral and linear factor augmenting trends) is well defined in the literature and so is not redefined in this paper.

Proxies for technological progress, such as Research and Development investment in energy efficient technologies, are not necessarily representative of the impact of technological progress. They do not capture the desire of technology developers to maximise the potential wealth of their energy saving concepts, and hence the potential lag between design and implementation. i.e. Developers may wish to delay the release of their product(s) to consumers until the existing technology has clearly peaked past the maturity stage of its product life cycle, when revenue streams show clear evidence of decline.

Linear trend approaches to proxy for technological progress are a useful and accessible start point for empirical modelling, however such trends are a pre-defined assumption. Moreover such an assumption is simply one of an infinite class of assumptions that could be made regarding the shape of the underlying technological progress trend that should be tested for, ideally from a very flexible form. This remainder of this section outlines two empirical approaches which offer more realistic representation for technological progress that can take non-linear forms.

(i) The (augmented) Almon Lag Model.

The first method applied to allow for a non-linear trend is derived from a dynamic econometric method, albeit applied within a static setting for the present example. An Almon lag specification is expressed in which the lag length is equal to the number of time periods in the dataset, however this lag is only applied to the model constant(s), not the slope coefficients, hence it is a heavily restricted form of the model. Essentially this produces a time varying constant within the model, which is formed as the sum of the intercept plus the trend. This therefore approximates the results of the Seemingly Unrelated Structural Time Series Model (detailed below) in which the constant is allowed to follow a stochastic process and is thus time varying. Hence the Almon approach is a deterministic approximation to a stochastic process, and is as such far more readily accessible to modellers. The necessary augmentations/restrictions are equivalent to expressing an ϕ^{th} order static polynomial function on a standard deterministic representation of technological progress i.e. $T=\Sigma(t^1+t^2+\ldots+t^{\phi})$ where ($0 < \phi < \infty$). For convenience the polynomial order is restricted to $\phi=4$, although could be set to any number (depending on the number of available degrees of freedom), and tested down accordingly. The specific form of the share equations is;

$$
\begin{aligned}
S_K &= \alpha_K + \gamma_{KK} \ln p_K + \gamma_{KL} \ln p_L + \gamma_{KE} \ln p_E + \gamma_{KM} \ln p_M + \sum_{\phi=1}^{\Phi} \beta_{T_K^1} T^1 + \ldots + \beta_{T_K^{\phi}} T^{\Phi} \\
S_L &= \alpha_L + \gamma_{LK} \ln p_K + \gamma_{LL} \ln p_L + \gamma_{LE} \ln p_E + \gamma_{LM} \ln p_M + \sum_{\phi=1}^{\Phi} \beta_{T_L^1} T^1 + \ldots + \beta_{T_L^{\phi}} T^{\Phi} \\
S_E &= \alpha_E + \gamma_{EK} \ln p_K + \gamma_{EL} \ln p_L + \gamma_{EE} \ln p_E + \gamma_{EM} \ln p_M + \sum_{\phi=1}^{\Phi} \beta_{T_E^1} T^1 + \ldots + \beta_{T_E^{\phi}} T^{\Phi} \\
S_M &= \alpha_M + \gamma_{MK} \ln p_K + \gamma_{ML} \ln p_L + \gamma_{ME} \ln p_E + \gamma_{MM} \ln p_M + \sum_{\phi=1}^{\Phi} \beta_{T_M^1} T^1 + \ldots + \beta_{T_M^{\phi}} T^{\Phi}
\end{aligned}
\qquad (1)
$$

Where T_i is used to denote the time trend for factor i, which proxies for the effects of technological progress. With $T^2 = \ldots = T^{\Phi} = 0$, this model reduces to the factor augmenting linear trend model, further restricting $T^1 = 0$ leads to the Hicks neutral representation of technology. Although in the present application the model contains no dynamic features, and hence to call it an Almon lag specification may be questionable, this name tag is retained on the basis that future work could include more general dynamic specifications being applied and tested. This model is estimated using Zellner's Seemingly Unrelated Regression.

(ii) The Seemingly unrelated structural time series model.

