Linear Programming, Shadow Prices and Environmental Taxes

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On several occasions in the literature on the economic aspects of pollution control, reference is made to the use of shadow prices as optimal pollution taxes. Thus for example, Kohn (1984), in a linear programming study of air pollution control obtains shadow prices for the various pollutants and claims that if emmission fees equal to these shadow prices are levied on producers, the market will yield a pollution optimising outcome without direct intervention. Again, Nordhaus (1977) in a study of CO_2 emissions and economic growth indicates that if the shadow price of carbon is applied as a unit tax on firm´s carbon emmissions, target levels of atmospheric CO_2 concentrations could be achieved over a long period. Whilst Nordhaus refers specifically to a unit tax on the pollutant, Kohn is unclear as to whether taxes should be imposed on pollutants or on output of polluting goods. The purpose of this note is to clarify the circumstances under which it is or is not valid to apply such taxes. In particular, the use of a simple unit tax on pollution is shown to be inappropriate except under largely unrealistic assumptions. An excess pollution tax is found to be superior in every case. Examples are used to demonstrate these points but no rigorous proofs are given.

Before proceeding it is perhaps worthwhile to restate briefly the chief advantages of taxation over more direct forms of pollution control, since this will justify the effort to find an appropriate taxation policy. We will asssume that the existence of externalities justifies intervention in the operation of a market in which pollution is produced, either on the grounds that bargaining between polluters and sufferers would not necessarily produce an optimal level of pollution or that transactions costs are so great as to make the market unworkable in the case of large numbers of sufferers facing a monopolistic polluter. Pollution taxes may be expected to achieve any given desired level of pollution at lower cost than physical controls, since they provide an incentive to firms to substitute pollution reducing techniques up to the point where marginal benefits equal marginal costs. They are preferable to subsidies on pollution control processes because of the encouragement which lump sum subsidies provide for an expansion of the polluting industry. They are better than enforcing standard techniques by way of investment grants to polluters since this discourages less capital intensive pollution reducing methods e.g. recycling. In certain circumstances they are preferable to auctioning pollution rights although this depends on the degree of uncertainty regarding the estimation of costs and benefits from pollution. Finally, only in emergency conditions (e.g. climate inversion) is there any clear case for supplementing taxes with more direct and immediate controls since time lags are inevitably involved in adjusting to cost changes. For further discussion of the advantages of pollution taxation see Fisher (1981).

In the following discussion we will treat pollution as a public good (bad) the desired level of which has been set through some political process very much in the same way as a defense budget. We are therefore concerned with the optimal use of resources within pollution constraints. This problem is particularly well suited to linear programming methods which seek to find

$$\begin{aligned} \max \quad & \mathbf{z} = \mathbf{cx} \\ \text{subject to} \quad & \mathbf{Ax} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where **z** is an objective function, **c** is a vector of unit values, **x** is a vector of outputs, including pollution emmissions, and **A** is a technology matrix whose elements a_{ij} are the amounts of good i required (produced in the case of the pollutant) per unit of output of good j. Here the j goods may either be distinct products, the output of different processes, or pollutant output levels. The models of Kohn, together with its input - output linkages, of Muller (1979) and the dynamic version of Nordhaus all fit in with this formulation. How then do shadow prices arise? For every linear programme (the PRIMAL) there is an associated (DUAL) programme which finds the optimal values (shadow prices) to be placed on the constraints of the PRIMAL. Mathematically, the shadow prices are the Lagrange multipliers (**p**) in the solution to the augmented problem

$$\max \mathbf{L(x,p)} = \mathbf{cx} + \mathbf{p(b - Ax)}.$$

In the situation where one of the constraints is a pollution limitation, the shadow price p_i shows the effect on the objective function of a marginal change in allowable pollution.

Intuitively therefore, a tax equal to **p** per unit of pollution would be just sufficient to deter production of the marginal unit of pollution at the preordained level, and thereby ensure that the constraint was not violated. No physical control of pollution output levels would be required to achieve this result.

