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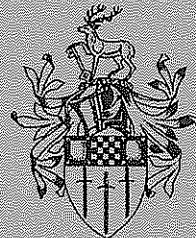
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A Model of Relative Price Elasticities from the Second Moments of Demand

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A MODEL OF RELATIVE PRICE ELASTICITIES FROM THE SECOND MOMENTS OF DEMAND

by Joseph G Hirschberg

1 Introduction

This article presents a model for estimating relative price elasticities from the second moments of the vector of the quantity demanded. The model will be referred to as the model of relative elasticities from the second moments of demand (RESMD). The development of the RESMD model was motivated by the observation of short-run variations in demand when most factors appear to be constant. Its underpinnings come from economic theory, which states that demand varies due to changes in the mix of preferences, prices, and income. If preferences and income remain constant in the short-run, then demand variation is assumed to be due to perceived price changes. The perceived price is defined as the product of the observed monetary price and a stochastic unobserved, or shadow, price. Once a generating process for the shadow prices and the form of the demand relationship are assumed, a set of relative price elasticities can be identified.

The RESMD model is tailored to the investigation of time-of-day (TOD) service demand and thus it relies on the availability of daily 24 hour usage levels. The model is based on the assumption that random shocks to the shadow price (i.e., the implicit opportunity cost) of employing a service at a particular time generate the stochastic component in the demand for TOD services as observed at a daily level of detail. For example, when an individual uses electricity to run an appliance, the price of using the electricity is composed of the price of the service and the shadow price at the time the usage

occurs. The shadow price is the implicit opportunity cost of using the service at that particular moment in time as compared to spending the time in some other activity. In this case, the shadow prices are assumed to be subject to stochastic shocks generated by the unforeseen hourly events that affect daily demand.

That the demand for TOD services is often measured by hour but the prices of these services do not vary by TOD are the characteristics that facilitate the application of the RESMD model to the analysis of these cases. This model is particularly useful in the analysis of the demand for such TOD services as shared computer services, communications networks, transportation, and public utility services. In most cases, the marginal cost of providing TOD services increases with aggregate demand. Thus TOD pricing can be a strategy for the implementation of marginal cost pricing when the system demand varies in a predictable TOD pattern. To implement such rates, however, the potential for inter-hour substitution that may occur when the rates are put into effect must be anticipated. The reactions to TOD rates can then be used to estimate the welfare effects of the rate, as was done using traditional regression analysis in Aigner and Hirschberg (1985).

Although many services are subject to some form of TOD pricing, in most cases these prices do not vary by hour but by blocks of hours. For example, most long-distance telephone services charge more for calls during the period from 8 a.m. to 5 p.m. than in off-peak hours. Given this type of data, traditional regression-based demand analysis techniques can only determine elasticities for blocks of hours and not for each hour. Applications of this model can be found in the modelling of TOD electricity demand (e.g. Hirschberg and Aigner 1983), and Aigner and Hirschberg 1985). Caves, Christensen, and Herriges (1987) extend this model further by relaxing the

assumption of strict separability between blocks of hours. They demonstrate that some cross-price elasticities can be identified for commodities in which no price variation is observed by using the symmetry of the Allen elasticities of substitution along with the ability to compute the income elasticities with only income variation. However, they are still unable to determine the complete set of hour-by-hour elasticities that can be obtained from the RESMD model.

The requirement for daily demand data by hour may seem highly restrictive at first but suppliers of TOD services often need to retain daily records to plan their capacity requirements. Monthly data can be employed as well. However, the longer time period increases the possibility that the month-to-month variation in hourly demand is due to systematic differences in preferences or income between the months. Systematic variation may also be present in daily observations and could be caused by calendar effects, exogenous secular trends, or seasonal factors. To ensure that the observed hour-to-hour variation in demand is due solely to variation in perceived price, a first-stage regression model is proposed to remove any possible systematic variation. These first-stage regressions are discussed in the applications presented below.

This article proceeds as follows. First, the literature on the use of second moments of demand is discussed. Next the RESMD model is developed by defining the relationship between the own- and cross-price elasticity matrix and the cross-demand covariance matrix. Then two applications of the RESMD model to TOD data are discussed. The first example concerns aggregate mainframe computer use, while the second example applies the RESMD model to the analysis of an individual household's TOD electricity consumption.

2 Earlier Studies of the Second Moments of Demand

Models that use the second-moments-of-demand equation residuals to estimate elasticity and substitution relationships have a long history. Allen and Bowley (1935, pp. 89-91) observed that the conditional correlation (conditioned on the level of total expenditure) between the level of demand for two goods indicates the nature of the substitution between the goods. Thus if the goods are perfect complements, the conditional correlation will be 1; if they are perfect substitutes, the conditional correlation will be -1. These conditional correlations are equivalent to the correlations of the residuals from Engel curves fit for each commodity, but they are not directly related to the Slutsky matrix of substitution.