Harvey's (1989) seemingly unrelated structural time series model (SUSTSM) is applied, where the state space representation and unique properties of the Kalman filter within the transition equation allow technological progress to take a stochastic and nonlinear form, previously only known to have been applied for this purpose by Harvey and Marshall (1991). One advantage of this method is that it is a more parsimonious approach to estimation compared to the deterministic non-linear approach discussed previously. In situations where the underlying technological progress is highly non-linear, the Almon lag approach could potentially consume many (often valuable) degrees of freedom. The estimated share equations of the SUSTSM are specified as;

$$\begin{aligned} S_K &= \mu_K + \gamma_{KK} \ln p_K + \gamma_{KL} \ln p_L + \gamma_{KE} \ln p_E + \gamma_{KM} \ln p_M \\ S_L &= \mu_L + \gamma_{LK} \ln p_K + \gamma_{LL} \ln p_L + \gamma_{LE} \ln p_E + \gamma_{LM} \ln p_M \\ S_E &= \mu_E + \gamma_{EK} \ln p_K + \gamma_{EL} \ln p_L + \gamma_{EE} \ln p_E + \gamma_{EM} \ln p_M \\ S_M &= \mu_M + \gamma_{MK} \ln p_K + \gamma_{ML} \ln p_L + \gamma_{ME} \ln p_E + \gamma_{MM} \ln p_M \end{aligned} \tag{2}$$

Where μ is a stochastic trend function representing the factor-augmenting technological progress. μ is defined for each time period and is expressed as $\mu_t = \mu_{t-1} + \xi_t$ and ξ_t is a normally distributed white noise error term. This estimation technology essentially delineates the effects of measurement error from the actual effects of the independent variables upon the dependent. Hence, given the theoretical grounding of the model, any effect that is not price induced is induced by technology and the measurement error arising from the estimation can be considered purely to be the unobservable component of technology.

For each of the four models estimated ((i) Hicks Neutral, (ii) linear, (iii) Almon and (iv) SUSTSM) the general assumption is that the production iso-surface is symmetric about any point, i.e. $\gamma_{KE} = \gamma_{EK}$ with similar relationships holding for all other pairs of factor inputs. As Turnovsky *et al* (1982) point out, this is equivalent to ensuring that linear homogeneity conditions are met.

The estimation results of the four alternative models are then used to derive the empirical elasticity of substitution between capital and energy. Although these elasticities are not directly comparable from one dataset to the next, due to the inconsistency in sector, time period and country, this will at least provide some bearing of the effect of differing assumptions on the form of technological progress. The most common measure of substitution used in empirical is the Allen Elasticity of Substitution (AES) which can be written in cross-price and own-price forms respectively as;

$$AES_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j}\ ;\ AES_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2}$$

The AES has been the most predominantly used elasticity in empirical research (see Broadstock *et al*, 2007), however it has been shown ***not*** to be an actual measure of the curvature of the production iso-surface, whereas the Morishima (1967) elasticity is, see Blackorby and Russell (1981). As a result of this, the AES will be evaluated (for consistency with the earlier studies) but will further be compared with the empirical value of the Morishima Elasticity of Substitution (MES) between capital and energy which is defined as;

$$MES_{ij} = S_j(AES_{ij} - AES_{jj})$$

i.e. as the Allen-Uzawa cross elasticity minus the Allen-Uzawa own price elasticity. This will exemplify the magnitude of the difference in policy conclusions that may be drawn from using the theoretically flawed AES compared to the MES. It should be noted that the as γ_{ij}= γ_{ji} i.e. the production functions parameters are symmetric, then AES_{ij}=AES_{ji} i.e. also symmetric. Although $MES_{ij} \neq MES_{ij}$ i.e. is not symmetric given its formulation, even though the parameters of the production function are. Thompson (2006) provides a recent account outlining the merits of measuring cross-price elasticities (CPE) rather than substitution elasticities, therefore these are presented in the results also. These is defined for the Translog function as;

$$CPE_{ij} = \frac{(\hat{\beta}_{ij} + S_i S_j)}{S_i}$$

3. The Data

The original data from the following papers are used for the empirical analysis; Berndt and Wood (1975, 1979) - US Manufacturing 1947-1971, Hunt (1984a, 1986) [3] - UK Industry 1960-1980, Norsworthy and Harper (1981) - US Manufacturing 1958-1977 and Jorgensen and Stiroh (2000)[4] - US Transport ordinance and equipment 1958-1996. The specific details of these datasets and their sources are explained in further detail in each of the respective papers.

