Catching on the Rebound:
Why Price Elasticities are Generally
Inappropriate Measures of
Rebound Effects

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School of Economics
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CATCHING ON THE REBOUND: WHY PRICE ELASTICITIES ARE GENERALLY INAPPROPRIATE MEASURES OF REBOUND EFFECTS

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ABSTRACT

Rebound effects occur when, due to behavioural responses by consumers to the resulting fall in the implicit price of energy services, energy efficiency improvements result in energy savings that are often less than those suggested by engineering calculations. In the absence of data on energy efficiency or on the energy services (such as heating or lighting) provided by the energy that is used to produce them, rebound effects are often estimated as the negative of own-price elasticities obtained from standard energy demand equations. Using a recently developed model of demand for energy services, which facilitates estimation of a much wider range of rebound effects than has been previously considered, this approach is shown to be inappropriate unless the energy demand equations are specified in a certain way, and even in that case, often only under somewhat heroic assumptions. Illustrative empirical analysis using UK time-series data indicates the extent to which rebound effects can differ from price elasticities.

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Catching on the Rebound:

Why Price Elasticities are Generally Inappropriate Measures of Rebound Effects

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1. Introduction

In the consumer context, when the price of a good or service is changed, there are two resulting effects. The first is the substitution effect, which refers to the response to the change in relative prices, holding utility constant, and which acts in the opposite direction of the price change, such as decreasing the quantity consumed of the good or service whose price has increased. The second is the income effect, which reflects the fact that a change in the price of one good or service with nominal income constant generally means that a different utility level will be attainable, and the change to this utility level will involve a rearrangement of the quantities that are consumed of various goods. A key implication then is that if the price of a good is decreased, consumption of that good would be expected to change – and most likely increase,[1] and consumption of other goods are also likely to change, and possibly increase. These effects are no different in the context of changes in the prices of energy sources, such as electricity or natural gas, than they are anywhere else. Within energy economics, however, rather than focusing on consumer responses to changes in the prices of these energy sources, attention

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[1] The actual outcome will depend on the sign and relative size of the income effect, but an own-price decrease will result in increased consumption unless the good in question is an inferior good where, with more income, less of it would be consumed.

has increasingly been concentrated on the effect of efficiency changes – that is, of changes that improve the efficiency with which energy services are delivered. Of course, such efficiency changes are tantamount to changes in the price of these energy services, and therefore also result in income and substitution effects. However, in the field of energy economics these effects are viewed with some consternation – since they tend to act in the opposite direction of the effect (reduced energy use) that was intended – so much so that they are jointly referred to as rebound effects.

Although precise definitions of what rebound effects actually are differ, and there are several different types of rebound effects (direct, indirect, economy-wide), as we describe later, in general a direct rebound effect is said to exist whenever the result of increased energy efficiency is an increase in consumption of the energy services that are now provided more efficiently. Thus, for example, if there is an increase in the efficiency of natural gas heating, so that the same amount of heat can be provided using less natural gas, then if consumption of heat changes (increases) as a result, there is said to be a direct rebound effect. Of course, under standard conditions on consumer preferences such an effect would be expected, since any relative price change (including prices of energy services) will result in a substitution effect that will increase consumption of the good or service whose relative price has decreased, while in many if not most cases the income effect will enhance this effect. If, for example, furnaces or boilers used to produce space heating become more efficient, thereby lowering the relative price of heating, demand for heating will likely increase. Depending on the extent of the efficiency increase, even with this increased consumption of heating there may still be a decrease in natural gas consumption compared to the level prior to the efficiency increase, or in an extreme case, natural gas consumption may actually increase, an effect known as 'backfire'. In any event, the

key issue is that natural gas consumption does not decrease as much as 'expected', where what is 'expected' in such cases is based on engineering calculations that assume that there will be no change in consumption of the energy services that are provided, in this case heating.[2]

There has been considerable interest in recent years in trying to determine the magnitude of these rebound effects, particularly – but not only – for transportation, since from a policy context their existence suggests that programs designed to reduce energy use by requiring – and often legislating – greater efficiency are likely to have their primary objective thwarted to some extent. However, probably because energy efficiency is not directly observed or measurable in many – if not most – cases, there is no standard approach for obtaining these estimates. Indeed, Sorrell and Dimitropoulos (2008) provide eight different definitions of direct rebound effects, most of which have been used to provide empirical estimates, and discuss the limitations and assumptions that each embodies. Our focus here, in part because of its widespread and increasing use – no doubt due in large part to the ease with which it can be calculated – is the own-price elasticity of demand for the energy source that produces the relevant energy service. With this approach (Definition 4 of Sorrell and Dimitropoulos, 2008), virtually any energy demand equation that is estimated provides empirical estimates of these price elasticities, and hence of direct rebound effects.[3]

The justification for this approach is provided later, but it is useful at the outset to note that for a number of reasons, Sorrell and Dimitropoulos (2008) conclude that "studies that use

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[2] In fact, under standard conditions on consumer preferences, the only way to get the engineering result is when the energy service is an inferior good, so that the resulting negative income effect resulting from a price decrease exactly outweighs the positive substitution effect.

[3] See, for example, the studies identified Sorrell and Dimitropoulos (2008, Section 4), and Greening et al., 2000.

the own-price elasticity of demand as a proxy for the direct rebound effect appear to be particularly flawed" (p. 646). In particular, they argue that these studies are likely to overestimate the magnitude of rebound effects due to such reasons as asymmetric responses to price changes, possible positive correlation between energy efficiency and capital costs, price-induced efficiency improvements, energy efficiency being endogenous, and perhaps a negative correlation between energy efficiency and time efficiency.

While, for any of these reasons, use of the own-price elasticity of energy demand for the relevant energy service as a measure of the direct rebound effect may be misguided, as we demonstrate in this paper, the primary problem is that the model on which such measures are usually based is inappropriate for this purpose. In much of the literature on the rebound effect, very little attention is paid to the consumer decision model from which rebound effects are likely to arise. In fact, even the alternative definitions of direct rebound effects provided by Sorrell and Dimitropoulos (2008) are based predominately on definitional relationships between energy services, energy efficiency and energy inputs, and between the cost per unit of energy services and the price of energy, and various derivatives of these relationships. This is somewhat analogous to basing demand analysis on the budget constraint and various derivatives of it, but ignoring the utility, cost, or other function that is being optimized.[4] As might be expected, what is required to examine rebound effects without using various contrivances that are unlikely to be correct is a model that takes account of consumer demand being for energy services rather than for energy sources, and therefore in which energy efficiency appears explicitly. Such a model was introduced by Hunt and Ryan (2014). Within such a model, the usefulness and limitations of

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[4] That said, the appendix to Sorrell and Dimitropoulos (2005) does introduce some fundamentals in terms of illustrative indifference curve analysis.

using own-price elasticities of various energy demands to measure direct rebound effects, and even of using various cross-price elasticities to measure (some) indirect rebound effects, becomes apparent. In particular, our findings indicate that to use such price elasticities as measures of rebound effects, it is necessary at a minimum to estimate them using a different model specification. More generally, however, it is shown that it is not possible to identify rebound effects from price elasticities except in either very aggregated energy demand relationships where the rebound estimates are of limited use anyway, or under some very strong assumptions that are unlikely to hold in practice.

The remainder of this paper is organized as follows. Section 2 discusses the background to the rebound effect in more detail, including the justification for the use of price elasticities as measures of rebound effects, and limitations to such use. Section 3 summarizes the model of utility maximization conditional on energy services, introduced by Hunt and Ryan (2014), while the relationship between rebound effects and price elasticities, based on the energy services demand framework, is explored in Section 4. Section 5 addresses issues of empirical implementation, including a discussion of approaches to modelling the unobserved energy efficiency that appears in the model in Section 3. Section 6 provides an empirical illustration in which the demands for energy sources that are derived from our energy services model are jointly estimated in a systems framework using the linearized version of the Almost Ideal Demand System (AIDS) specification with time series data on energy expenditures and prices for the UK. Furthermore, the implications of these results for rebound effects are investigated and explored. A brief summary and conclusion is presented in Section 7.

2. Rebound Effects

Typically, three types of rebound effects are delineated.[5] The first is the direct rebound effect, where improved efficiency for a particular energy service results in increased consumption of that service. The second is the indirect rebound effect, which refers to increased demand for other goods and services that also require energy for their provision. The third category is economy-wide rebound effects, which occur when a decrease in the real price of energy services leads to adjustments in the economy that result in expansion of the energy-intensive sectors relative to those that are less energy-intensive.[6]

As shown by Sorrell and Dimitropoulos (2008), use of the own-price elasticity of demand as a measure of the direct rebound effect is based on the following two definitional relationships:

(1) $\varepsilon = S/E$ and

(2) $p_S = p_E/\varepsilon$

In (1), efficiency (ε) is defined as the units of the energy service produced (S) per unit of the energy source (E), in other words, the number of units of energy services provided by one unit of the energy source.[7] Thus, if more energy services can be produced using the same amount of

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[5] See, for example, Sorrell and Dimitropoulos (2008).

[6] As Sorrell (2009) notes, with indirect rebound effects distinctions are also sometimes made between embodied energy (such as the energy required to produce the energy efficiency improvement) and secondary effects that result from the energy efficiency improvement.

[7] Note that this definition of efficiency abstracts from how efficient any particular equipment that is used to convert energy to energy services may be relative to other equipment that might be used to perform the same task. This definition is usually associated with appliances associated with households and firms as distinct from the arguments around economy-wide aggregate energy efficiency and energy intensity (see for example, Filippini and Hunt, 2011).

energy, then efficiency has increased. In (2), the cost per unit of energy services, referred to as the per-unit price of such services, is defined as the cost of energy required to produce one unit of those services, that is, the price per unit of the energy source divided by energy efficiency. Jointly, (1) and (2) imply:

(3) $p_S S = p_E E,$

that is, that expenditure on energy services is equal to expenditure on the energy source(s) used to provide these services.

According to Sorrell (2009), the generally accepted measure of the direct rebound effect is given by the elasticity of the demand for energy services with respect to energy efficiency, $\eta_\varepsilon(S)$, defined as:

(4) $\eta_\varepsilon(S) = \left(\partial S / \partial \varepsilon\right)\left(\varepsilon / S\right),$

Differentiating (1) with respect to energy efficiency, ε, yields:

(5) $\eta_\varepsilon(S) = \left(\partial E / \partial \varepsilon\right)\left(\varepsilon / E\right) + 1 = \eta_\varepsilon(E) + 1,$

where $\eta_\varepsilon(E)$ is the elasticity of the demand for energy with respect to energy efficiency, or the efficiency elasticity of the demand for energy. Thus, if $\eta_\varepsilon(S) = 0$ $(\eta_\varepsilon(E) = -1)$ there is no rebound effect; if $0 < \eta_\varepsilon(S) < 1$ $(-1 < \eta_\varepsilon(E) < 0)$ then there will be a rebound effect; and if $\eta_\varepsilon(S) \geq 1$ $(\eta_\varepsilon(E) \geq 0)$ then there will be backfire.