Theil (1952) applied Allen and Bowley's concept in an investigation of the demand for quality versus quantity of a good. He interpreted the negative correlation of residuals from two Engel curve functions as an indication of the substitution of quality for quantity. One function was estimated for a proxy for the quality defined as the average cost for all types of a differentiated product, and the other was estimated for the quantity as defined by the total amount of the differentiated product purchased.

In 1958, Theil and Neudecker proposed a direct link between the preference structure and observed covariance of Engel curves. To generate a stochastic demand, they assumed that the linear portion of a quadratic utility function is a random variable, thus implying an additive random component to marginal utility. However, they concluded that they had insufficient information to identify the preference structure without making an assumption concerning the covariance structure of the shocks to the marginal utility.

In later work, Theil (1967, p. 232) suggested that the matrix of price substitution effects multiplied by a negative scalar will be directly related to the

covariance of the random shocks to the marginal utility identified in his 1958 paper with Neudecker. (According to Barten (1968), this concept first appeared in a 1964 working paper by Theil.) Theil (1967) employs this relationship between the substitution matrix and the covariance of random shocks to the marginal utility to compare the cross-equation covariances of the residuals from an estimated demand system with the theoretical covariance matrix implied by the weighted estimated price substitution matrix. He concluded that there are similarities between the two matrices but made no attempt to incorporate these restrictions on the cross-equation covariance directly into the system estimation.

Under the term "rational random behavior" Theil (1971, 1974) generalized this optimization problem to a second-order Taylor Series expansion of any objective function. In his 1974 paper, Theil showed the relationship between the mean square of the residuals from a system of estimated demand equations and the negative of the estimated coefficients of substitution from the same equations.

Patterns in the cross-equation covariance matrix of a system-of-demand equations have been examined by Phlips (1971, 1974) and Phlips and Rouzier (1972) for indications of model misspecification. These applications are in the spirit of Theil (1967) but use different demand specifications. Phlips further assesses the nature of substitution patterns through the use of principal component analysis of the cross-equation covariance.

3 Model of Relative Elasticities from the Second Moments of Demand (RESMD) Model

In this section a model of relative elasticities from the second moments of demand (RESMD) based on the logarithmic demand equation is defined.

Although it is a general model, the type of data needed to apply it are most prevalent for services is sold by TOD. Thus the model is defined in terms of a TOD service.

Assume that TOD prices include a stochastic component. Thus the log of the perceived price (P) is defined as:

$$P = P_o + P_s + \epsilon \quad (1)$$

where $P_s = \log$ of an unobserved shadow price assumed to be equal to one for every hour and thus $P_s = 0$ for every hour; $P_o = \log$ of the observed monetary price (as defined by existing TOD rates); and $\epsilon =$ a random error in the perception of the log shadow price, in which $E[\epsilon_s] = 0$, $E[\epsilon_s^2] = \sigma^2$, and $E[\epsilon_s \epsilon_t] = 0$ when $s \neq t$. A logarithmic demand relationship for 24-hour services is given as:

$$X = PE \quad (2)$$

where $n =$ the number of days in the sample, X is an n -by-24 matrix of the log of the hourly demand for services, P is an n -by-24 matrix of the log of the perceived price, and E is a 24-by-24 matrix of price elasticities composed of elements denoted by e_{ji} which are the Marshallian or uncompensated price elasticity of commodity j with respect to price i .

The elasticity version of the Slutsky equation provides the relationship:

$$e_{ji} w_j + m_j w_i w_j = e_{ij} w_i + m_i w_i w_j \quad (3)$$

where m_j is the income elasticity of commodity j and w_j is the expenditure share of commodity j . It is assumed that the income elasticities for the same good at different times of the day are equal to one another. (This assumption can be relaxed if values for the income elasticities are available.) This assumption allows the definition of $h_{ij} = e_{ij} w_i$ and $h_{ji} = e_{ji} w_j$. Thus $h_{ij} = h_{ji}$, and the n -by- n

symmetric matrix H with elements h_{ij} is defined as:

$$H = E \text{diag}(w) \quad (4)$$

where $\text{diag}(w)$ is a matrix with the cost shares on the diagonal, note that when all TOD prices are equal the cost shares are the proportions of demand. H can also be written as a function of the Slutsky matrix (S):

$$H = \text{diag}(p) S \text{diag}(p) m - \text{diag}(w) M \text{diag}(w) \quad (5)$$

where $\text{diag}(p)$ is a matrix with the prices on the diagonal, m is the income, and M is a matrix with the income elasticities for commodity i in every element of row i . Under the assumption that $m_i = m_j, (\forall i \neq j)$ M is a matrix with equal elements; thus H is symmetric. Therefore we can define S as:

$$S = \text{diag}(p^{-1}) (H + \text{diag}(w) M \text{diag}(w)) \text{diag}(p^{-1}) \quad (6)$$

Consequently, for S to be negative semidefinite it is necessary, but not sufficient, for H be negative semidefinite.