The first case concerns a polluter with two goods or processes (x_1 and x_2) with differing profitabilities i.e. objective function coefficients (c_1 and c_2), and unit resource requirements (inequality 1.1), but with identical unit pollution characteristics (inequality 1.2). The firm faces a general constraint on output of 10 units of capacity (1.1), and the community would wish to impose a pollution output constraint of 8 units of pollution (1.2). The firm's problem, in the presence of the pollution constraint is to

$$\text{maximise } z_1 = x_1 + 2x_2$$

$$\begin{aligned} \text{subject to}\quad x_1 + 3x_2 &\leq 10 \text{ (capacity)} && (1.1)\\ x_1 + x_2 &\leq 8 \text{ (pollutant output)} && (1.2)\\ x_1, x_2 &\geq 0 \end{aligned}$$

As may be verified the optimal solution is $x_1 = 7$ and $x_2 = 1$ which yields an objective function value z of 9. The shadow prices of capacity and of pollution are both 1/2. This is the result which would be expected when physical controls on pollution production are imposed. Now consider the situation arising from the imposition of a unit tax on pollution output (x_3) equal to the shadow price of pollution (1/2). The problem for the polluter now becomes

$$\max \quad z_2 = x_1 + 2x_2 - 1/2x_3$$

$$\begin{array}{llll} \text{subject to} & x_1 + 3x_2 & \leq 10 & (1.1) \\ & x_1 + x_2 - x_3 & \leq 0 & (2.3) \\ & x_1, x_2, x_3 & \geq 0 & \end{array}$$

where constraint (2.3) expresses the joint product relationship between the goods (x_1 and x_2) and their associated pollution production namely that the total amount of pollution is by assumption at least equal to the sum of the outputs of the two goods. Here the optimal solution is $z_2 = 5$, $x_1 = 0$, $x_2 = 3\ 1/3$ and $x_3 = 3\ 1/3$. Two things should be noted regarding this solution - it clearly satisfies both the pollution constraint ($x_3 = 3\ 1/3 < 8$) and the capacity constraint. Furthermore only at this unit tax level of 1/2 is the pollution constraint satisfied by decentralised decision making on the part of the firm. This is shown by varying the tax rate around 1/2 and investigating the results.

Tax	z	x_1	x_2	x_3 (Pollution)
0.4	6	10	0	10*
0.49	5.1	10	0	10*
0.6	4 2/3	0	3 1/3	3 1/3

* Pollution constraint violated

Setting tax at .4, .49 and .6 is sufficient to show that for any tax below the shadow price, the pollution constraint would be violated, while for any tax above the shadow price a lower value of the objective function is achieved resulting in a welfare loss to the producer (although the sum of profits and tax revenues remains constant).

Is this result however optimal from society´s point of view? An alternative tax regime would be to charge a pollution tax equal to the shadow price purely on pollution over and above the desired level - an excess pollution tax. It is necessary to introduce a new variable x_4 to represent pollution output in excess of 8 units with an objective function coefficient of -1/2. The non taxed pollution, x_3, is restricted to not more than 8 units. The original variables x_1 and x_2 are then expressed as $x_1 = x_{11}+x_{21}$ and $x_2 = x_{12}+x_{22}$ to represent output associated with pollution levels x_3 and x_4 respectively. The problem facing the firm is now

$$\max z_3 = x_{11}+2x_{12}+0x_3+x_{21}+2x_{22}-1/2x_4$$

$$\begin{aligned}
\text{subject to}\quad & x_{11}+3x_{12}+0x_3+x_{21}+3x_{22}+0x_4 && \leq 10 && (3.1)\\
& x_{11}+3x_{12}-x_3 && \leq 0 && (3.2)\\
& x_3 && \leq 8 && (3.3)\\
& x_{21}+3x_{22}-x_4 && \leq 0 && (3.4)
\end{aligned}$$

The optimal solution is $x_{11} = 7$, $x_{12} = 1$, $x_3 = 8$, and $x_4, x_{21}, x_{22} = 0$. Total pollution output is $x_3+x_4 = 8$ which satisfies the pollution constraint. Here z_3 is of course identical with the value of the objective function in the original, physically constrained problem and at 9 units is greater than the value of z in the pollution tax solution. It follows that an excess pollution tax is superior to a unit pollution tax from the firms´

point of view. From society's point of view also the excess pollution tax is preferred since the sum of profits and tax revenues is greater (9 compared with 6 2/3).