To relate the rebound effect, $\eta_\varepsilon(S)$, to the own-price elasticity of demand, use is made of (2) and the assumptions that $S = S(p_S)$ and that the price of energy, p_E, does not depend on the efficiency with which energy services are delivered, ε. Specifically,[8]

(6) $\partial E / \partial \varepsilon = \frac{\partial}{\partial \varepsilon}\left(\frac{S}{\varepsilon}\right) = \left[\frac{1}{\varepsilon}\left(\frac{\partial S}{\partial \varepsilon}\right) - \frac{S}{\varepsilon^2}\right] = \left[\frac{1}{\varepsilon}\left(\frac{\partial S}{\partial p_S}\frac{\partial p_S}{\partial \varepsilon}\right) - \frac{E}{\varepsilon}\right] = \frac{1}{\varepsilon}\left[\frac{\partial S}{\partial p_S}\left(\frac{-p_E}{\varepsilon^2}\right) - E\right],$

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[8] See Sorrell and Dimitropoulos (2008).

so that

$$(7) \quad \eta_\varepsilon(E) = \left(\frac{\partial E}{\partial \varepsilon}\right)\left(\frac{\varepsilon}{E}\right) = \frac{1}{E}\left[\frac{\partial S}{\partial p_S}\left(\frac{-p_S}{\varepsilon}\right) - E\right] = \left[\frac{\partial S}{\partial p_S}\left(\frac{-p_S}{S}\right) - 1\right]$$

$$= -\eta_{p_S}(S) - 1,$$

where $\eta_{p_S}(S)$ is the elasticity of the demand for energy services with respect to the price of those services. Since, from (1) and (2), $S = E\varepsilon$, and $p_S = \frac{p_E}{\varepsilon}$, then if ε is treated as constant, $\eta_{p_S}(S)$ simplifies to: $\left[\frac{\partial E}{\partial p_E}\left(\frac{p_E}{E}\right)\right] = \eta_{p_E}(E)$, that is, the own-price elasticity of demand for energy. Hence, from (4), (5) and (7),

$$(8) \quad \eta_\varepsilon(S) = \left(\frac{\partial S}{\partial \varepsilon}\right)\left(\frac{\varepsilon}{S}\right) = \eta_\varepsilon(E) + 1 = -\eta_{p_S}(S) = -\eta_{p_E}(E).$$

Thus, under these (restrictive) conditions, the rebound effect is just equal to the negative of the own price elasticity of energy demand. For example, if the own-price elasticity was –0.5, this would mean that a 1% decrease in price would result in a 0.5% increase in energy demand. Hence (from (8)), a 1% increase in energy efficiency would result in a 0.5% increase in demand for energy services, in which case there would be a 50% "take-back" of the energy savings resulting from energy efficiency improvements due to the direct rebound effect. Although this equivalence in (8) has been used by a number of authors, as noted previously, there are a number of caveats to its use. One of the most problematic would seem to be the requirement in the last step (following (7)), of treating energy efficiency, ε, as constant, which seems at odds with determining the efficiency elasticity of the demand for energy services, which is defined as the percentage change in demand for energy services resulting from a percentage change in energy

efficiency.[9] Second, in view of its derivation, the definition is only appropriate when energy demand refers to a single energy service. Third, while – subject to these caveats – the negative of the own-price elasticity may provide a measure of the direct rebound effect, the derivation provides no information about how corresponding measures of indirect rebound effects might be determined.

A clue to the type of models that might help with some of these issues is provided by (8) and consideration of the forms of functional expressions that would give rise to equality between $\eta_\varepsilon(S)$ and $-\eta_{p_E}(E)$. For example, if $S = f(p_S, \dots) = f(\frac{p_E}{\varepsilon}, \dots)$, then $\eta_\varepsilon(S) = -f' . \frac{p_E}{\varepsilon} . \frac{1}{S}$, and if, in view of (1), $E = \frac{1}{\varepsilon} f(\frac{p_E}{\varepsilon}, \dots)$, then $-\eta_{p_E}(E) = -\frac{1}{\varepsilon} . f' . \frac{p_E}{\varepsilon} . \frac{1}{E} = -f' . \frac{p_E}{\varepsilon} . \frac{1}{S} = \eta_\varepsilon(S)$, where f' is the derivative of f with respect to $\left(\frac{p_E}{\varepsilon}\right)$. So, this suggests that in order to use the negative of the price elasticity as a measure of the rebound effect, energy service demand and energy demand would both need to be functions of the price of the energy source (p_E) relative to energy efficiency, (ε). This is the type of specification derived by Hunt and Ryan (2014) in their development of empirically estimable models of energy service demand, suggesting that such models might be a useful way to determine more generally the relationship between price elasticities and various rebound effects.

The energy services demand model that is specified in the following section of this paper helps clarify the relationship between price elasticities and rebound effects. By focusing on energy services, determination of the direct rebound effect in the model is straightforward, and the specification indicates how some indirect rebound effects can be analogously obtained.

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[9] Of course, it could be argued that energy efficiency is fixed at the time energy consumption decisions are made, but even in this case in order to examine the rebound effect there must be the possibility that changes in energy efficiency cause changes in consumption of energy services.

Conversion of energy services demands to energy source demands reveals the generally complex nature of the relationship between rebound effects and price elasticities. It also helps address arguments that have been made about why estimates of rebound effects that are based on own-price elasticity estimates are likely to overstate the magnitudes of the rebound effects. For later use we briefly summarize two of these key arguments here.

Asymmetric price responses to energy price changes

First, there is evidence from many empirical studies (see, for example, Gately and Huntington, 2002) that price elasticities are higher when energy prices are rising than when they are falling. Essentially, the main rationalizations for this finding are: (i) when energy prices are rising there are technical improvements made in energy efficiency that are not reversed when energy prices subsequently fall; (ii) investments in measures such as insulation that are made when prices are rising are unlikely to be reversed when prices subsequently fall; (iii) higher energy efficiency requirements that may be imposed through regulation when energy prices rise are unlikely to be repealed if these prices subsequently fall. In terms of the implications of this for using price elasticities as estimates of rebound effects, the argument that has been made is that increases in efficiency are 'equivalent' to decreasing energy prices, so that the relevant price elasticities for estimating rebound effects would be those obtained when energy prices are falling (Sorrell and Dimitropulos, 2008).

However, even if demand responses to energy price changes are asymmetric, the argument that the elasticity estimates that should be used to measure rebound effects are those from a period when energy prices are falling seems counter-intuitive. If energy prices are rising, households have incentives to improve energy efficiency and reduce their energy use, whereas

no such incentives exist when energy prices are falling. In any event, the key point is that the arguments for the existence of asymmetry pertain to increasing energy efficiency when energy prices are rising compared to when they are falling. This suggests that if a model of energy demand could capture these efficiency effects separately from the price effects, then the price elasticity may be a more appropriate measure of the rebound effect, regardless of the direction in which prices have been moving. This is one of the features of the model developed in Section 3.

Price-induced energy efficiency

A second explanation of why estimates of rebound effects that are based on own-price elasticity estimates are likely to overstate the magnitudes of these effects pertains to energy efficiency being endogenous and/or induced by energy prices. As summarized by Sorrell and Dimitropoulos (2008), there appear to be two main components to this argument. First, energy efficiency may be expected to be a function of current and historical energy prices. Ignoring this effect is likely to result in price elasticities that overstate the magnitude of the rebound effect. Second, consumers with a higher demand for energy services may be more likely to choose appliances with higher energy efficiency in order to minimize their energy costs.

These, as well as other, aspects of efficiency are also taken into account in the model developed in the following section. By explicitly including energy efficiency in the model of consumer behaviour and modelling its determinants, the roles of past values of energy prices are recognized. It is not clear whether current energy prices should play a role in determination of current efficiency since, at least in the household context, there is typically a lag between choice and installation of the equipment that embodies a particular level of energy efficiency, and the energy services that are demanded from this equipment. Regardless, a variety of different

specifications of the determinants of energy efficiency is consistent with the energy services model presented in the next section. In addition, the general specification of the determinants of efficiency that is used in the model would allow such efficiency to depend on past usage levels, and thereby capture the potential effect of increased energy usage inducing increased efficiency. More generally, the specification would also allow the effects of exogenous changes – such as those imposed through legislation or minimum efficiency performance standards – to be taken into account.[10]

3. A Model of the Demand for Energy Services

The preceding analysis and discussion indicates that although the negative of estimated own (and cross) price elasticities of energy demand have frequently been used as estimates of the rebound effect, the basis for this is very weak. In particular, it is based upon an *ad hoc* approach that does not explicitly consider the demand for energy services, and hence the efficiency variables that drive the rebound effects, and relies on overly restrictive assumptions. To address these drawbacks, we utilize a consumer utility maximization model that is defined over energy services, as developed in Hunt and Ryan (2014). In such a model it is assumed that utility is derived from energy services rather than energy itself, as well as other non-energy goods. The conceptual approach in which utility is derived from energy services rather than energy (or

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[10] By utilizing different specifications of the determinants of the energy efficiency terms, the model presented in the next section can readily deal with other issues that in a standard model have been thought to result in price elasticities yielding over-estimates of rebound effects. These include the possible positive correlation between energy efficiency and the capital costs of the equipment that embodies such efficiency, and the idea that as time becomes more valuable, consumers will substitute energy services for time.

energy sources) itself appears to have a long history in energy economics, as many authors refer to derived demands for energy (see for example, Howarth, 1997; Hunt et al., 2003; Haas et al., 2008; Fouquet and Pearson, 2012). However, as far as is known, apart from studies by Walker and Wirl (1993), Haas and Schipper (1998) and Goerlich and Wirl (2012), there have been no previous attempts to model this explicitly, and none have used such a framework to analyze rebound effects.

For generality we consider the case where there are n energy sources, indexed $i, j = 1, \ldots, n$, each of which may produce more than one of n^* energy services, indexed $m, q = 1, \ldots, n^*$. Following Hunt and Ryan (2014), for notational convenience energy services are defined separately according to the energy source that is used to produce them. Thus, rather than considering heating as an energy service that can be produced using electricity or natural gas, or some other energy source or combination of energy sources, heating produced by electricity is considered to be a different energy service to heating produced by natural gas.[11]

As explained in Section 2, it is standard in this type of set up to define efficiency as the units of the service produced per unit of the energy source.[12] Thus, using x_i to denote the quantity of the ith energy source or good, and v_m to denote the quantity of the mth energy service, the efficiency of energy source i used to produce energy service m is given by ε_m, where:

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[11] Although in both cases the end result is heating, it is often the case that efficiency improvements are for a particular energy service–energy source combination. Note that this way of modelling energy services does not preclude more than one energy source being used at the same time to produce these energy services, such as a boiler that produces heat using both electricity and natural gas.

[12] It is not necessary to be specific about the definition of a unit of each of these energy services, since the units of measurement of the efficiency variables will adjust to accommodate alternative choices of units of services.