Combining (2) with (1) the expected value of the level of usage is given as

$$E(X) = (P_o + P_s) E \quad (7)$$

thus

$$X - E(X) = \varepsilon E \quad (8)$$

Equation (4) can be solved for E :

$$E = H \text{diag}(w)^{-1} \quad (9)$$

The covariance of the observed X is given by:

$$\text{cov}(X) = \sigma^2 \text{diag}(w)^{-1} H^T H \text{diag}(w)^{-1} \quad (10)$$

To estimate H , we can employ the eigenvalue decomposition of a matrix formed by pre- and post-multiplication of the estimated covariance matrix by the diagonal matrix of the mean cost shares.

$$\text{diag}(\bar{w}) \text{cov}(X) \text{diag}(\bar{w}) = L \Lambda L^T \quad (11)$$

And thus a symmetric and negative semidefinite estimate of σH is given by:

$$\hat{\sigma} \hat{H} = L \Lambda^{1/2} L^T \quad (12)$$

where L = matrix of eigenvectors for $\text{cov}(X)$ as each column and $\Lambda^{1/2}$ = diagonal matrix of the square roots of the eigenvalues for $\text{cov}(X)$. Theil and Neudecker (1958) showed that (12) provides a unique solution for $\hat{\sigma} \hat{H}$. Thus \hat{H} satisfies a necessary condition for the second-order conditions of a system of demand equations. Note that these estimates can be formed even if the estimated covariance is not of full rank. Alternatively, one can use other covariance estimates such as the estimate proposed by Theil and Fiebig (1984). Thus (12) can be used to compute estimates with relatively few observations, however the precision of the estimates will be lower with small sample sizes. The application presented below uses Efron's bootstrap technique (1982) to estimate the covariance of the estimates of $\hat{\sigma} \hat{H}$.

The estimate of the price-elasticity matrix up to a scalar multiple $\hat{\sigma} \hat{E}$, is given by

$$\hat{\sigma} \hat{E} = \hat{\sigma} \hat{H} \text{diag}(w) \quad (13)$$

Because E can be identified only up to a scalar multiple, the relative elasticities are all that are referred to in the applications discussed below.

4 Aggregate Demand for Computing Services

This application computes the substitution patterns for aggregate TOD demand for shared computer services. Before estimating the relative elasticities, it is first necessary to remove the systematic factors that affect the daily data. The presence of systematic calendar effects will influence the results of a direct application of the RESMD if these non-random changes in daily usage patterns are not removed. To remove these effects, a set of regressions is computed in which the log of the service demand is a function of dummy variables for days of the week and a cubic in time. This conditioning is designed to reduce the potential for cross-hour positive correlation in the residuals induced by secular trends in demand over all hours of the day and across days.

The computer usage data were collected over a three-month period for the academic computer at the Bradfield Computer Center, Southern Methodist University. The operating system (an IBM VM/CMS) has very little capacity for batch or background processing. The data consist of 51 daily counts of active users by hour. The period of data collection included the beginning of a semester, which may have induced a trend in the data.

For the purposes of this analysis, no attempt was made to determine the implicit cost of usage at various times due to congestion factors, as proposed by Gale and Koenker (1984). An important assumption is that the congestion-induced reduction in system performance is incorporated in the preferences of the individuals who use the system.

The equation for an hour's demand can be written as:

$$x_i = (P_o + P_s + \varepsilon)e_i + Q_i D + u_i \quad (14)$$

where x_i is n -by-1 vector; P_o , P_s , and ε are n -by-24 matrices; e_i is a 24-by-1 i th column of E ; Q_i is an n -by- k matrix of dummy variables for each day of the

week and a cubic function in time and D_i are the corresponding k -by-1 vector of coefficients (in this case $n = 51$). This equation can be rewritten as:

$$x_i = (P_o + P_s) e_i + Q_i D_i + \varepsilon e_i + u_i \quad (15)$$

To facilitate estimation, it is assumed that $u_i = 0$; thus all the error in the model is contained in the εe_i term. The relaxation of this assumption may be possible when special assumptions are made concerning the form of E , i.e., that the elasticities for all hours from 1 a.m. to 4 a.m. are the same.