4. Results

This section outlines the empirical results emanating from this study. The results reported omit information on significance values for the Translog parameters to avoid making presentation overly cumbersome, and also on the premise the reported coefficients are the 'correct' values given the data available, and the formalised theory that underpins their estimation. Given that this study seeks to exemplify the range of policy implications that may arise from uncontested assumptions, specifically within the context of the Translog cost function, it does not serve a purpose to present them in this table. However in the analysis of the derived substitution/cross-price elasticities, the standard errors will be reported in order to help identify the robustness of the co-relationships that exist between the factors over time. i.e

[3] Hunt's (1984, 1986) data for the UK contains no information on materials, though this is not the only empirical work in this area of literature which estimates a ***KLE*** rather than ***KLEM*** function.

[4] This dataset contains 35 separate industries, of which only one is chosen in the interest of pedagogy.

whether two factors have consistently been complements (or substitutes) and whether this has been of a consistent magnitude over the period of analysis. Given the ultimate desire to shed light on the relationship between energy and capital, this section also omits discussion of the results relating to labour and materials.

The general results are given in Table 1 and reveal that the alternative representations for technological progress impart an observable impact upon the estimated parameters. For instance the four alternative model specifications applied to the BW 1975/1979 data produce results for γ_{KE} that range from +0.02 to +0.06. Of greater interest is the impact of the four alternative methods upon γ_{KL} from the Hunt 1984/1986 data which range from -0.04 to +0.04, thus indicating that changes in sign and magnitude can arise dependent on the assumption made. It can therefore be inferred that there is some potential bias being imparted on the estimated parameters by not appropriately accounting for the effects of technological progress. There does not appear to be any clear pattern as to whether these apparent biases are systematically over/under-estimating the production functions main parameters.

		SK				**SL**			**SE**		**SM**
		γ_{KK}	γ_{KL}	γ_{KE}	γ_{KM}	γ_{LL}	γ_{LE}	γ_{LM}	γ_{EE}	γ_{EM}	γ_{MM}
BW (1975)	***Hicks***	0.03	0.00	-0.01	-0.02	0.08	0.00	-0.07	0.02	0.00	0.09
	Linear	0.04	0.02	0.00	-0.04	0.12	0.03	-0.14	0.01	-0.04	0.21
	ALMON	0.04	0.01	-0.01	-0.04	0.13	0.03	-0.15	0.02	-0.05	0.20
	SUSTM	0.03	0.01	0.00	-0.05	0.05	0.03	-0.05	0.06	-0.12	0.17
Hunt (1986)	***Hicks***	-0.02	0.04	-0.01	-	-0.03	-0.01	-	0.02	-	-
	Linear	0.03	-0.04	0.01	-	0.09	-0.05	-	0.04	-	-
	ALMON	0.02	-0.04	0.01	-	0.09	-0.06	-	0.04	-	-
	SUSTM	0.04	-0.02	0.01	-	0.05	-0.04	-	0.04	-	-
NH (1981)	***Hicks***	0.07	-0.01	0.00	-0.05	-0.01	0.00	0.02	0.02	-0.02	0.06
	Linear	0.07	-0.02	0.00	-0.05	0.20	0.01	-0.19	0.02	-0.02	0.27
	ALMON	0.07	-0.03	0.00	-0.03	0.20	-0.01	-0.15	0.00	0.01	0.17
	SUSTM	0.07	-0.02	0.00	-0.04	0.08	-0.01	-0.10	0.03	-0.02	0.22
JS (2000)	***Hicks***	-0.01	0.03	-0.01	-0.01	-0.09	0.03	0.00	0.03	-0.09	0.17
	Linear	-0.01	0.02	0.00	-0.01	0.00	0.01	-0.01	0.00	-0.02	0.02
	ALMON	-0.01	0.02	0.00	-0.01	0.01	0.00	-0.01	0.00	-0.01	0.01
	SUSTM	0.00	0.01	0.00	0.00	-0.03	0.00	0.01	0.01	-0.03	0.05