(9) $\varepsilon_m = \frac{v_m}{x_{im}}$,

where x_{im} is the quantity of energy source i that is used to produce energy service m, so that:

(10) $x_i = \sum_{m\epsilon i} x_{im}$,

where $\sum_{m\epsilon i}(.)$ refers to the sum over all energy services (m) that are provided by energy source i. With this formulation, the (unobserved) cost per unit, or price, of an energy service (p_m^*), which given the above specification is provided uniquely by a single energy source, is a function of the corresponding efficiency measure. For example, if ε_m is twice as great, this means that it would take half as much of energy source i to produce the same amount of the energy service m, so that the (variable) cost (that is, the cost of the energy) would be halved.[13] Thus, the price of energy service (p_m^*) that is produced using energy source i is related directly to the price of that energy source (p_i):

(11) $p_m^* = \frac{p_i}{\varepsilon_m}$, $i = 1, \dots, n$; $m = 1, \dots, n^*$; $m\epsilon i$.

Total expenditure on energy source i is the sum of expenditure on all the services produced by that energy source, so that:

(12) $p_i x_i = \sum_{m\epsilon i} p_i x_{im} = \sum_{m\epsilon i} p_m^* v_m$, $i = 1, \dots, n$; $m = 1, \dots, n^*$; $m\epsilon i$.

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[13] Of course there would also be a capital cost associated with obtaining increased heating efficiency (e.g., a new furnace or boiler, or increased insulation), but it is assumed that this capital decision has been made in advance of the utilization decision. The analysis could be generalized to include capital equipment purchasing decisions, but we do not consider this extension here.

Hence, the budget constraint for the case where the household derives utility from consumption of energy services (and other goods) becomes $\sum_{i=1}^{n} p_i x_i = \sum_{m=1}^{n^*} p_m^* v_m = Y$, where Y is the household budget.[14] The consumer's problem is therefore to determine v_m , $m = 1, ..., n^*$, to:

(13) $Max\{U(v_1, v_2, ..., v_{n^*})\}$ subject to $\sum_{m=1}^{n^*} p_m^* v_m = Y$,

which yields derived demand equations for the services/goods given by:

(14) $v_m = v_m(p_1^*, p_2^*, ... , p_{n^*}^*, Y) = v_m(\frac{p_1}{\varepsilon_1}, ..., \frac{p_n}{\varepsilon_{n^*}}, Y)$.

Multiplying both sides of (14) by p_m^* and dividing by Y yields expenditure shares for each energy service, $s_m^* = \frac{p_m^* v_m}{Y} \left(= \frac{p_i x_{im}}{Y} \right)$, which have the form:

(15) $s_m^* = s_m^*(p_1^*, p_2^*, ... , p_{n^*}^*, Y) = s_m^*(\frac{p_1}{\varepsilon_1}, ..., \frac{p_n}{\varepsilon_{n^*}}, Y)$, $m = 1, ..., n^*$.

If data were available on the expenditure shares, $s_m^* = \frac{p_m^* v_m}{Y}$, as well as the prices, p_m^*, for each energy service, m, (15) could be estimated directly, after specifying a functional form for $s_m^*(.)$ that is consistent with the consumer's optimization problem, such as the commonly-estimated equations of the Almost Ideal Demand System (AIDS), introduced by Deaton and Muellbauer (1980). Of course, s_m^* will typically be unobserved unless each energy source i is used to produce only a single energy service. However, summing over all energy services produced by energy source i yields the expenditure share for that energy source, $s_i = \frac{p_i x_i}{Y} = \sum_{m \epsilon i} s_m^*$, which is observed. Applying this process to (15) yields the following system of expenditure share equations for the various energy sources:

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[14] If attention is limited to spending on energy services (sources), Y would be total spending on these energy services (sources), but consumption of other (non-energy based) goods and services could also be included in the consumer's optimization problem in (13) (yielding expenditure equations for these goods and services as well), in which case Y would be total expenditure on all goods and services.

(16) $\quad s_i = \frac{p_i x_i}{Y} = \sum_{m \epsilon i} s_m^* = s_i(\frac{p_1}{\varepsilon_1}, ..., \frac{p_n}{\varepsilon_{n^*}}, Y), \quad , \quad i = 1, ..., n.$

Thus, the fundamental difference from the typical system of energy specification that is estimated is that in (16) all the energy source price terms are expressed in efficiency units, that is, the price of each energy source, which appears (potentially) more than once in each expenditure share function, in each case is divided by a particular efficiency term. For example, the price of natural gas may appear divided by natural gas heating efficiency, while the price of electricity may appear multiple times, alternatively divided by electricity heating efficiency, electricity lighting efficiency, etc.[15] Obviously, if all these efficiency terms equal one, that is, there is a one-to-one relationship between consumption of the energy source and consumption of energy services – this will simplify to the usual energy demand specification where efficiency variables do not appear. Note, however, that estimation of equations such as (16) require data over time or over individuals, with an additional subscript added to all variables. Thus, to simplify to the usual specification it would be necessary that all the efficiency terms equalled one for all observations, which seems unlikely.[16] We address the issue of the requirement for data on the energy efficiency terms for each energy service in a later section. First, we consider the implications of this specification for the identification of rebound effects using price elasticities.

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[15] The model here can be simplified for the case where there is a single (aggregate) energy source and a single energy service. In such a model, which is considered in detail in Hunt and Ryan (2014), the key feature again is that energy demand depends on the energy price relative to energy efficiency.

[16] Since there is no set definition of a unit of heating or of lighting, the scaling of the ε_j is arbitrary, so it may be preferable to view the model as reducing to the usual specification if the ε_j are the same for each service, for all observations, which seems equally unlikely.

4. Price Elasticities and Rebound Effects

As we show below, when multiple energy sources can provide the same service (such as electricity or natural gas providing space or water heating), and multiple services can be provided by the same energy source (such as natural gas providing space heating and cooking), the relationship between the rebound effects and the price elasticities is necessarily complex. In particular, it will not generally be possible to identify specific direct rebound effects directly from the own-price elasticities. Intuitively this result is obvious: if, for example, there are three energy sources but five energy services, there would be three own-price elasticities but five direct rebound effects, in which case it would clearly not be possible to uniquely identify each direct rebound effect from each estimated own-price elasticity. Rather, in such situations, as we show below, each price elasticity will involve a weighted sum of certain rebound effects. However, the question remains as to how rebound effects differ from price elasticities, and under what, if any, circumstances estimates of rebound effects can be recovered from price elasticities.

Given that the expenditure share of the i$^{\text{th}}$ energy source is defined as $s_i = p_i x_i / Y$, so that $x_i = Y s_i / p_i$, it follows that the own- and cross-price elasticities can be written as:

$$(17) \quad \eta_{ij} = \eta_{p_j}(x_i) = \left(\frac{\partial x_i}{\partial p_j}\right)\left(\frac{p_j}{x_i}\right) = -\delta_{ij} + \left(\frac{1}{s_i}\right)\left(\frac{\partial s_i}{\partial ln p_j}\right), i, j = 1, \dots, n,$$

where $\delta_{ij} = 1$ if $i = j$, and $= 0$ otherwise.

From (4), the direct $(m = q)$ and indirect $(m \neq q)$ rebound effects for the energy service demands in (14) are given by:

$$(18) \quad \eta^*_{mq} = \eta_{\varepsilon_q}(v_m) = \left(\frac{\partial v_m}{\partial \varepsilon_q}\right)\left(\frac{\varepsilon_q}{v_m}\right), \qquad m, q = 1, \dots, n^*.$$

Since, from (11), ε_q only appears in the denominator of p_q^*, where $\frac{\partial ln p_q^*}{\partial ln \varepsilon_q} = -1$, then using the chain rule the rebound effects can be written as:

(19) $\quad \eta^*_{mq} = \left(\frac{\partial v_m}{\partial p^*_q}\right)\left(\frac{\partial p^*_q}{\partial lnp^*_q}\right)\left(\frac{\partial lnp^*_q}{\partial ln\varepsilon_q}\right)\left(\frac{\partial ln\varepsilon_q}{\partial \varepsilon_q}\right)\left(\frac{\varepsilon_q}{v_m}\right) = -\left(\frac{\partial v_m}{\partial p^*_q}\right)\left(\frac{p^*_q}{v_m}\right), \quad m,q = 1, ..., n^*.$

To simplify we note that s^*_m, the expenditure share of the mth energy service is defined as $\frac{p^*_m v_m}{Y}$,

so that $v_m = \frac{Y s^*_m}{p^*_m}$, and (19) can be written as:

(20) $\quad \eta^*_{mq} = -\left(\frac{\partial v_m}{\partial p^*_q}\right)\left(\frac{p^*_q}{v_m}\right) = \delta_{mq} - \left(\frac{1}{s^*_m}\right)\left(\frac{\partial s^*_m}{\partial lnp^*_q}\right), \quad m,q = 1, ..., n^*,$

where $\delta_{mq} = 1$ if $m = q$, and $= 0$ otherwise.

To determine the relationship between the price elasticities in (17) and the rebound

effects in (20), we note that in (17) $s_i = \sum_{m\epsilon i} s^*_m$, where $m\epsilon i$ refers to all energy services (m)

that are provided by energy source i, so that:

(21) $\quad \frac{\partial s_i}{\partial lnp_j} = \sum_{m\epsilon i} \frac{\partial s^*_m}{\partial lnp_j} = \sum_{m\epsilon i}\left[\sum_{q\epsilon j}\left(\frac{\partial s^*_m}{\partial lnp^*_q}\right)\left(\frac{\partial lnp^*_q}{\partial lnp_j}\right)\right], \quad i,j = 1, ..., n; \; m,q = 1, ..., n^*,$

where the jth energy source may be used to produce a number of energy services, so that its price,

p_j, may be related to several prices of energy services, via (11). Since, from (11), $\frac{\partial lnp^*_q}{\partial lnp_j} = 1$, for

$q\epsilon j$, then substituting (21) in (17) yields:

(22) $\quad \eta_{ij} = \eta_{p_j}(x_i) = -\delta_{ij} + \left(\frac{1}{s_i}\right)\sum_{m\epsilon i}\left[\sum_{q\epsilon j}\left(\frac{\partial s^*_m}{\partial lnp^*_q}\right)\right], \quad i,j = 1, ..., n; \; m,q = 1, ..., n^*.$

Substituting for $\frac{\partial s^*_m}{\partial lnp^*_q}$ from (20) now yields:

(23) $\quad \eta_{ij} = \eta_{p_j}(x_i) = -\delta_{ij} - \sum_{m\epsilon i}\left(\frac{s^*_m}{s_i}\right)\left[\sum_{q\epsilon j}(\eta^*_{mq} - \delta_{mq})\right], i,j = 1, ..., n; \; m,q = 1, ..., n^*.$

To simplify, we note that if $i \neq j$, then $\delta_{ij} = 0$, and since $m\epsilon i$ and $q\epsilon j$, then $m \neq q$, so that