The regression is of the form:

$$x_i = \alpha_i + Q_i D_i + \zeta_i \quad (16)$$

where $\alpha_i = (P_o + P_s) e_i$ and $\zeta_i = \varepsilon e_i$. The residuals from the set of all regressions can now be used to construct the covariance matrix from which H can be estimated as in (11) using the result in (12):

$$\text{diag}(\bar{w}) \text{cov}(\hat{Z}) \text{diag}(\bar{w}) = \sigma^2 H^\top H = L \Lambda L^\top \quad (17)$$

where \hat{Z} is the 24-by- n matrix of residuals from the set of 24 regressions fit to (16). An element of $\hat{\sigma}^2$ is chosen as the divisor (in the present case, the first row, first column). The relative elasticity matrix (RE) is defined as

$$RE = \frac{\hat{\sigma}^2}{\hat{\sigma}_{11}} \quad (18)$$

The estimate of RE obtained from (18) is a point estimate. To obtain variance estimates, Efron's bootstrap (1982) was applied to the covariance of the

residuals $[\text{cov}(\hat{\theta})]$. Beran and Srivastava (1985) have shown that confidence intervals of functions of the covariance matrix can be found from the use of a bootstrap on the covariance matrix. One of the advantages of this model is the low computational expense for the calculation of $RE^{\hat{e}}$, which facilitates the use of the bootstrap technique.

The bootstrap was performed with a balanced resampling procedure (see Davison, Hinkley, and Schechtman 1986) by recomputing the regression on 500 resamplings of the original data in which each observation has exactly the same probability of being redrawn. (Very little change was noticed from runs using only 100 resamplings.) Once the regression was estimated the residuals were redrawn to compute the values of $RE^{\hat{e}}$. This form of the bootstrap, commonly referred to as a nonconditional bootstrap, was used because here the statistics of interest are a function of the residuals themselves rather than functions of the regression parameters as in the cases discussed in Freedman and Peters (1984). To check for the assumption of homoscedasticity needed to apply a bootstrap, the residuals from the hourly regression models were examined for autocorrelation, no autocorrelation was found. The program used to compute these results was written in the PROC IML language of the SAS[®] computer program.

A plot of the relative own-price elasticities with a 90% confidence interval based on the empirical distribution of the bootstrap estimates appears in Figure 1. Although confidence bounds based on the percentage method of calculating confidence intervals have been shown to be flawed for purposes of hypothesis testing (see Efron and Tibshirani 1986), they are used here to convey the general degree of variation in the estimates of the relative own-price elasticities. In Figure 1, the own-price relative elasticities are quite low in the early morning and then rise steadily until 5 p.m. The late-evening peak may be due to the phenomenon of end-of-day jobs that are run to back up disk data sets on tape and to perform accounting functions.

To show the general characteristics of the off-diagonal terms in the RE matrix, the means of the off-diagonal relative cross-price elasticities by hour from the diagonal were computed. These values for cross-price elasticities are relative to the own-price elasticities. For any particular hour, the diagonal value was divided into the values in all the other columns in the row; then, depending on how far in time those relative cross-price elasticities are from the time of the diagonal, they are assigned to a particular number of hours away from the present hour. The mean and a bootstrap 90% confidence interval are shown in Figure 2. Thus the 1st through 12th hour are average relative values of the

cross-price elasticity for the price of the hours following the present hour, and the hours from 12 to 23 are the hours preceding the present hour.

Figure 2 is of interest because a number of researchers have introduced prices of other periods, in a demand specification as a simple weighted average of the prices in other hours. These weights are assumed to be a function of how long a period will pass until that hour or how long a period has passed since the hour. This procedure assumes that periods separated in time have lower levels of potential substitution than proximate hours. The model for household electricity usage developed by Gallant and Koenker (1984), which uses a trigonometric weight for the shape of all off-diagonal values of an hourly preference structure makes such an assumption. Another variant of this assumption is found in Hirschberg and Aigner (1987), where the weight is the inverse of the number of hours to the period of the other price.

An estimated relative cross-price elasticity matrix was computed as the average matrix from the bootstrapped \hat{H} . An overview of the relative cross price elasticity matrix is given in Figure 3 and 4. Cross-price elasticities declined as the price period was further from the time of the demand, and almost all the relative cross-price elasticities were positive (the same sign as the own-price elasticity for the first hour), indicating that computer use in one hour was complementary to computer use in all other hours. This result was influenced

by the ability of the exogenous terms in the regression to remove all the secular trends and the influence of other nonprice effects in the data. The misspecification of (16) will result in the possibility of systematic variation in daily usage, which will lead to a covariance structure exhibiting only positive covariances. The pattern of cross-price elasticities in the hours between 8 a.m. and 9 p.m. indicates a relationship between hours of the day in which most computer users are active relative to the other hours of the day. These results indicate that assumptions of block substitution, which are often necessitated by the structure of available TOD price data, may indeed be supported by the elasticities computed here.