Table 1: Translog Cost Function – Key Parameters

The technological progress parameters are omitted from Table 1 as it is arguably more useful to inspect them visually, Figure 1 therefore graphs the estimated technological progress trends derived from the capital (K) and energy (E) share equations. A downward (upward) sloping trend reflects a factor-saving (using) effect of technological progress i.e. the effects of technological progress over time have led to a reduction (increase) in the demand for that factor. The details of these trends are given further consideration in the following discussions of the particular results relating to each dataset;

Berndt and Wood (1975, 1979);

The results generated using the Berndt and Wood data would tend to imply that technological progress is largely speaking a linear function of time. The results depicted in Figure 1 reveal that although the SUSTSM generally supports the notion that the underlying effect of technological progress is linear, it takes a different slope. The gradient using the deterministic approaches is consistently steeper. Wald tests are used to determine if the SUSTSM outperforms the models with 'linear technological progress' and 'no technological progress' given that these are restricted forms of the SUSTSM. The Almon lag approach is not strictly nested within the SUSTSM, therefore it is not directly compared to the SUSTSM[5], though the Almon lag approach is compared to the linear trend and no trend models respectively also using Wald tests as these are also restricted forms of the Almon model. The results for the Wald tests, see Table 2, seek to understand (i) whether the addition of technological progress benefits the models overall performance (ii) if this is better done using non-linear trends compared to linear ones. Given that this is not a full general to specific econometric exercise and that more general Translog specifications and/or alternative functional forms could be applied, these tests are indicative of potential avenues for further research, rather than providing a conclusive answer(s) as to the most appropriate model(s) for the data at hand.

[5] This is justifiable given that the Wald tests will reveal whether or not linear technological progress is outperformed by non-linear alternatives.

Hence, that the SUSTSM and Almon models are not directly compared, is secondary to the finding that any non-linear trend model is preferred to a linear or no trend model.

Wald test results

	BW (1975)	Hunt (1986)	NH (1981)	JS (2000)
From SUSTSM				
to linear trend	Unable to reject	Reject	Reject	Reject
to no trend	Unable to reject	Reject	Reject	Reject
From Almon				
to linear trend	Unable to reject*	Reject	Reject	Reject
to no trend	Reject	Reject	Reject	Reject

** p-value=0.052, therefore marginal*

Table 2: Wald restriction tests from general non-linear models

The results of these tests indicate that for the Berndt and Wood data, the SUSTSM is *not* preferred to either the linear trend or no trend models, while conversely the Almon lag model shows some evidence of being preferred. From a qualitative perspective, the fact that a linear technology seems to prevail is perhaps not surprising given that the results are found upon a sample of data from a stable sector within a developed economy.

Hunt (1984a, 1986);

As with the Berndt and Wood data, Hunt's (1984a, 1986) data is again generally representative of a stable sector in a stable economy, although the sample period does incorporate the oil price shocks of the 1970's as well as a period of recession towards the end of the 1970's also. There is some evidence of these events manifesting in the empirical results of the SUSTSM, where the factor augmenting technological progress seemingly shifts slope following 1973/74. For capital, the effects of technology seem to dissipate, whereas for energy the effect of technology results in greater, albeit less stable, energy use.