$\delta_{mq} = 0$. Conversely, if $i = j$, then $\delta_{ij} = 1$, and $\sum_{q\epsilon j}\delta_{mq} = 1$, so that since $\sum_{m\epsilon i}\left(\frac{s^*_m}{s_i}\right) = 1$, it

follows that:

(24) $\quad \eta_{ij} = \eta_{p_j}(x_i) = -\sum_{m\epsilon i}\left(\frac{s^*_m}{s_i}\right)\left[\sum_{q\epsilon j}\eta^*_{mq}\right], i,j = 1, ..., n; \; m,q = 1, ..., n^*.$

This result indicates that the own- and cross-price elasticities (η_{ij}) are weighted linear combinations of various direct and indirect rebound effects(η^*_{mq}), where the weights are (the negative of) the ratios of the expenditure shares of an energy source used to provide a particular energy service to the total expenditure share (across all uses) of that energy source.[17] For example, suppose that one of the energy sources is natural gas (g), and that this produces natural gas heating (h_g) and cooking (c_g). Then if $s^*_{h_g}$ denotes the (usually unobserved) expenditure share of gas used for heating, and $s^*_{c_g}$ denotes the (usually unobserved) expenditure share of gas used for cooking, where $s^*_{h_g} + s^*_{c_g} = s_g$, with s_g being the (observed) expenditure share of gas used for all purposes, then from (24),

$$(25) \quad -\eta_{gg} = \frac{s^*_{h_g}}{s_g}\left(\eta_{\varepsilon_{h_g}}(h_g) + \eta_{\varepsilon_{c_g}}(h_g)\right) + \frac{s^*_{c_g}}{s_g}\left(\eta_{\varepsilon_{h_g}}(c_g) + \eta_{\varepsilon_{c_g}}(c_g)\right)$$

where $\eta_{\varepsilon_{h_g}}(h_g)$ = elasticity of h_g (heating with gas) with respect to efficiency of <u>heating</u> with gas, $\eta_{\varepsilon_{c_g}}(h_g)$ = elasticity of h_g with respect to efficiency of <u>cooking</u> with gas, $\eta_{\varepsilon_{h_g}}(c_g)$ = elasticity of c_g (cooking with gas) with respect to efficiency of <u>heating</u> with gas, and $\eta_{\varepsilon_{c_g}}(c_g)$ = elasticity of c_g with respect to efficiency of <u>cooking</u> with gas.

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[17] Note that (21), and hence the subsequent derivation of (24), only applies if prices of energy sources (p_j) only appear in the energy service expenditure share equations in (15) as the numerators of the p^*_q terms. An example where this would not apply, which we consider in Section 5, occurs if the Linearized version of the AIDS model is used for (15), where the predetermined Stone price index term included in this specification would typically be defined just using the observed prices of energy sources. In this case, the relationship between the η_{ij} and η^*_{mq} can still be determined, but it will differ from (24), as we show later.

It is possible (but not necessary) that the two indirect rebound effects in (25) are zero, $\eta_{\varepsilon_{c_g}}(h_g) = \eta_{\varepsilon_{h_g}}(c_g) = 0$, but even in this case it is still not possible to identify the two direct rebound elasticities, $\eta_{\varepsilon_{h_g}}(h_g)$ and $\eta_{\varepsilon_{c_g}}(c_g)$, from the own-price gas elasticity, η_{gg}. Of course, if gas only produced a single energy service, say natural gas heating, then $s_{h_g}^* = s_g$, and $s_{c_g}^* = 0$, so that $-\eta_{gg} = \eta_{\varepsilon_{h_g}}(h_g)$, and from (25) the natural gas heating rebound effect would be the negative of the natural gas own-price elasticity. In fact, this result is more general. If each energy source produced only a single energy service, and each energy service was produced by only a single energy source then it would be possible to identify all rebound effects, direct and indirect, from the own- and cross-price elasticities obtained using the type of model we have specified in (16). In such a case, since each energy source would be identified with a unique energy service, then $n = n^*$, $m = i$, and $q = j$, $(i, j = 1, \dots, n)$ so that (24) would reduce to:

(26) $-\eta_{ij} = -\eta_{p_j}(x_i) = \eta_{ij}^* = \eta_{\varepsilon_j}(v_i)$, $i, j = 1, \dots, n$.

To consider a concrete example, which we use later in our empirical illustration, suppose that there are three energy sources – electricity (with price p_e), natural gas (with price p_g), and oil products – or other energy (with price p_o).[18] In this simplified example we suppose that "lighting and power for appliances" (la), with price p_{la}, is a single energy service (which means that everything within this category has the same energy efficiency) that is produced only by electricity (e), and this single energy service is all that electricity produces. In addition, we suppose that "cooking and natural gas heating" (cgh), with price p_{cgh}, (again, everything within

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[18] Here we have restricted the analysis to energy expenditures, but as noted earlier, the energy services model in Section 3 is more general, and could include consumption of other goods, in which case the consumption of those goods and the services provided by them would be equated, with the corresponding efficiency term defined throughout as equalling 1.0.

this category would also have the same energy efficiency), is also a single energy service that is produced only by natural gas (g), and that natural gas produces no other energy services. Finally, we suppose that only oil products – or other energy (o), produces "other heating" (oh) with price p_{oh}, and nothing else. As a result, expenditure on "cooking and natural gas heating", $\mathrm{p}_{cgh}cgh$, equals expenditure on natural gas, $\mathrm{p}_g g$, expenditure on "lighting and power for appliances", $\mathrm{p}_{la}la$, equals expenditure on electricity, $\mathrm{p}_e e$, and expenditure on "other heating", $\mathrm{p}_{oh}oh$, equals expenditure on oil products, $\mathrm{p}_o o$. In this case, with three energy sources there are nine own- and cross-price elasticities, and nine direct and indirect rebound effects, and these can be determined as shown in Table 1:

Table 1: Relationship between Price Elasticities and Rebound Effects in the Simplified Example

Energy Source	Price of Electricity	Price of Natural gas	Price of Oil products
Electricity	$-\eta_{p_e}(e) = \eta_{\varepsilon_{la}}(la)$	$-\eta_{p_g}(e) = \eta_{\varepsilon_{cgh}}(la)$	$-\eta_{p_o}(e) = \eta_{\varepsilon_{oh}}(la)$
Natural gas	$-\eta_{p_e}(g) = \eta_{\varepsilon_{la}}(cgh)$	$-\eta_{p_g}(g) = \eta_{\varepsilon_{cgh}}(cgh)$	$-\eta_{p_o}(g) = \eta_{\varepsilon_{oh}}(cgh)$
Oil Products	$-\eta_{p_e}(o) = \eta_{\varepsilon_{la}}(oh)$	$-\eta_{p_g}(o) = \eta_{\varepsilon_{cgh}}(oh)$	$-\eta_{p_o}(o) = \eta_{\varepsilon_{oh}}(oh)$

Hence, in this simplified example, $\eta_{\varepsilon_{cgh}}(cgh) = -\eta_{p_g}(g)$, so that the direct rebound effect for increased efficiency in cooking and natural gas heating, that is, the elasticity of demand for cooking and natural gas heating with respect to a change in the energy efficiency of cooking and natural gas heating, is equal to the negative of the own-price elasticity of demand for natural gas. Similarly, the direct rebound effect for increased efficiency in lighting and appliances is equal to the negative of the own-price elasticity of demand for electricity and the direct rebound effect for other heating would be equal to the negative of the own-price elasticity of demand for oil products.

In addition to direct rebound effects, however, in this simplified example we also obtain a variety of indirect rebound effects. Here these refer to changes in demand for particular energy services that result from an increase in the efficiency of a different energy service. To continue with the above example, this could be the change in the demand for lighting and appliances because of increased cooking and natural gas heating efficiency. This could arise, for example, because the increased cooking and natural gas heating efficiency might increase the heated area in a house, making more space usable and thus requiring increased lighting. In this case, the indirect rebound effect, $\eta_{\varepsilon_{cgh}}(la)$, would be given by the cross-price elasticity between electricity and natural gas, that is, $-\eta_{p_g}(e)$. Other indirect rebound effects are found similarly, according to the relationships in Table 1.

One interesting aspect of the energy services model formulation and the relationship between price elasticities and rebound effects is that in the simplified example where each energy source produces a single energy service, it is also possible with a particular choice of variables to obtain certain measures of economy-wide rebound effects. In particular, suppose that the previous example with three energy sources is expanded to include another category, referred to as all other goods (oth). In the energy services model, the efficiency of providing these other goods, which are not energy services, would be defined as unity, so that the "consumption services" provided by other goods would just be equated to the actual amount of other goods that are purchased. Now, in addition to the nine elasticities and corresponding direct and indirect rebound effects identified in Table 1, there would be an additional seven terms. Of these, three would be price elasticities of the various energy sources with respect to the price of other goods, and one would be the own-price elasticity of the demand for other goods. However, the other three would be price elasticities of the demand for other goods with respect to each of

the three energy sources, and the negative of each of these price elasticities would be the rebound effect of a change in the efficiency of providing that energy service on the demand for the other goods.

While this analysis, especially concerning direct rebound effects, appears to provide a rationalization for the common practice of using (the negative of) own-price elasticities as measures of these direct rebound effects, there are two very important caveats. First, it is necessary that in the models that are estimated, the efficiency of providing each energy service must be explicitly included; in our formulation, the prices of each energy source appear relative to the efficiency of the energy service that they provide, but other specifications could be used. In previous empirical studies that have used the price elasticity in determining the rebound effect, this is typically not the case.[19] Second, the results derived here only hold where each energy source provides a single energy service, and each energy service is provided by only a single energy source. However, this one to one correspondence between energy services and energy sources is unlikely to hold in practice.[20] For example, heating can be produced using natural gas, or electricity, or combinations of both, or possibly from other energy sources.

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[19] See, for example, Berkhout et al (2000), who seem to suggest that any estimates of price elasticities can be used. Sorrell and Dimitropoulos (2008) also appear to support this view, if the price elasticity-based definition of the rebound effect is used.

[20] Of course, it will hold in a model that focuses on total demand for energy, since in that case there is only one energy source (total energy) and (implicitly) one aggregate energy service, so that (24) will simplify to the usual rebound-price elasticity relationship in (8). However, for this relationship to be appropriate, the demand equation for total energy must include the ratio of the energy price to energy efficiency. Further, there will be an implicit assumption in such a model that any efficiency increase applies to *all* the various energy services that total energy delivers, which seems extremely unrealistic, and would make use of rebound estimates from such a model seemingly unreliable.

Natural gas can produce space heating, cooking, and water heating (which may or may not be separate from space heating), and electricity can be used to produce lighting (as in the simple model), heating, refrigeration, power for small appliances, etc.