5 Household Electricity Demand

In this section, the relative price elasticities for a household whose electricity consumption was monitored by TOD, are estimated. The household was selected from the control group in an experiment conducted by the Los Angeles Department of Water and Light to measure the impact of TOD prices on the demand for electricity. Thus the electricity demand of this household was measured as though they were on a TOD rate. (See Hirschberg 1989 for details of these data.)

As with the computer usage data, it was first necessary to remove the systematic factors that affect the daily data. To this end, the model specified

in (16) was applied to the daily observations for 89 days beginning on Dec 1, 1976. Winter observations were chosen to lessen the impact of the demand for cooling, which would have a strong influence on summer observations.

A plot of the relative own-price elasticities, the diagonal elements of the matrix $RE^{\hat{e}}$ defined in (18) is shown in Figure 5 (it corresponds to Figure 1). From this plot, it can be seen that the implied own-price elasticity was twice as high for early-morning electricity usage than for the 1 a.m. period elasticity which was used as the denominator. This contrasts with the early evening usage which appears to have a significantly lower own-price elasticity than the 1 a.m. period elasticity. Interestingly, these results indicate that the elasticity of electricity demand was least elastic during the late afternoon period, which is the period of the system peak for many utilities.

Relative cross-price elasticities are shown in Figure 6. It corresponds to Figure 2 for the computer services data. In this case however, there is much less complementarity between the hour of consumption and the hours after and before. This may be due to the difference between the smooth nature of aggregate activity and the discrete character of individual actions. Furthermore, the relationship appears to be considerably more asymmetrical in this case than in the computer usage case in that substitution was less in following hours than in preceding hours. Also, the scale is different on this

plot, and the cross elasticities were three times less than those for the computer case.

Differences in the elasticities of substitution are even more striking in Figures 7 and 8. The level of cross-price elasticities was much less pronounced for the household's electricity usage. The matrix of relative elasticities appears to show bands of elasticities for sets of two-hour intervals. This regularity may be due to the use of a particular appliance.

6 Conclusions

The RESMD model proposed and estimated in this article provides a model for the interpretation of the variation observed in the demand for a service as an indication of elasticity of substitution for the same service at different times of the day. The primary result of this model is the ability to estimate a set of relative price elasticities when the low level of observed price variation would heretofore have made this impossible when employing traditional regression based techniques.

The simplicity of the RESMD model and the large number of estimated parameters make this type of analysis a convenient first step in the consideration of any pricing proposal involving the institution of differentiated prices for a service that had not been previously subject to such pricing. As

shown in the applications above, this analysis can be applied to either aggregate or individual data.

In many instances where TOD prices are in effect, they are usually not defined for every hour of the day. To evaluate the impact of changes in the price period definition or to combine data from rates defined over time periods, it may be necessary to estimate an hourly model as in the case of combining the results of the various TOD electricity studies done by Aigner and Leamer (1984). To use the RESMD model in this case one might use the estimated $RE^{\hat{e}}$ as a scaling matrix by which the actual cross-price matrix may be found as a function of $RE^{\hat{e}}$. For example, the elasticity matrix E might be defined as a polynomial in the matrix $RE^{\hat{e}}$ in the form:

$$E = \sum_{i=1}^k \gamma_i RE^{\hat{e}^i} \quad (19)$$

where γ_i is a scalar that, given sufficient price variation for the order of the polynomial specified in $RE^{\hat{e}}$, could be estimated.

An alternative strategy is to use $RE^{\hat{e}}$ and its estimated covariance to define a formal prior distribution in a Bayesian model as suggested by Paulus (1975). This strategy may also prove fruitful when little price variation is present. The estimated parameters and covariance could be used to form a complete prior in such an application where the estimation of a large system of demand equations is hampered by the multicollinearity in the prices.

The log-log demand model is limited by the assumption of constant elasticities. Because this model can be estimated with a relatively short time series, estimates of RE can be used as the first stage in a series of models to investigate time variation in elasticities. These models may also form the first stage in an hierarchical model with a second-stage model to explain the variation in elasticities as a function of individual characteristics. For example, a panel of household data of hourly electricity consumption could be studied by first estimating values of RE for each month and individual household. Then, in a secondary analysis model, these values can be modelled as a function of household characteristics and monthly weather using a regression model.

The RESMD model can be viewed in the spirit of Leontief's input-output model of an economy: a simple but detailed analysis. As shown in the examples presented here, the analysis of TOD service demand may furnish the most appropriate application of this technique due to its low computational cost, the large number of parameters to be estimated, the need for detailed demand data in small time increments, and the relatively small variation or no variation in the observed price.