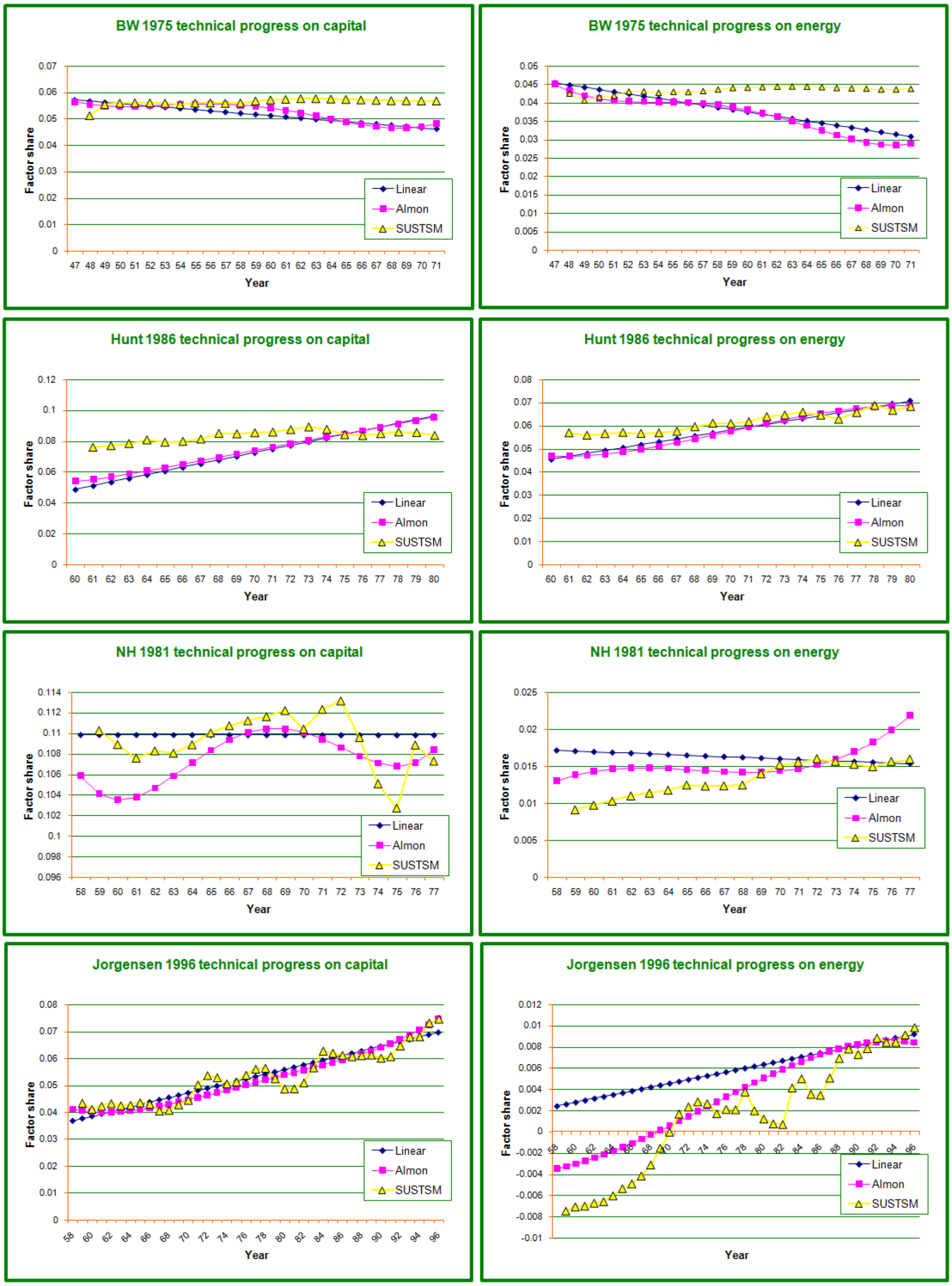

Figure 1: Factor Augmenting Technological progress on Energy and capital;

The Wald tests clearly support that the use of a non-linear trend is preferred, with both the deterministic and stochastic versions being preferred to the linear trend and no trend models. Although the trends depicted in figure one may appear upon an initial cursory inspection to be quite linear on average, the added explanation of the variation increases the model performance sufficiently enough for it to be a preferred approach. Focussing for instance upon the underlying trend of technological progress effects on the share of capital in the factor mix, a number of observations can be made. Prior to 1973, the effect of technology was steadily increasing, but for some mild variation in and around 1965 which persisted for a few years. Post 1973, these effects dissipated and would tend to imply that a structural shift in the impact of technological progress occurred following the oil shock[6]. A similar albeit less obvious pattern appears in the energy share equation, with increased volatility in the trend post 1973. Although this variation appears small and not far removed from the linear trend, the ability to accurately capture the non-linearities proves to be a significant advantage to model specification. This might be a result of the order of the model specified, and the situation that a fourth order polynomial is insufficiently able to capture the true underlying trend.