In the simplified example described above, the energy services categories have been contrived to conform to this requirement of a one to one correspondence between energy services and energy sources, and this could be maintained in empirical work, as we illustrate subsequently. However, such a specification imposes very strict requirements on efficiency improvements that are unlikely to hold in practice. In particular, within any one energy service, for rebound effects to be able to be equated to negative price elasticities, there must be a uniform energy efficiency within that energy service. In our simplified example we combined energy services from lighting and appliances (provided by electricity), so that this does not allow for changes in energy efficiency of lighting separate from changes in energy efficiency of appliances. Similarly, we combined energy services from natural gas heating and cooking (provided by natural gas), so that this does not allow for changes in energy efficiency of natural gas heating separate from changes in energy efficiency of natural gas cooking. As soon as this implausible assumption is relaxed, the relationship between price elasticities and rebound effects reverts to (24), and without further information, it is impossible to determine rebound effects from price elasticities. However, in this case the price elasticities will still provide information about combinations of rebound effects, and if these elasticity estimates can be supplemented with information about the shares of an energy source that are used for different purposes, it may be possible to provide some conditional information about specific rebound effects.

5. Empirical Implementation

Next we consider empirical implementation of (16) with multiple energy sources and energy services using typically available data that pertain just to consumption and prices of energy sources, with a view to showing empirically how rebound effects differ from the price elasticities calculated using typical energy demand models. Due to its frequent use in empirical analysis, the specification considered here is the Linearized version of the Almost Ideal Demand System (LAIDS) introduced by Deaton and Muellbauer (1980), although similar analysis could be undertaken using alternative functional formulations. Consistent with the model in Section 3, we allow different energy services to be provided by the same energy source, so these face the same price for that energy source (but possibly have different efficiencies), where unobserved expenditure shares for different energy services provided by a specific energy source sum to the observed expenditure share for that energy source. In this framework, specification of a LAIDS model for the expenditure shares for each energy service, $s_m^* = \frac{p_m^* v_m}{Y} = \frac{p_i x_{im}}{Y}$, yields the following system of expenditure share equations, one for each energy source:

$$(27) \quad s_m^* = \alpha_m^* + \sum_q \gamma_{mq}^* ln(p_q^*) + \beta_m^*(ln\,Y - ln\,P^*), \quad m, q = 1, ..., n^*,$$

where $ln\,P^*$ is the predetermined Stone Price Index:

$$(28) \quad ln\,P^* = \sum_{i=1}^n \sum_{m \epsilon i} s_m^* ln\,p_i = \sum_{i=1}^n ln\,p_i \left(\sum_{m \epsilon i} s_m^* \right) = \sum_{i=1}^n s_i ln\,p_i = ln\,P,$$

where $s_i = \frac{p_i x_i}{Y} = \sum_{m \epsilon i} s_m^*$, so that $ln\,P^*$ simplifies to $ln\,P$, the usual Stone Price Index defined over energy sources.[21] As shown by Hunt and Ryan (2014), (27) can be re-written as:

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[21] Note that in (28), $ln\,P^*$ could be defined as: $ln\,P^* = \sum_{m=1}^{n^*} s_m^* ln\,p_m^* = ln\,P - \sum_{m=1}^{n^*} s_m^* ln\,\varepsilon_m$. This would preserve the relationship between rebound effects and price elasticities as in (24). Typically, since the reason for using the Stone Price Index, and LAIDS, is as an approximation that linearizes the share equations, and since introducing unobserved efficiency terms into (28) would mean that $ln\,P^*$ could not

(29) $\quad s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i (\ln Y - \ln P) - \sum_{m\epsilon i}(\sum_q \gamma^*_{mq} \ln \varepsilon_q), \quad i,j = 1, ..., n,$

where

(30) $\quad \alpha_i = \sum_{m\epsilon i} \alpha^*_m; \; \beta_i = \sum_{m\epsilon i} \beta^*_m; \; \gamma_{ij} = \sum_{m\epsilon i} \sum_{q\epsilon j} \gamma^*_{mq}; \; s_i = \sum_{m\epsilon i} s^*_m, \quad i,j = 1, ..., n,$

and $\sum_{m\epsilon i}(.)$ refers to the sum over all energy services (m) that are provided by energy source (i),

where $i, j = 1, ..., n$ index the n energy sources. Note that the usual adding-up conditions (since

the sum of the budget shares is 1), along with homogeneity and symmetry, which apply to the

energy services share equations in (27), have implications for the parameters in the energy

sources share equation (29). Specifically, adding-up in (27) requires:

(31) $\quad \sum_m \alpha^*_m = 1; \; \sum_m \gamma^*_{mq} = 0 \;$ for all $q; \; \sum_m \beta^*_m = 0; \quad m, q = 1, ..., n^*,$

while homogeneity and symmetry require:

(32) $\quad \sum_q \gamma^*_{mq} = 0 \;$ for all $m; \;$ and $\; \gamma^*_{mq} = \gamma^*_{qm}, \quad m, q = 1, ..., n^*.$

Incorporating these requirements on the parameters in (29), as defined in (30), implies that:

(33) $\quad \sum_i \alpha_i = 1; \; \sum_i \beta_i = 0; \; \sum_j \gamma_{ij} = \sum_i \gamma_{ij} = 0,$ where $\gamma_{ij} = \gamma_{ji}, \quad i,j = 1, ..., n,$

which are the usual conditions applied to a specification like (29). Of course, for (29) to satisfy

adding up it is also necessary that $\sum_i \sum_{m\epsilon i}(\sum_q \gamma^*_{mq} \ln \varepsilon_q) = 0,$ which can be shown to hold given

(31) and (32).

Thus, compared to a typical model in which consumers determine demands for energy

sources rather than for energy services, when energy services might be provided by multiple

energy sources, and any particular energy source may provide several different types of energy

services, the only change with the LAIDS model is that each expenditure share equation also

includes the last term in (29). This extra expression, involves the $\ln \varepsilon_q$, terms that (in natural

be determined prior to estimation and would therefore result in all current period energy service shares

appearing as explanatory variables in each share equation, it would not be defined in this way.

logarithms) reflect the energy efficiency of the various energy sources when used to produce particular energy services. When there are differences in efficiency and variation over time, as would be expected, then estimation of (29) without including these efficiency terms will result in omitted variables bias, and hence potentially misleading estimates of price elasticities and other measures that would be determined from the estimated parameters of such a system.

With the LAIDS model in (29), the price elasticities for the various energy sources can be calculated from the estimated parameters using the relationship (Buse, 1994):

$$(34) \quad \eta_{ij} = \eta_{p_j}(x_i) = \frac{\gamma_{ij}}{s_i} - \beta_i\left(\frac{s_j}{s_i}\right) - \delta_{ij}, \qquad i, j = 1, \dots, n.$$

where $\delta_{ij} = 1$ if $i = j$, and $= 0$ otherwise. Of course, as noted previously, given that demand is for energy services and not energy sources, these price elasticities will be biased unless energy efficiency is explicitly incorporated in the model (via the last term in (29)). Further, in the absence of these energy efficiency terms in (29), a relationship of the type derived earlier in (24), between price elasticities and rebound effects will not apply. With the LAIDS model, because of the definition of $\ln P$ in (28) where the efficiency terms do not appear, p_j and ε_j do not only appear in ratio form. Therefore, the relationship between the negative of the price elasticity and a combination of rebound effects for this model will differ from (24).[22] Specifically, the rebound effects for (27), using $\ln P^*$ as defined in (28), are calculated using (20) as:

$$(35) \quad \eta^*_{mq} = \eta_{\varepsilon_q}(v_m) = \frac{-\gamma^*_{mq}}{s^*_m} + \delta_{mq}, \qquad m, q = 1, \dots, n^*.$$

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[22] However, in the nonlinear AIDS model of Deaton and Muellbauer (1980), which does not utilize the Stone Price Index as an approximation, p_j and ε_j only appear in ratio form, so that (24) will hold for that model.

Thus, rather than (24), by using (30) and the analysis whereby (24) was obtained from (23), the relationship between η_{ij} and η_{mq}^* for the LAIDS model is given by:

$$(36) \quad \eta_{ij} + \beta_i \left({s_j}/{s_i} \right) = - \Sigma_{m\epsilon i} \left(\frac{s_m^*}{s_i} \right) [\Sigma_{q\epsilon j} \eta_{mq}^*], \ i,j = 1, \dots, n; \ m, q = 1 \dots, n^*.$$

The main feature that (36) illustrates is that provided energy efficiency terms are included in the energy service, and hence energy source, demand equation specification, even if energy source prices do not appear only in ratio form (that is, divided by particular energy services efficiency terms), it is still possible to find a relationship between price elasticities and rebound effects. In such circumstances, estimates of price elasticities, possibly used in conjunction with various other parameters and variables, can be used to obtain estimates of functions of rebound effects. If energy source prices always appear relative to particular energy services efficiency terms, then (24) will hold. If not, then some other relationship analogous to (36) will hold. The key point is that efficiency terms must be included in energy demand source equations to be able to relate price elasticities to rebound effects, and in general, there will not be a direct 1:1 relationship between a particular price elasticity and a particular rebound effect. Thus, the relationship in (8), appealed to in a large number of studies that have used price elasticities to calculate rebound effects, is misleading, and should not be used to obtain estimates of rebound effects.

Of course energy efficiency of various energy services are typically not observed, and this needs to be dealt with in any empirical implementation. For notational convenience we include a time of observation subscript on the efficiency variables, so the issue to be considered is how to specify $\ln \varepsilon_{qt}$ ($q = 1,...,n^*$, t=1, …, T) in the equations that are to be estimated. The

approach we use, following Hunt and Ryan (2014), is to approximate these terms using functions of observed variables:

$$(37) \quad ln \, \varepsilon_{qt} = f_q(Z_{1t}, Z_{2t}, \cdots, Z_{Rt}),$$

where the function may differ for different energy services. In the LAIDS model, the functional expression for (functions of) the rebound effects in (36) does not depend explicitly on the ε_q, but only on the parameters that appear directly in (34). In particular, these rebound effects will not depend explicitly on the particular variables included in these efficiency functions, and their corresponding estimated coefficients, although these aspects of the specifications will of course potentially affect the values of all the other estimated parameters in the model and the estimated shares, and therefore will affect the numerical values of the rebound effects.[23]

In our empirical example in the next section we estimate (29) first without any energy efficiency terms, which is therefore similar to the form of many energy demand equations that are estimated. In subsequent estimations we represent the term $[-\sum_{m\epsilon i}(\sum_q \gamma^*_{mq} ln \, \varepsilon_q)]$ in (29) by $\sum_r \varphi_{ir} Z_{rt}$, where the Z_{rt} are chosen to reflect some of the different issues that have been raised in the literature concerning the determinants of energy efficiency.[24] Of course there are many other choices that could be made, and others may be preferable to those we have chosen here for illustrative purposes.[25]

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[23] Of course, this may differ if an alternative functional specification is adopted. For example, in the elasticity formula for the nonlinear AIDS model the ε_j would appear explicitly, so that with that specification, the particular variables included in the efficiency functions, and their corresponding estimated coefficients, will affect the elasticities directly.