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FIGURES 1 - 8

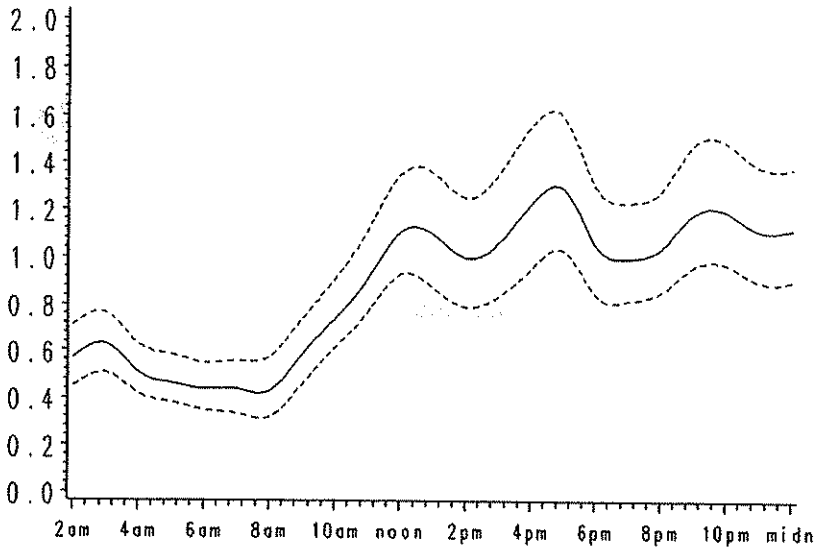


Figure 1. Relative Own-Price Elasticities for Computer Services.

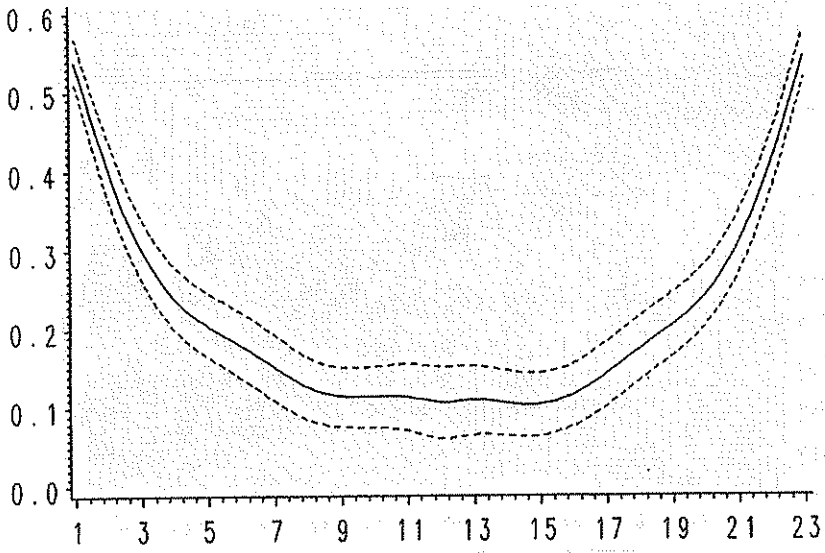


Figure 2. The Relative Cross-Price Elasticities for Computer Services.

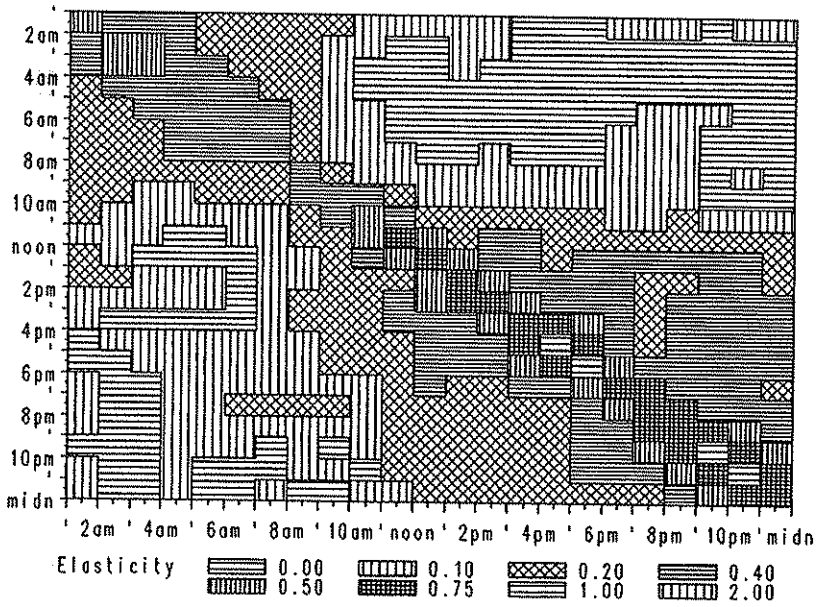


Figure 3. The Matrix of Relative Price Elasticities for Computer Services.