In the above case, the polluting characteristics of the two goods were identical although their profitability varied. A more interesting situation is one where the objective function coefficients are identical as are the unit resource requirements but the goods have different pollution coefficients. Here input substitution is available as a means of reducing pollution. The following case also illustrates a weakness in the Kohn cost minimising formulation. The problem facing the firm is

$$\max \quad z_4 = 2x_1+2x_2$$

$$\begin{array}{llll} \text{subject to} & x_1+x_2 \leq 10 & \text{(capacity)} & (4.1) \\ & 1/2x_1+x_2-x_3 \leq 0 & \left.\right\}\text{(pollution)} & (4.2) \\ & x_3 \leq 4 & \left.\right\} & (4.3) \\ & x_1, x_2, x_3 \geq 0 & & \end{array}$$

In order to be binding it should be noticed that the pollution constraint (4.3) has been made tighter. Here the shadow price of pollution is found to be 4 with $x_1 = 8$, $x_2 = 0$, $x_3 = 4$ and $z_4 = 16$.

Dropping the pollution constraint (4.3) and taxing pollution at the shadow price of 4 per unit produces a revised programme of

$$
\begin{aligned}
\max z_5 &= 2x_1+2x_2-4x_3 \\
\text{subject to } & x_1+x_2 \leq 10 && (5.1) \\
& 1/2x_1+x_2-x_3 \leq 0 && (5.2) \\
& x_1,\ x_2,\ x_3 > 0
\end{aligned}
$$

with a solution of $x_1 = x_2 = 0$ and $z_5 = 0$. Not surprisingly, at zero output levels the pollution constraint is satisfied. The advantage of an excess pollution tax is clear in this case. Letting x_4 represent pollution levels exceeding 4 units, and taxing x_4 at 4 per unit the problem is now

$$
\begin{aligned}
\max z_6 &= 2x_{11}+2x_{12}+0x_3+2x_{21}+2x_{22}-4x_4 \\
\text{subject to } & x_{11}+x_{12} \quad +x_{21}+x_{22} \leq 10 && (6.1) \\
& .5x_{11}+x_{12}-x_3 \leq 0 && (6.2) \\
& .5x_{21}+x_{22}-x_4 \leq 0 && (6.3) \\
& x_3 \leq 4 && (6.4)
\end{aligned}
$$

with an optimal solution of $x_{11} = 8$, $x_3 = 4$, $x_{12}, x_{21}, x_{22} = 0$, and $F = 16$. No excess pollution is produced and the solution is identical to the physical control solution.

These results have implications as has been intimated for the approach of Kohn and others. Kohn takes not only the maximum permitted pollution output as given but also the output of the entire set of pollution control processes in order to minimise total firm costs. The resulting shadow price of pollution would, certainly, if incorporated in the linear programme as a unit tax, yield the optimal solution by decentralised decision making. This result is however solely dependent on a prespecification of all output levels. If, alternatively, firms were free to vary their output levels within a pollution constraint, no such unique solution can be guaranteed under linear programming. Where, for example, the slope of the objective function in the two good case is identical to that of a constraint it is possible for multiple optima to arise. This is the case in the previous example where the firm´s profit maximising solution with the simple unit pollution tax is not unique. For all x_1 values between 0 and 8, profits are identical. Kohn´s method would have yielded a unique solution by determining output levels in advance. The divergence between cost minimising and profit maximising solutions is quite a common phenomenon in economics. The benefits of the profit maximising approach coupled with an excess pollution tax are that fewer restrictive assumptions are necessary and that an improved solution in terms of social welfare is obtained.

The intention of this note has been to show that a simple use of pollution shadow prices as unit pollution taxes is inappropriate in the control of pollution. In the linear programming framework of the examples considered an excess pollution tax based on the same shadow prices emerges as superior both from the point of view of the polluter and of society as a whole. The hypothesis that such a tax will produce a solution consistent with society´s preference for pollution (or rather the lack of it) while avoiding the general costs associated with administered control of output is borne out by the examples cited. We make no comment on the merits of production versus pollution taxes in controlling pollution since, in the examples above, both methods would yield identical results. The advantages of the less restrictive profit maximisation assumptions over those of the cost minimising cases usually considered in the environmental economics´ literature are also stressed.

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