[24] Note that the theoretical model places no restrictions on the coefficients of these variables, other than the usual adding-up conditions, whereby $\sum_i \varphi_{ir} = 0$, for all *r=1, ..., R*.

[25] Some of these specifications are also considered in Hunt and Ryan (2014).

Energy Efficiency Dependent on Time

One possibility is that energy efficiency simply follows a time trend, reflecting the effects of technological progress. The use of time trends in systems of demand equations to represent technical progress has a long history (see Hunt et al. (2003) for a summary of the debate in the literature about the use of a linear versus nonlinear trend), and such an approach here would therefore be consistent with many previous studies. Allowing for possible nonlinearities, and since a stochastic trend is difficult to incorporate in a systems framework, the specification that we consider in this category, which we denote as Model A1, is:

(38) $\quad \sum_r \varphi_{ir} Z_{rt} = b_{i,t} t + b_{i,tt} t^2.$

Energy Efficiency Dependent on Past Energy Prices

A drawback of (38) is the assumption that improvements in energy efficiency are 'exogenous', and not driven by prices. If the prices of energy sources are increasing, there is presumably more motivation for technical change to improve efficiency of the delivery of various household energy services. In the case of natural gas heating, for example, such technical change could be embodied in the furnace or boiler that is used for the space heating, or it could be reflected in additional insulation that is installed in the house. Either of these factors would be expected to increase the amount of service (heat) produced per unit of the energy source, that is, the energy efficiency of natural gas heating. Of course since households can change the energy source(s) they use for some particular purposes – although not instantly – it is likely that energy efficiency will be a function of past prices of various energy sources, not necessarily just the one that is used to provide the energy services being considered. Alternative choices for the lagged energy source price terms could include relative prices or real prices, or growth rates. In our

illustrative empirical application (Model A2) we use three- and five-year growth rates of real

prices of three energy sources (electricity, natural gas, and other), which we supplement with

time trends as in (38), to allow for the possibility that not all efficiency is price-induced:

(39) $\quad \sum_r \varphi_{ir} Z_{rt} = \sum_{j=1}^{n} \theta_{i,j} g3_{j,t-1} + \sum_{j=1}^{n} \phi_{i,j} g5_{j,t-1} + b_{i,t}t + b_{i,tt}t^2,$

where $g3_{j,t-1}$ is the once-lagged three year growth rate for the real price of the jth energy source,

$g3_{j,t} = ln(\frac{p_{jt}}{p_{jt-3}}) - ln(\frac{pd_t}{pd_{t-3}})$ and pd_t is a general price deflator, and $g5_{j,t-1}$ is the once-lagged

five-year growth rate for the real price of the jth energy source, defined analogously.

Energy Efficiency Dependent on Past Energy Price Components

A number of authors have estimated energy demand models that allow for asymmetric

responses to price changes. A rationalization for the existence of such asymmetric responses is

that when energy prices rise, households make irreversible energy efficiency investments that

reduce their responsiveness to energy demand, even if energy prices were subsequently to

decrease. In the energy services model developed here, these energy efficiency investments are

explicitly recognized rather than being the implicit cause of an effect captured through allowing

different coefficients on different components of energy prices. Of course, the types of terms

used to allow for asymmetric effects, as considered by Gately and Huntington (2002) (among

others), could be used to summarize past behaviour of energy prices that affects energy

efficiency. Specifically, what matters in determining energy efficiency might not just be past

prices of energy sources, but the actual sequence of these prices. In other words, energy

efficiency might only be expected to increase if there is a sustained period of energy price

increases. Thus, rather than use lagged prices, the past maximum price (provided it is not too far

in the past), along with cumulative price recoveries and cumulative price decreases, might be an

appropriate way to summarize the effects of price on energy efficiency. To consider this possibility, in Model A3 we specify:

$$(40) \quad \sum_r \varphi_{ir} Z_{rt} = \sum_{j=1}^n \theta_{i,j} PMAX_{j,t-1} + \sum_{j=1}^n \phi_{i,j} PREC_{j,t-1} + \sum_{j=1}^n \psi_{i,j} PCUT_{j,t-1} + b_{i,t} t + b_{i,tt} t^2,$$

where $PMAX_{j,t-1}$, $PREC_{j,t-1}$, and $PCUT_{j,t-1}$ are the components of the Gately and Huntington (2002) real (logarithmic) price decomposition of the jth energy source, lagged one period, and linear and quadratic time trends are included for similar reasons as in Model A2.

6. Empirical Application

To illustrate the approach developed here, we estimate the various specifications described above using annual UK data for the residential (domestic) sector for the period 1964 to 2012, from the UK Office for National Statistics (ONS) online database,[26] for electricity, gas and 'other' (oil and solids) expenditures, with the implied deflators used for the price indices. We compare results for a 'base' benchmark model – the LAIDS model without the extra terms that result from explicitly incorporating energy services in the consumer utility maximizing framework – with augmented Models A1, A2, and A3 that alternately capture the effect of efficiency using (38), (39), and (40), respectively. The models that we estimate have the form:

$$(41) \quad s_{it} = \alpha_i + \sum_j \gamma_{ij} ln\, p_{jt} + \beta_i (\ln Y_t - \ln P_t) + \theta_i JT_t + \sum_r \varphi_{ir} Z_{rt} + \lambda_1 s_{it-1}$$

$$+\lambda_2 s_{it-2} + e_{it}, \qquad\qquad i,j = 1, ..., n,\ t=3,..., T,$$

where: $i,j = 1, ..., n$ represents the n (=3) energy sources (i,j =1 for electricity, =2 for natural gas, =3 for other (oil and solids); s_{it} is the expenditure share on the ith energy source in period t,

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[26] Available from www.statistics.gov.uk.

Y_t is total per-capita expenditure on the three energy sources, p_{jt} is the price of the jth energy source, $j = 1, ..., n$, $ln P_t$ is the Stone Price Index, where $ln P_t = \sum_j s_{jt} \ln p_{jt}$, JT_t is a weather-related variable (typically included in residential energy demand equations) – here, average January temperature (degrees Celsius), and e_{it} is a random error term. One- and two-period lagged shares are included to deal with possible autocorrelation. Adding up, homogeneity, and symmetry conditions are imposed, so that we jointly estimate two of the three expenditure share equations ('other' is omitted, and the parameters of this equation are retrieved via the adding up conditions).

The difference between the various models arises through the term $\sum_r \varphi_{ir} Z_{rt}$. In the Base Model this term does not appear. Thus, this base model could be viewed as a typical energy demand share equation specification where efficiency is ignored. In such a specification, the negative of the price elasticities cannot be equated to rebound effects, although this has often been done in practice. We retrieve the price elasticities from this model for comparison with the other specifications. In the three augmented models A1, A2, and A3, $\sum_r \varphi_{ir} Z_{rt}$ is replaced with (38), (39) or (40), respectively. For these models we retrieve the price elasticities using (34) and the combinations of rebound effects using (36).

Table 2 displays the log-likelihood value (LL) and the Schwarz Bayesian information criterion (BIC) for the base specification and the three alternative specifications. Likelihood Ratio (LR) tests show that all of the augmented specifications are preferred to the base specification, showing the importance of modelling energy services and attempting to capture the energy efficiency impacts. LR tests also indicate that models with various forms of lagged prices (A2 and A3) are preferred to specification A1 that only includes t and t^2. Although specifications A2 and A3 are not nested, and therefore cannot be formally tested against each

other, specification A3 that encapsulates asymmetric price responses gives the highest LL and lowest BIC.

Table 2: Results of Estimated Specifications

Specification	Parameters	LL	BIC	LR Test against Base Specification	LR Test against Specification $A1$
Base	11	269.985	-245.486	N/A	N/A
A1	15	289.615	-256.207	Reject	N/A
A2	27	301.597	-241.463	Reject	Reject
A3	33	342.099	-268.603	Reject	Reject

Note: Tests are conducted using a 5% significance level.

Estimated own-price elasticities, η_{ii}, obtained from (34) with $i=j$, and denoted subsequently as E_{ii} (where $i=1$ for electricity, $i=2$ for natural gas, and $i=3$ for other fuels) for the Base Specification as well as for Augmented Specifications A1, A2 and A3, are shown for all years in Figure 1 (electricity), Figure 2 (natural gas), and Figure 3 (other).[27] Since empirical studies often only include average values, we also present average values of the elasticities in the columns labelled $\overline{E_{ii}}$ in Table 3.

Table 3 Average Estimated Price Elasticities ($\overline{E_{ii}}$) and Rebound Effects ($\overline{R_{ii}}$)

Specification	$\overline{E_{11}}$	$\overline{R_{11}}$	$\overline{E_{22}}$	$\overline{R_{22}}$	$\overline{E_{33}}$	$\overline{R_{33}}$
Base	-0.82		-1.01		-0.87	
A1	-0.68	0.70	-0.62	0.72	-0.74	0.62
A2	-0.60	0.66	-0.67	0.71	-0.63	0.54
A3	-0.43	0.72	-0.92	0.75	-0.75	0.63

Note: Rebound Effects are obtained for the simplified example presented in Section 4.

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[27] Based on the estimated elasticities and their corresponding estimated standard errors, all own-price elasticities are significantly different from zero, except where indicated for other fuels in Figure 3.

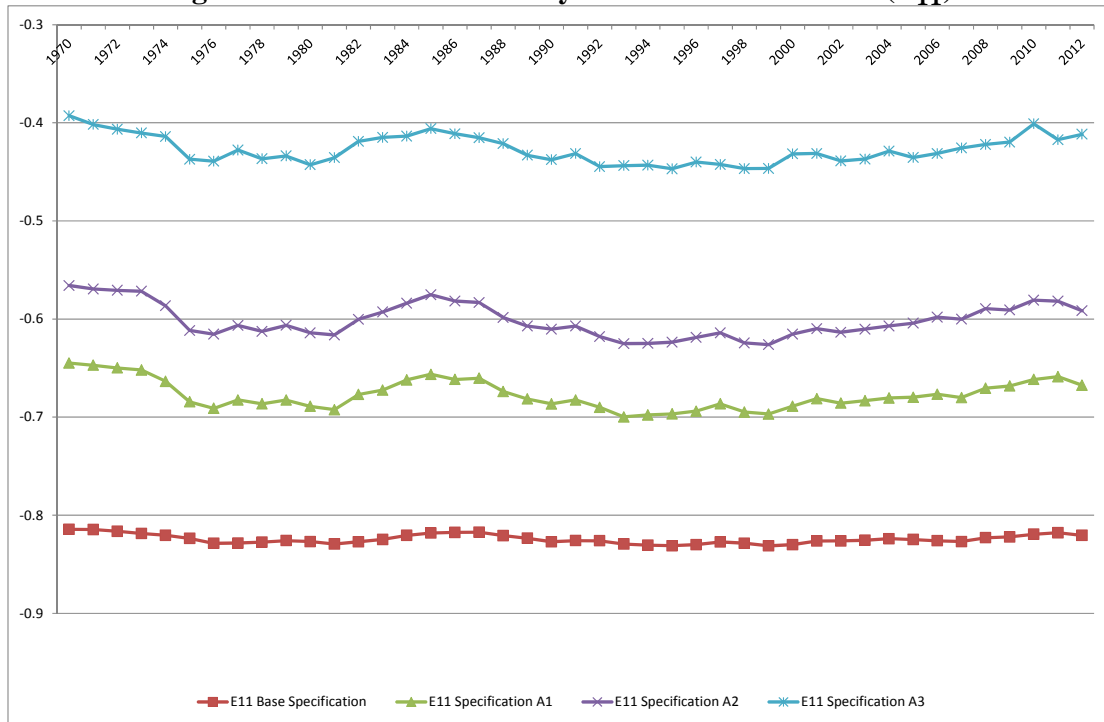# Figure 1: Estimated Electricity Own-Price Elasticities (E_{11})