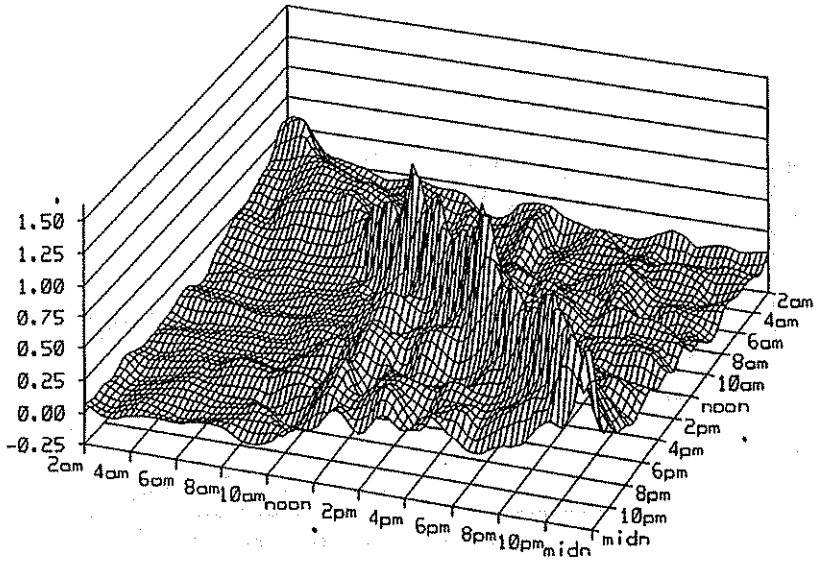


Figure 4. The Profile Plot of Relative Price Elasticity Matrix for Computer Services.

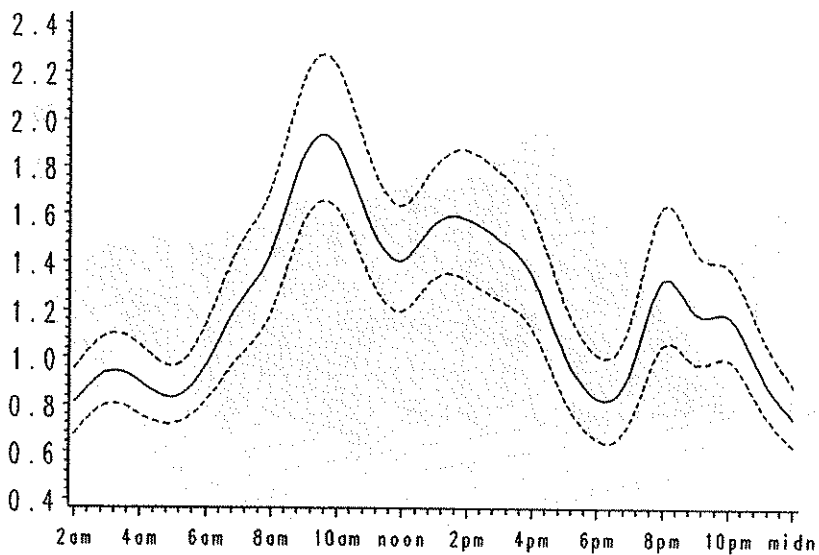


Figure 5. Relative Own-Price Elasticities for Household Electricity Usage.

Source: Author's calculations based on data from the U.S. Department of Energy, Energy Information Administration, "Electricity Delivery and Losses: Annual Report 2002" (Washington, DC: EIA, 2003), Table 2.1.1.

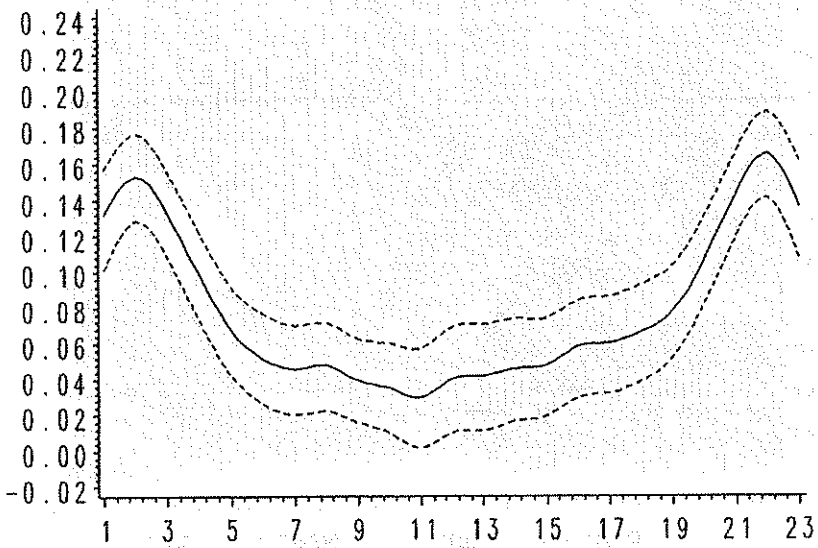


Figure 6. Relative Cross-Price Elasticities for Household Electricity Usage.