Norsworthy and Harper (1981);

This data set represents a generally stable economy, but during a less stable time period than compared to the Berndt and Wood data i.e. the sample incorporates the same oil price crises that were present in Hunt's (1984a, 1986) data. The underlying technological progress trends generated by the non-linear models are more profoundly departed from a linear trend than with the previous two examples. Both the Almon lag approach and the SUSTSM suggest an almost sinusoidal underlying technological progress for capital, although the SUSTSM shows a less fluctuating trend pre 1973 and more severely fluctuating trend post 1973. For energy the trends from the two approaches largely follow the same pattern but with differing

[6] Hunt (1984b) estimated a Translog cost function for this data allowing for the possibility of a structural break following the oil price shock and found evidence of a statistically significant structural break in the data generation process.

magnitudes, though post 1973 the two trends seem to separate, with the SUSTSM suggesting no substantial effect of technology on factor use, whereas the Almon lag approach implies factor using technological progress. The Wald tests reveal that the non-linear trends are preferred to the linear trends.

The linear trend model results imply that for energy the effect of technological progress is energy saving, whereas the preferred non-linear models would rather tell a different story. Using the non-linear trends, the implication is that the effects of technological progress have been mostly energy consuming over the period in question

Jorgensen and Stiroh (2000);

The implied technology trends for the US Transport ordnance and equipment sector when using the non-linear trend models imply that the oil price shock of the early 1970's had a pronounced impact upon the effectiveness of new technology implementation, both during the period of the shock and for some time thereafter. This is not too surprising given that the estimation is for the transport ordnance and equipment sector, where output will be predominantly governed by output/activity in the transport sector. Hence the volatility in the technology trend is a by-product of a fluctuating demand for oil resulting from the price shock/crisis during this period. The non-linear models are preferred based upon the Wald test results.

Elasticities of substitution;

With respect to the general implications for energy-capital substitutability, Table 3 identifies no clear-cut patterns[7] as to whether they are complements or substitutes, though this is not entirely unexpected given the unique nature of each of the datasets. The Morishima elasticity

[7] Even though some of the Translog parameters show little variation, as for instance in the Norsworthy and Harper data, which shows four identical σ_{ke} values. This is because of the addition and multiplication by factor shares and/or the inclusion of own-price parameters in the calculation of the final elasticities, as in the MES.

favours substitutability between capital and energy (and vice versa), while conversely the more widely used Allen-Uzawa elasticity tends to suggest substitutability between capital and energy but complementarity between energy and capital.