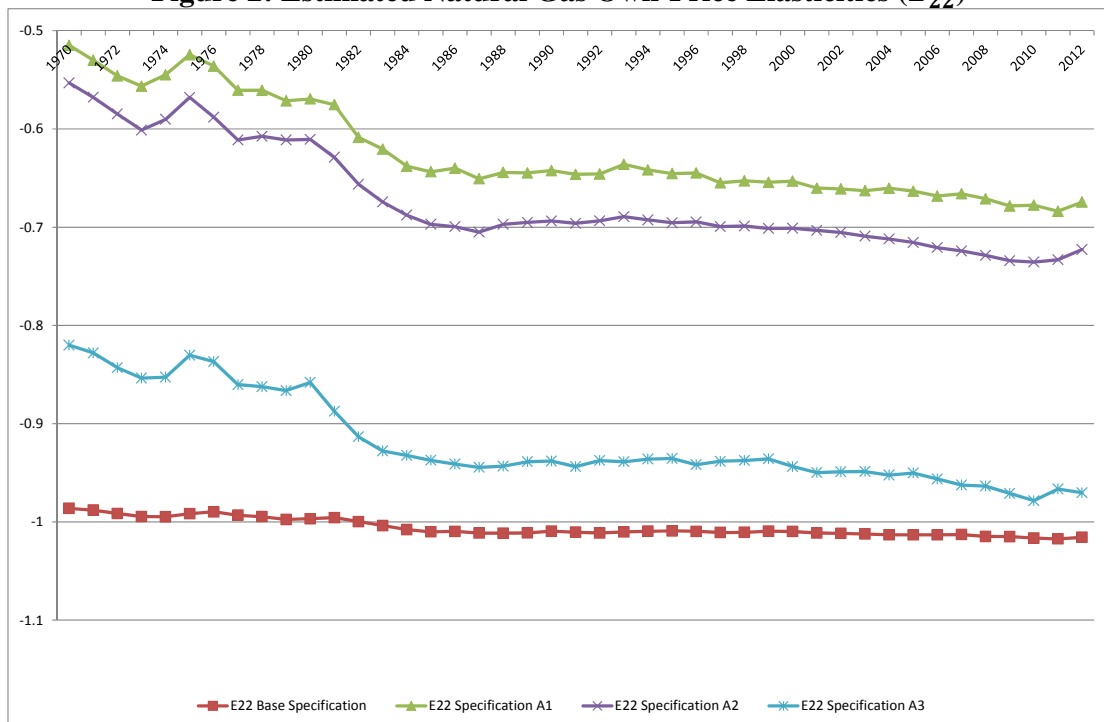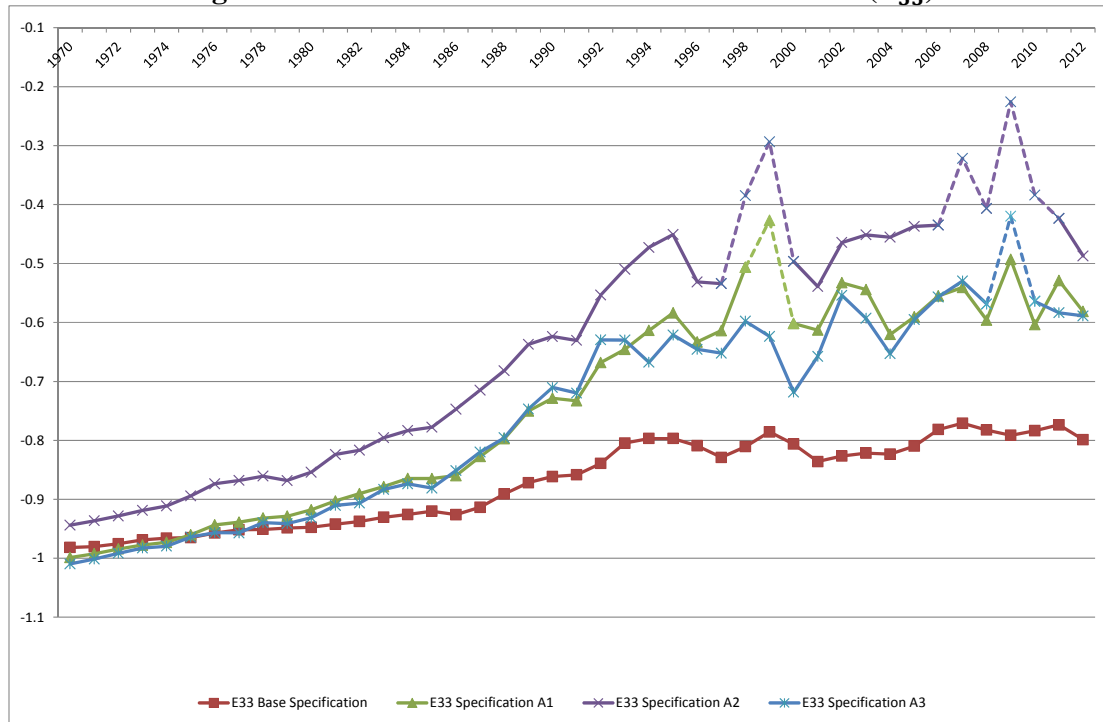# Figure 2: Estimated Natural Gas Own-Price Elasticities (E_{22})

Figure 3: Estimated 'Other' Own-Price Elasticities (E_{33})

Note: Dotted lines indicate that the estimates are not significantly different from zero at the 5% level.

Figures 1 to 3 and the $\overline{E_{ii}}$ averages in Table 3 clearly show that there is a noticeable difference between both the level and variation over time in the own-price elasticities obtained when no account is taken of energy efficiency (the Base specification) compared to any of the other models. This is most noticeable for electricity, but is also apparent for natural gas. This illustrates that the explicit modelling of energy services impacts on the estimated energy source own-price elasticities, indicating that results obtained from estimation of standard 'energy demand' relationships that do not consider energy service demand are likely to be biased, confirming Hunt and Ryan's (2014) result that was based on aggregate data. Moreover, the 'bias' can vary. For electricity, the estimated average own price elasticity ($\overline{E_{11}}$) from the base

model is about –0.82, which is almost double the –0.43 obtained from specification A3 (and this finding is consistent throughout the estimation period). The estimated standard errors for the individual own-price elasticities for electricity are approximately 0.06 for both the Base Model and Model A3, so these elasticities are significantly different from each other. For natural gas, the estimated average own price elasticity ($\overline{E_{22}}$) from the base model is about –1.0, while for A3 it is -0.92, and with estimated standard errors for individual years being approximately 0.04 to 0.05 for the base model and 0.06 to 0.091 for A3, these own-price elasticities do not generally appear to be significantly different from each other. However, as Figure 2 shows, the difference is greater in 1970 and reduces over the estimation period. Further, for specifications A1 and A2, the natural gas own-price elasticities do appear to differ significantly from those for the base model. For 'other', the estimated average own price elasticity ($\overline{E_{33}}$) from the base model is about –0.87, which again is larger in absolute value than the averages for the other models. However, as Figure 3 shows, the difference is smaller in 1970 and increases over the estimation period as elasticity estimates from A1, A2 and A3 decrease (in absolute terms) in line with 'other' falling as a share of consumer expenditure on energy as a whole.[28]

As discussed earlier, in view of the general result in (24), or the specific result for the LAIDS model in (36), it is not possible to obtain estimates of direct rebound effects (η^*_{mq}) using the estimated price elasticities (η_{ij}) since there is not a one to one correspondence between these two measures. However, as described in the simplified example presented in Section 4, if the energy services categories are artificially contrived to conform to the requirement of a one to one correspondence between energy services and energy sources, then estimated rebound effects can

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[28] Estimated elasticities for 'other' are insignificant for some observations beyond 1997 with specifications A1, A2, and A3, but this is not observed with the base model.

be obtained from the price elasticities, as shown in Table 1 for the general case. It is worth emphasizing that an underlying, and unrealistic, requirement for this procedure is that within each aggregated energy service there must be a single uniform energy efficiency. With this proviso, it is then possible to obtain a direct rebound effect for a (uniform) increase in the efficiency with which energy services are provided by electricity, by natural gas, and by other fuels, as well as various indirect rebound effects.

Using this approach, for the LAIDS model estimated direct rebound effects, η_{ii}^*, obtained from (36) by setting $i=j=m=q$, and denoted subsequently as R_{ii} (where $i=1$ for services provided by electricity, $i=2$ for services provided by natural gas, and $i=3$ for services provided by other fuels) for Augmented Specifications A1, A2 and A3, are shown for all years in Figure 4 (electricity services), Figure 5 (natural gas services), and Figure 6 (energy services from other fuels).[29] Rebound effects cannot be calculated for the Base Specification, since no efficiency terms are included there, but for comparison purposes, Figures 4, 5, and 6 include the negative of the corresponding own-price elasticity from this Base specification, which has often been used as a measure of the rebound effect in other studies. In addition, for comparison with the estimated average elasticities, the estimated average rebound effects for the Augmented Specifications are presented in the columns labelled $\overline{R_{ii}}$ in Table 3.

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[29] As with the elasticities in Figures 1, 2 and 3 the rebound effects are significantly different from zero, except where indicated for other fuels in Figure 6.