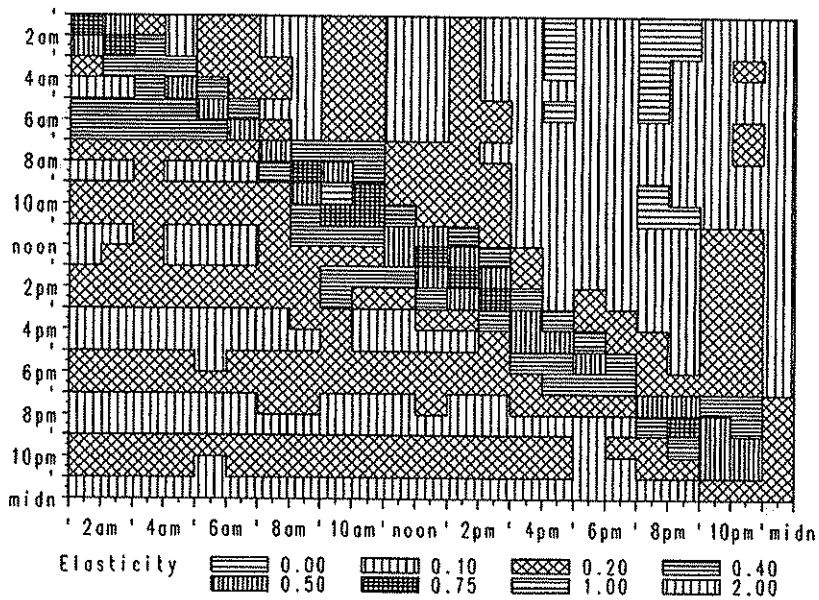


Figure 7. Matrix of Relative Price Elasticities for Household Electricity Usage.

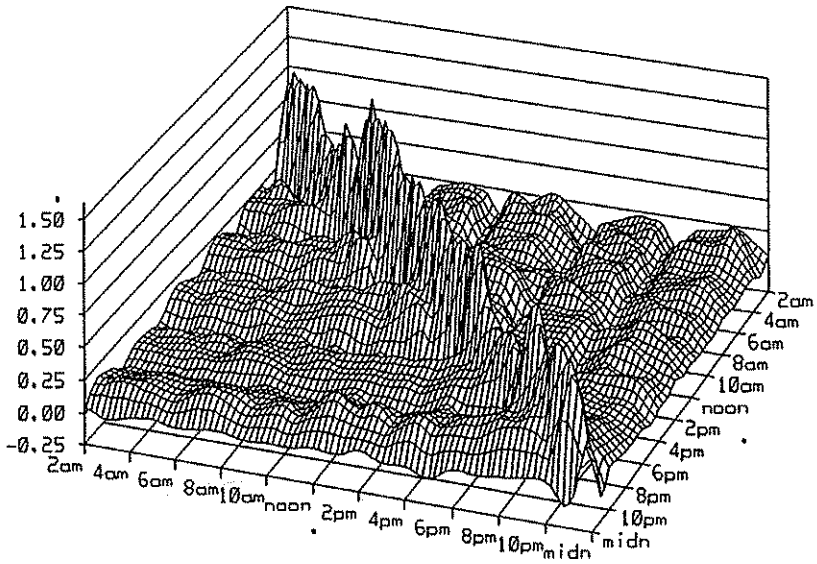
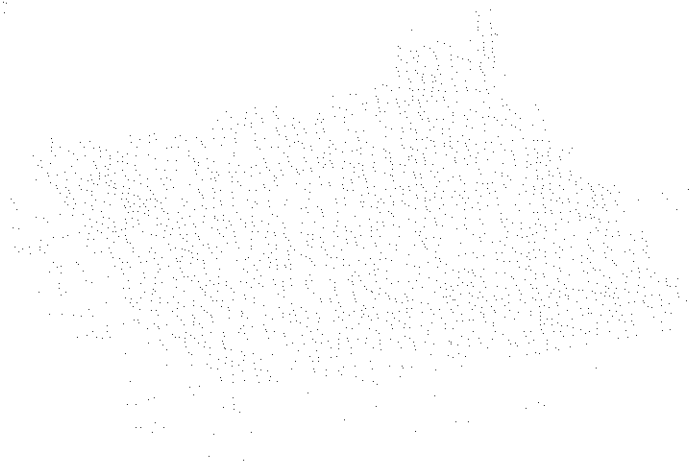


Figure 8. The Profile Plot of Relative Price Elasticity Matrix for Household Electricity Usage.

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