Form of Technical progress	BW (1975) σ_{KE}	BW (1975) σ_{EK}	Hunt (1986) σ_{KE}	Hunt (1986) σ_{EK}	NH (1981) σ_{KE}	NH (1981) σ_{EK}	JS (2000) σ_{KE}	JS (2000) σ_{EK}
Allen-Uzawa elasticities								
Hicks	-3.33		-1.66		-0.55		-22.29	
	(0.446)		(0.437)		(0.166)		(8.069)	
Linear	-1.04		2.68		-0.36		-6.46	
	(0.211)		(0.276)		(0.146)		(2.583)	
ALMON	-2.17		3.30		-1.25		-4.88	
	(0.327)		(0.378)		(0.241)		(2.036)	
SUSTSM	0.77		2.04		-0.19		-0.07	
	(0.023)		(0.170)		(0.128)		(0.369)	
Morishima elasticities								
Hicks	0.39	0.11	0.47	1.08	-0.33	0.18	1.51	0.57
	(0.032)	(0.059)	(0.046)	(0.026)	(0.201)	(0.068)	(0.144)	(0.188)
Linear	0.49	0.10	0.46	0.70	-0.44	0.21	1.66	0.94
	(0.027)	(0.059)	(0.058)	(0.039)	(0.220)	(0.069)	(0.126)	(0.079)
ALMON	0.51	0.13	0.47	0.89	-0.46	0.14	1.40	1.01
	(0.027)	(0.060)	(0.059)	(0.026)	(0.219)	(0.056)	(0.079)	(0.079)
SUSTSM	-0.35	0.42	0.44	0.58	1.16	0.81	1.24	1.10
	(0.093)	(0.048)	(0.057)	(0.049)	(0.031)	(0.015)	(0.041)	(0.045)
Cross price elasticities								
Hicks	-0.15	-0.18	-0.11	-0.13	-0.01	-0.06	-0.21	-0.77
	(0.017)	(0.016)	(0.022)	(0.026)	(0.002)	(0.018)	(0.068)	(0.133)
Linear	-0.05	-0.06	0.17	0.21	-0.01	-0.04	-0.06	-0.22
	(0.008)	(0.009)	(0.009)	(0.011)	(0.002)	(0.016)	(0.022)	(0.043)
ALMON	-0.10	-0.11	0.21	0.26	-0.02	-0.14	-0.04	-0.17
	(0.012)	(0.012)	(0.013)	(0.014)	(0.003)	(0.030)	(0.017)	(0.035)
SUSTSM	0.03	0.04	0.13	0.16	0.00	-0.02	0.00	0.00
	(0.003)	(0.005)	(0.006)	(0.008)	(0.002)	(0.013)	(0.003)	(0.016)

standard errors in parentheses
if $\sigma > 0$ then substitutes
if $\sigma < 0$ then complements

Table 3: Allen-Uzawa and Morishima Elasticities

The Morishima elasticities of substitution are much closer to values which one might find empirically acceptable, whereas some of the Allen-Uzawa results provide elasticities which are far more extreme. In particular the AES's for the Jorgensen and Stiroh data are extremely high. The application of different assumptions on the form of underlying technological progress is seen (in Table 3) to result in different values for the elasticity of substitution, mostly in terms of magnitude, though sometimes in terms of sign also. Although this conclusion is more pronounced for the Allen elasticities, it holds true even for the Morishima elasticities albeit with a much smaller absolute variation in the magnitude of the results. The standard errors provided in Table 3 reinforce the conclusion that the non-linear trend models are better capable of producing robust results for less-stable economies, as for the BW and Hunt data there is an increase in the size of the standard errors as the trend function becomes arguably unnecessarily more complex. However for the NH and JS datasets, which include more pronounced variation, the increased flexibility in the trend function seems to enhance the robustness of the results. These results are confirmed by the increased robustness of the CPE.

5. Discussion and Conclusions.

With respect to the core research hypothesis, this paper has provided empirical evidence to suggest that technological progress is not necessarily a linear function of time and subsequently that empirical studies need to test for the alternatives. Making the wrong assumption could lead to bias in the estimated production function parameters and hence the derived elasticity of substitution between capital and energy may be based upon an incorrectly specified model. Arguably the SUSTSM further provides a more econometrically efficient estimation method than the alternative non-linear deterministic approach considered. This is largely because it allows for non-linear technological progress to feature in the model without adding excess variables/parameters to the model as an Almon lag structure might. Similarly, factor augmenting linear deterministic trends are a restricted form of the SUSTSM and can be easily tested and chosen if econometrically preferred.

It is recommended that a general to specific approach to estimation is advocated whereby the most general form should be the SUSTSM, which allows the underlying technological progress term to take *any* shape. Although the Almon lag representation is not strictly nested within this specification, the empirical results would lend themselves to the conclusion that the Almon lag is an approximation to the results of the SUSTSM. To this extent it may be reasonable to suggest that the two methods are substitutes rather than anything else. In less stable/developing economies, it may simply be infeasible to apply a technological progress term which accurately reflects the underlying effects of technology in the sector. Given its potential ability to parsimoniously specify accurate non-linear trends the SUSTSM would also be a recommended model for small or unstable data samples.

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