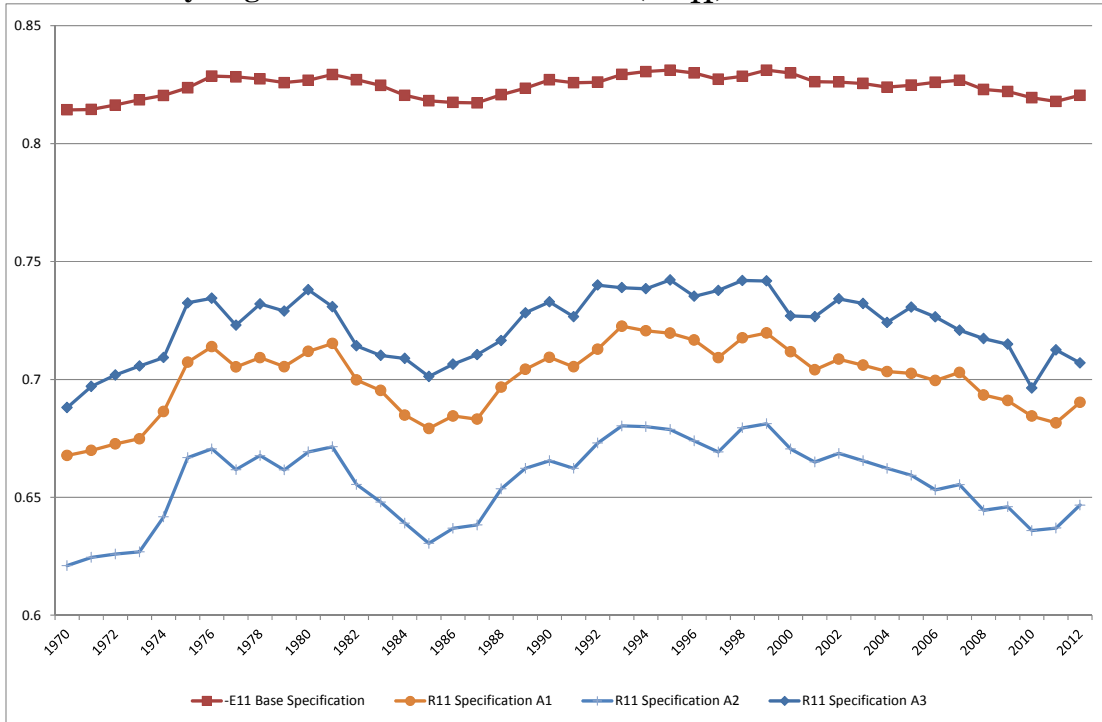**Figure 4: Electricity Negative Own-Price Elasticities ($-E_{11}$) and Direct Rebound Effects (R_{11})**

Figure 5: Natural Gas Negative Own-Price Elasticities ($-E_{22}$) and Direct Rebound Effects (R_{22})
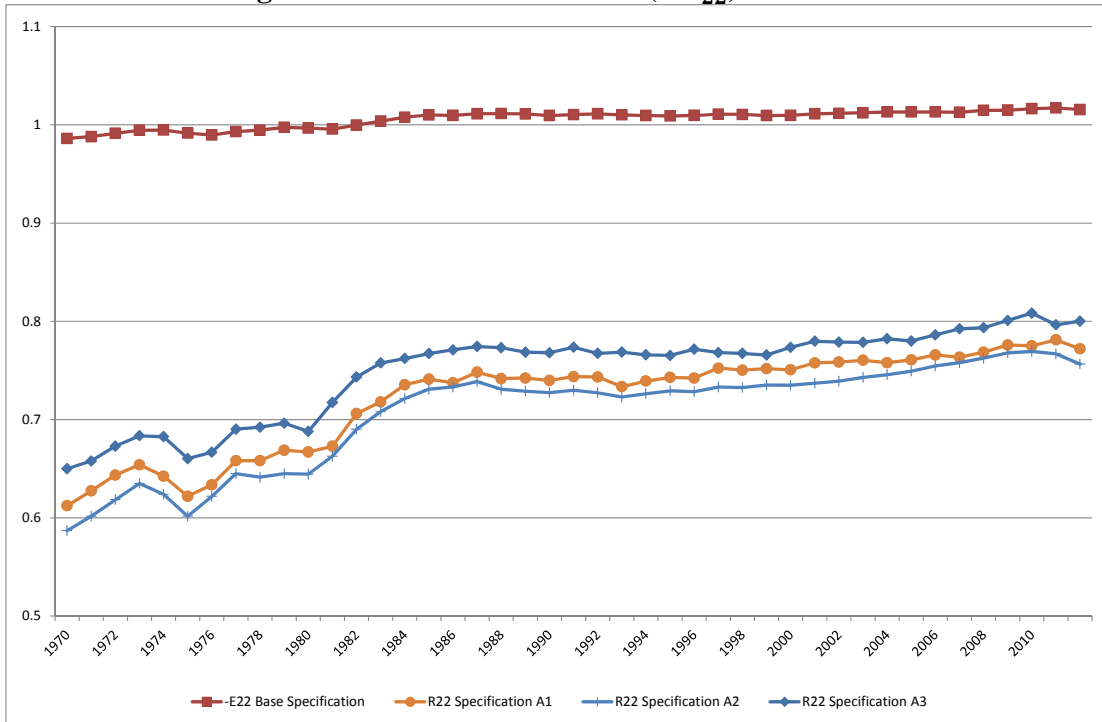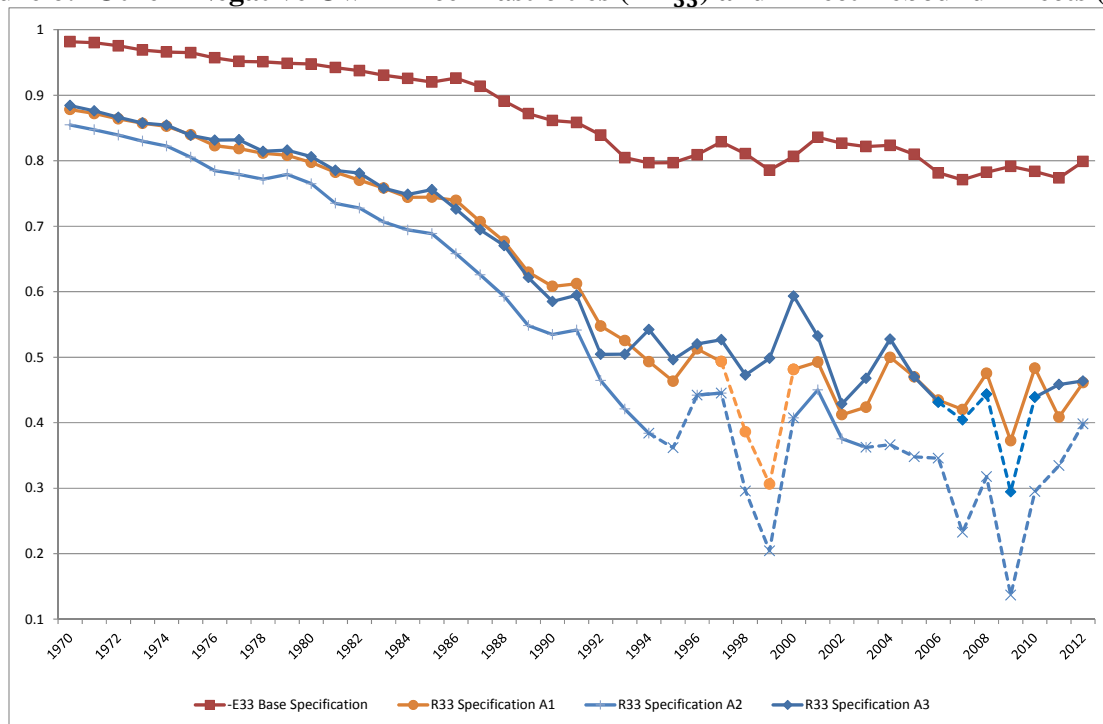
Figure 6: 'Other' Negative Own-Price Elasticities ($-E_{33}$) and Direct Rebound Effects (R_{33})

Note: Dotted lines indicate that the estimates are not significantly different from zero at the 5% level.

Figures 4 to 6 and the $\overline{R_{ii}}$ averages in Table 3 clearly show that the estimated rebound effects differ somewhat from the (negative) of the own price elasticities of demand from the base model, clearly highlighting the problems with previous research that used the negative of the own price elasticity of energy demand as an estimate of the (direct) rebound elasticity. Moreover the 'bias' can vary. For electricity, the estimated average rebound elasticity ($\overline{R_{11}}$) is about 0.7, which is less than the negative of the estimated own-price elasticity ($-\overline{E_{11}}$) from the base model of about 0.8 (and the differences are consistent over the estimation period as illustrated in Figure 4). For natural gas, the estimated average rebound elasticity ($\overline{R_{22}}$) is again about 0.7 which is somewhat less than the negative of the estimated own-price elasticity ($-\overline{E_{22}}$) of about 1.0 from

the base model. However, as Figure 5 shows, the difference between these rebound elasticities and $-E_{22}$ for the base model diminishes over the estimation period, although the difference at the end of the period is still noteworthy. For 'other', the estimated average rebound elasticity (R_{33}) is about 0.5 to 0.6 which is less than the negative of the estimated own-price elasticity ($-\overline{E_{33}}$) from the base model value of about 0.9. Moreover, Figure 6 shows that for 'other' the difference between the estimated rebound elasticity (R_{33}) and the estimated (negative) own price elasticity ($-E_{33}$) from the base model increases over the estimation period. Again, this highlights potential problems with viewing the negative of the estimated energy demand own-price elasticity from a model that does not explicitly take account of energy service demand as a measure of the rebound effect.

7. Summary and Conclusions

This paper focuses on the empirical measurement of rebound effects in energy economics, and on the particular question of whether the size of rebound effects can be measured by negative estimated own-price elasticities obtained from standard energy demand equations (as has been done in numerous previous studies, using various functional forms, without the inclusion of energy efficiency terms). Building on a recently developed approach to modelling energy services demand and energy sources demand, we demonstrate that this is generally inappropriate unless the energy demand equations are specified in a certain way and even in that case, often only under somewhat heroic assumptions.

Specifically, we show that once it is acknowledged that demand is for energy services rather than energy sources, the efficiency of providing each energy service must be explicitly

included in the energy demand models that are estimated. In our formulation, the prices of each energy source appear relative to the efficiency of the energy service that they provide, but other specifications could be used. While this energy services demand formulation has the benefit of facilitating the empirical determination of rebound effects, it also identifies some serious limitations to the use of estimated price elasticities for this purpose. In particular, we show that a consequence of recognizing that a particular energy service can be produced by any one (or a combination) of a number of energy sources, and that each energy source can produce many energy services, is that empirical estimates of direct (and indirect) rebound effects for particular energy services cannot be individually obtained using price elasticities for energy sources – regardless of the model that is estimated. Rather, these price elasticities yield only a weighted sum of a number of rebound effects. If, however, energy services categories are artificially contrived to conform to the requirement of a one to one correspondence between energy services and energy sources, then estimated rebound effects for each of these aggregated energy services can be obtained from the price elasticities. Nevertheless, an underlying, and unrealistic, requirement for this procedure is that within each aggregated energy service there must be a single uniform energy efficiency. Even in this case, our illustrative empirical analysis using UK time-series data shows that direct rebound effects could be significantly different from the negative of standard estimated own-price elasticities. It would therefore appear to be unwise to persist with interpreting price elasticities as meaningful representations of specific rebound effects.
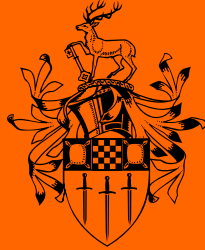
Acknowledgements

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